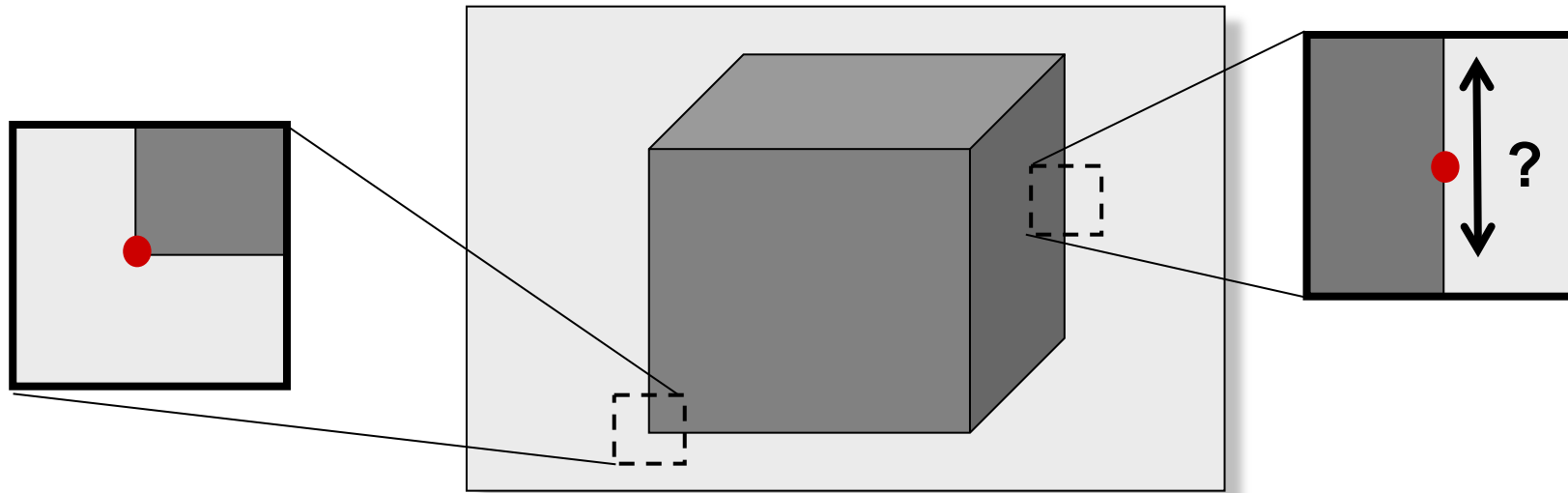


# Keypoint detection

- Many applications benefit from features localized in  $(x,y)$
- Edges well localized only in one direction → detect corners?

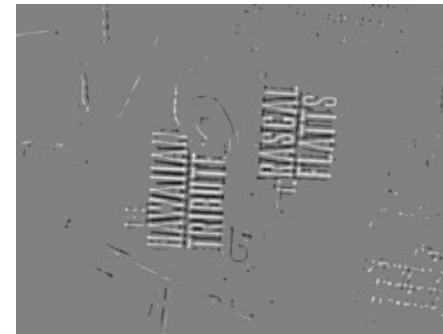
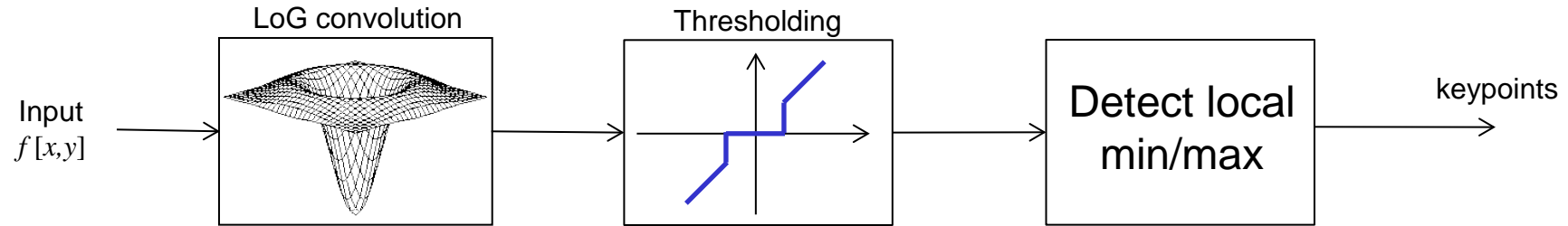


- Desirable properties of keypoint detector
  - Accurate localization
  - Invariance against shift, rotation, scale, brightness change
  - Robustness against noise, high repeatability

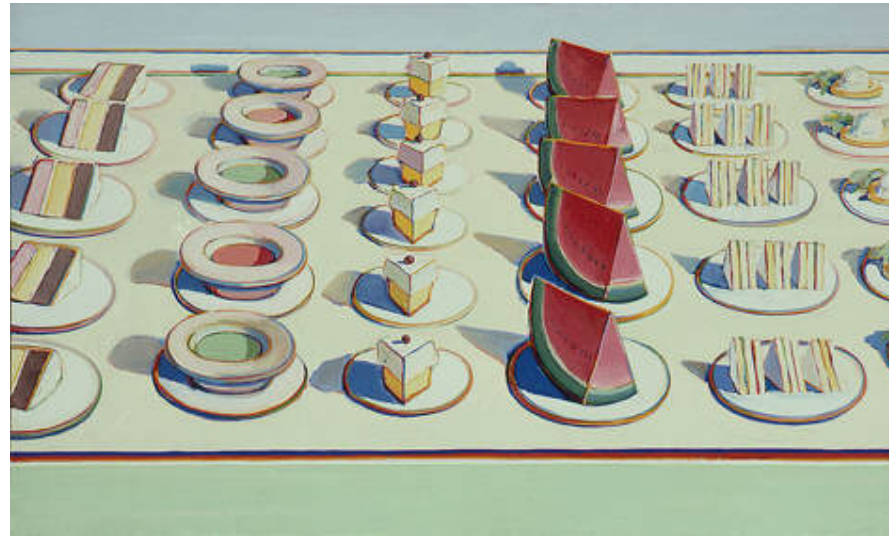
# Keypoint detection

- Laplacian detector
- Determinant of Hessian detector
- Harris detector
- FAST detector

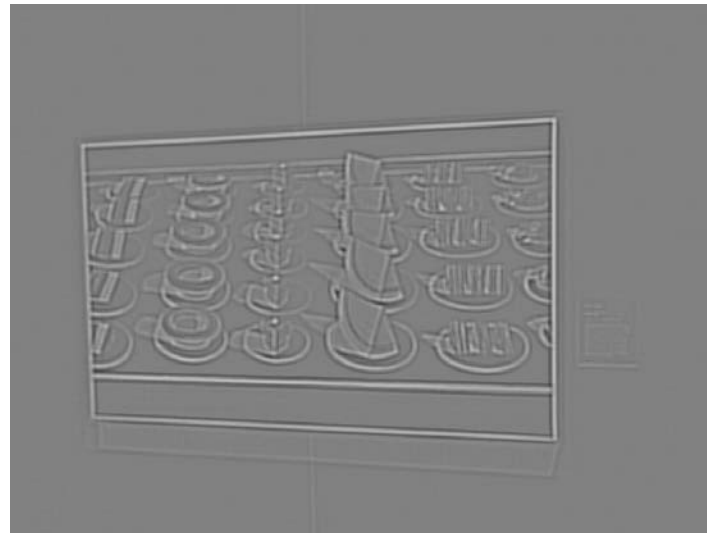
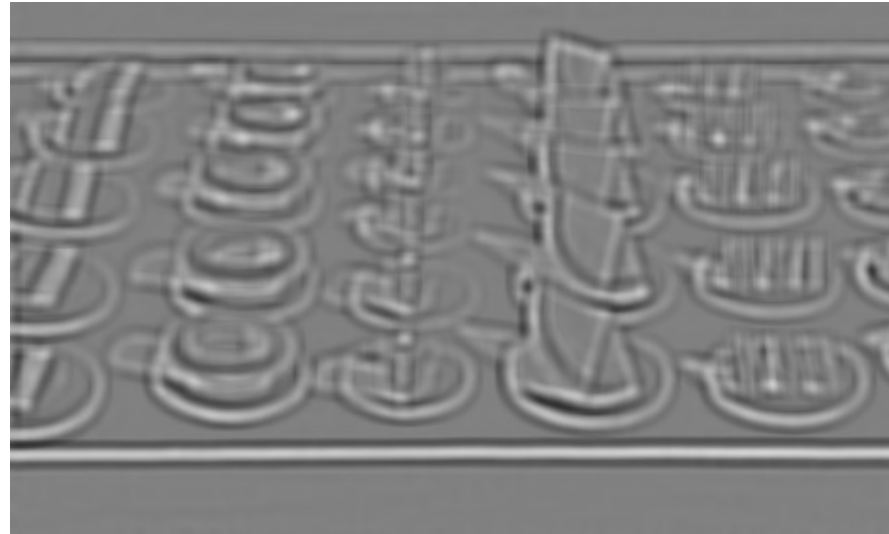
# Laplacian keypoint detector



