Edge detection

- Gradient-based edge operators
  - Prewitt
  - Sobel
  - Roberts
- Laplacian zero-crossings
- Canny edge detector
- Hough transform for detection of straight lines
- Circle Hough Transform

Gradient-based edge detection

- Idea (continous-space): local gradient magnitude indicates edge strength

$$\left|grad\left(f(x,y)\right)\right| = \sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2}$$

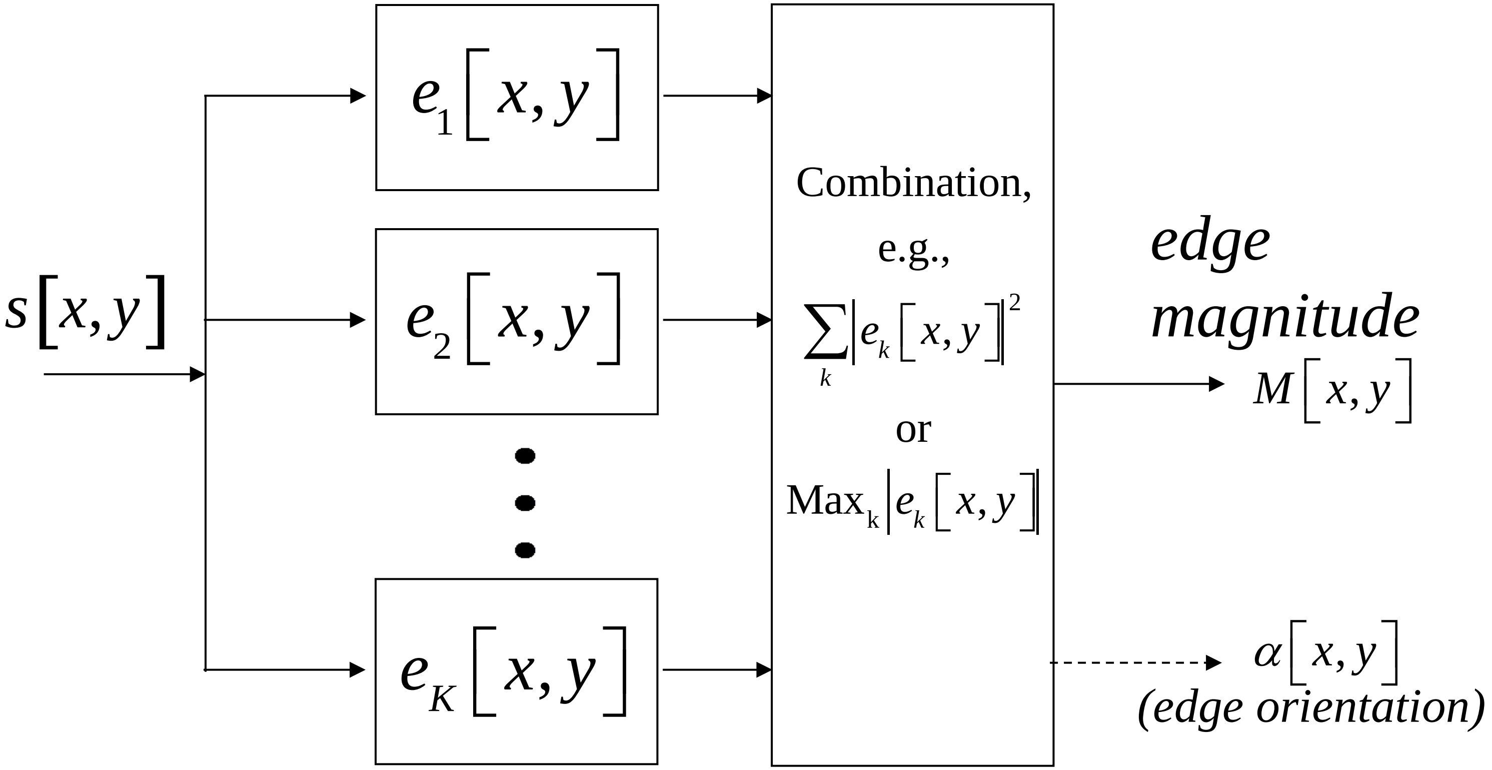
- Digital image: use finite differences to approximate derivatives

- Edge templates

difference	$\begin{pmatrix} -1 & 1 \end{pmatrix}$
central difference	$\begin{pmatrix} -1 & [0] & 1 \end{pmatrix}$
Prewitt	$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$
Sobel	$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix}$

Practical edge detectors

- Edges can have any orientation
- Typical edge detection scheme uses $K=2$ edge templates
- Some use $K>2$



Gradient filters (K=2)

Central Difference
$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & [0] & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & [0] & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Roberts
$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

Prewitt
$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Sobel
$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -2 & -1 \\ 0 & [0] & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

Kirsch operator (K=8)

$$\text{Kirsch}\quad \begin{pmatrix} +5 & +5 & +5 \\ -3 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & +5 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & -3 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & +5 \\ -3 & +5 & +5 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & -3 \\ +5 & +5 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & +5 & -3 \end{pmatrix} \begin{pmatrix} +5 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & -3 & -3 \end{pmatrix} \begin{pmatrix} +5 & +5 & -3 \\ +5 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix}$$

Prewitt operator example

Original
1024x710

Magnitude of image filtered with

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

(log display)

Magnitude of image filtered with

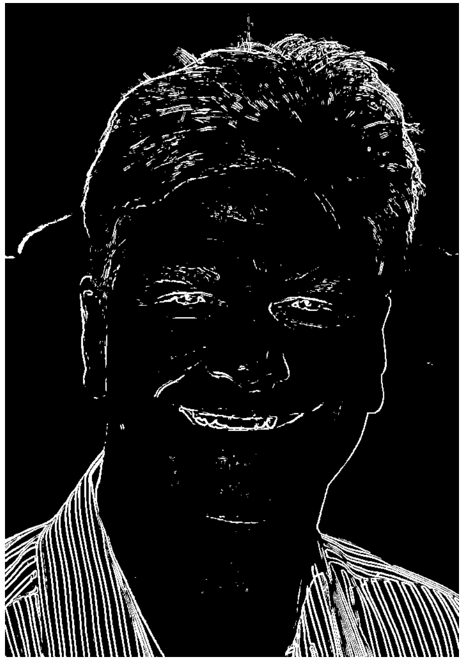
$$\begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

(log display)

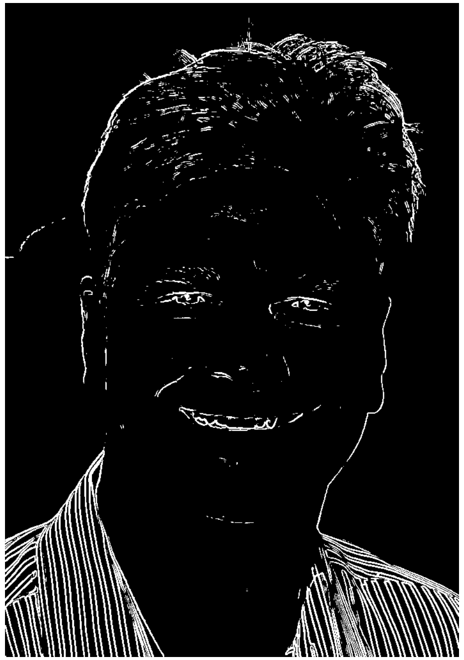
Prewitt operator example (cont.)

Sum of squared horizontal and vertical gradients (log display)

threshold = 900



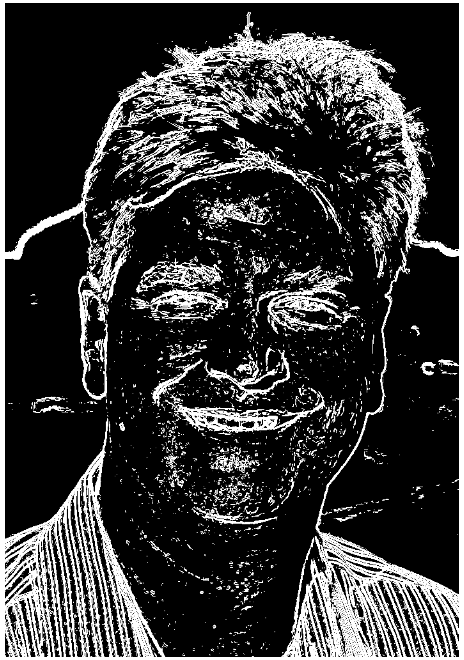
threshold = 4500



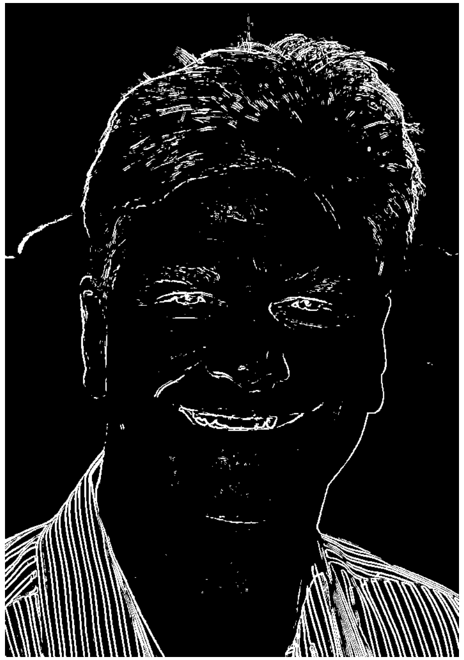
threshold = 7200

Sobel operator example

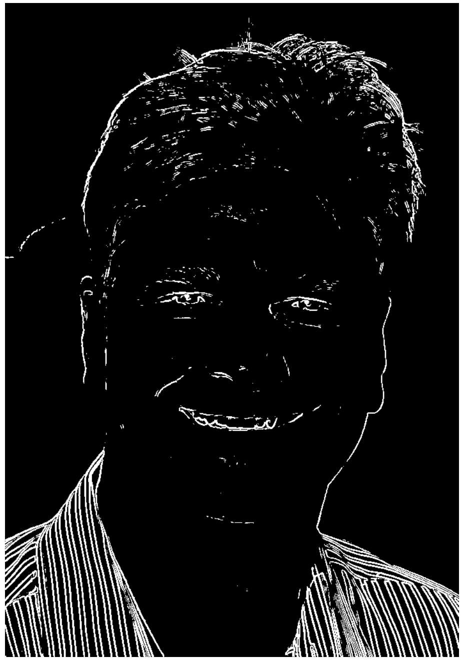
Sum of squared horizontal and vertical gradients (log display)



threshold = 1600



threshold = 8000



threshold = 12800

Roberts operator example

Original
1024x710

Magnitude of image filtered with

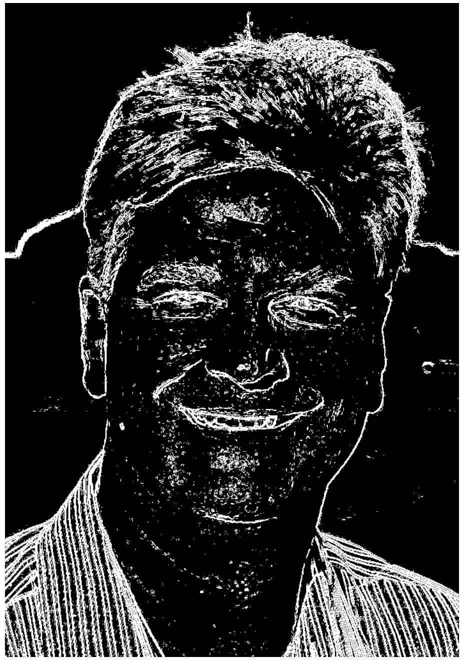
$$\begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

