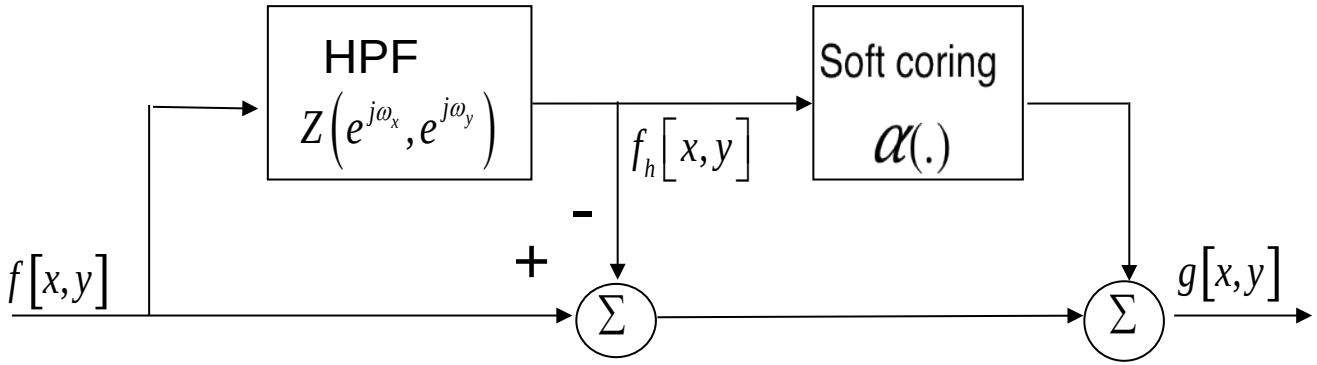
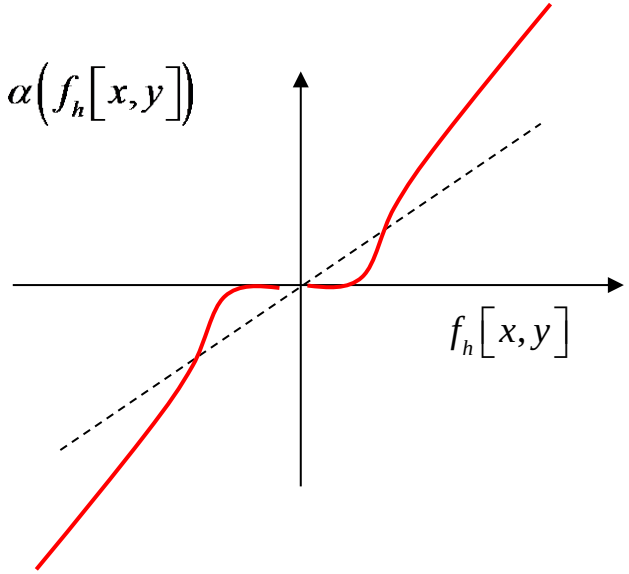
Nonlinear noise reduction/sharpening