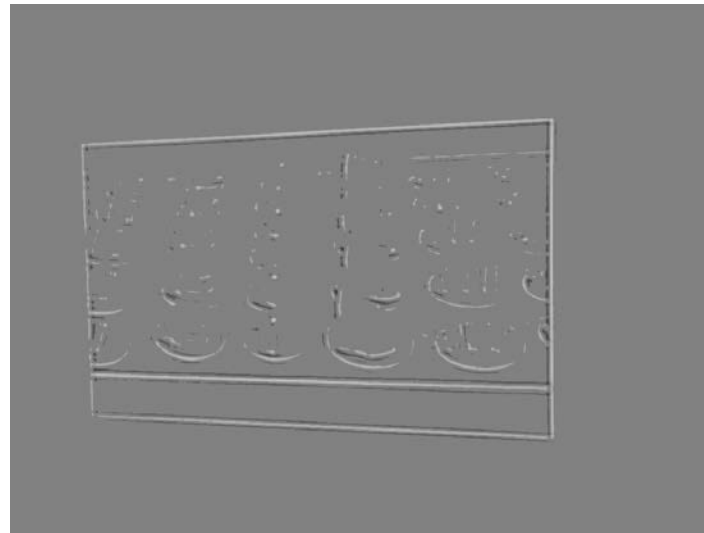
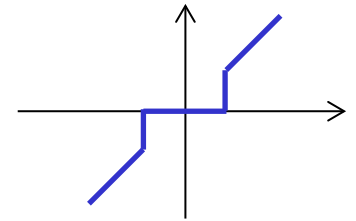
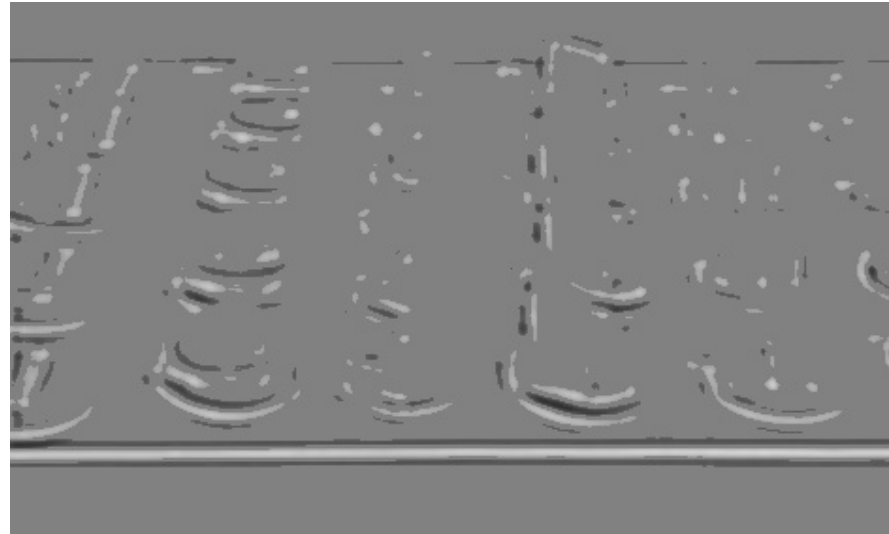
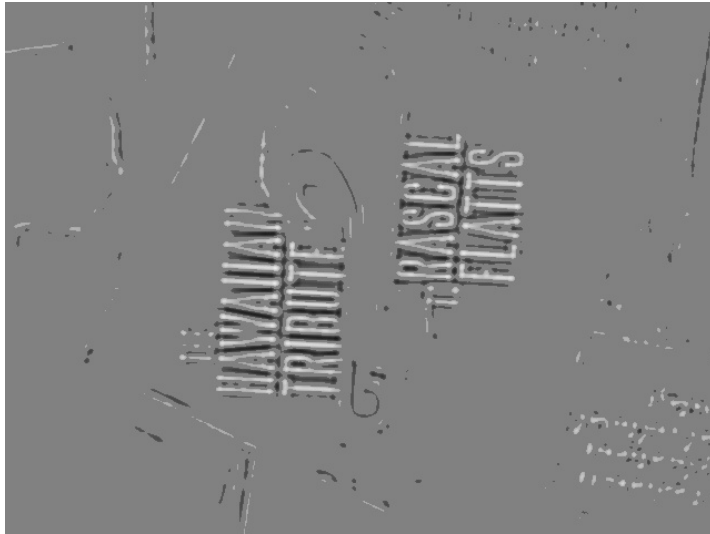
# Input images



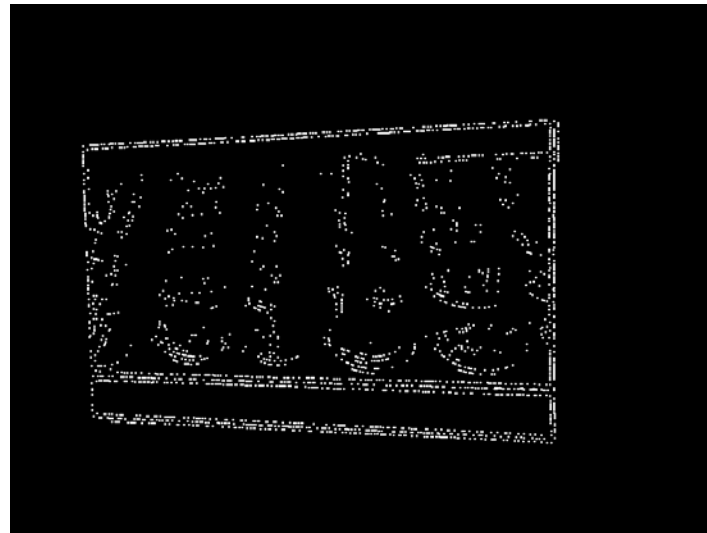
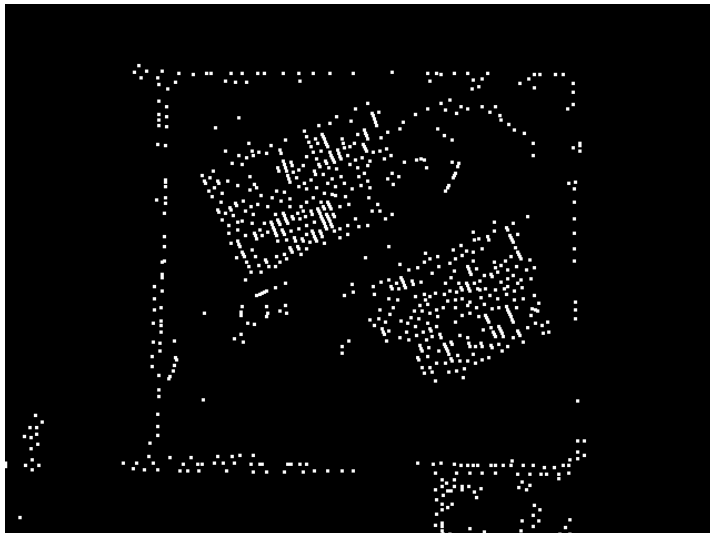
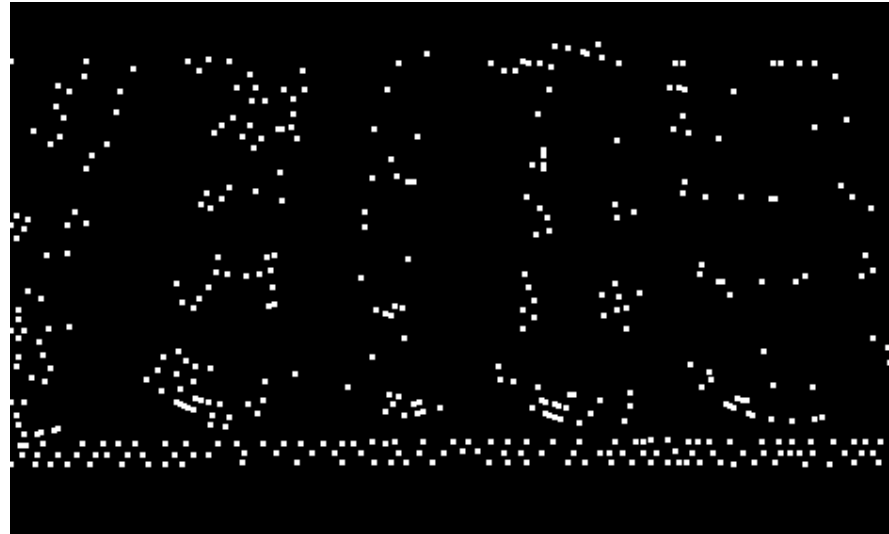
# LoG response