Magnitude of image filtered with

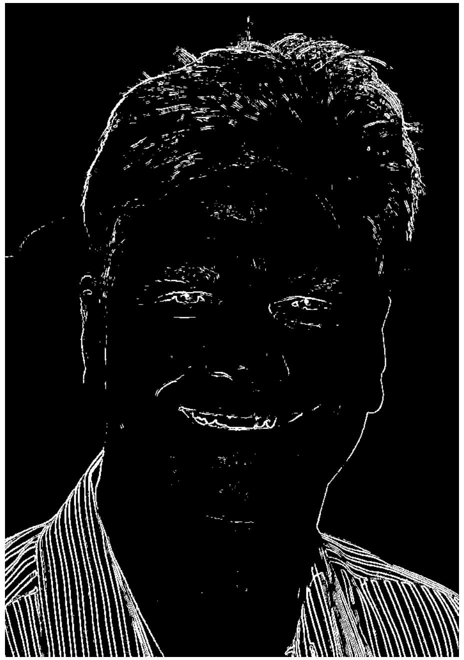
$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix}$$

Roberts operator example (cont.)

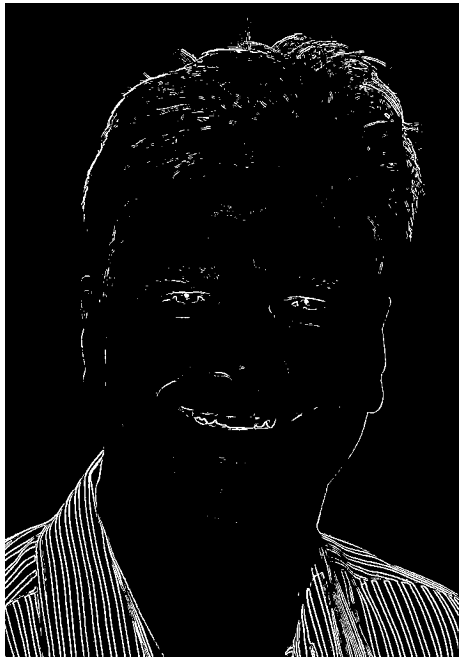
Sum of squared diagonal gradients (log display)



threshold = 100



threshold = 500

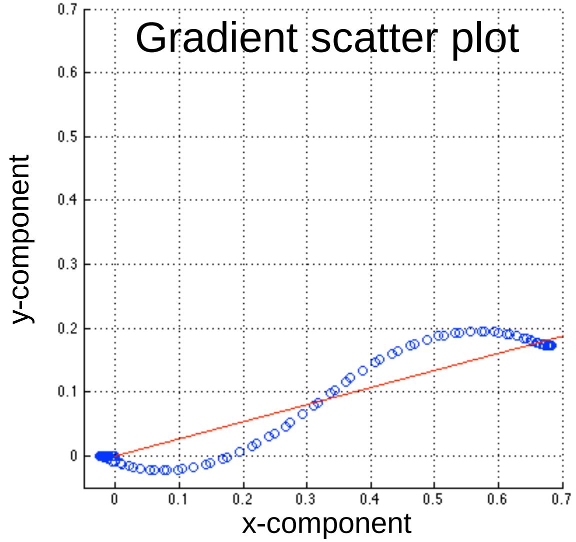


threshold = 800

Edge orientation

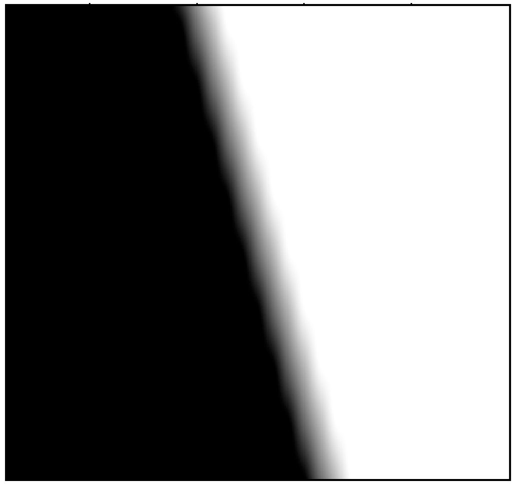
Central Difference

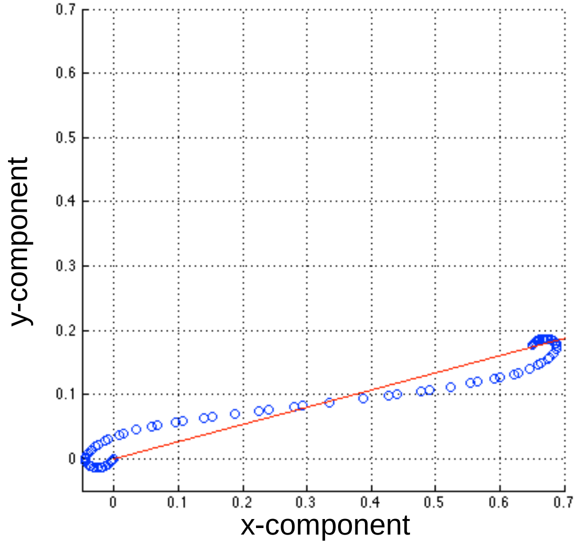
$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & [0] & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & [0] & 0 \\ 0 & 1 & 0 \end{pmatrix}$$



Roberts

$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$



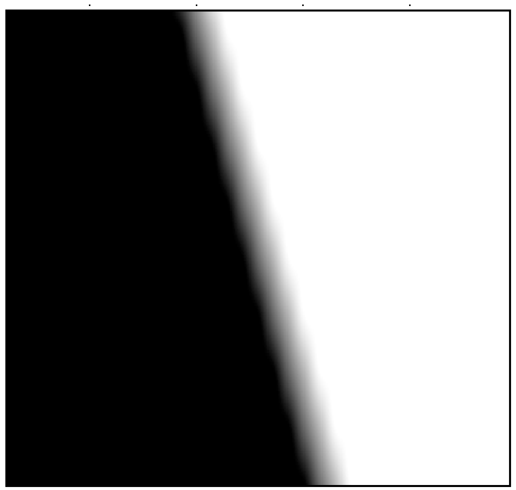


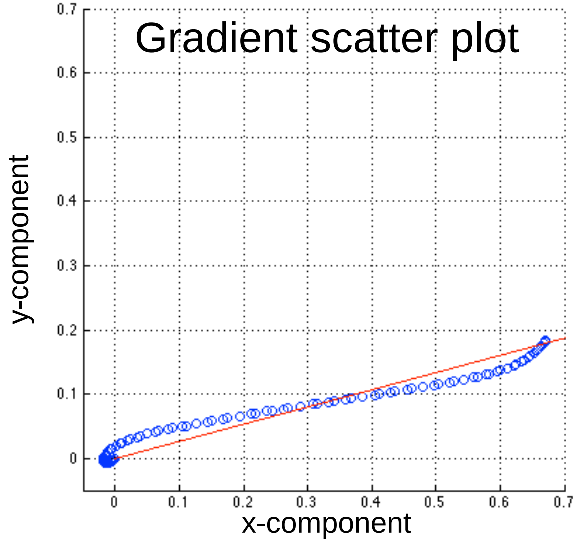
Edge orientation

Prewitt

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

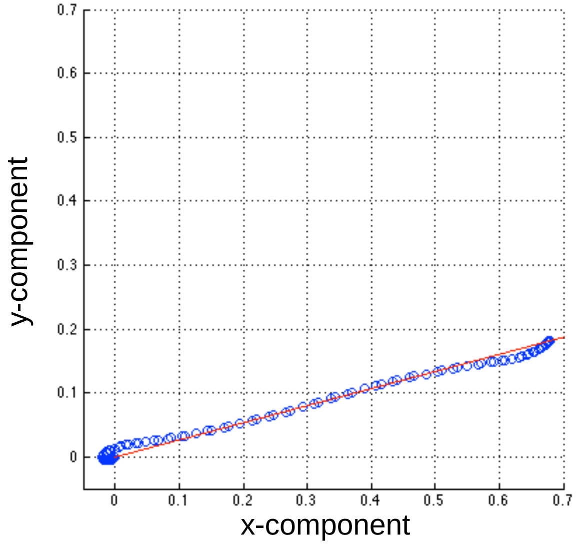




Sobel

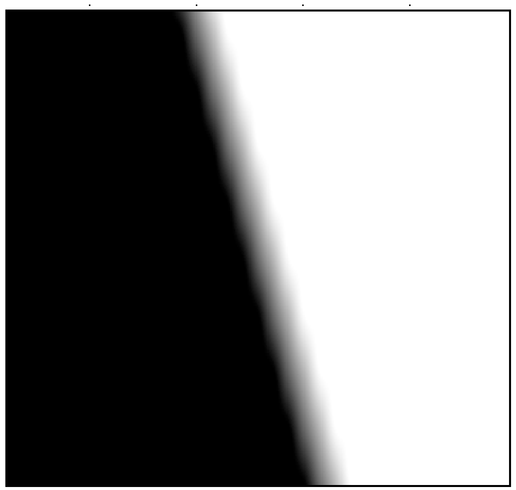
$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix}$$

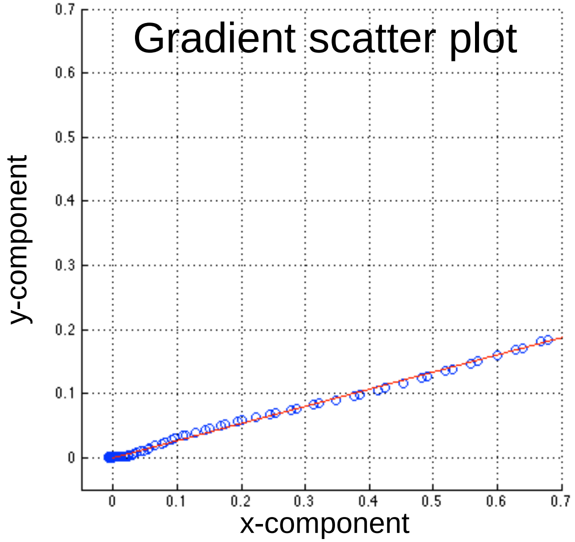
$$\begin{pmatrix} -1 & -2 & -1 \\ 0 & [0] & 0 \\ 1 & 2 & 1 \end{pmatrix}$$



