Scale-space representation of images (cont.)

- Non-creation of local extrema (for $f(x,y)$ and all of its partial derivatives) since $g^t(x,y) \geq 0$ and unimodal.
- Solution to diffusion equation (heat equation)

$$\frac{\partial}{\partial t} f^t(x,y) = \frac{1}{2} \nabla^2 f^t(x,y)$$

$$\begin{aligned}
\frac{\partial}{\partial t} F^t(\omega_x,\omega_y) &= \frac{\partial}{\partial t} G^t(\omega_x,\omega_y) F(\omega_x,\omega_y) \\
&= \frac{\partial}{\partial t} \exp\left(-\frac{t}{2}(\omega_x^2+\omega_y^2)\right) F(\omega_x,\omega_y) \\
&= -\frac{1}{2}(\omega_x^2+\omega_y^2) \exp\left(-\frac{t}{2}(\omega_x^2+\omega_y^2)\right) F(\omega_x,\omega_y) \\
&= -\frac{1}{2}(\omega_x^2+\omega_y^2) F^t(\omega_x,\omega_y)
\end{aligned}$$

$$\frac{\partial}{\partial t} f^t(x,y) = \frac{1}{2}\nabla^2 f^t(x,y)$$

t = 0.07 sec