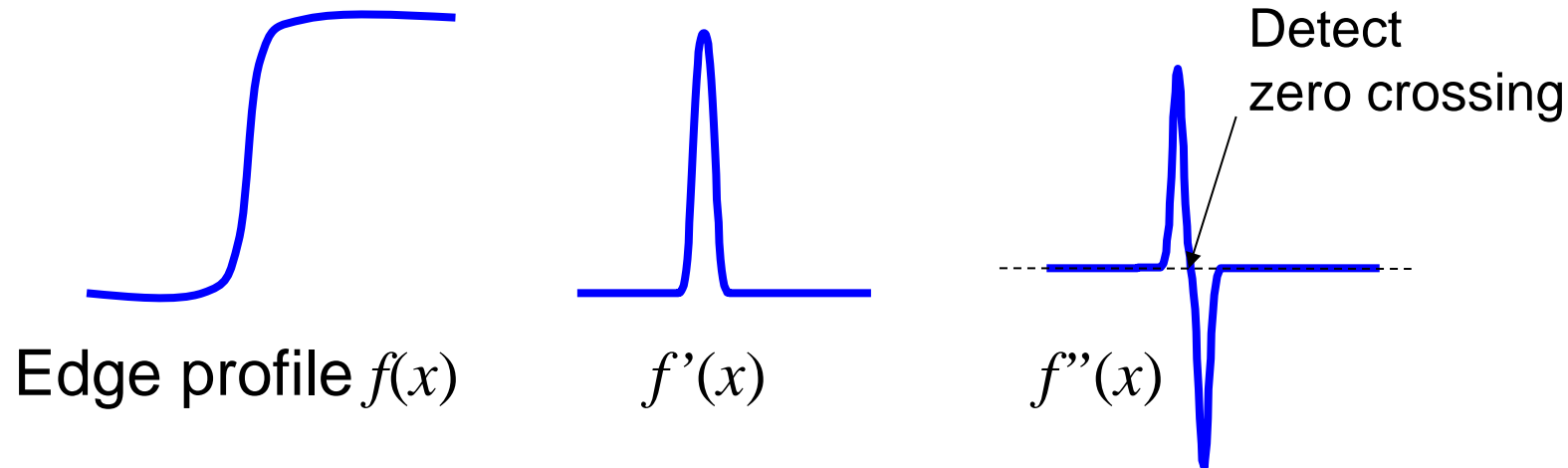


Laplacian operator

- Detect edges by considering second derivative

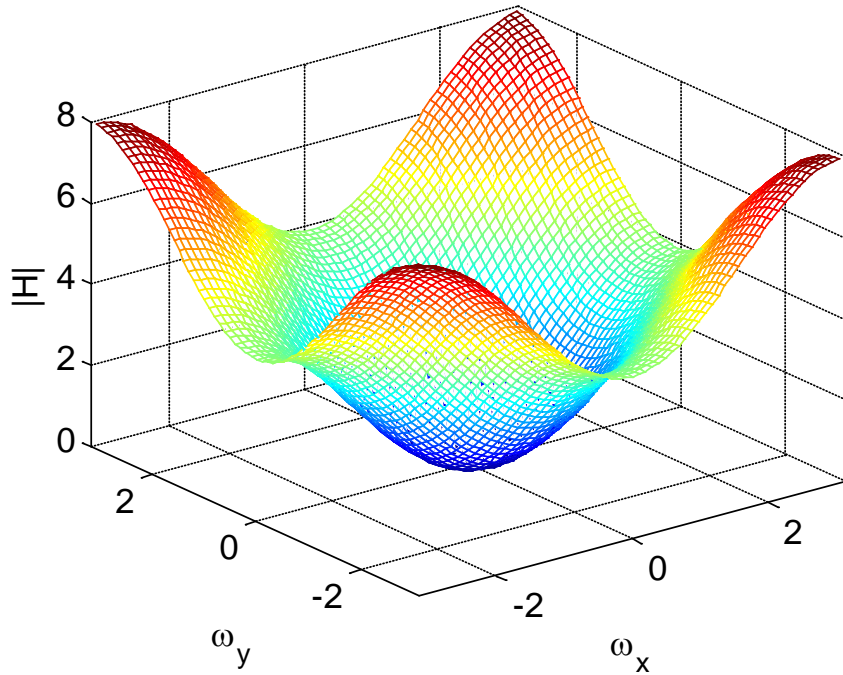
$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