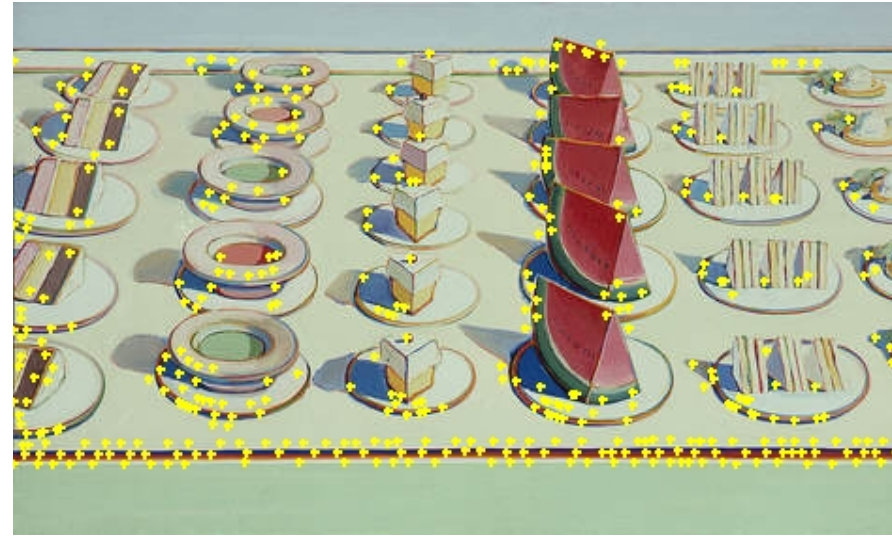
# Thresholded LoG response



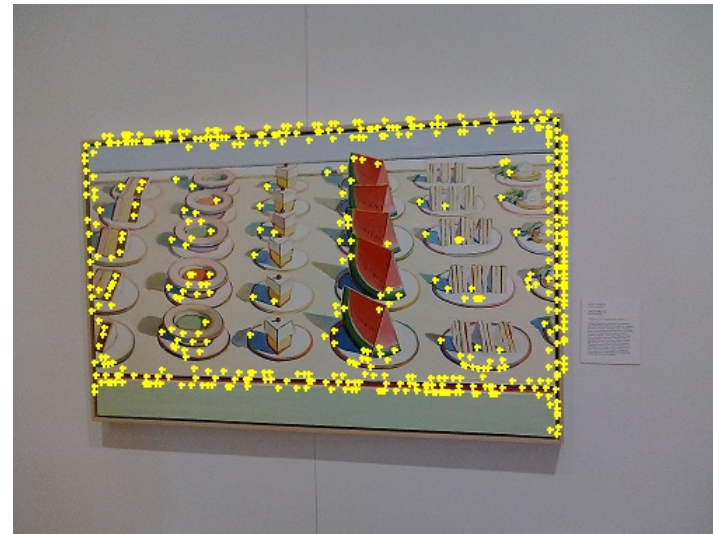
# Local extrema of thresholded LoG response



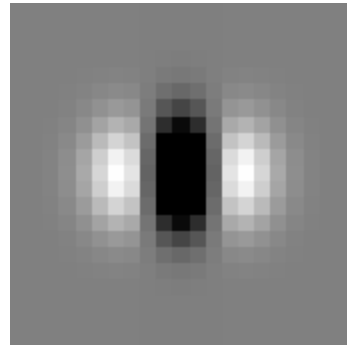
# Superimposed LoG keypoints



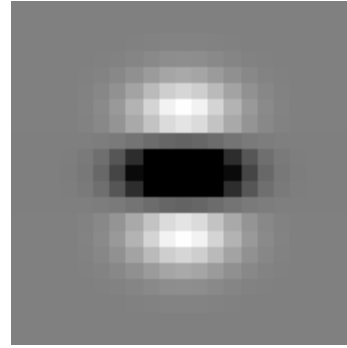
500 strongest keypoints



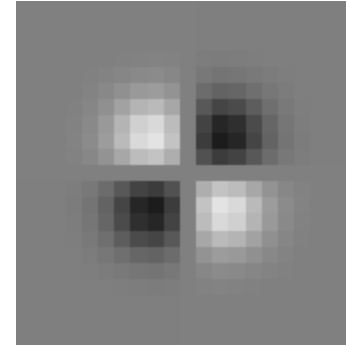
# Determinant of Hessian keypoint detector



$D_{xx}$



$D_{yy}$



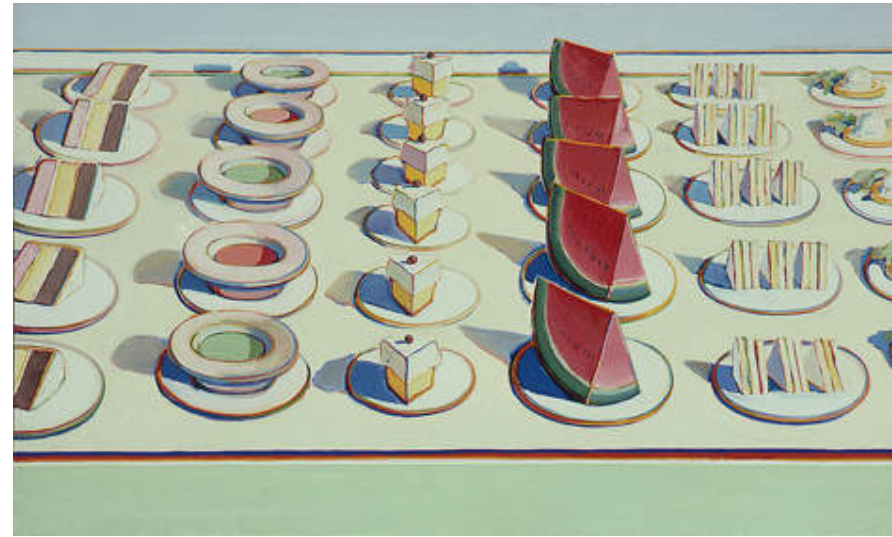
$D_{xy}$

$$\mathbf{H}[x,y] = \begin{bmatrix} f_{xx}[x,y] & f_{xy}[x,y] \\ f_{xy}[x,y] & f_{yy}[x,y] \end{bmatrix}$$
$$= \begin{bmatrix} D_{xx}[x,y] * f[x,y] & D_{xy}[x,y] * f[x,y] \\ D_{xy}[x,y] * f[x,y] & D_{yy}[x,y] * f[x,y] \end{bmatrix}$$

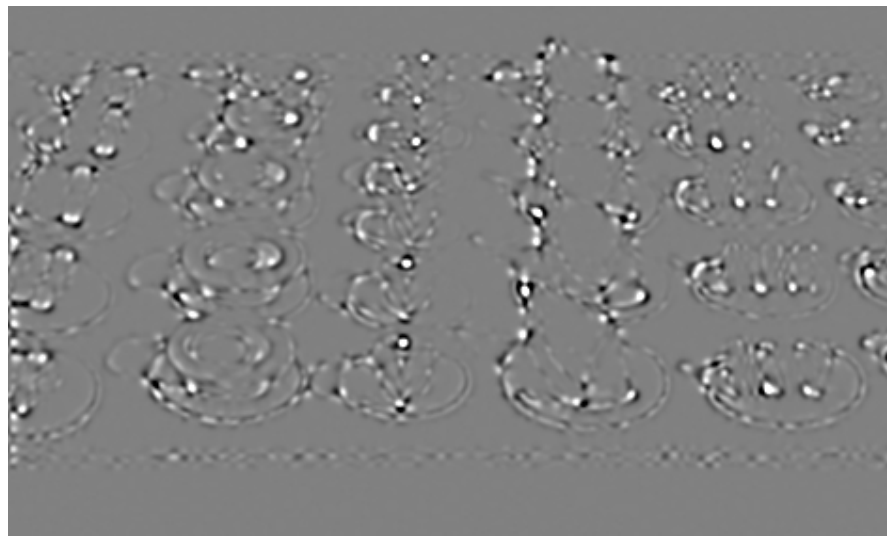
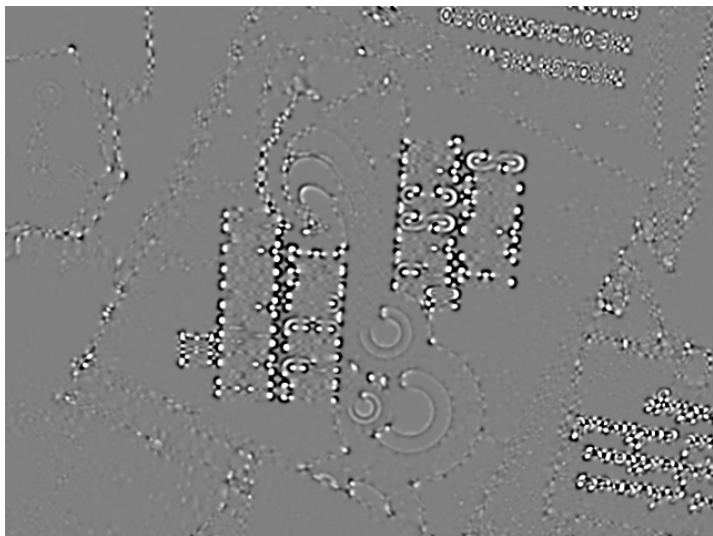
$$\det \mathbf{H}[x,y] = f_{xx}[x,y]f_{yy}[x,y] - (f_{xy}[x,y])^2$$



