

What patterns can be localized most accurately?

- Local displacement sensitivity (assuming continuous $f(x,y)$)

$$S(\Delta x, \Delta y) = \sum_{(x,y) \in \text{window}} [f(x,y) - f(x + \Delta x, y + \Delta y)]^2$$

- Linear approximation for small $\Delta x, \Delta y$

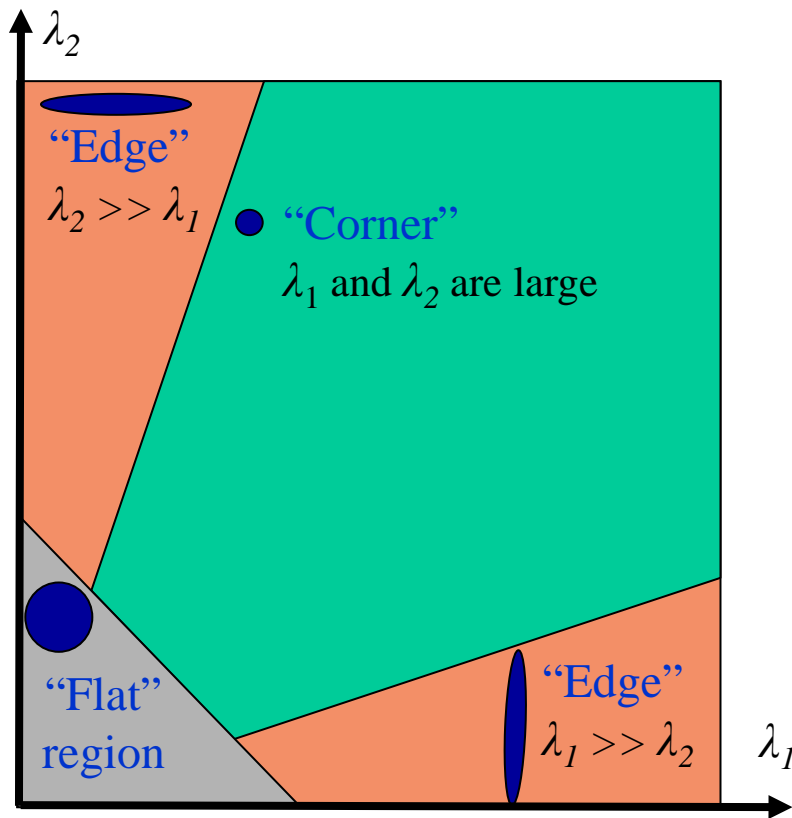
$$f(x + \Delta x, y + \Delta y) \approx f(x, y) + f_x(x, y)\Delta x + f_y(x, y)\Delta y$$

$f_x(x, y)$ – horizontal image gradient
 $f_y(x, y)$ – vertical image gradient

$$\begin{aligned} S(\Delta x, \Delta y) &\approx \sum_{(x,y) \in \text{window}} \left[\begin{pmatrix} f_x(x, y) & f_y(x, y) \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \right]^2 \\ &= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \left\{ \sum_{(x,y) \in \text{window}} \begin{bmatrix} f_x^2(x, y) & f_x(x, y)f_y(x, y) \\ f_x(x, y)f_y(x, y) & f_y^2(x, y) \end{bmatrix} \right\} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \\ &= \begin{pmatrix} \Delta x & \Delta y \end{pmatrix} \mathbf{M} \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \end{aligned}$$

- Iso-sensitivity curves are ellipses

Harris detector



Based on eigenvalues λ_1, λ_2 of “structure matrix”
(aka “normal matrix” aka “second-moment matrix”)