- Isotropic (rotationally invariant) operator
- Zero-crossings mark edge location

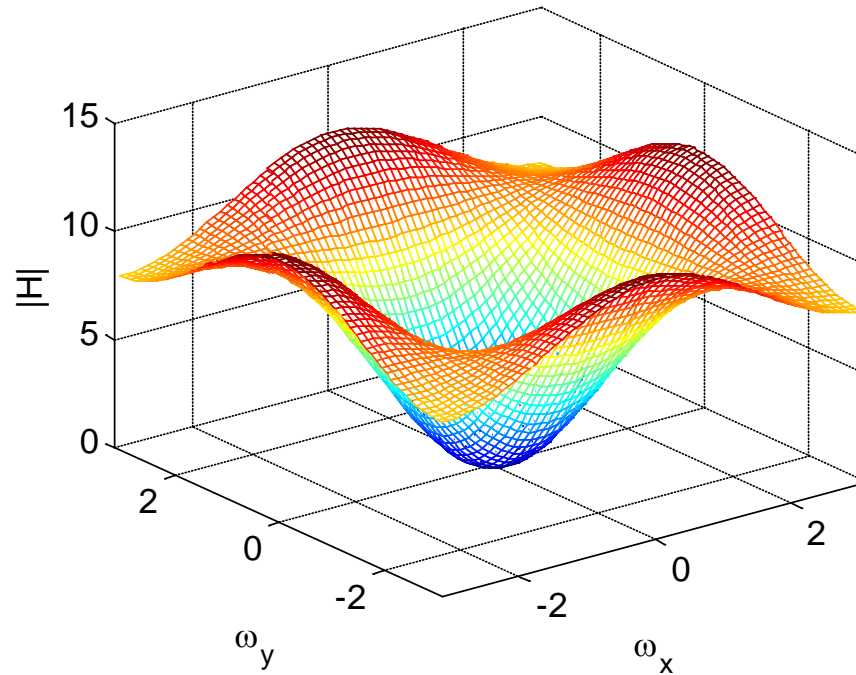


Approximations of Laplacian operator by 3x3 filter

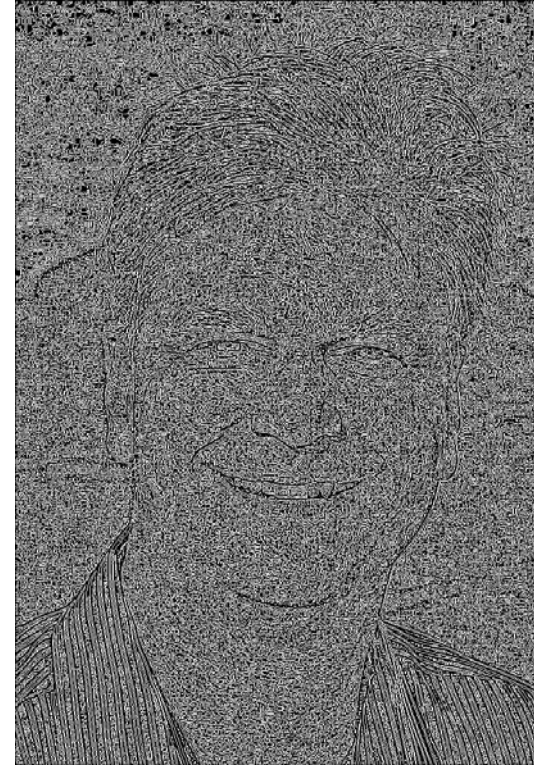
$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & [-4] & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & [-8] & 1 \\ 1 & 1 & 1 \end{pmatrix}$$