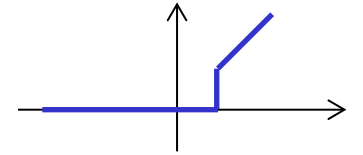
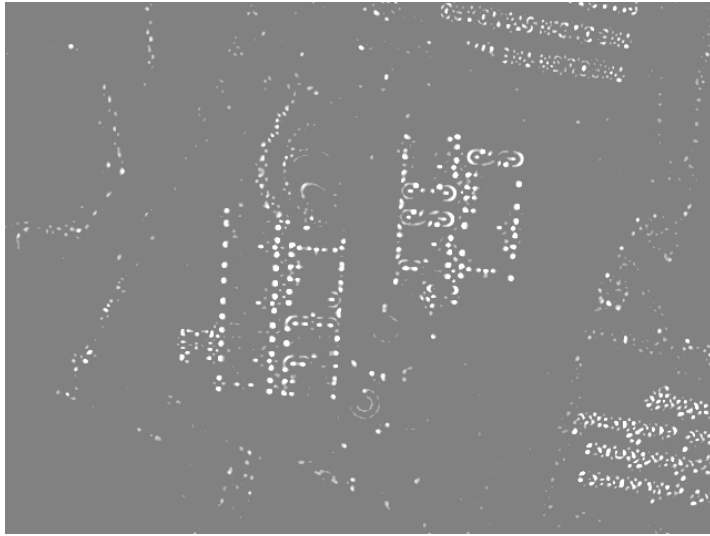
# Input images



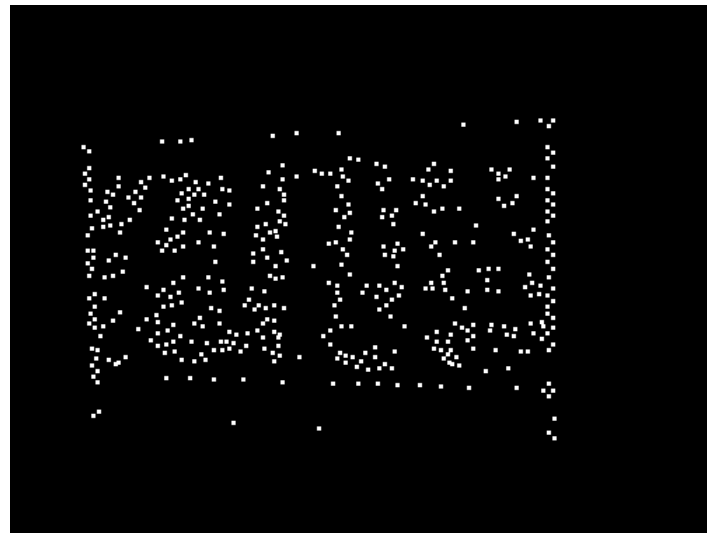
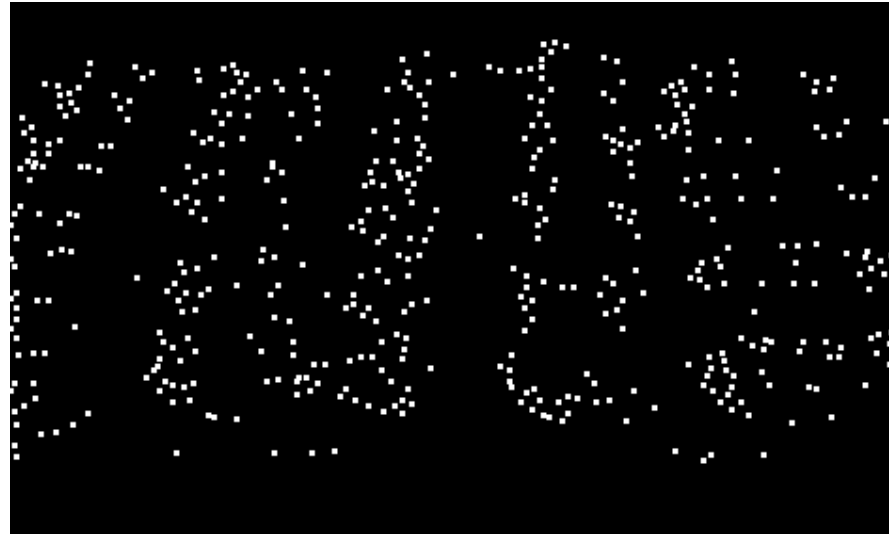
# DoH response



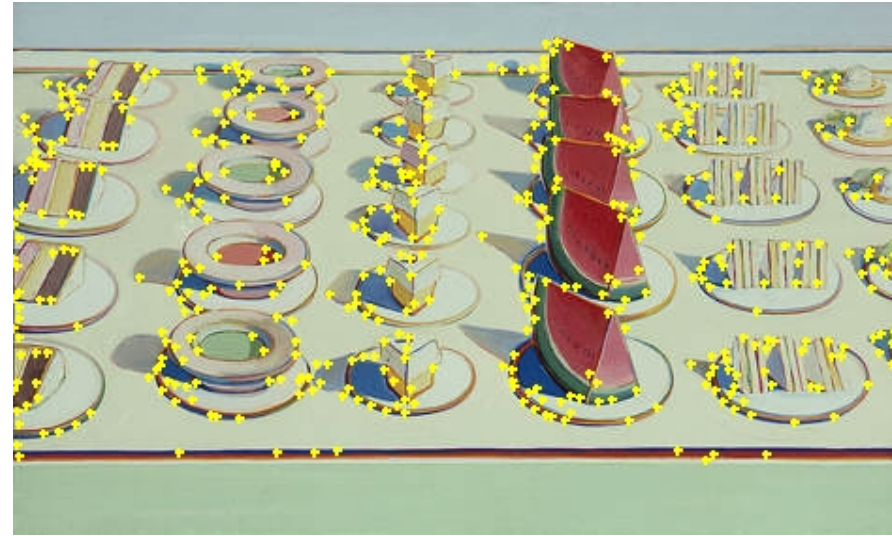
# Thresholded DoH response



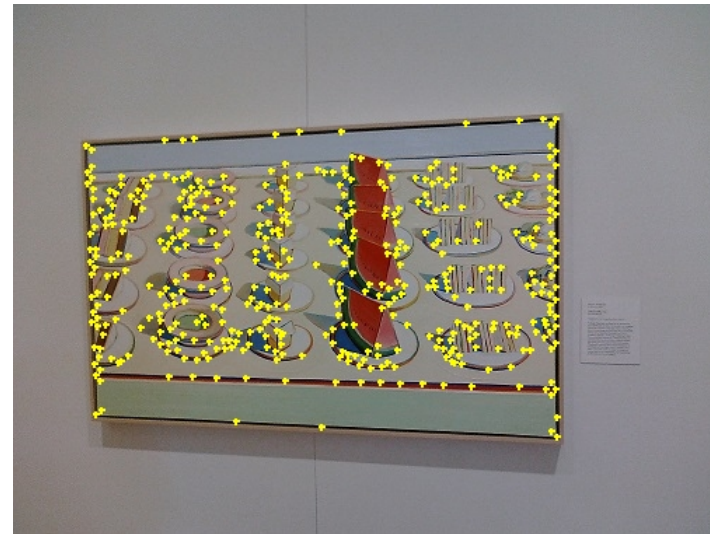
# Local maxima of DoH response



# Superimposed DoH keypoints

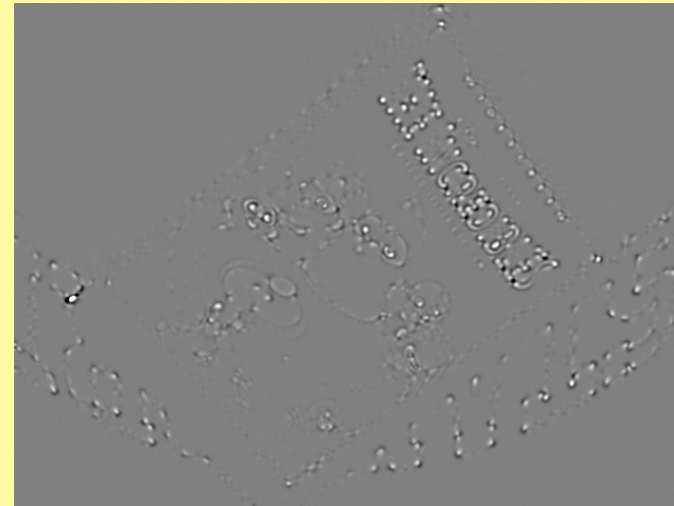


500 strongest keypoints





Which of the following images is the Laplacian of Gaussian and which is the Determinant of Hessian of the above image?



