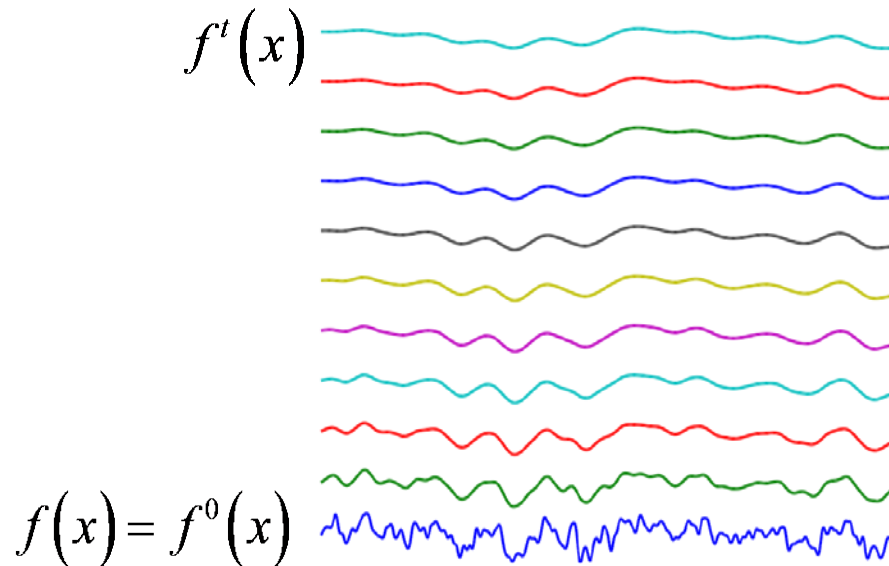
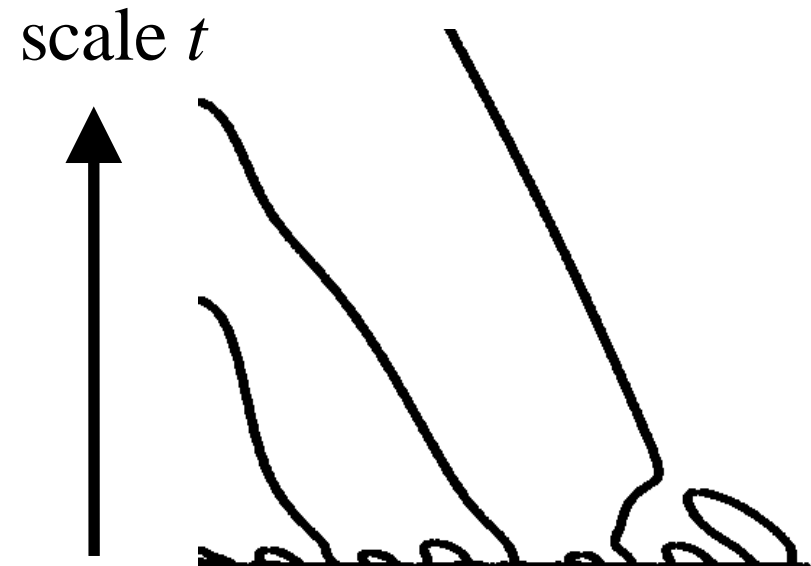


# Scale-space representation of a signal

Parametric family of signals  $f^t(x)$  where fine-scale information is successively attenuated



Successive smoothing  
with a Gaussian filter



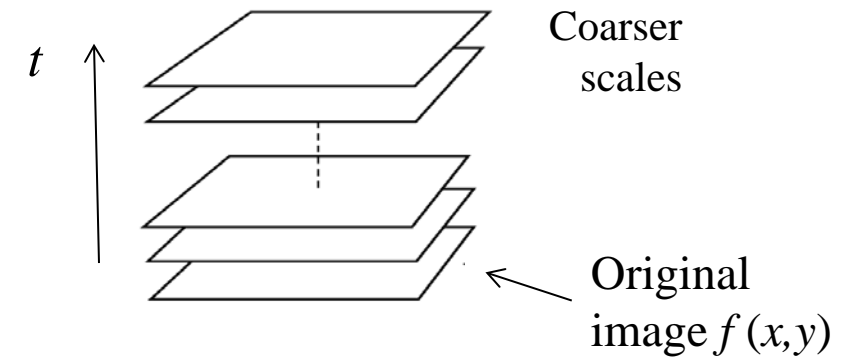
Zero-crossings of 2<sup>nd</sup> derivative  $f''(x)$   
Fewer edges at coarser scales



# Scale-space representation of images

- Parametric family of images smoothed by Gaussian filter

$$f^t(x, y) = g^t(x, y) * f(x, y); t \geq 0 \quad \text{with} \quad g^t(x, y) = \frac{1}{2\pi t} \exp\left(-\frac{x^2 + y^2}{2t}\right)$$
$$F^t(\omega_x, \omega_y) = G^t(\omega_x, \omega_y) F(\omega_x, \omega_y) \quad \text{with} \quad G^t(\omega_x, \omega_y) = \exp\left(-\frac{t}{2}(\omega_x^2 + \omega_y^2)\right)$$



- Shift-invariance

$$f^t(x - \Delta x, y - \Delta y) = g^t(x, y) * f(x - \Delta x, y - \Delta y)$$

- Rotation-invariance

$$f^t(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta) = g^t(x, y) * f(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$$

# Scale-space representation of images (cont.)

- Commutative semigroup property

$$\begin{aligned} f^{t_1+t_2}(x, y) &= g^{t_1}(x, y) * f^{t_2}(x, y) = g^{t_2}(x, y) * f^{t_1}(x, y) \\ &= g^{t_1}(x, y) * g^{t_2}(x, y) * f(x, y) \end{aligned}$$

- Separability

$$\begin{aligned} g^t(x, y) &= \frac{1}{2\pi t} \exp\left(-\frac{x^2 + y^2}{2t}\right) = \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{x^2}{2t}\right) \cdot \frac{1}{\sqrt{2\pi t}} \exp\left(-\frac{y^2}{2t}\right) \\ G^t(\omega_x, \omega_y) &= \exp\left(-\frac{t}{2}(\omega_x^2 + \omega_y^2)\right) = \exp\left(-\frac{t}{2}\omega_x^2\right) \exp\left(-\frac{t}{2}\omega_y^2\right) \end{aligned}$$