Edge orientation

5x5 "consistent" gradient operator [Ando, 2000]

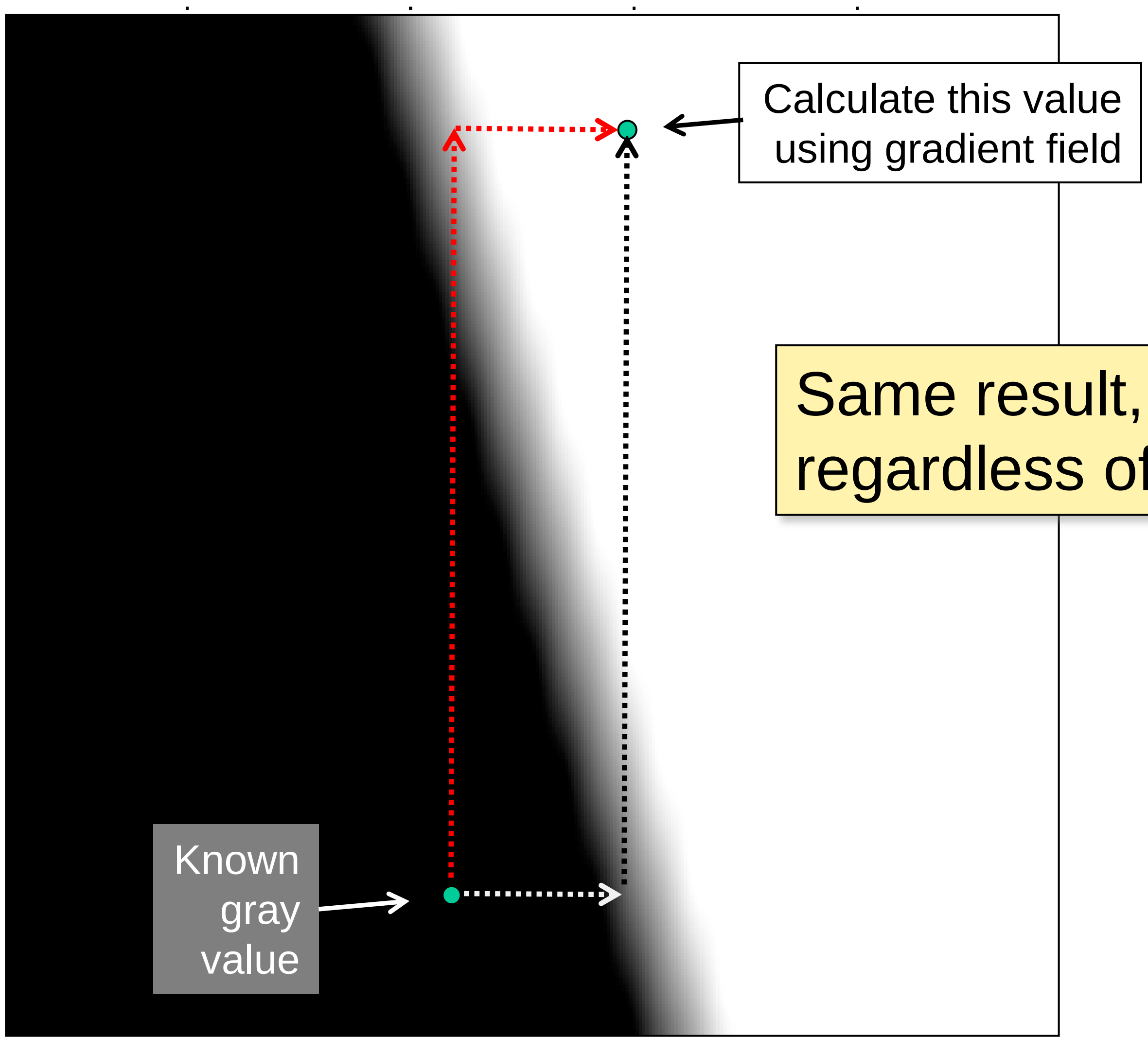




-0.0604	-0.1632	0	0.1632	0.0604
-0.4286	-1.1335	0	1.1335	0.4286
-0.7448	-1.9612	[0]	1.9612	0.7448
-0.4286	-1.1335	0	1.1335	0.4286
-0.0604	-0.1632	0	0.1632	0.0604

-0.0604	-0.4286	-0.7448	-0.4286	-0.0604
-0.1632	-1.1335	-1.9612	-1.1335	-0.1632
0	0	[0]	0	0
0.1632	1.1335	1.9612	1.1335	0.1632
0.0604	0.4286	0.7448	0.4286	0.0604

Gradient consistency problem



Same result, regardless of path?

In order to make the Sobel and Prewitt edge detection results very similar, threshold values must be chosen as

$$\theta(\text{Sobel}) = \frac{16}{9}\theta(\text{Prewitt})$$

Which of the following ratios would make Sobel and Central Difference (CD) edge detection results most similar?

(a) $\theta(\text{Sobel}) = \theta(CD)$

(b) $\theta(\text{Sobel}) = 4 \cdot \theta(CD)$

(c) $\theta(\text{Sobel}) = 8 \cdot \theta(CD)$

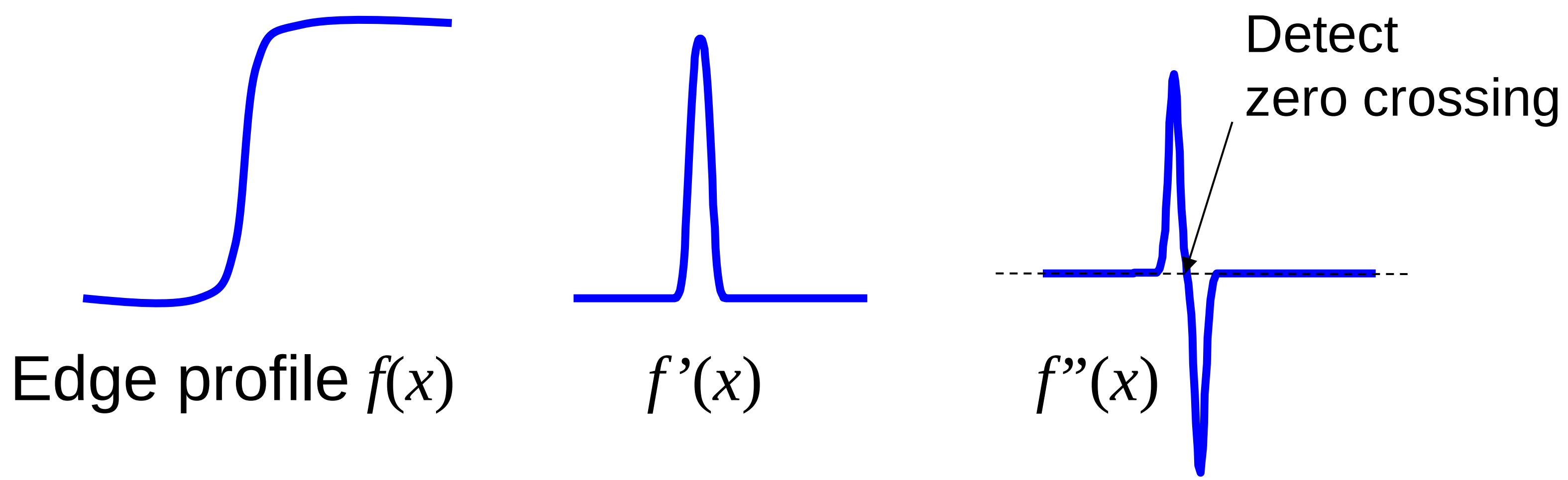
(d) $\theta(\text{Sobel}) = 16 \cdot \theta(CD)$

Laplacian operator

- Detect edges by considering second derivative

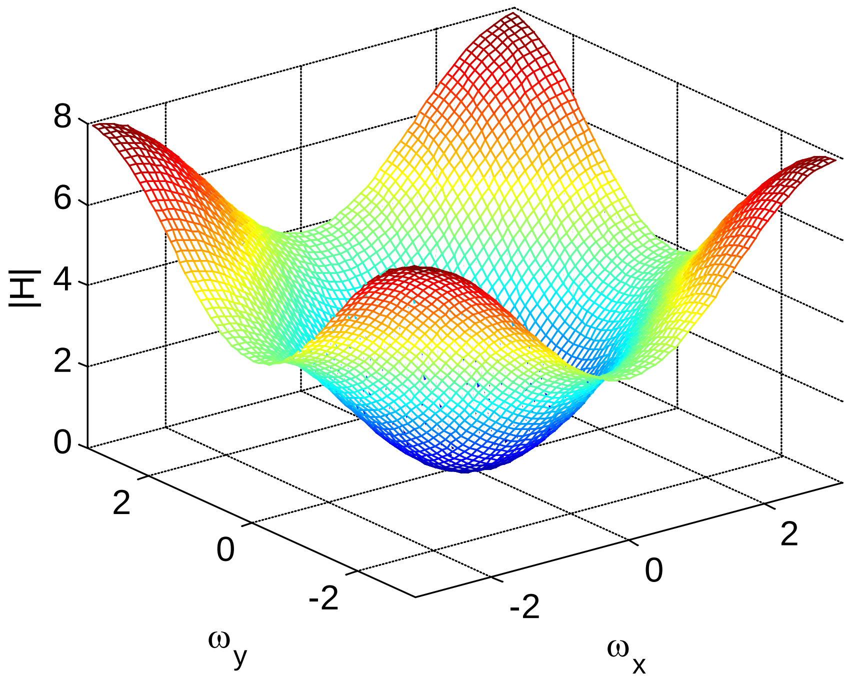
$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

- Isotropic (rotationally invariant) operator
- Zero-crossings mark edge location