# What patterns can be localized most accurately?

- Local displacement sensitivity (assuming continuous  $f(x,y)$ )

$$S(\Delta x, \Delta y) = \sum_{(x,y) \in \text{window}} [f(x,y) - f(x + \Delta x, y + \Delta y)]^2$$

- Linear approximation for small  $\Delta x, \Delta y$

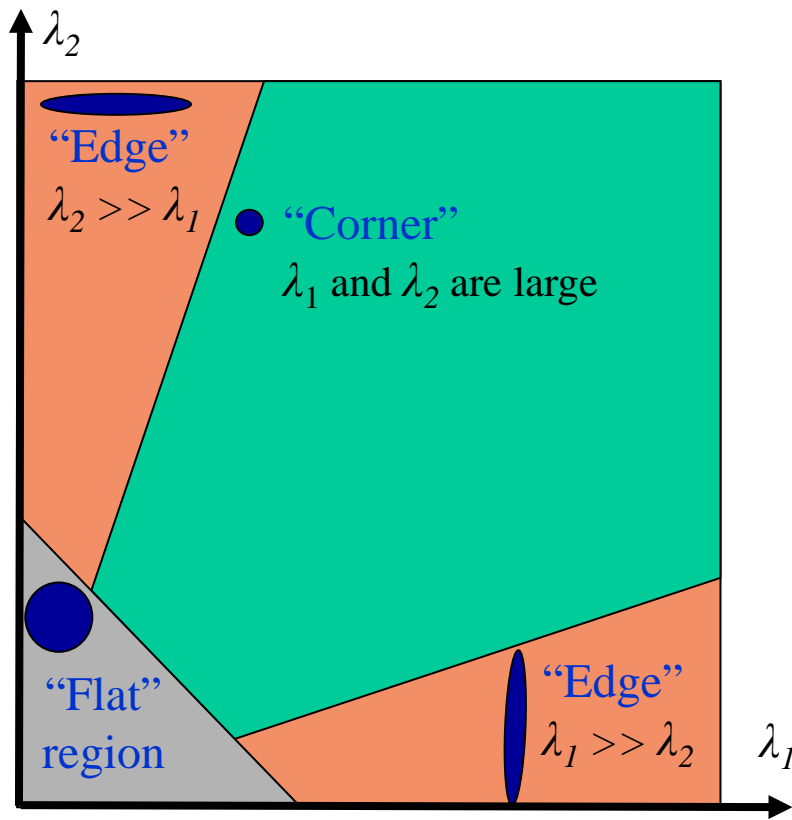
$$f(x + \Delta x, y + \Delta y) \approx f(x,y) + f_x(x,y)\Delta x + f_y(x,y)\Delta y$$

$f_x(x,y)$  – horizontal image gradient  
 $f_y(x,y)$  – vertical image gradient

$$\begin{aligned}
 S(\Delta x, \Delta y) &\approx \sum_{(x,y) \in \text{window}} \left[ \begin{pmatrix} f_x(x,y) & f_y(x,y) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right]^2 \\
 &= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \left\{ \sum_{(x,y) \in \text{window}} \begin{bmatrix} f_x^2(x,y) & f_x(x,y)f_y(x,y) \\ f_x(x,y)f_y(x,y) & f_y^2(x,y) \end{bmatrix} \right\} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \\
 &= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \mathbf{M} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}
 \end{aligned}$$

- Iso-sensitivity curves are ellipses

# Harris detector



Based on eigenvalues  $\lambda_1, \lambda_2$  of “structure matrix”  
(aka “normal matrix” aka “second-moment matrix”)

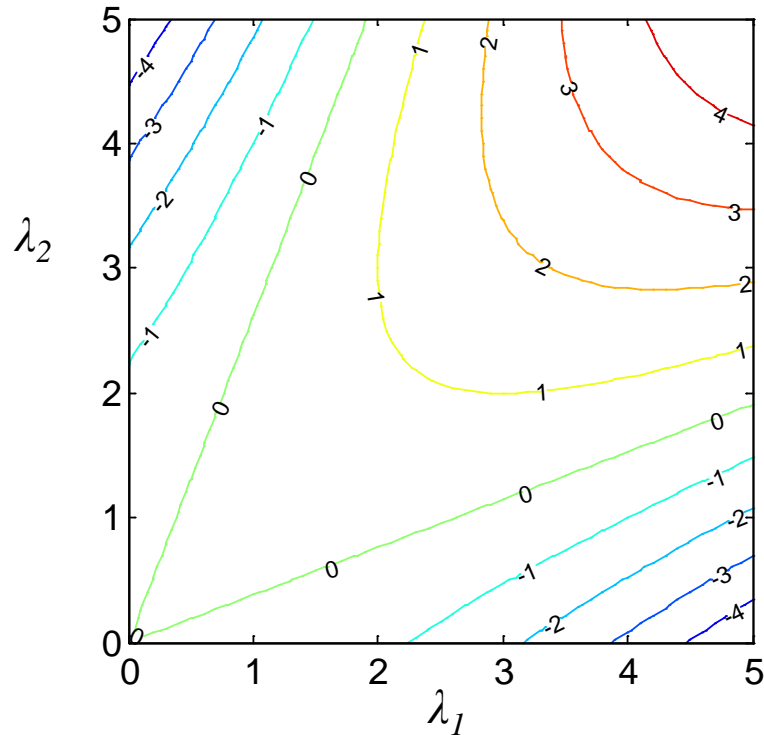
$$\mathbf{M} = \begin{bmatrix} \sum_{[x,y] \in \text{window}} f_x^2[x,y] & \sum_{[x,y] \in \text{window}} f_x[x,y]f_y[x,y] \\ \sum_{[x,y] \in \text{window}} f_x[x,y]f_y[x,y] & \sum_{[x,y] \in \text{window}} f_y^2[x,y] \end{bmatrix}$$

$f_x[x,y]$  – horizontal image gradient  
 $f_y[x,y]$  – vertical image gradient

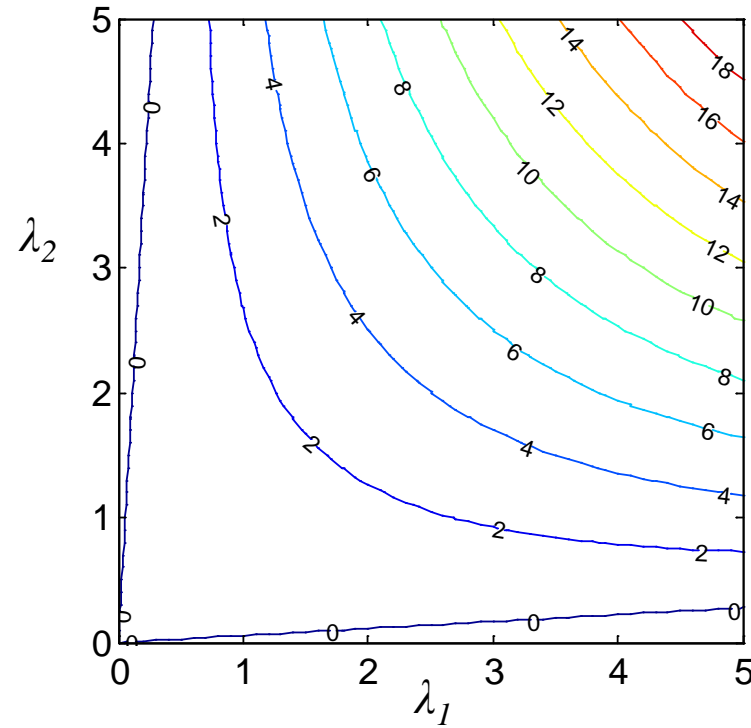
# Harris cornerness

$$C = \det(\mathbf{M}) - k \cdot (\text{trace}(\mathbf{M}))^2 = \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2$$