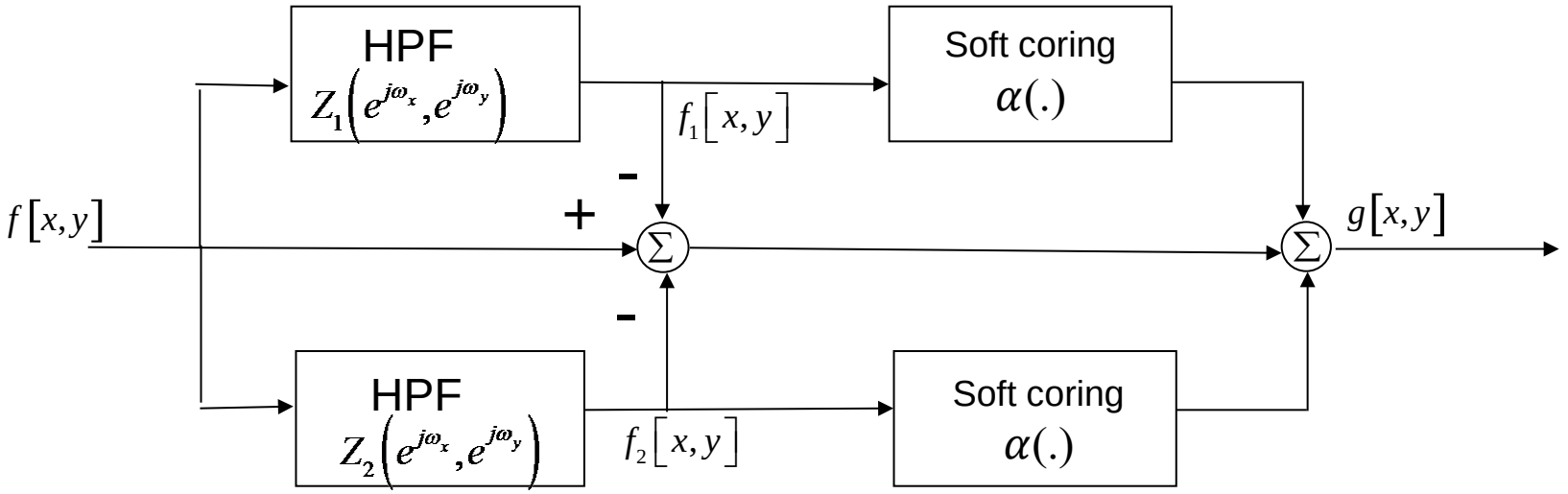
- Noise reduction: smooth the image, lowpass filtering
- Deblurring: sharpen edges, highpass filtering
- How can both be achieved simultaneously?
- Key insight: large amplitude of highpass filtered image indicates presence of edge

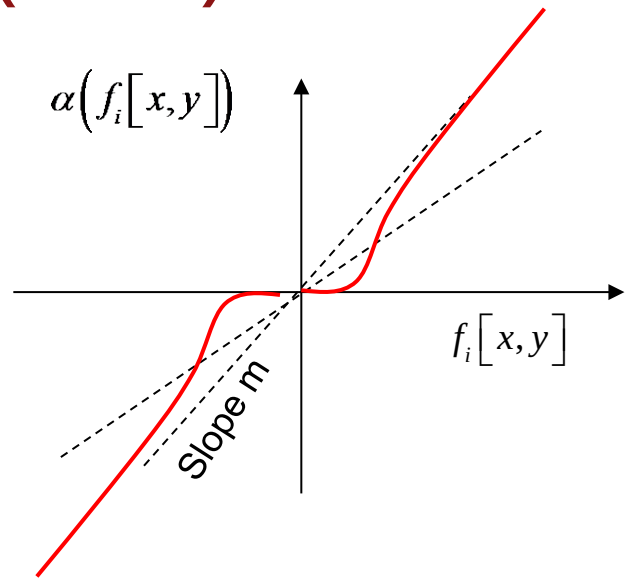




- Can be extended to multiple HPFs

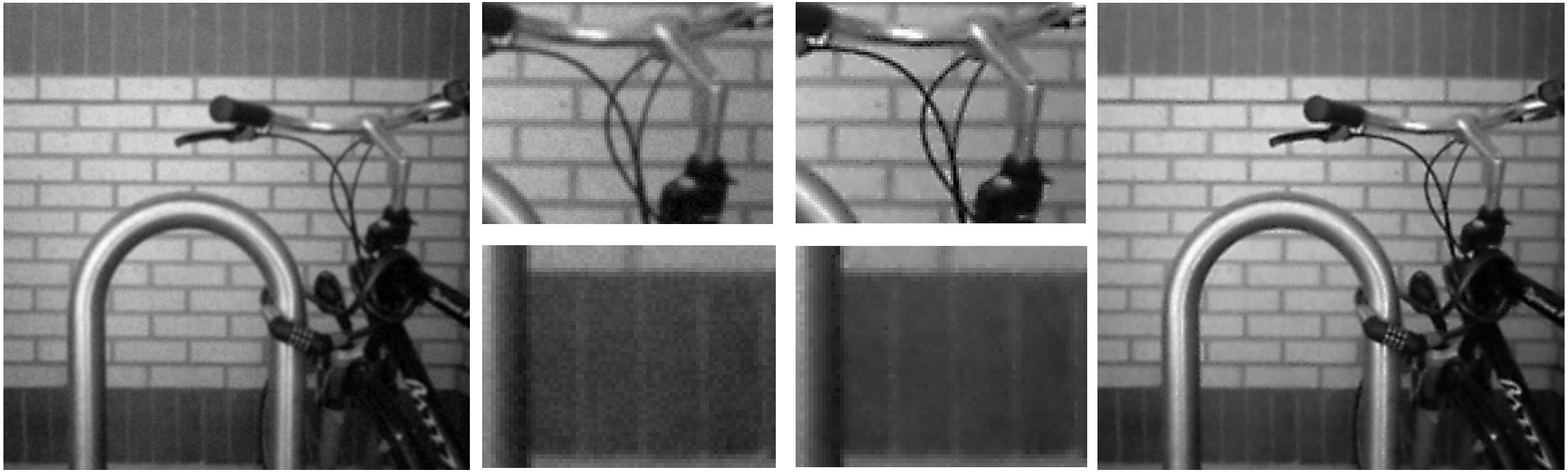
Nonlinear noise reduction/sharpening (cont.)