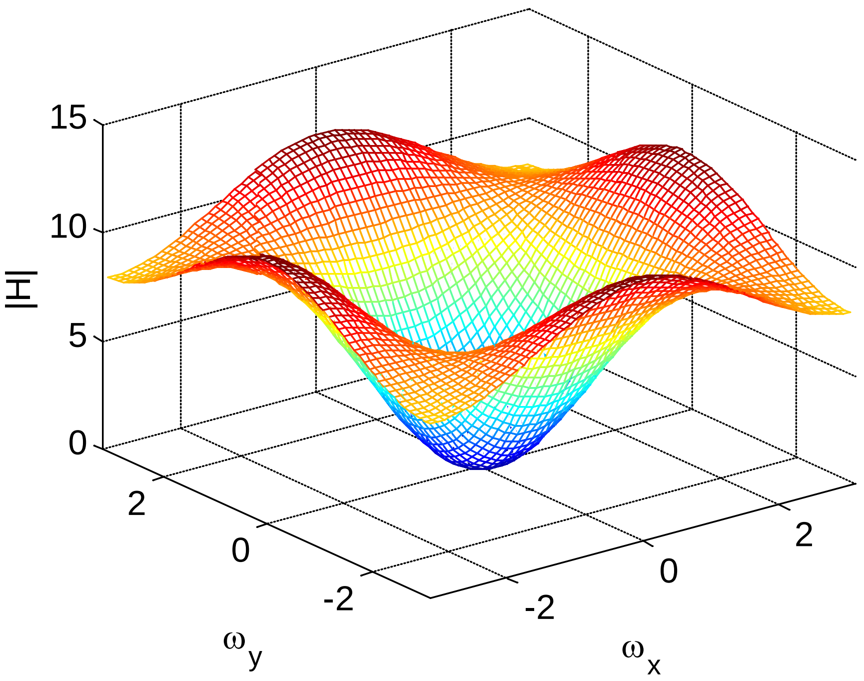


Approximations of Laplacian operator by 3x3 filter

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & [-4] & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & [-8] & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



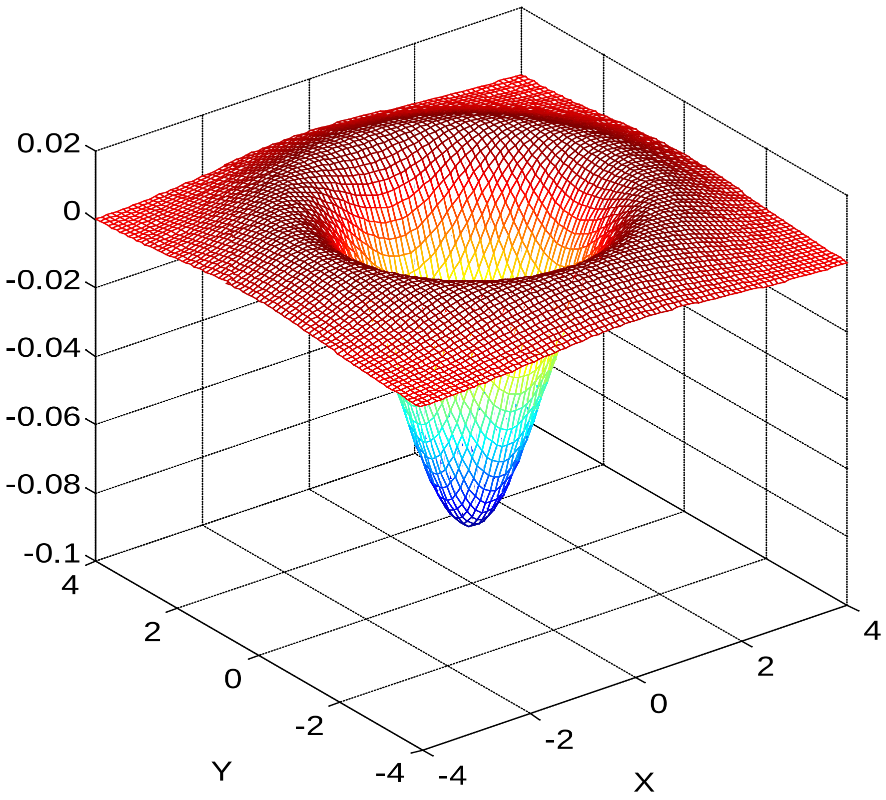
Zero crossings of Laplacian

- Sensitive to very fine detail and noise ➔ blur image first
- Responds equally to strong and weak edges
  ➔ suppress zero-crossings with low gradient magnitude

Laplacian of Gaussian

- Filtering of image with Gaussian and Laplacian operators can be combined into convolution with Laplacian of Gaussian (LoG) operator

$$LoG(x,y) = -\frac{1}{\pi\sigma^4}\left(1-\frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$

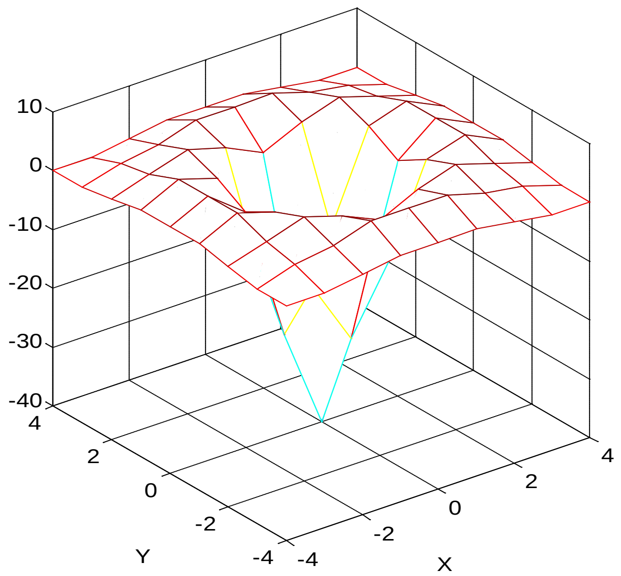


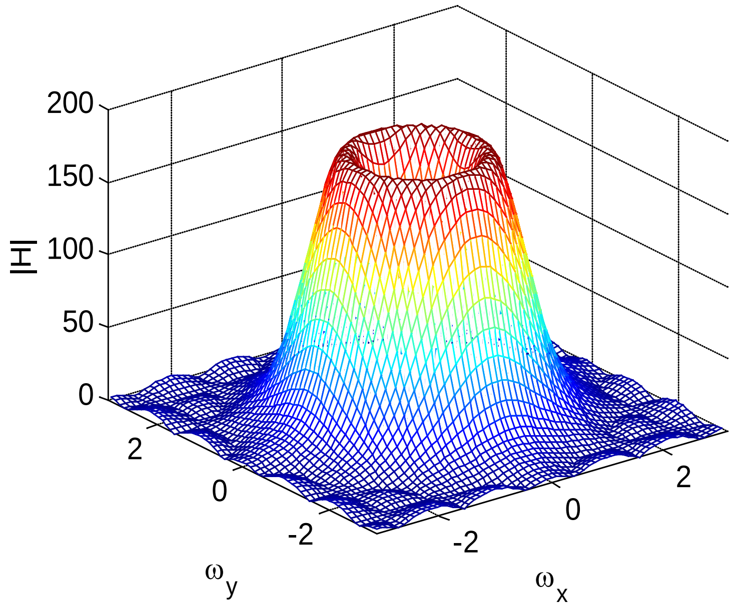
$\sigma = \sqrt{2}$

Discrete approximation of Laplacian of Gaussian

$$\sigma = \sqrt{2}$$

0	0	1	2	2	2	1	0	0
0	2	3	5	5	5	3	2	0
1	3	5	3	0	3	5	3	1
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
1	3	5	3	0	3	5	3	1
0	2	3	5	5	5	3	2	0
0	0	1	2	2	2	1	0	0



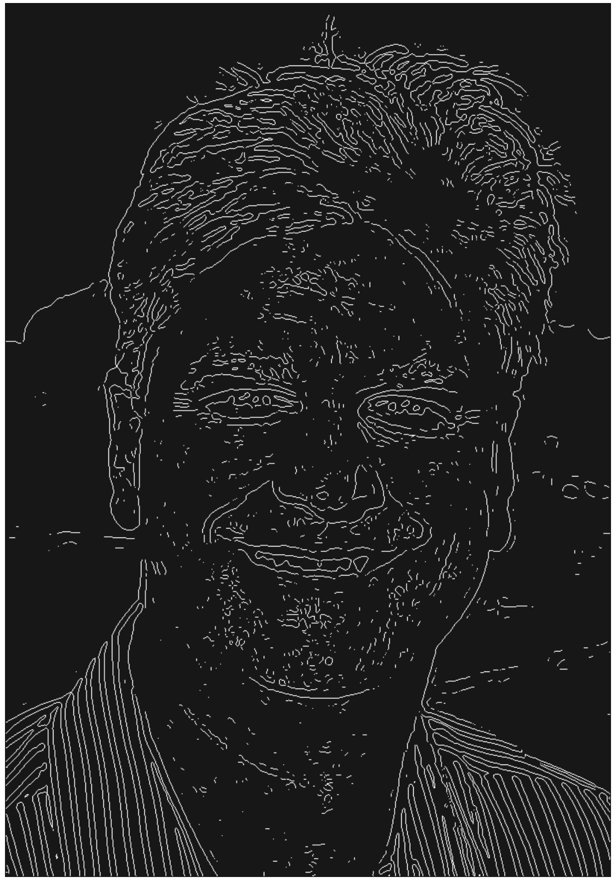


Zero crossings of LoG

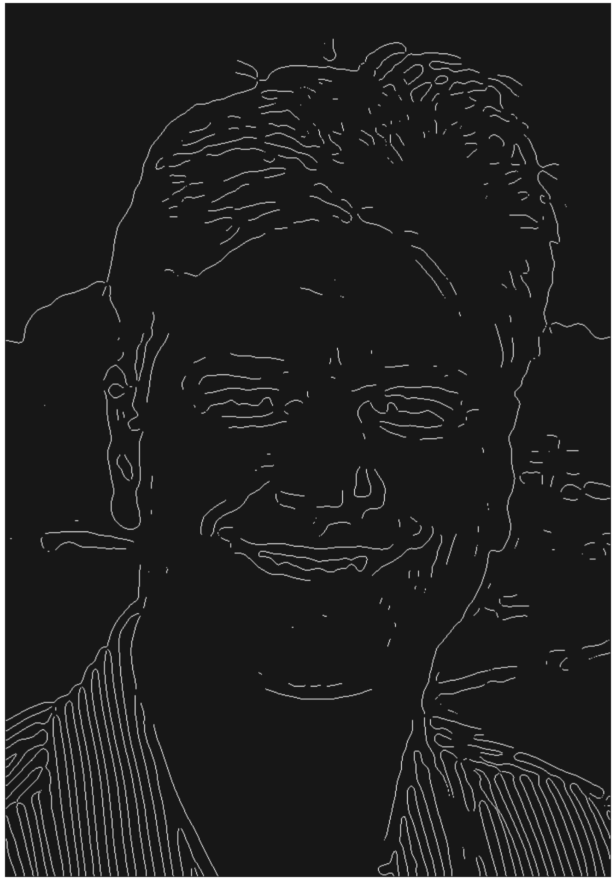
$\sigma = \sqrt{2}$ $\sigma = 2\sqrt{2}$ $\sigma = 4\sqrt{2}$ $\sigma = 8\sqrt{2}$

Zero crossings of LoG – gradient-based threshold

$\sigma = \sqrt{2}$



$\sigma = 2\sqrt{2}$



$\sigma = 4\sqrt{2}$

$\sigma = 8\sqrt{2}$

Canny edge detector

1. Smooth image with a Gaussian filter
2. Approximate gradient magnitude and angle (use Sobel, Prewitt . . .)

$$M[x,y] \approx \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

$$\alpha[x,y] \approx \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

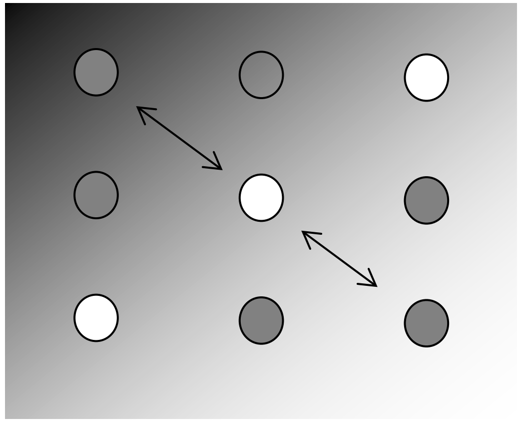
3. Apply nonmaxima suppression to gradient magnitude
4. Double thresholding to detect strong and weak edge pixels
5. Reject weak edge pixels not connected with strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]

Canny nonmaxima suppression

- Quantize edge normal to one of four directions:
  horizontal, -45^o, vertical, $+45^o$
- If $M[x,y]$ is smaller than either of its neighbors in edge normal direction

  ➔ suppress; else keep.



