Gradient-based edge detection

- Idea (continous-space): local gradient magnitude indicates edge strength

$$\left|grad\left(f(x,y)\right)\right| = \sqrt{\left(\frac{\partial f(x,y)}{\partial x}\right)^2 + \left(\frac{\partial f(x,y)}{\partial y}\right)^2}$$

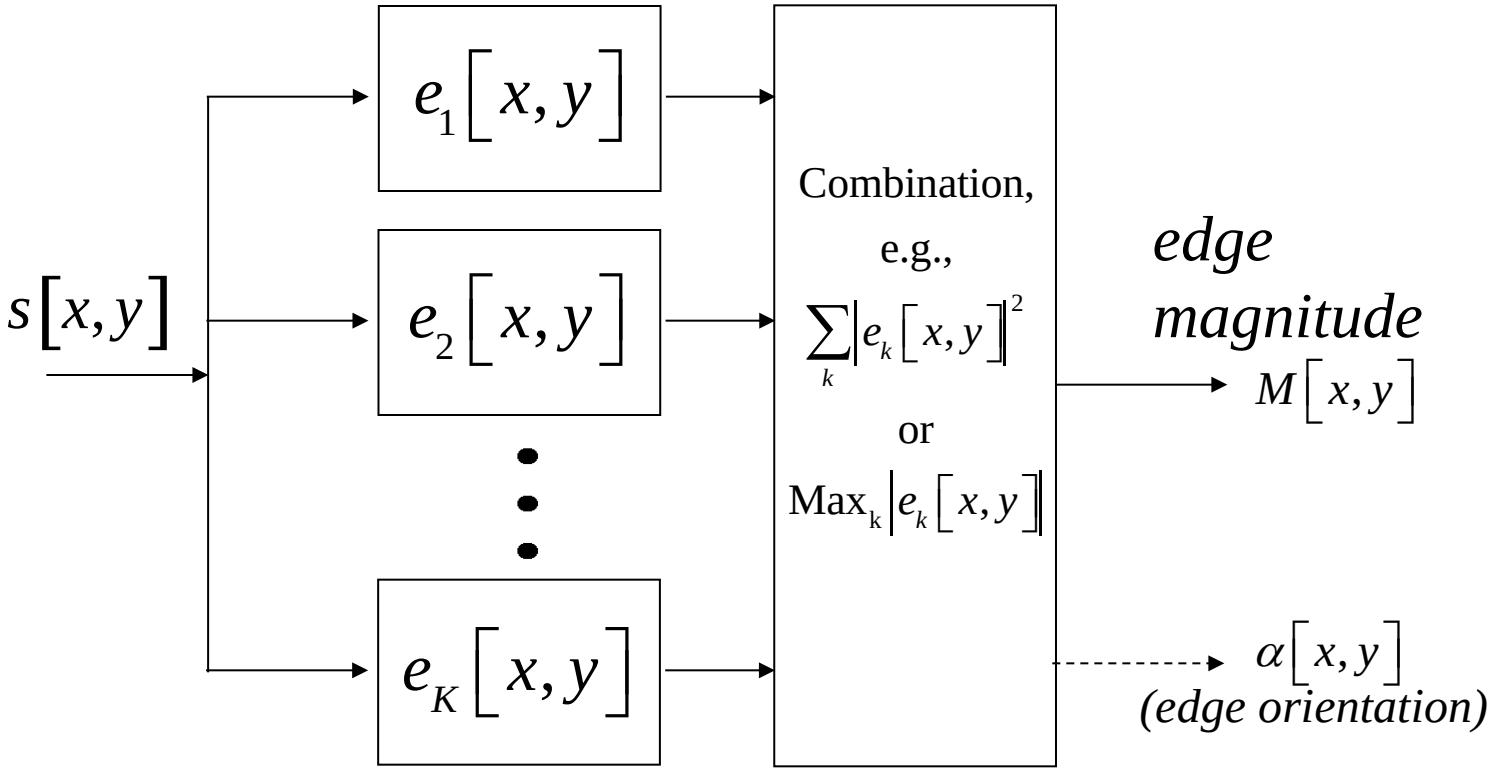
- Digital image: use finite differences to approximate derivatives