Zero crossings of Laplacian



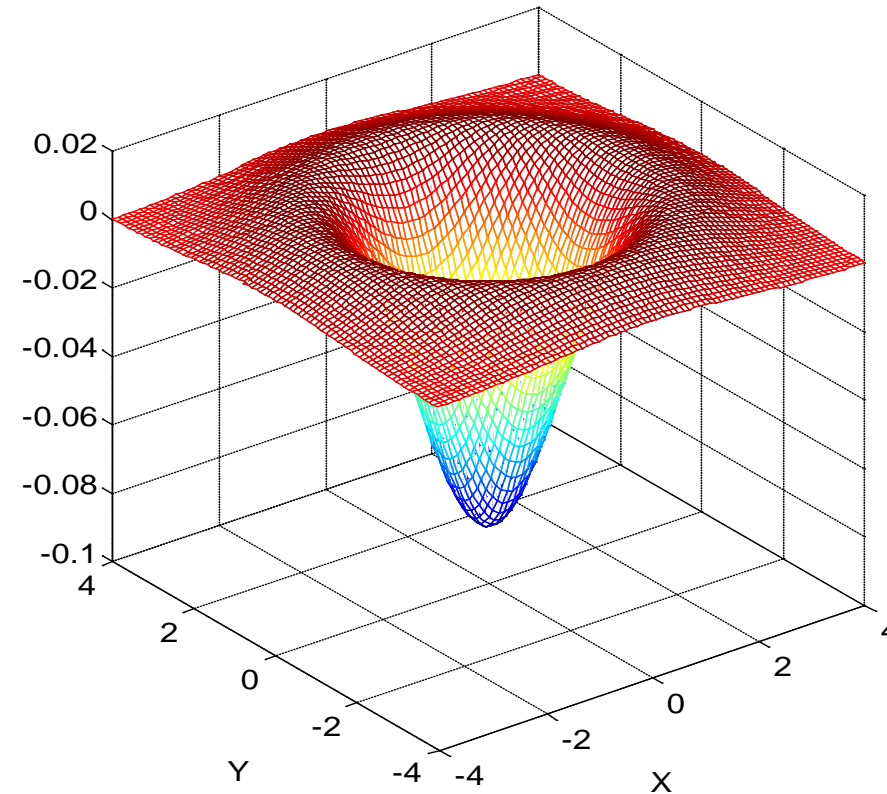
- Sensitive to very fine detail and noise → blur image first
- Responds equally to strong and weak edges
→ suppress zero-crossings with low gradient magnitude



Laplacian of Gaussian

- Filtering of image with Gaussian and Laplacian operators can be combined into convolution with Laplacian of Gaussian (LoG) operator

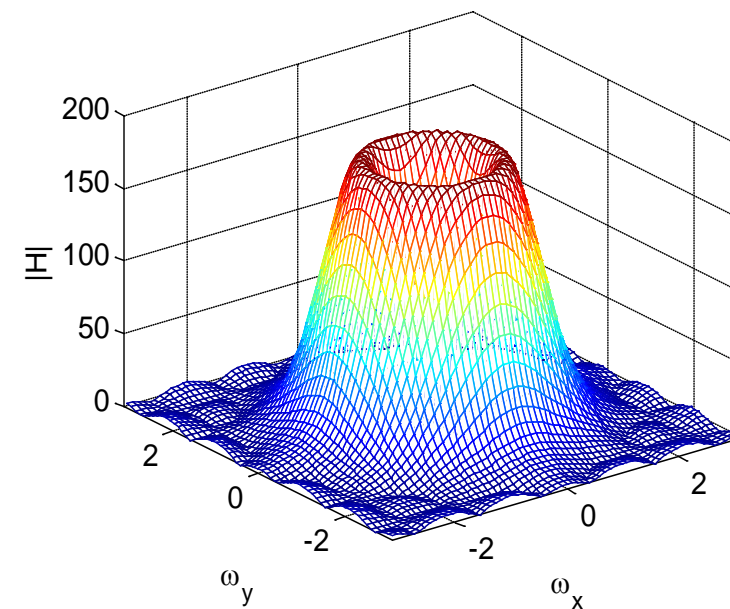
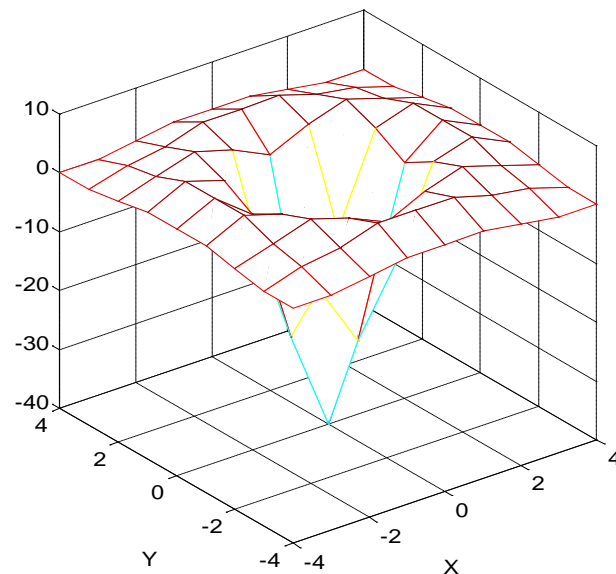
$$LoG(x, y) = -\frac{1}{\pi\sigma^4} \left(1 - \frac{x^2 + y^2}{2\sigma^2} \right) e^{-\frac{x^2 + y^2}{2\sigma^2}}$$