[Canny, IEEE Trans. PAMI, 1986]

Canny thresholding and suppression of weak edges

- Double-thresholding of gradient magnitude

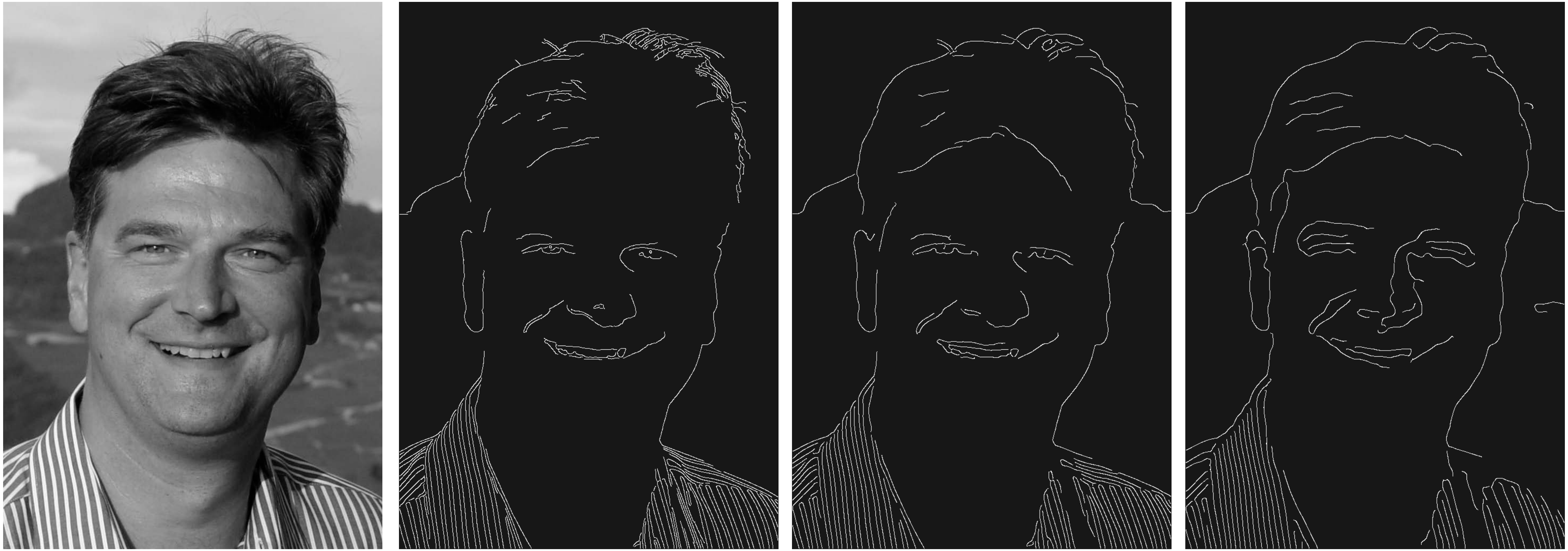
$$\text{Strong edge:} \quad M[x,y] \geq \theta_{high}$$

$$\text{Weak edge:} \quad \theta_{high} > M[x,y] \geq \theta_{low}$$

- Typical setting: $\theta_{high} / \theta_{low} = 2...3$
- Region labeling of edge pixels
- Reject regions without strong edge pixels

[Canny, IEEE Trans. PAMI, 1986]

Canny edge detector



$$\sigma = \sqrt{2}$$

$$\sigma = 2\sqrt{2}$$

$$\sigma = 4\sqrt{2}$$

True or false? The Laplacian of Gaussian filter …

(a) responds more strongly to a vertical or horizontal edge than to an edge at 45 degrees

(b) is sensitive to changes in image brightness

(c) has a zero at $(\omega_x,\omega_y) = (0,0)$ in the frequency response

True or false? The Canny edge detector …

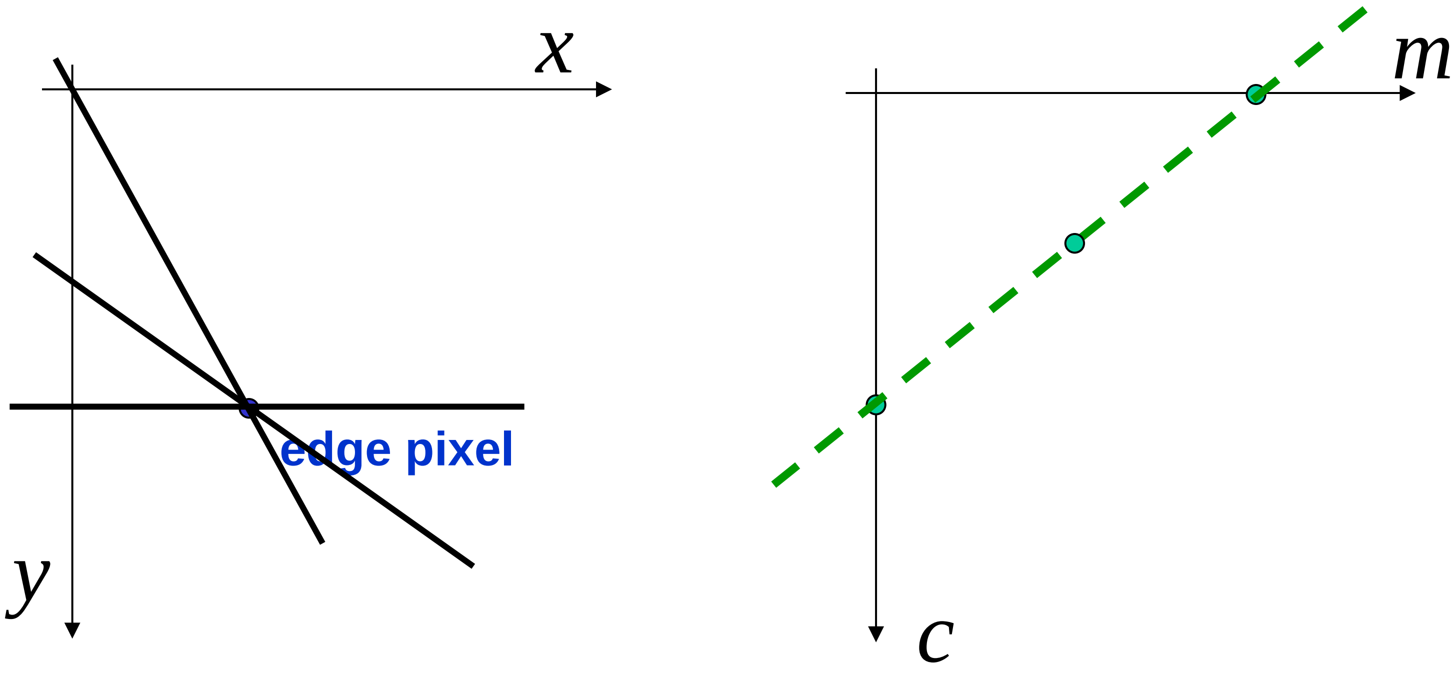
(a) is both linear and shift-invariant

(b) can detect edges at different scales by varying σ of the Gaussian

(c) produces edges that are 1 pixel wide

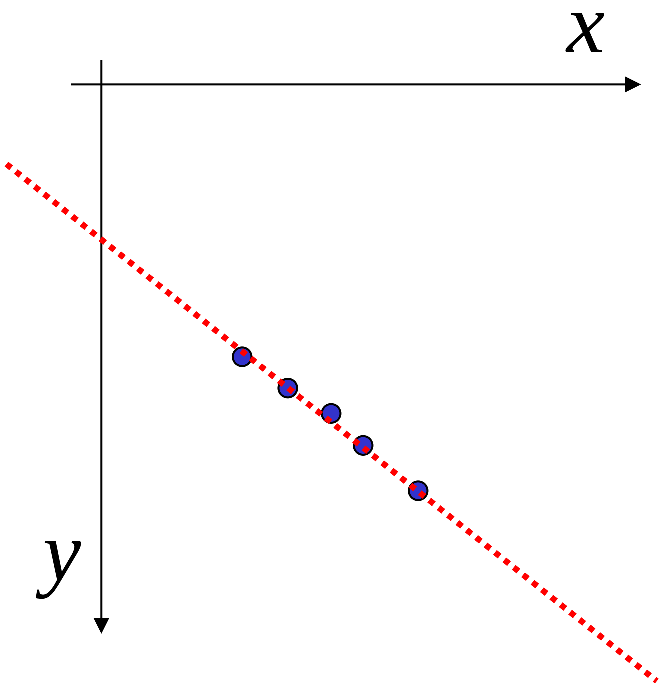
Hough transform

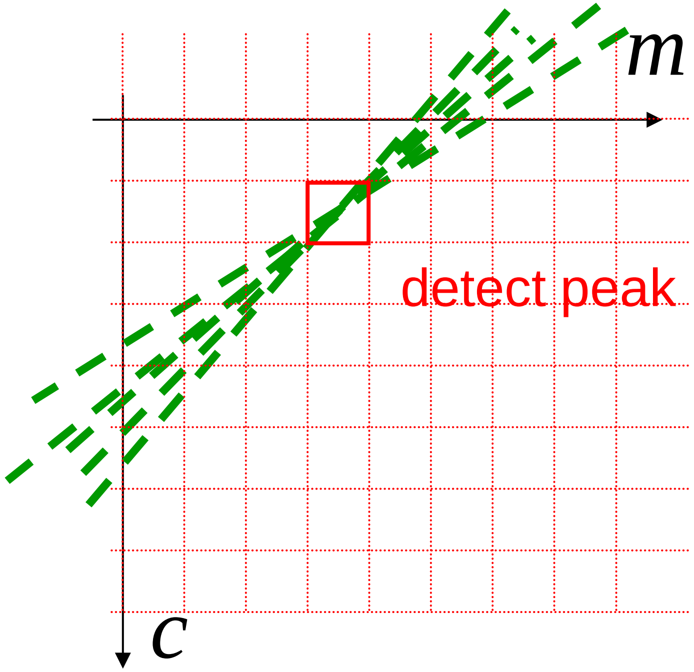
- Problem: fit a straight line (or curve) to a set of edge pixels
- Hough transform (1962): generalized template matching technique
- Consider detection of straight lines $y = mx + c$



Hough transform (cont.)