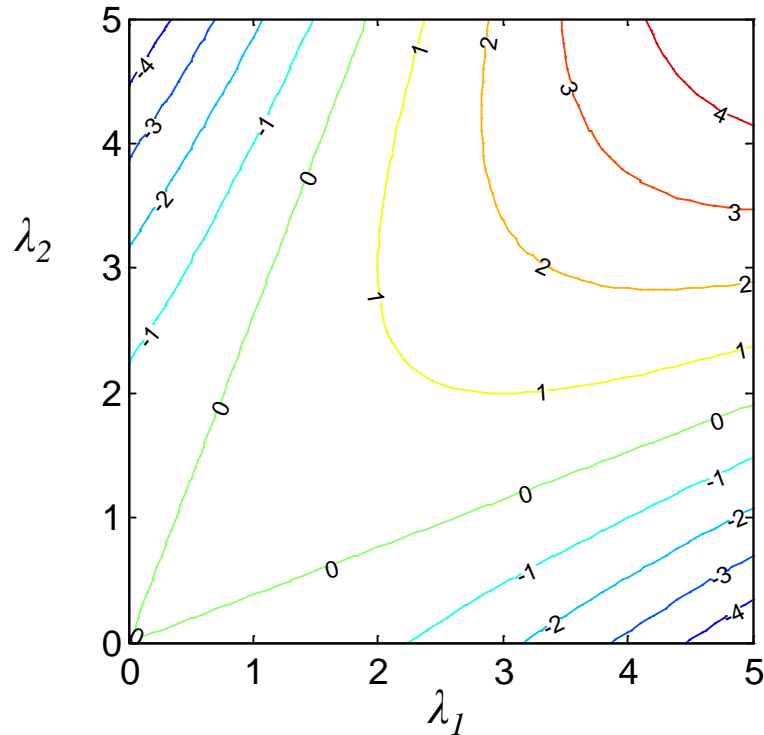
$$\mathbf{M} = \begin{bmatrix} \sum_{[x,y] \in \text{window}} f_x^2[x,y] & \sum_{[x,y] \in \text{window}} f_x[x,y]f_y[x,y] \\ \sum_{[x,y] \in \text{window}} f_x[x,y]f_y[x,y] & \sum_{[x,y] \in \text{window}} f_y^2[x,y] \end{bmatrix}$$

$f_x[x,y]$ – horizontal image gradient
 $f_y[x,y]$ – vertical image gradient

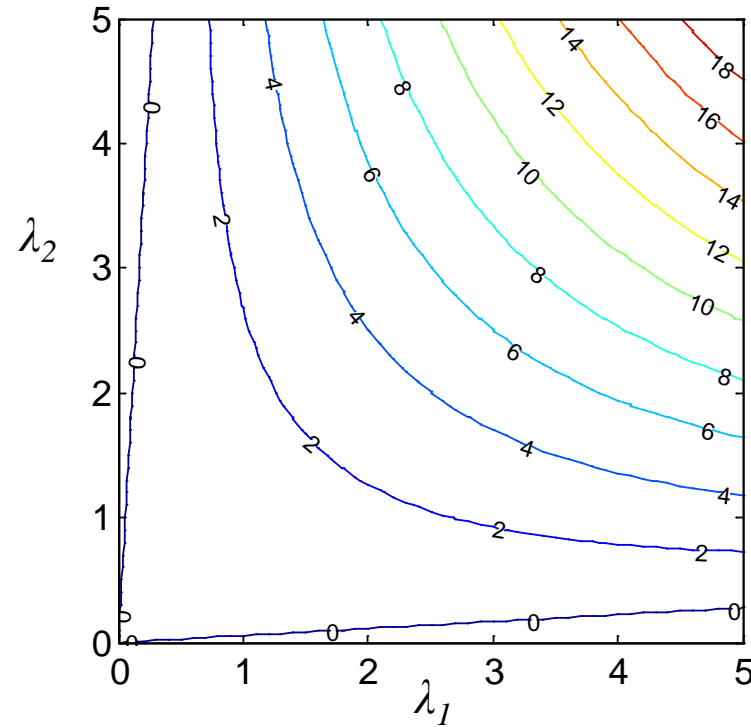
Harris cornerness

$$C = \det(\mathbf{M}) - k \cdot (\text{trace}(\mathbf{M}))^2 = \lambda_1 \lambda_2 - k \cdot (\lambda_1 + \lambda_2)^2$$