- Edge templates

difference	$\begin{pmatrix} -1 & 1 \end{pmatrix}$
central difference	$\begin{pmatrix} -1 & [0] & 1 \end{pmatrix}$
Prewitt	$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$
Sobel	$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix}$

Practical edge detectors

- Edges can have any orientation
- Typical edge detection scheme uses $K=2$ edge templates
- Some use $K>2$



Gradient filters (K=2)

Central Difference

$$\begin{pmatrix} 0 & 0 & 0 \\ -1 & [0] & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -1 & 0 \\ 0 & [0] & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Roberts

$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix} \quad \begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

Prewitt

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

Sobel

$$\begin{pmatrix} -1 & 0 & 1 \\ -2 & [0] & 2 \\ -1 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & -2 & -1 \\ 0 & [0] & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

Kirsch operator (K=8)

Kirsch

$$\begin{pmatrix} +5 & +5 & +5 \\ -3 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & +5 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & -3 \end{pmatrix} \begin{pmatrix} -3 & -3 & +5 \\ -3 & [0] & +5 \\ -3 & -3 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & +5 \\ -3 & +5 & +5 \end{pmatrix}$$

$$\begin{pmatrix} -3 & -3 & -3 \\ -3 & [0] & -3 \\ +5 & +5 & +5 \end{pmatrix} \begin{pmatrix} -3 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & +5 & -3 \end{pmatrix} \begin{pmatrix} +5 & -3 & -3 \\ +5 & [0] & -3 \\ +5 & -3 & -3 \end{pmatrix} \begin{pmatrix} +5 & +5 & -3 \\ +5 & [0] & -3 \\ -3 & -3 & -3 \end{pmatrix}$$

Prewitt operator example

Original
1024x710

Magnitude of
image filtered with

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & [0] & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

(log display)

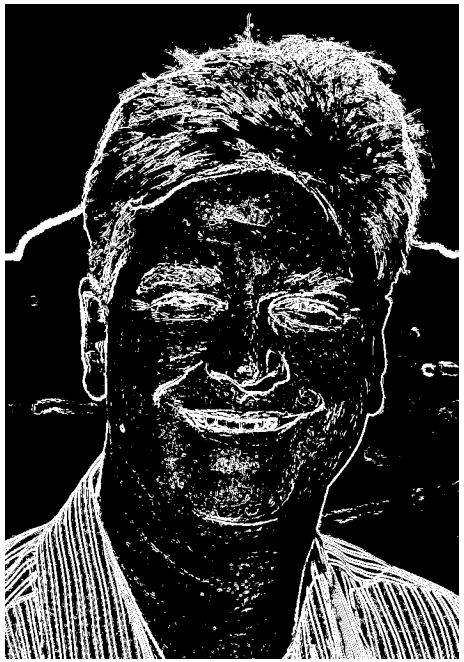
Magnitude of
image filtered with

$$\begin{pmatrix} -1 & -1 & -1 \\ 0 & [0] & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

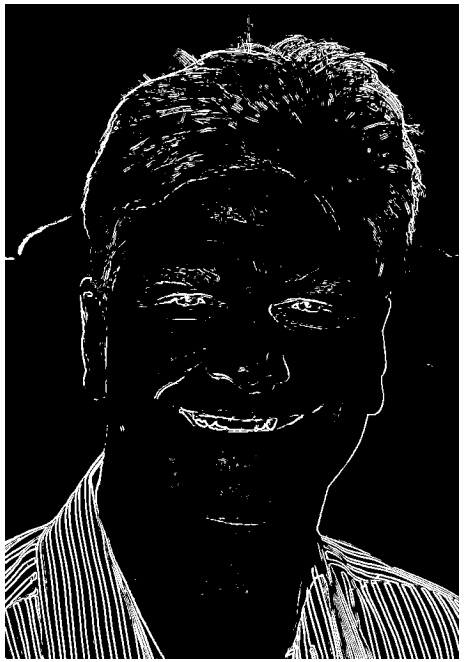
(log display)

Prewitt operator example (cont.)