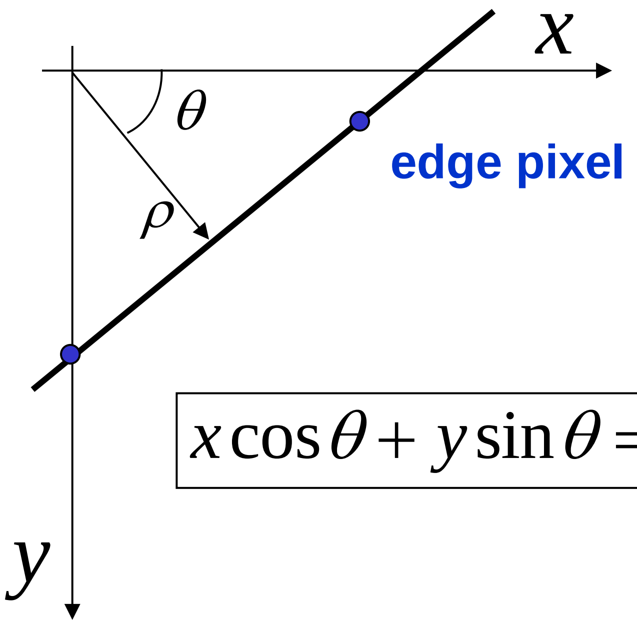
- Subdivide (m,c) plane into discrete "bins," initialize all bin counts by 0
- Draw a line in the parameter space $[m,c]$ for each edge pixel $[x,y]$ and increment bin counts along line.
- Detect peak(s) in $[m,c]$ plane



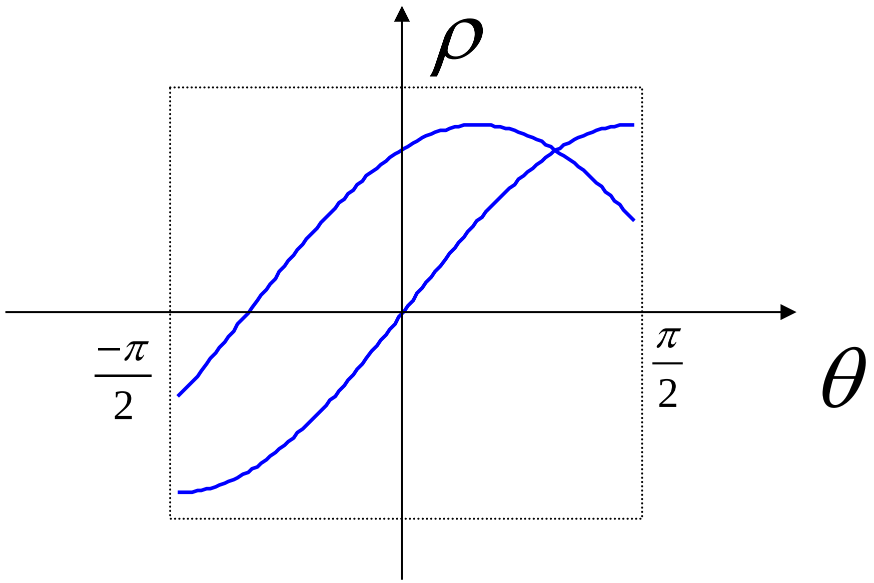


Hough transform (cont.)