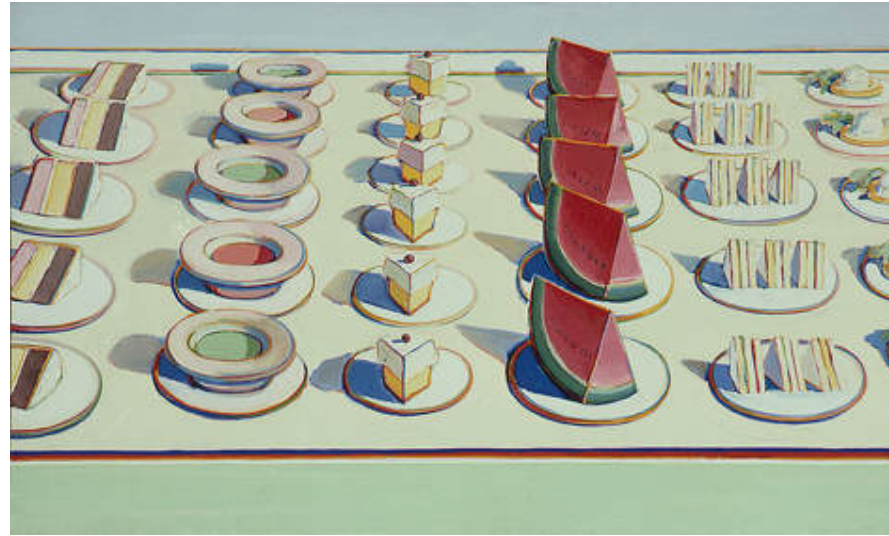
$k = 0.2$



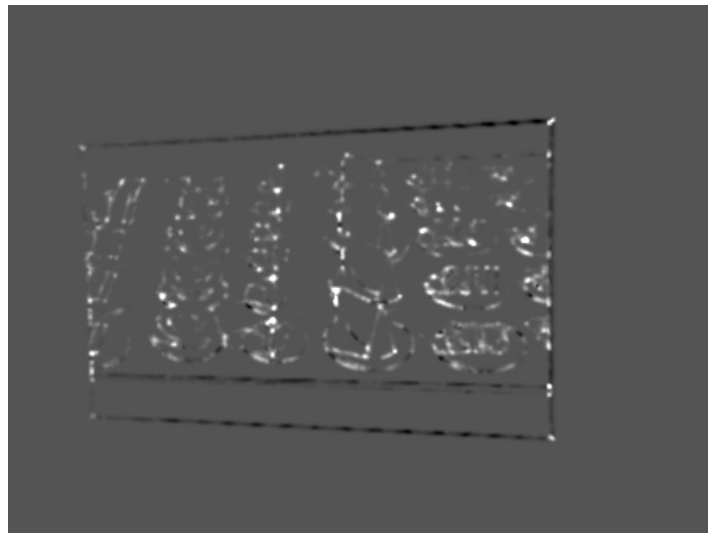
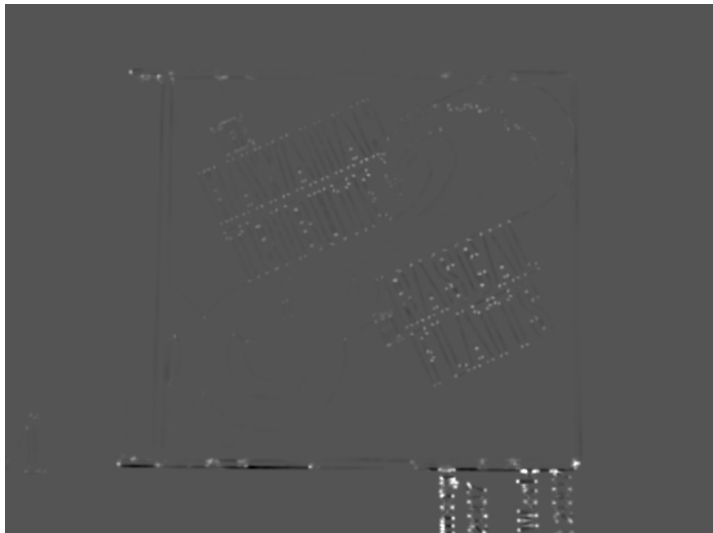
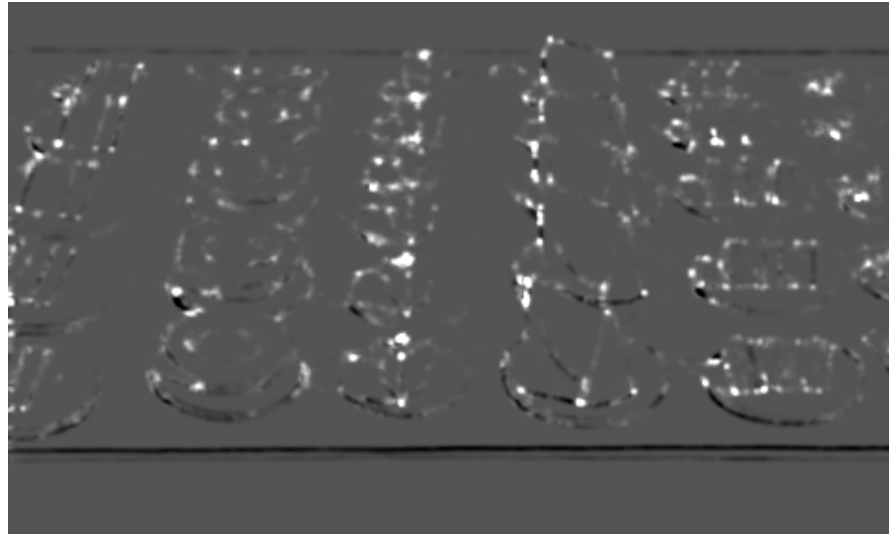
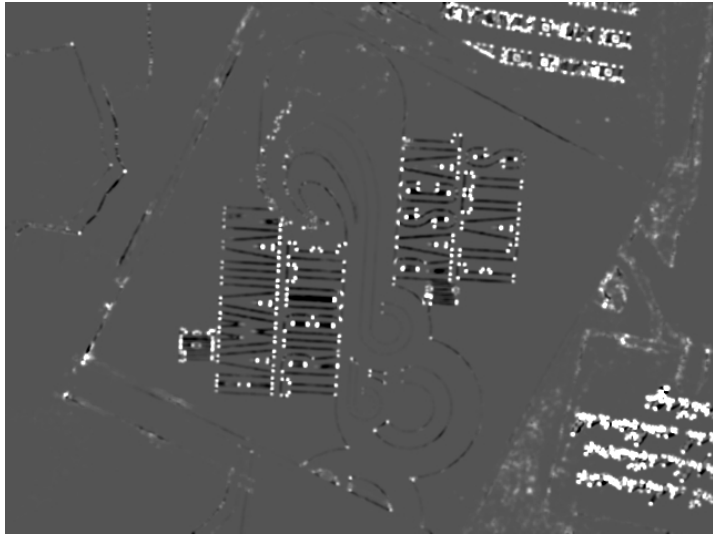
$k = 0.05$



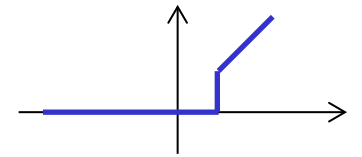
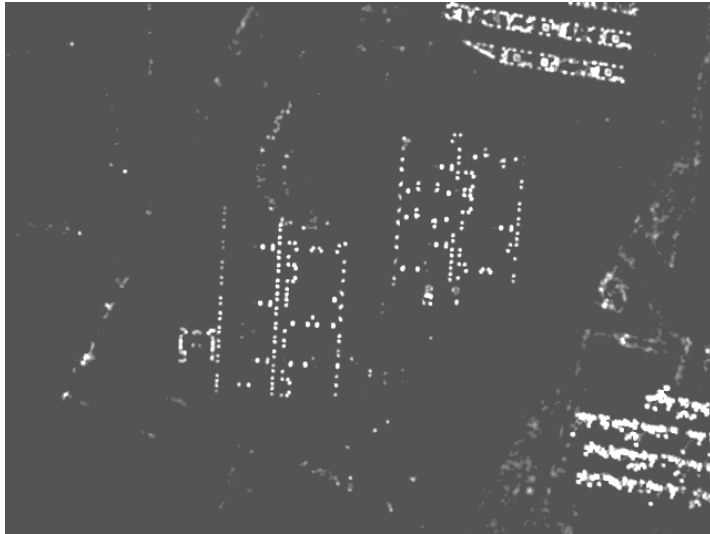
# Input images



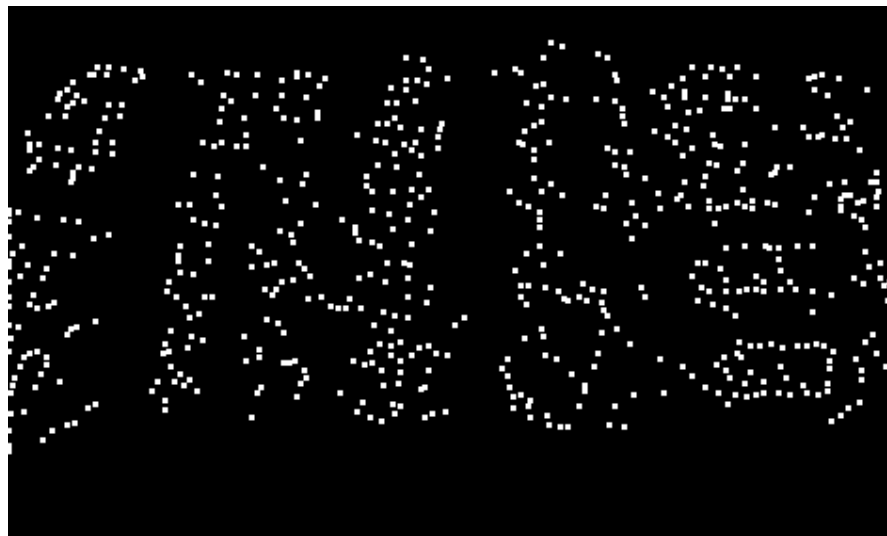
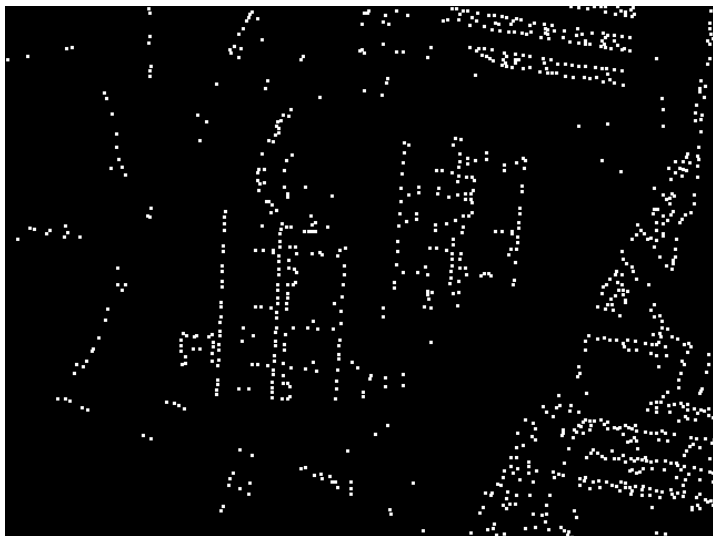
# Harris cornerness