- Flat areas: $\alpha \approx 0 \rightarrow H\left(e^{j\omega_x}, e^{j\omega_y}\right) \approx 1 - Z_1\left(e^{j\omega_x}, e^{j\omega_y}\right) - Z_2\left(e^{j\omega_x}, e^{j\omega_y}\right)$
- f_1 small; f_2 large: $H\left(e^{j\omega_x}, e^{j\omega_y}\right) \approx 1 - Z_1\left(e^{j\omega_x}, e^{j\omega_y}\right) + \left(m-1\right) Z_2\left(e^{j\omega_x}, e^{j\omega_y}\right)$
- f_1 large; f_2 small: $H\left(e^{j\omega_x}, e^{j\omega_y}\right) \approx 1 + \left(m-1\right) Z_1\left(e^{j\omega_x}, e^{j\omega_y}\right) - Z_2\left(e^{j\omega_x}, e^{j\omega_y}\right)$
- Both large: $H\left(e^{j\omega_x}, e^{j\omega_y}\right) \approx 1 + \left(m-1\right)\left[Z_1\left(e^{j\omega_x}, e^{j\omega_y}\right) + Z_2\left(e^{j\omega_x}, e^{j\omega_y}\right)\right]$

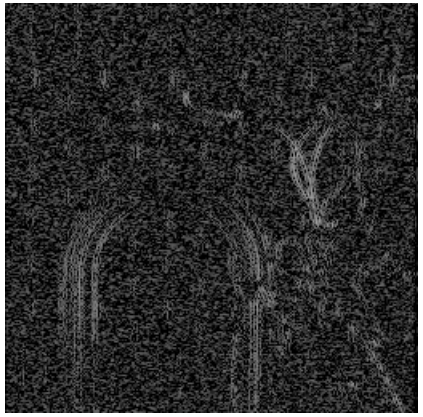
Nonlinear noise reduction/sharpening example