- Alternative parameterization avoids infinite-slope problem

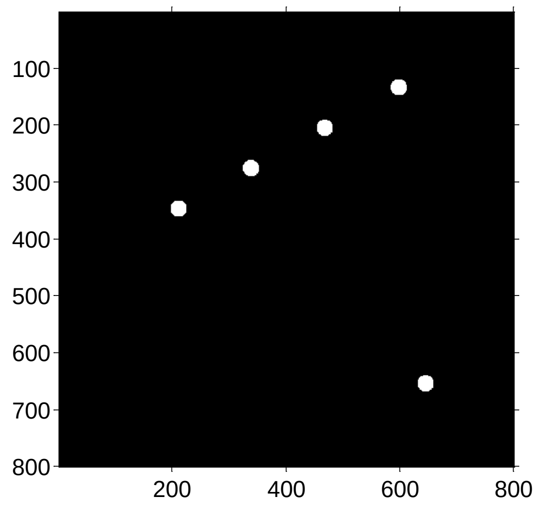


$$x\cos\theta + y\sin\theta = \rho$$

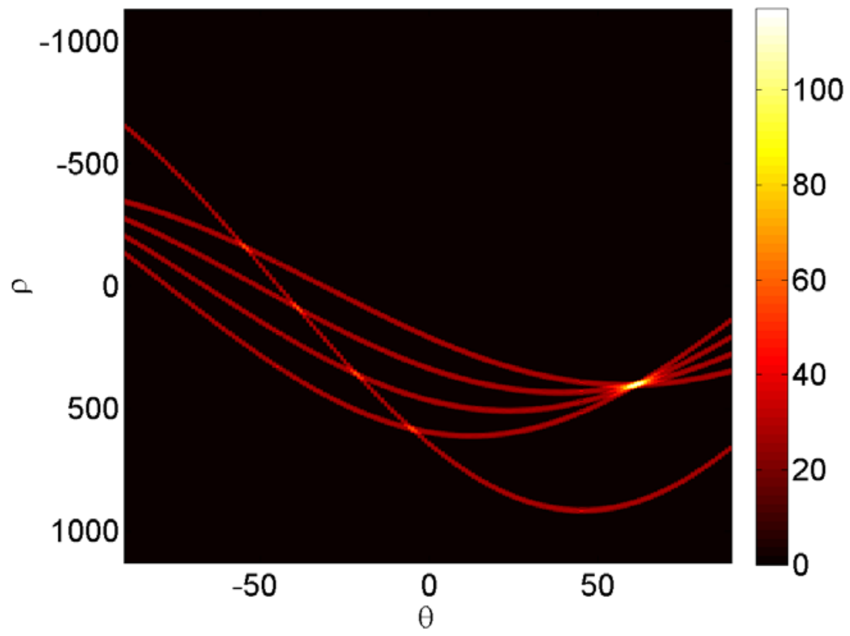


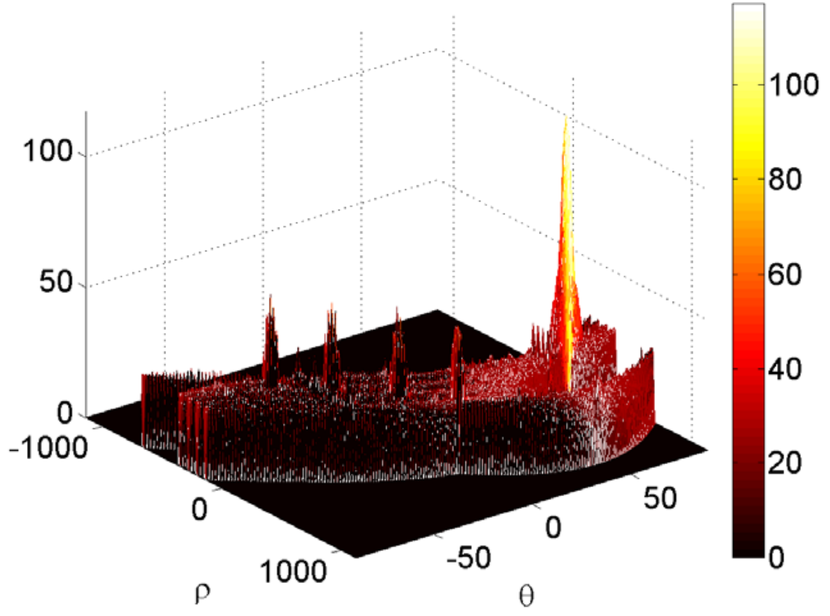
- Similar to Radon transform

Hough transform example

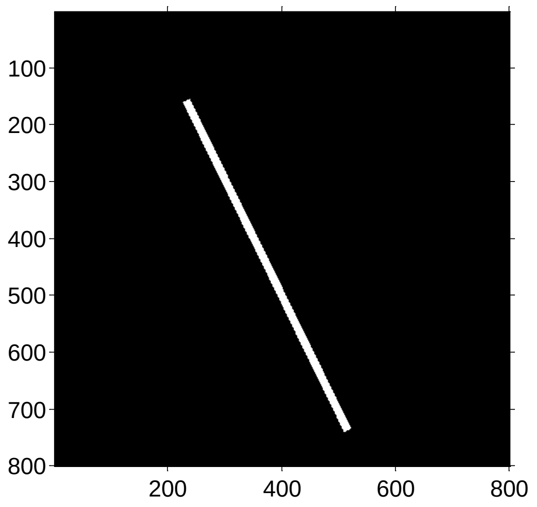


Original image

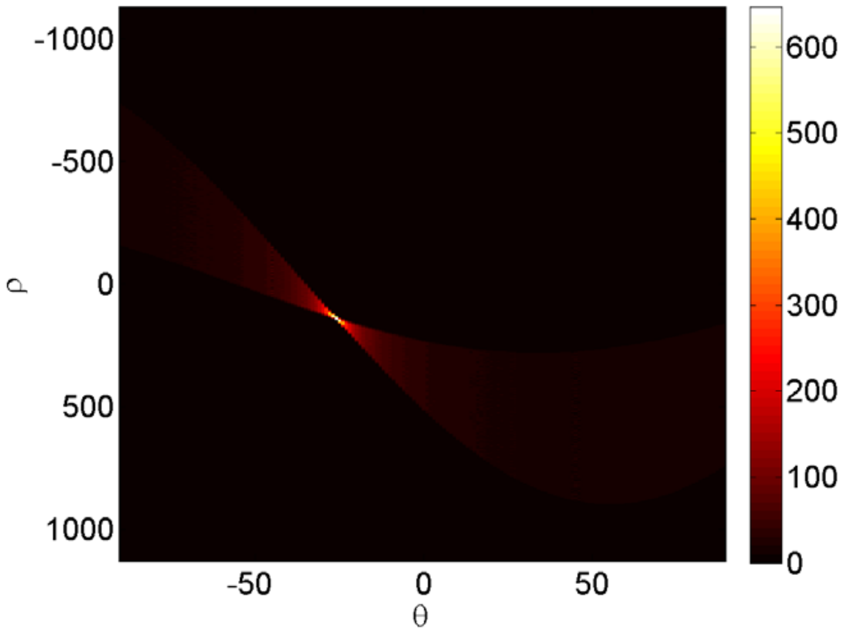


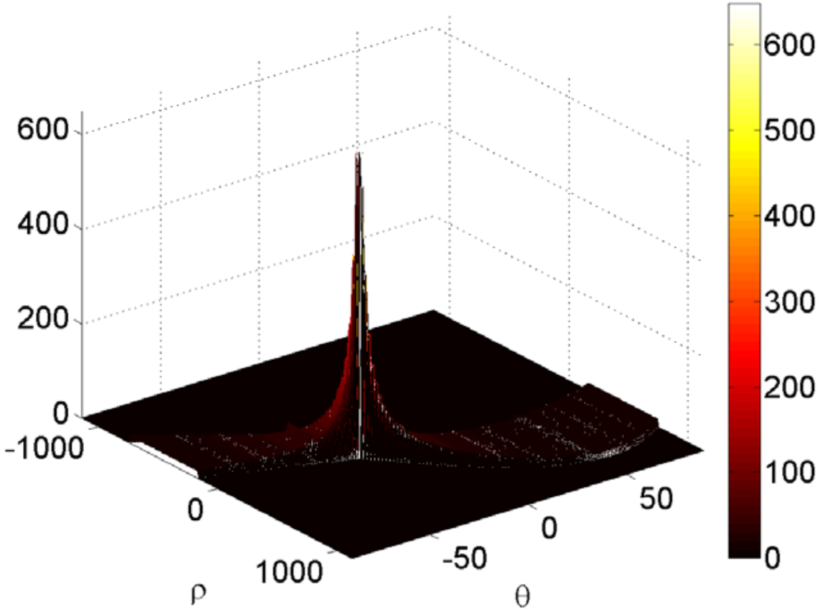


Hough transform example

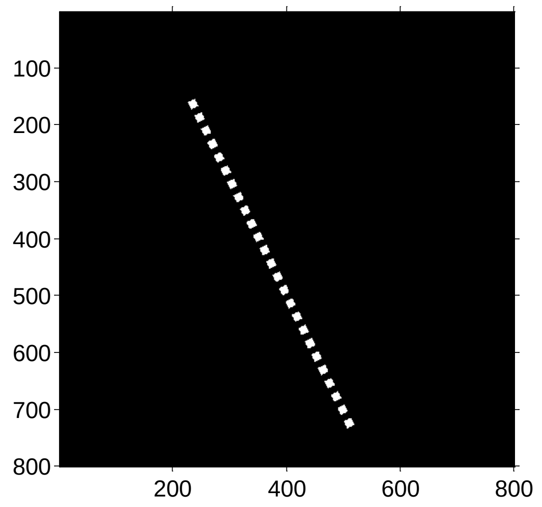


Original image

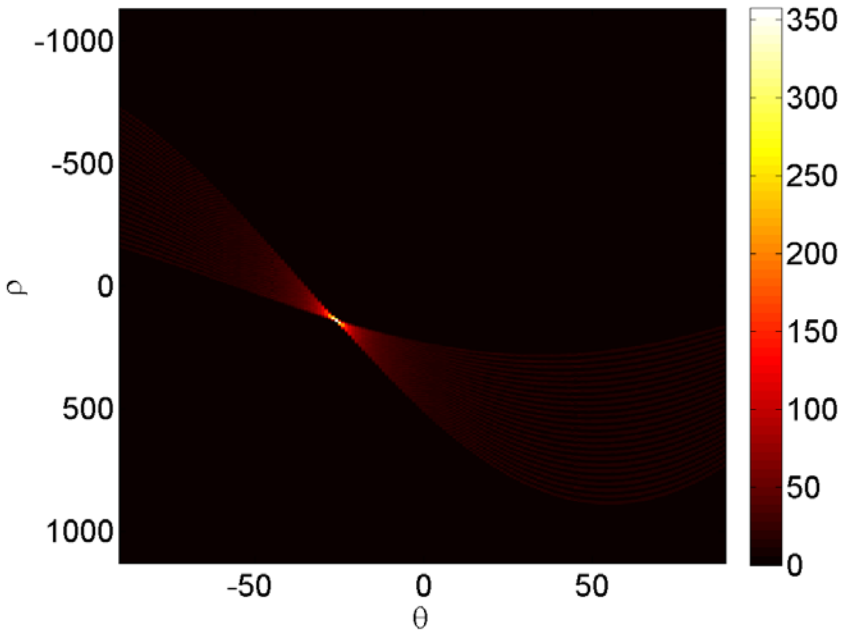


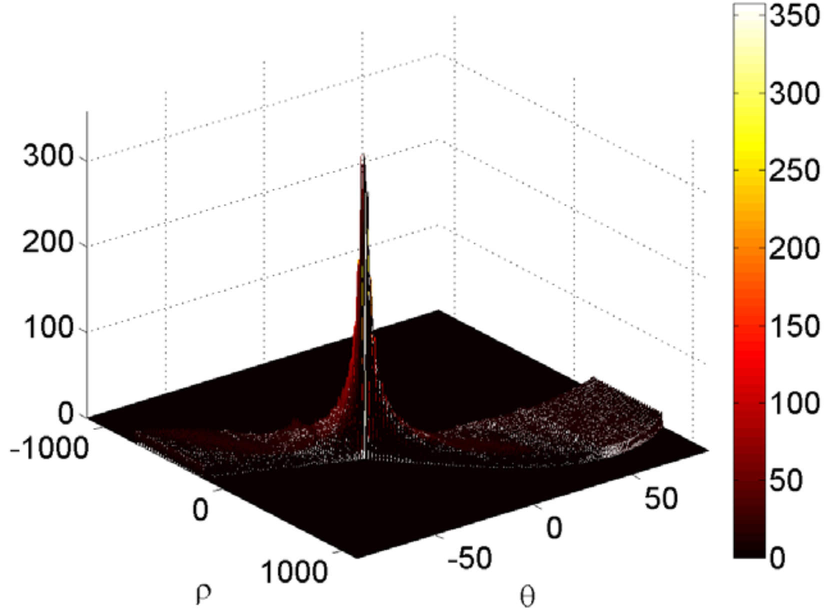


Hough transform example

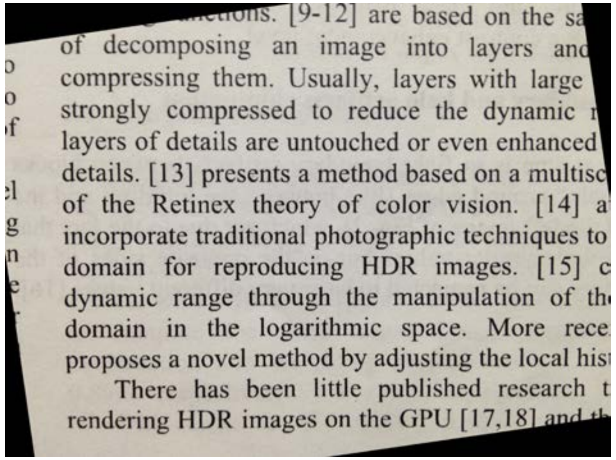


Original image

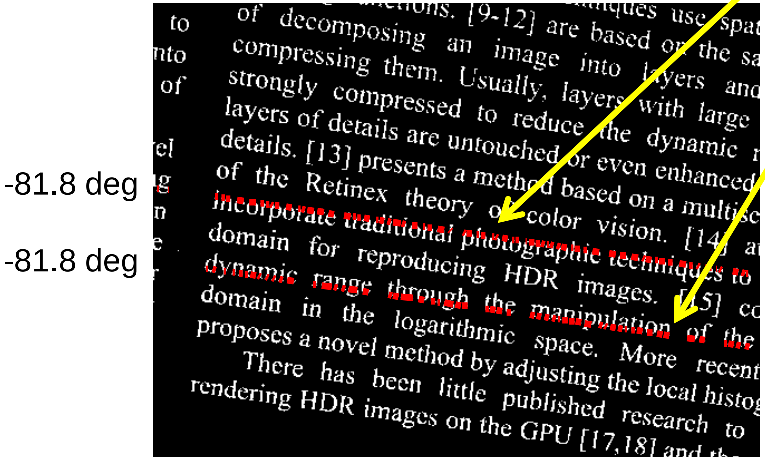




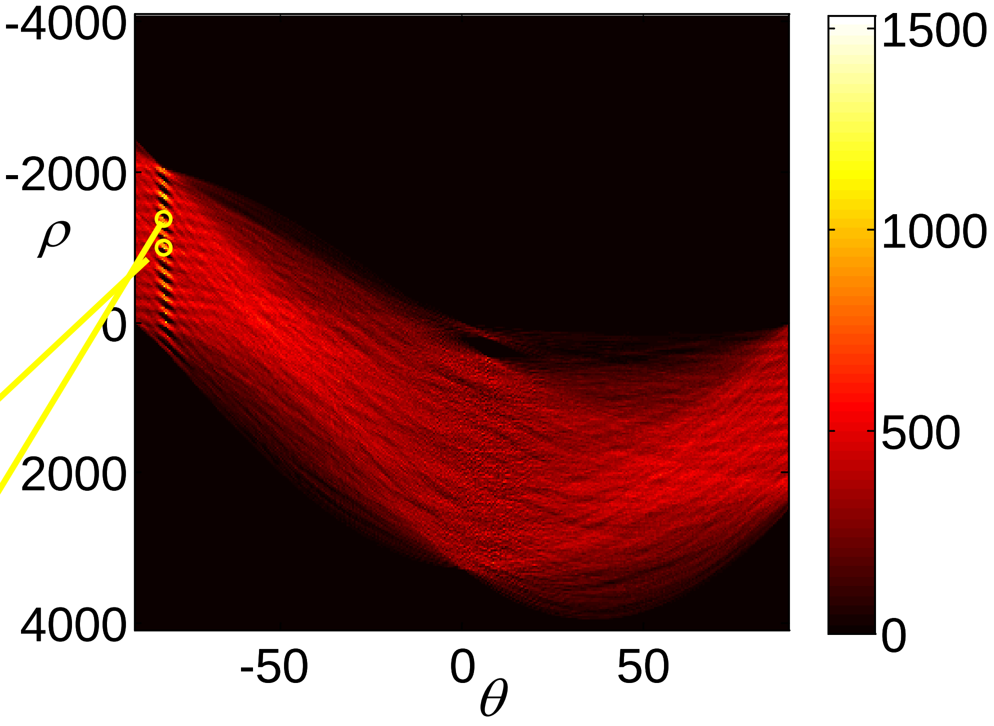
Hough transform example


...ctions. [9-12] are based on the sa
of decomposing an image into layers and
compressing them. Usually, layers with large
strongly compressed to reduce the dynamic
layers of details are untouched or even enhanced
details. [13] presents a method based on a multisc
of the Retinex theory of color vision. [14] a
incorporate traditional photographic techniques to
domain for reproducing HDR images. [15] c
dynamic range through the manipulation of th
domain in the logarithmic space. More rece
proposes a novel method by adjusting the local hist
There has been little published research t
rendering HDR images on the GPU [17,18] and th

