Eigenmatrix of the autocorrelation matrix

Definition: eigenmatrix Φ of autocorrelation matrix R_{ff}

- Φ is unitary
- The columns of Φ form a set of eigenvectors of R_{ff}, i.e.,

$$R_{ff}\Phi = \Phi\Lambda$$

← Λ is a diagonal matrix of eigenvalues λ_i

$$\Lambda = \begin{pmatrix} \lambda_0 & & & 0 \\ & \lambda_1 & & \\ & & \ddots & \\ 0 & & & \lambda_{N-1} \end{pmatrix}$$

- R_{ff} is normal matrix, i.e., $R_{ff}^H R_{ff} = R_{ff} R_{ff}^H$,
  hence unitary eigenmatrix exists („spectral theorem“)
- R_{ff} is symmetric nonnegative definite, hence $\lambda_i \geq 0 \quad \text{for all } i$

Karhunen-Loève transform

- Unitary transform with matrix

$$A = \Phi^H$$

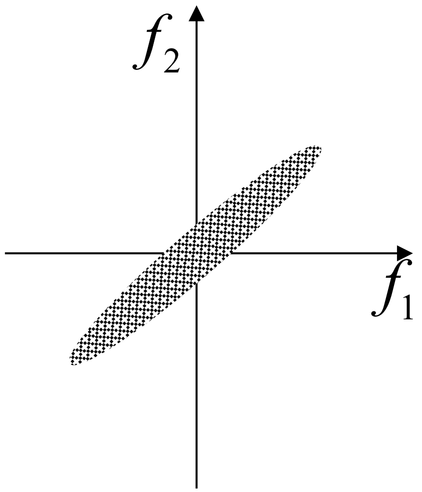
- Transform coefficients are pairwise uncorrelated

$$R_{cc} = A R_{ff} A^H = \Phi^H R_{ff} \Phi = \Phi^H \Phi \Lambda = \Lambda$$

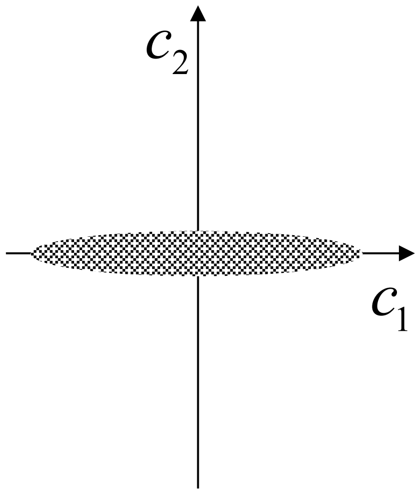
- Columns of $\mathbf{\Phi}$ are ordered according to decreasing eigenvalues.
- Energy concentration property:
  - No other unitary transform packs as much energy into the first J coefficients.
  - Mean squared approximation error by keeping only first J coefficients is minimized.
  - Holds for any J.

Illustration of energy concentration

Strongly correlated samples,
equal energies



$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



<u>After KLT:</u>
uncorrelated samples,
most of the energy in
first coefficient

Basis images and eigenimages

- For any transform, the inverse transform

$$\vec{f} = A^{-1}\vec{c}$$

  can be interpreted in terms of the superposition of columns of A^{-1} („basis images“)
- For the KL transform, the basis images are the eigenvectors of the autocorrelation matrix R_{ff} and are called „eigenimages.“
- If energy concentration works well, only a limited number of eigenimages is needed to approximate a set of images with small error. These eigenimages span an optimal linear subspace of dimensionality J.