Discrete approximation of Laplacian of Gaussian

$$\sigma = \sqrt{2}$$

0	0	1	2	2	2	1	0	0
0	2	3	5	5	5	3	2	0
1	3	5	3	0	3	5	3	1
2	5	3	-12	-23	-12	3	5	2
2	5	0	-23	-40	-23	0	5	2
2	5	3	-12	-23	-12	3	5	2
1	3	5	3	0	3	5	3	1
0	2	3	5	5	5	3	2	0
0	0	1	2	2	2	1	0	0



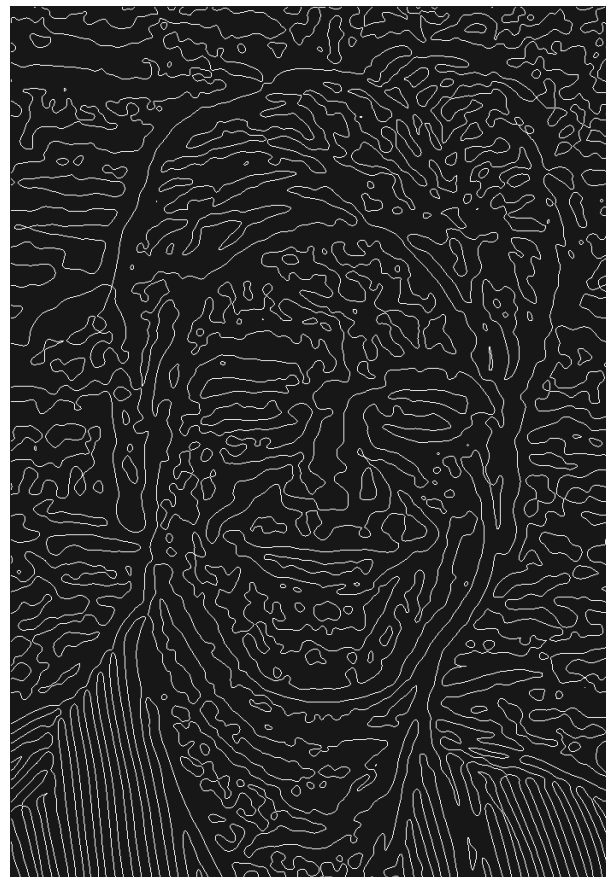
Zero crossings of LoG



$$\sigma = \sqrt{2}$$