De-skewed Paper

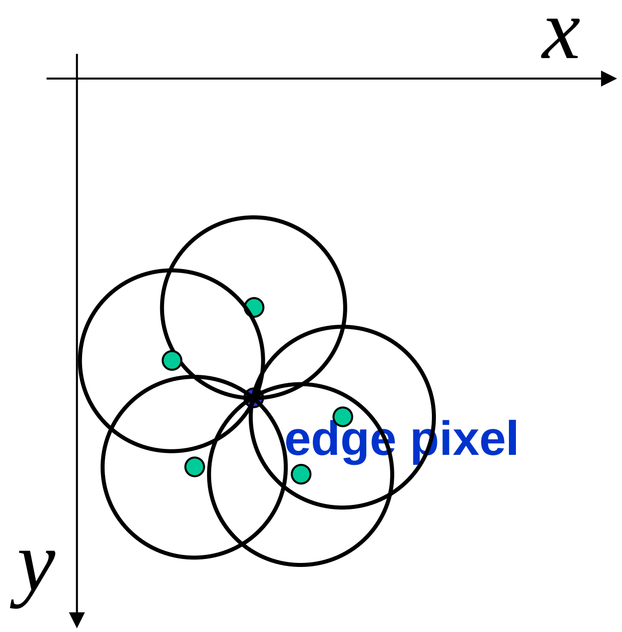


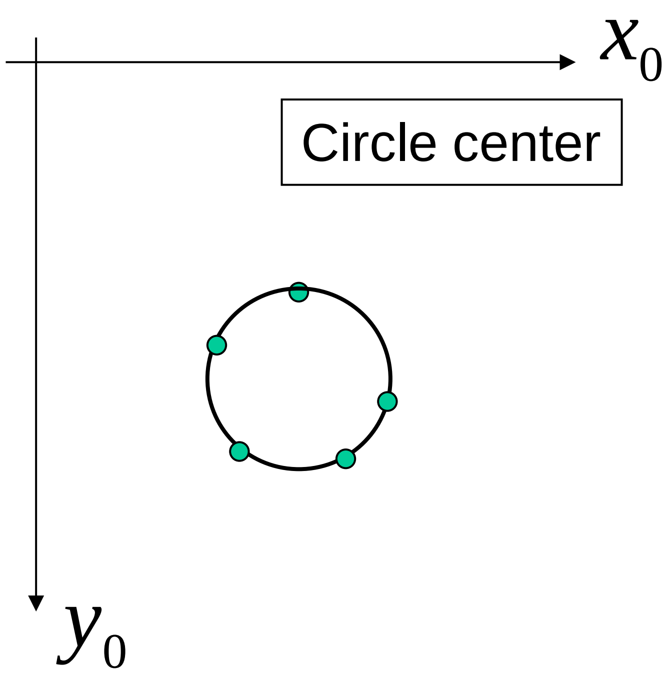
Global thresholding



Circle Hough Transform

- Find circles of fixed radius r

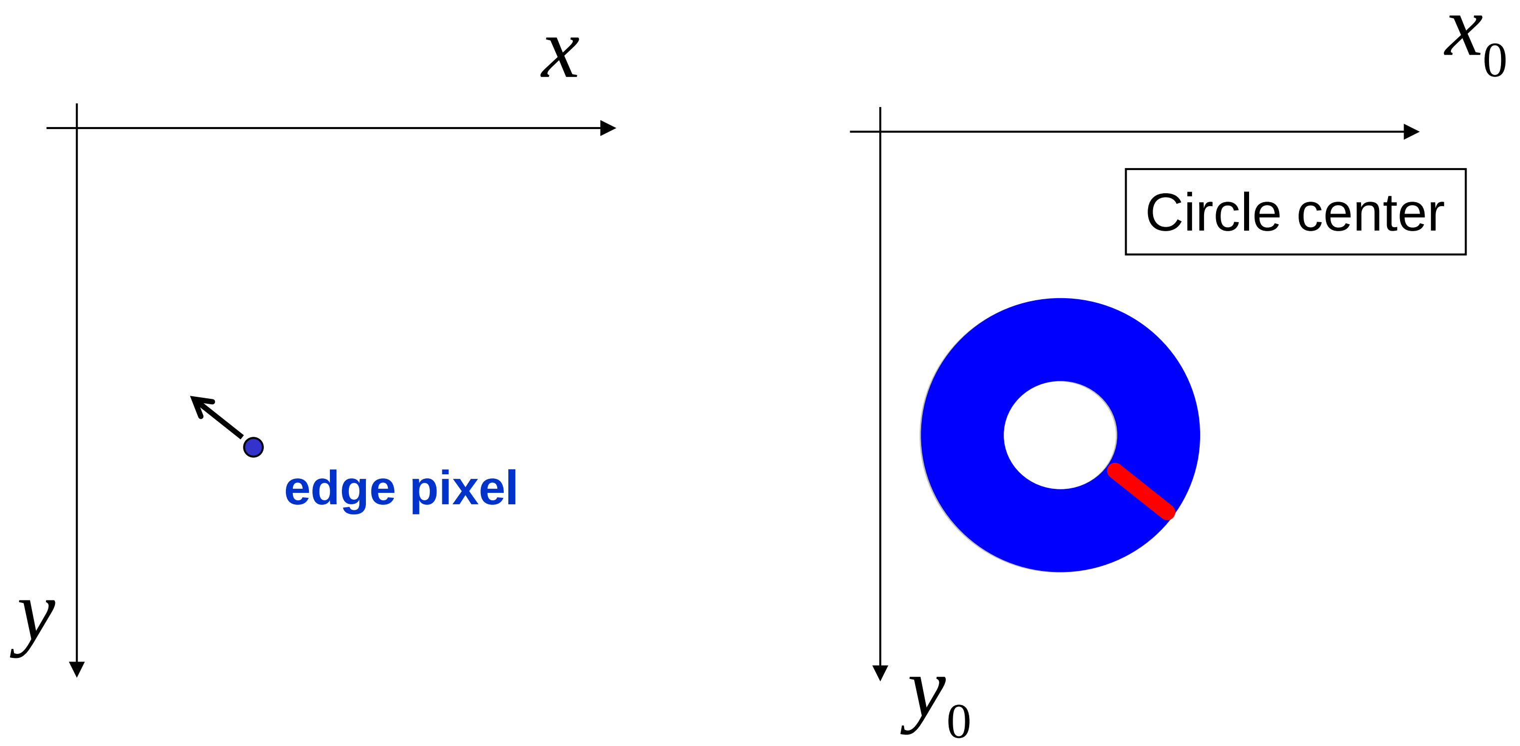




- Equivalent to convolution (template matching) with a circle

Circle Hough Transform for unknown radius

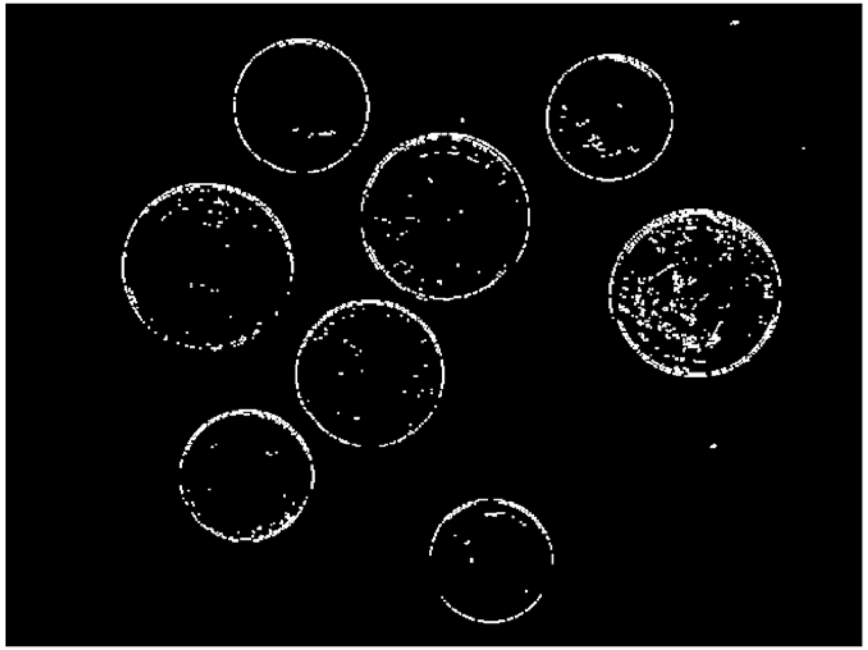
- 3-d Hough transform for parameters (x_0, y_0, r)
- 2-d Hough transform aided by edge orientation ➔ “spokes” in parameter space



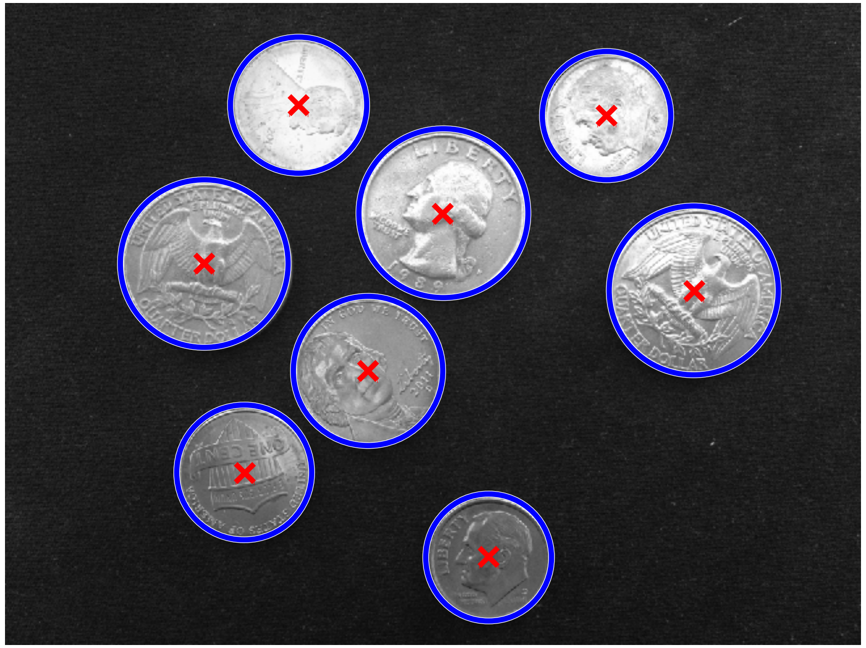
Example: circle detection by Hough transform

Original
coins image

Prewitt edge detection



Detected circles



Suppose we are trying to detect a L-shape of known size and orientation, but unknown center location (x_0, y_0) using a specialized Hough transform. For the given edge pixel shown in the x-y space, what should be drawn in the x_0-y_0 space?