blurred, noisy image

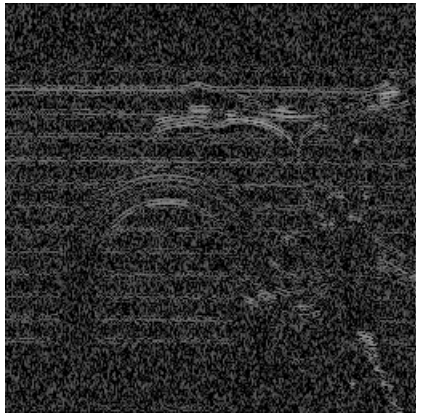
noise-reduced
and sharpened

Highpass filtered images



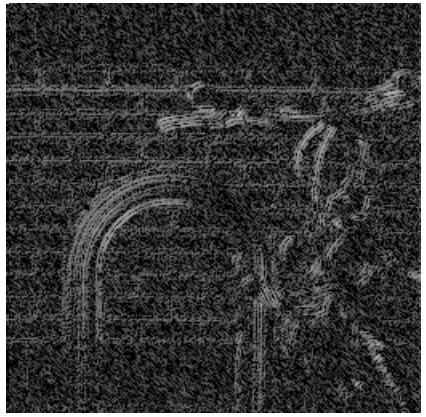
log magnitude of image filtered with

$$\begin{pmatrix} 0 & 0 & 0 \\ -0.5 & [1] & -0.5 \\ 0 & 0 & 0 \end{pmatrix}$$



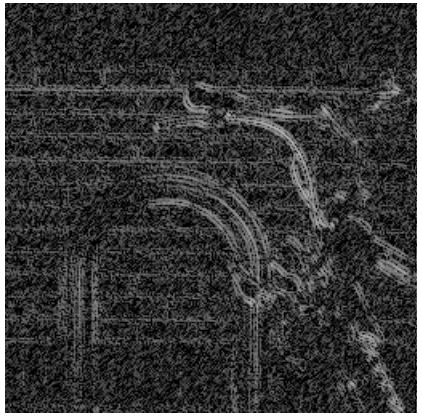
log magnitude of image filtered with

$$\begin{pmatrix} 0 & -0.5 & 0 \\ 0 & [1] & 0 \\ 0 & -0.5 & 0 \end{pmatrix}$$



log magnitude of image filtered with

$$\begin{pmatrix} -0.5 & 0 & 0 \\ 0 & [1] & 0 \\ 0 & 0 & -0.5 \end{pmatrix}$$

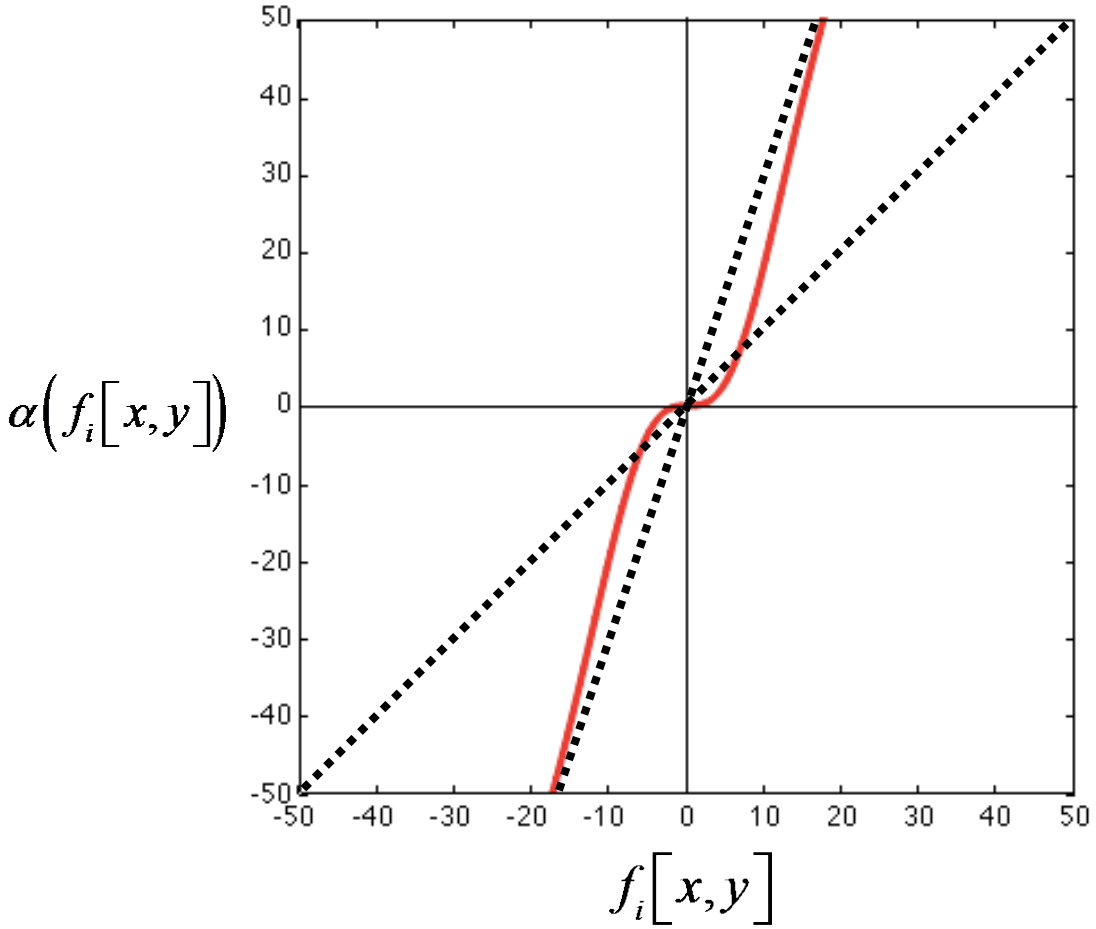