$$\sigma = 2\sqrt{2}$$



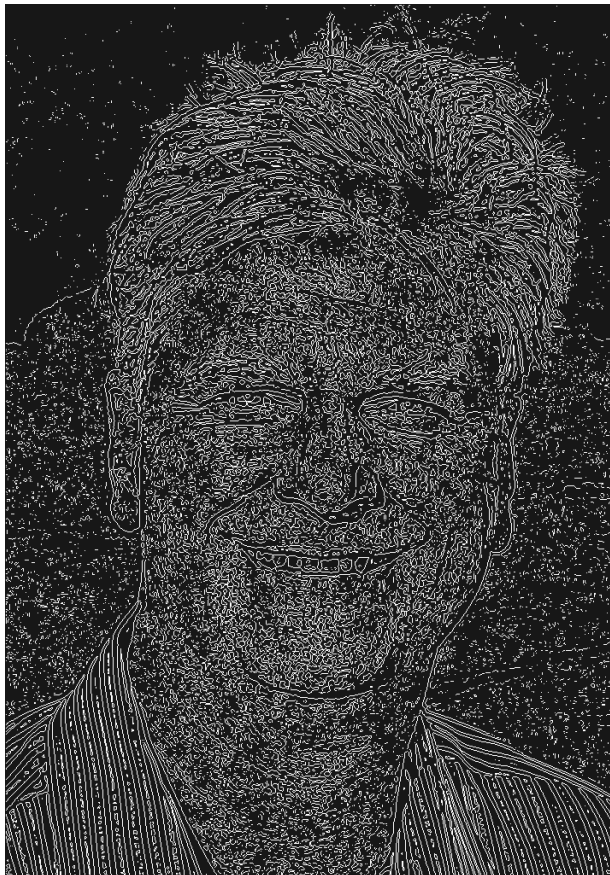
$$\sigma = 4\sqrt{2}$$



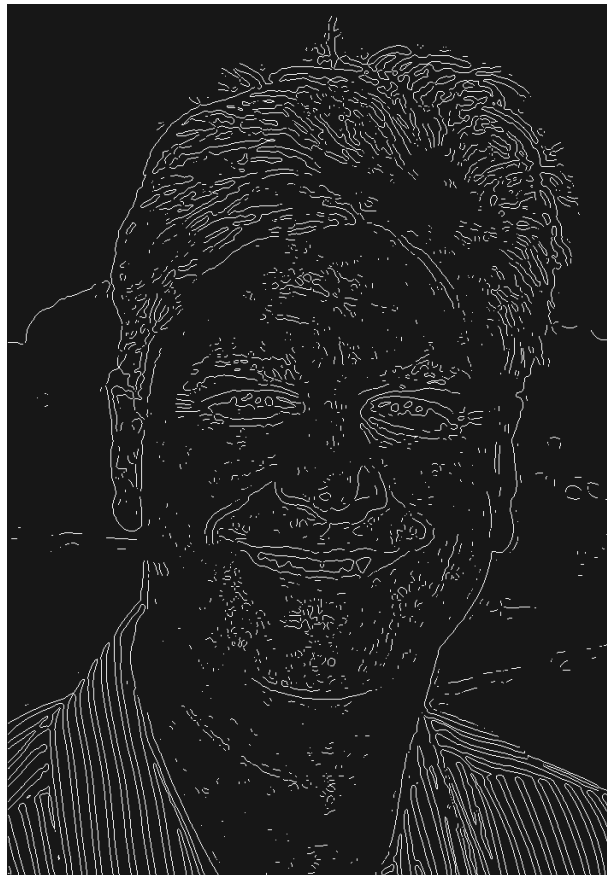
$$\sigma = 8\sqrt{2}$$



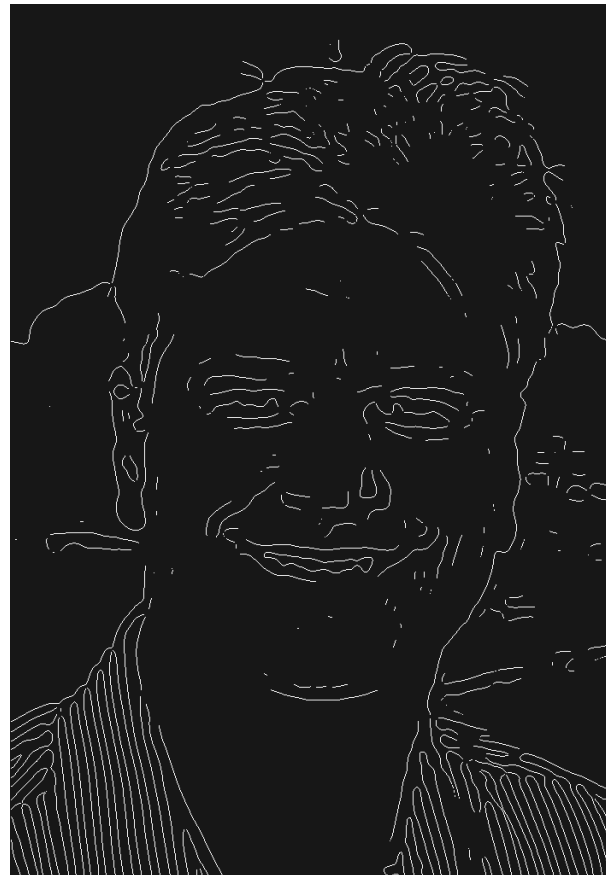
Zero crossings of LoG – gradient-based threshold



$$\sigma = \sqrt{2}$$



$$\sigma = 2\sqrt{2}$$



$$\sigma = 4\sqrt{2}$$



$$\sigma = 8\sqrt{2}$$