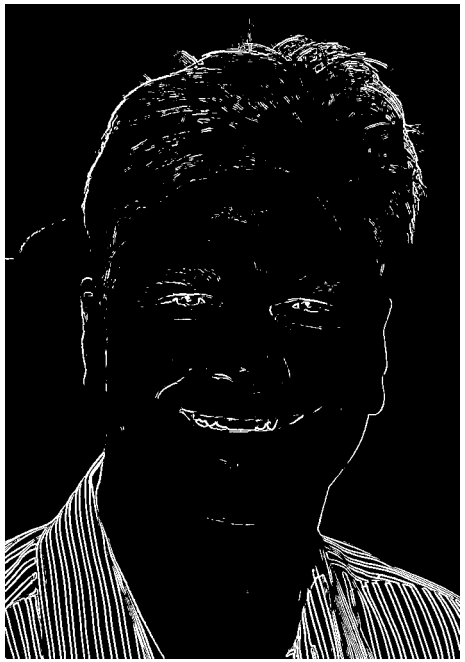
Sum of squared horizontal and vertical gradients (log display)



threshold = 900



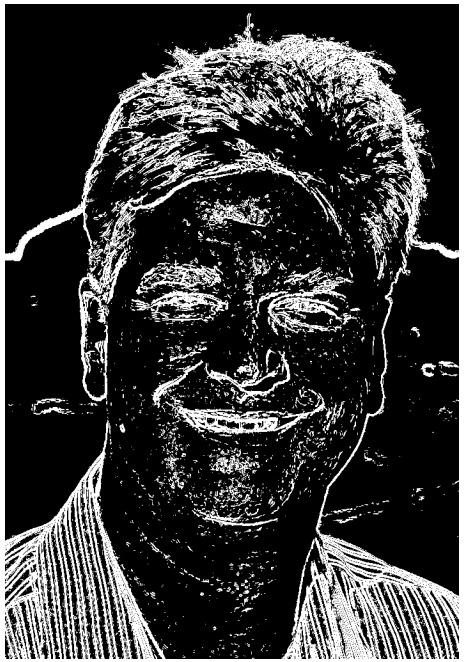
threshold = 4500



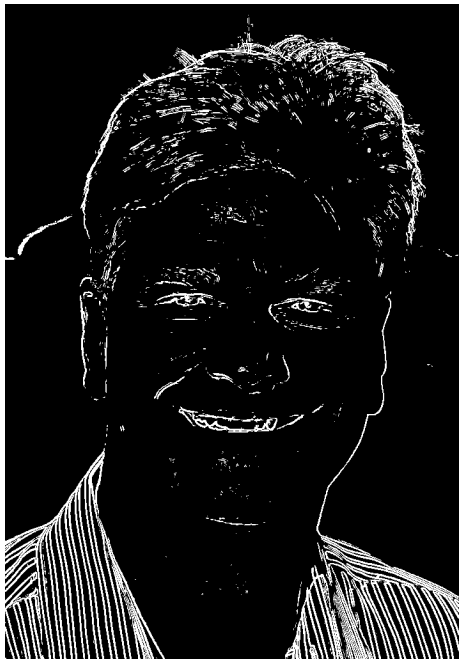
threshold = 7200

Sobel operator example

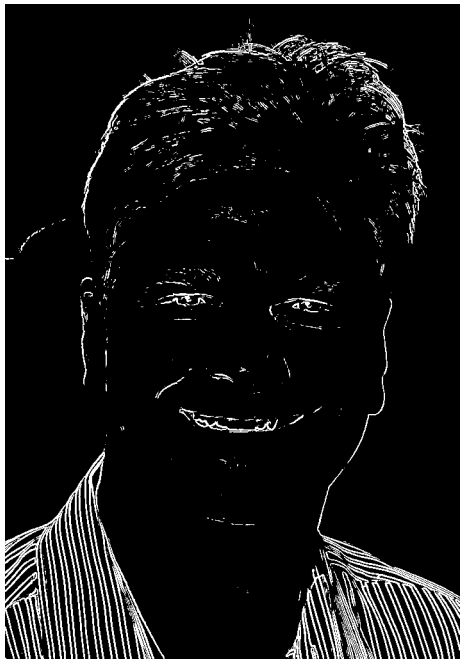
Sum of squared horizontal and vertical gradients (log display)