log magnitude of image filtered with

$$\begin{pmatrix} 0 & 0 & -0.5 \\ 0 & [1] & 0 \\ -0.5 & 0 & 0 \end{pmatrix}$$

Soft coring function

$$\alpha\left(f_i[x,y]\right) = m \cdot f_i[x,y] \cdot \left(1 - e^{-\left|\frac{f_i[x,y]}{\tau}\right|^{\gamma}}\right)$$



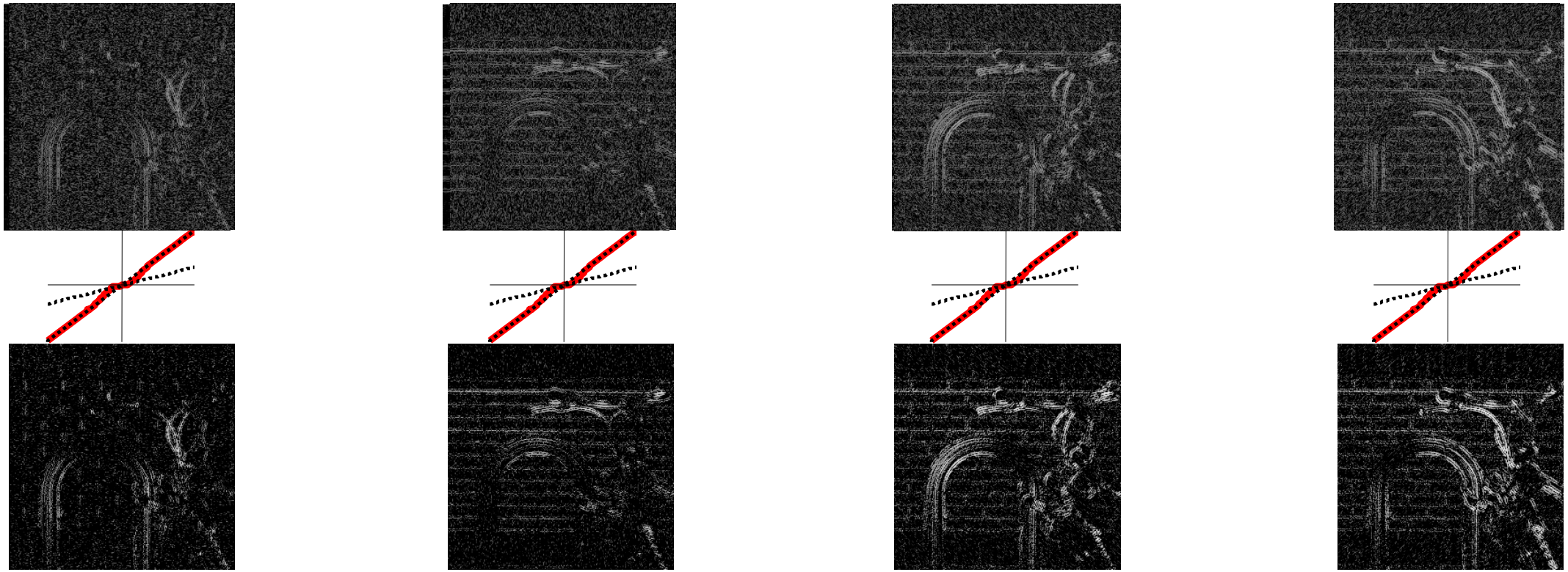
Example:

$m = 3$

$\gamma = 2$

$\tau = 10$

Soft coring of highpass filtered images



Linear vs. nonlinear noise reduction/sharpening

Noise reduction by lowpass filter (linear)

Sharpening by highpass filter (linear)

Combined noise reduction and sharpening (nonlinear)