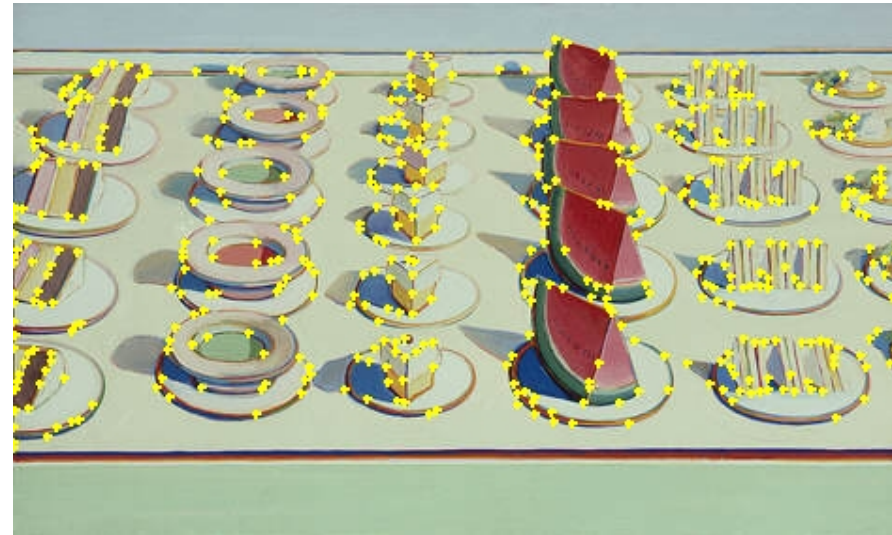
# Thresholded cornerness



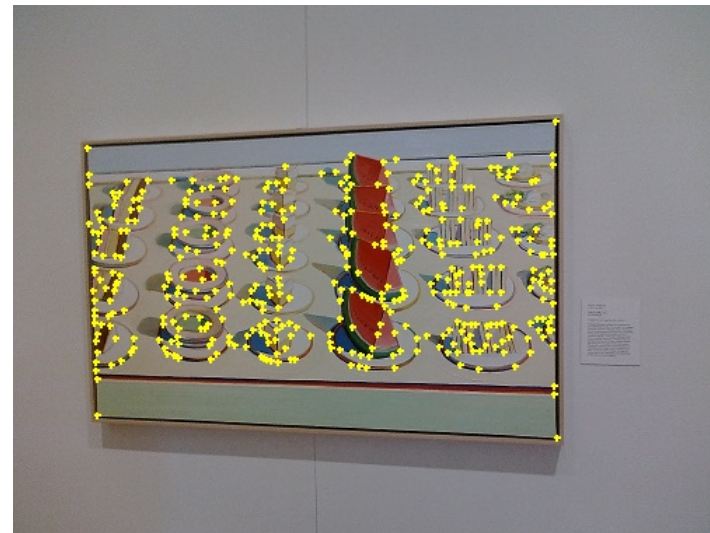
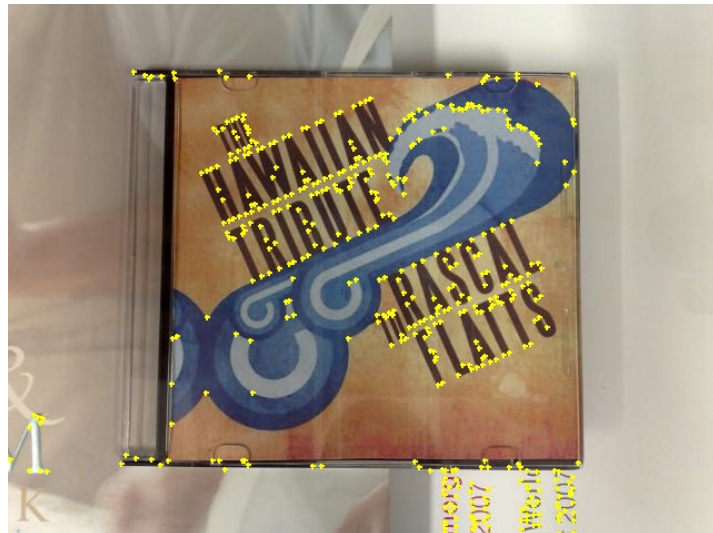
# Local maxima of cornerness



# Superimposed Harris keypoints



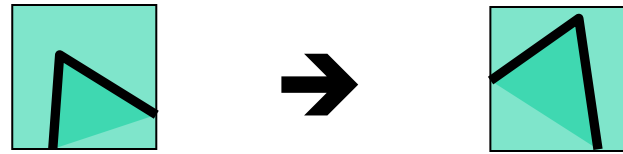
500 strongest keypoints



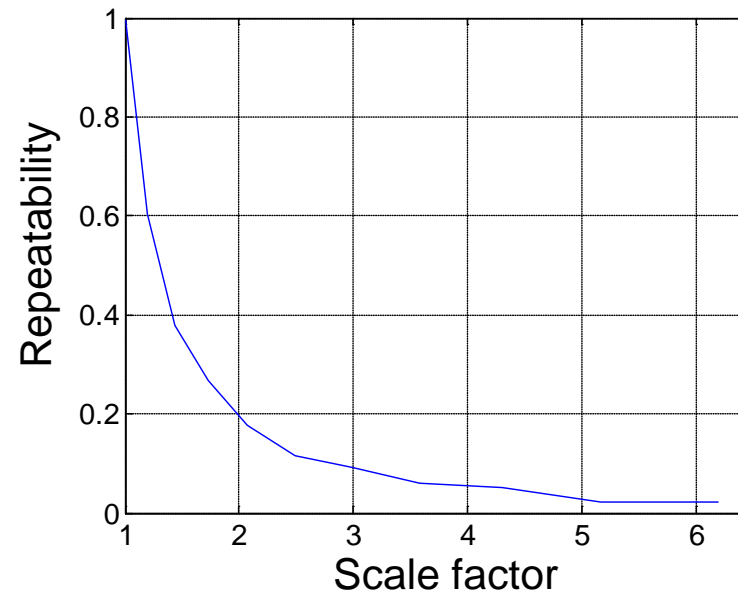
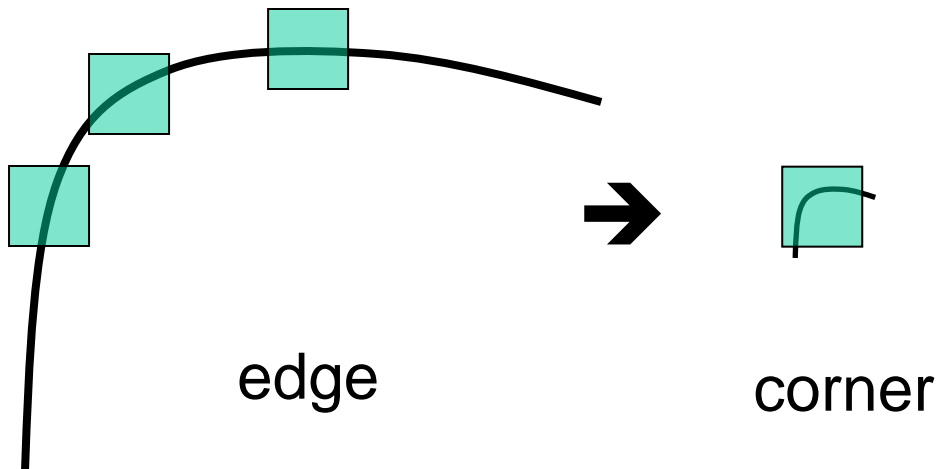
# Robustness of Harris detector