threshold = 1600



threshold = 8000



threshold = 12800

Roberts operator example

Original
1024x710

Magnitude of
image filtered with

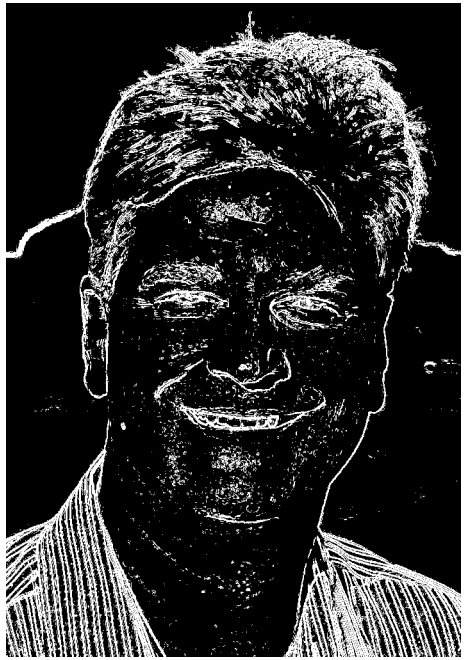
$$\begin{pmatrix} [1] & 0 \\ 0 & -1 \end{pmatrix}$$

Magnitude of
image filtered with

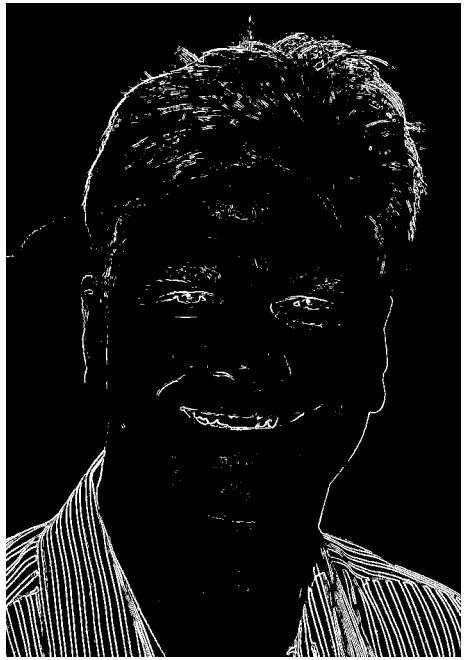
$$\begin{pmatrix} [0] & 1 \\ -1 & 0 \end{pmatrix}$$

Roberts operator example (cont.)

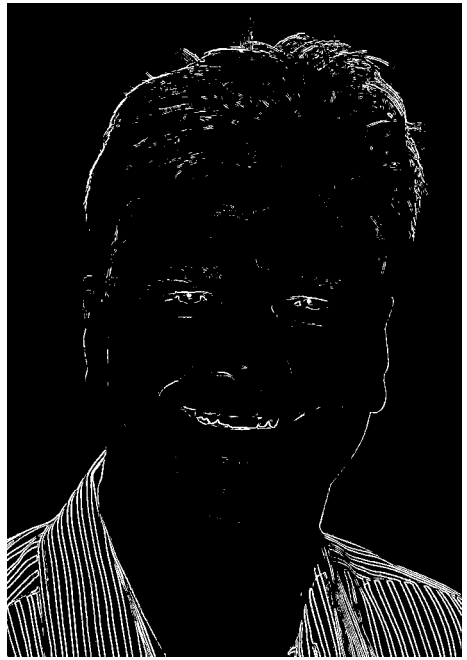
Sum of squared diagonal gradients (log display)



threshold = 100



threshold = 500



threshold = 800