- Invariant to brightness offset:  $f[x,y] \rightarrow f[x,y] + c$

- Invariant to shift and rotation



- Not invariant to scaling



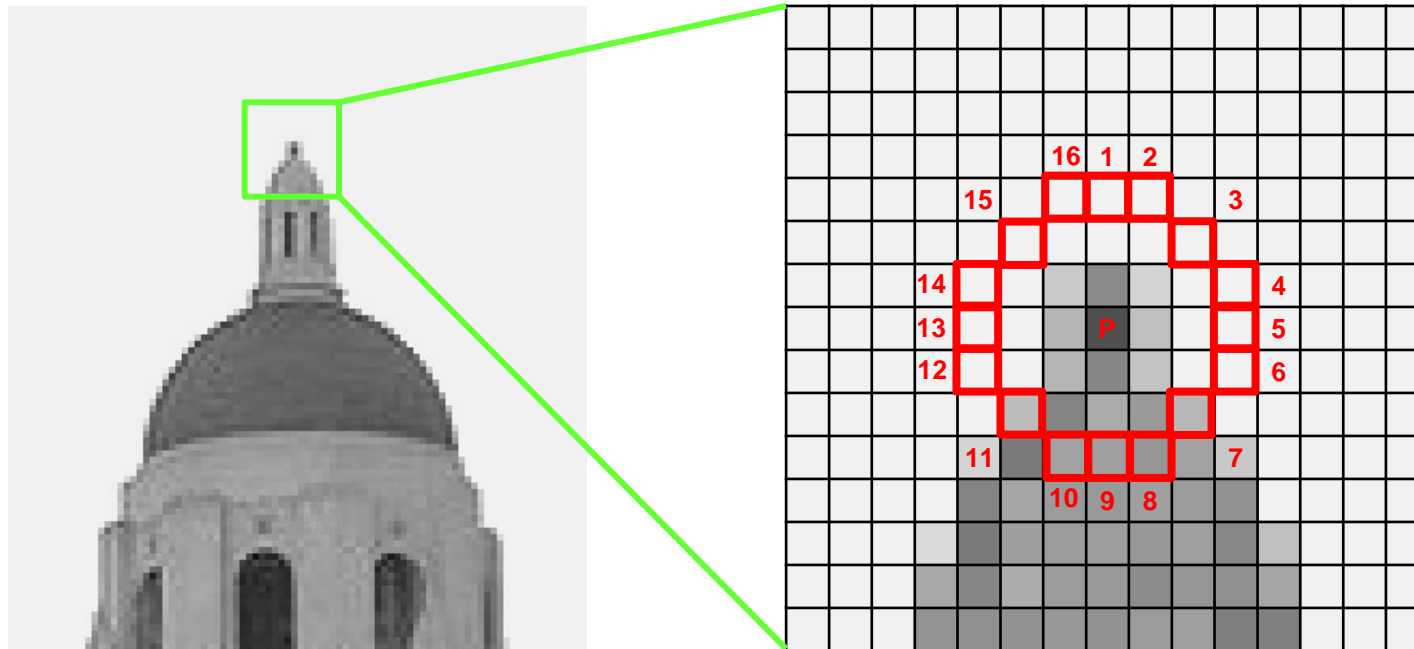
The following structure matrices are examples of the three different cases “Flat”, “Edge” and “Corner” of the Harris detector. Which is which?

$$M = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}$$

$$M = \begin{bmatrix} 12 & 0.1 \\ 0.1 & .003 \end{bmatrix}$$

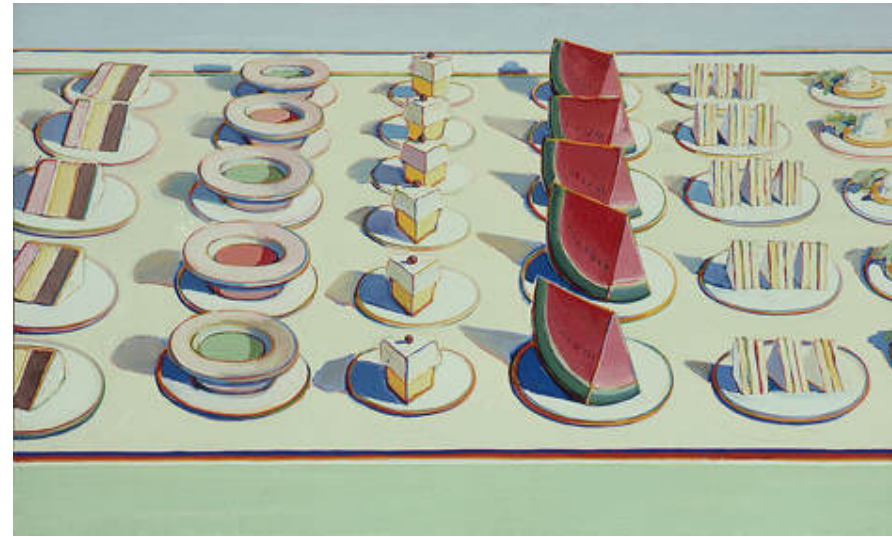
$$M = \begin{bmatrix} .05 & -0.01 \\ -0.01 & 0.02 \end{bmatrix}$$

# Features from Accelerated Segment Test (FAST)

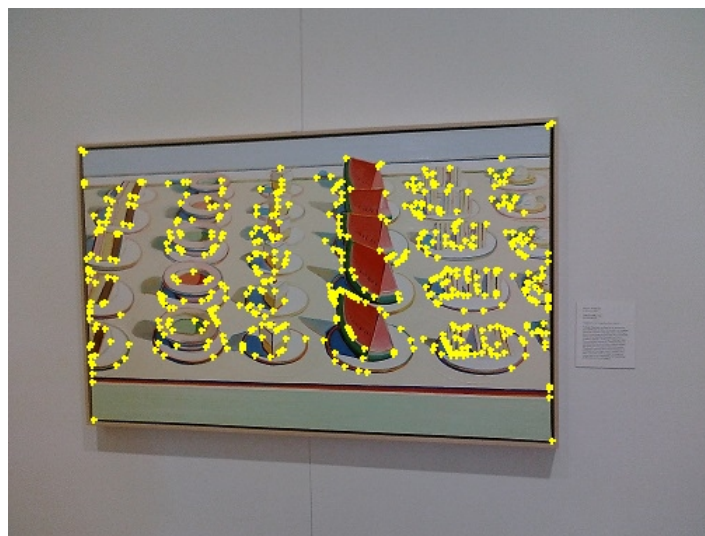
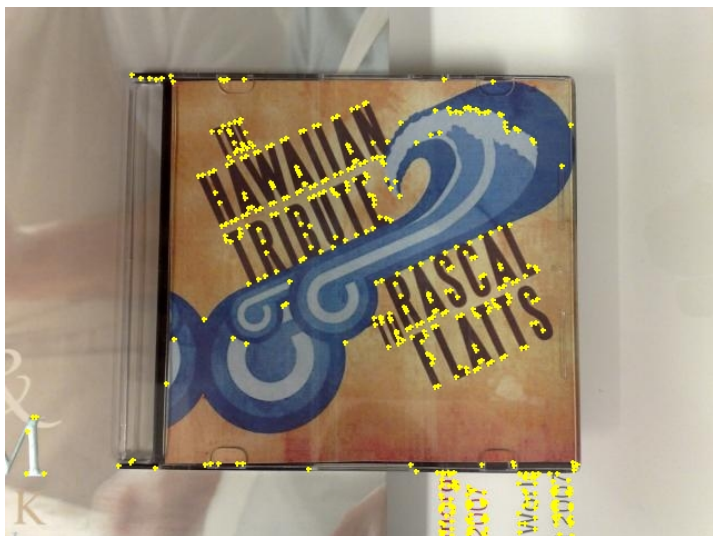
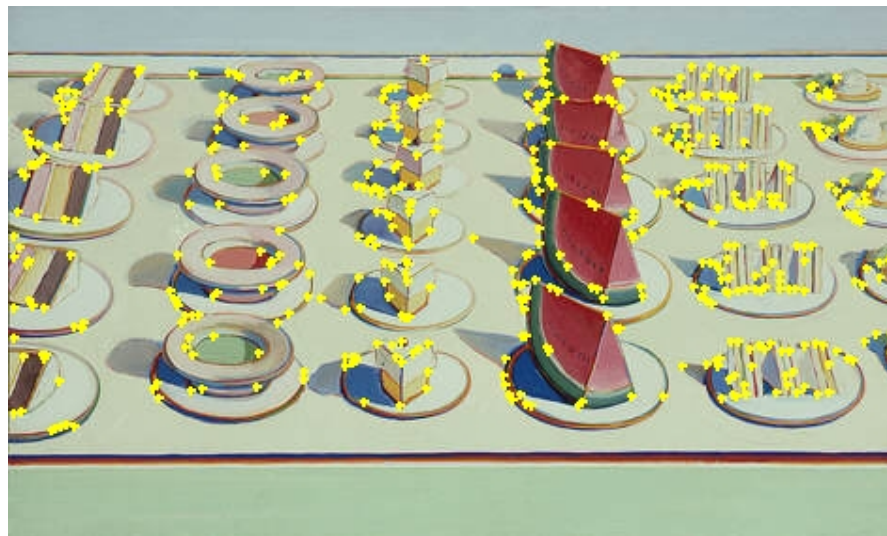


- Compare “nucleus”  $p$  to circle of sixteen pixels
- Nucleus is feature point, iff at least  $n=9$  contiguous circle pixels are either all brighter, or all darker, by  $\theta$
- Optimize pixel comparisons to reject non-corners early

# Input images



# FAST corners superimposed



# FAST corner detection on smartphone

# FAST keypoint tracking on smartphone