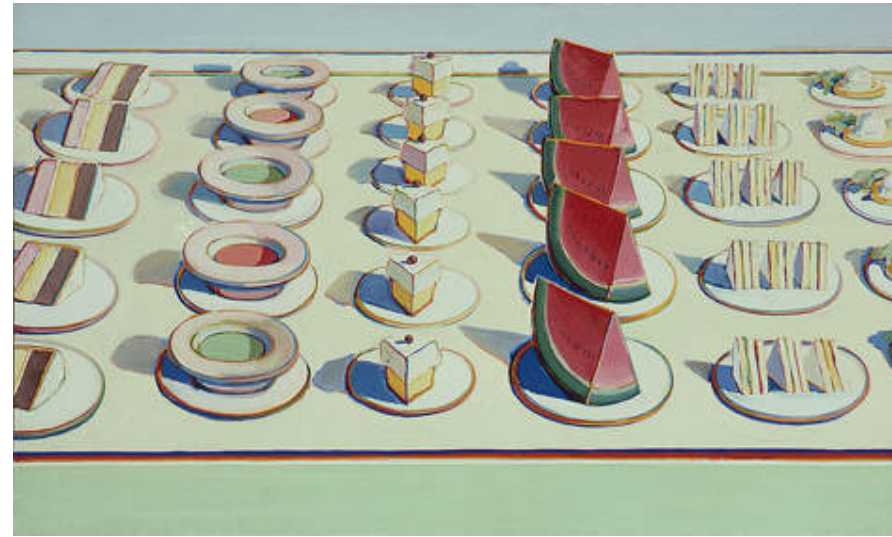
$k = 0.2$



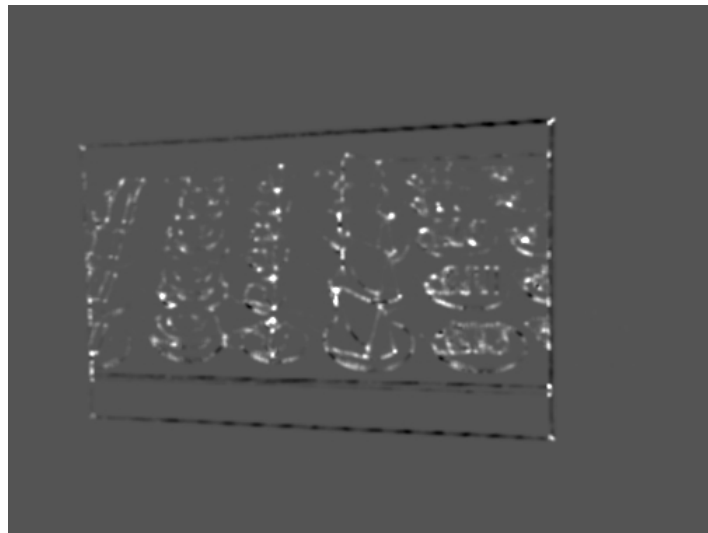
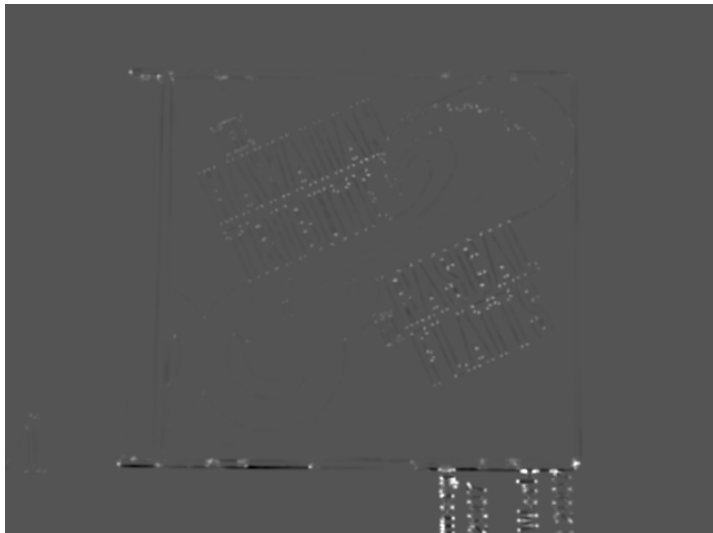
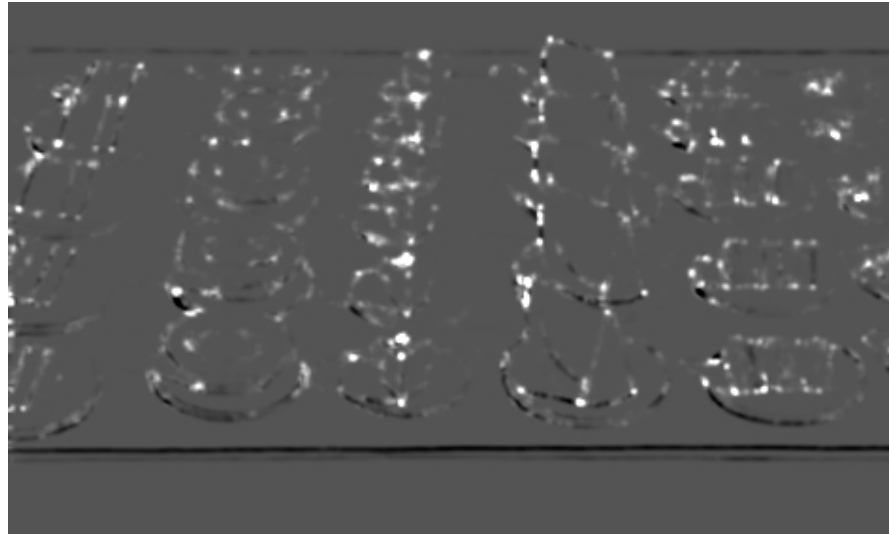
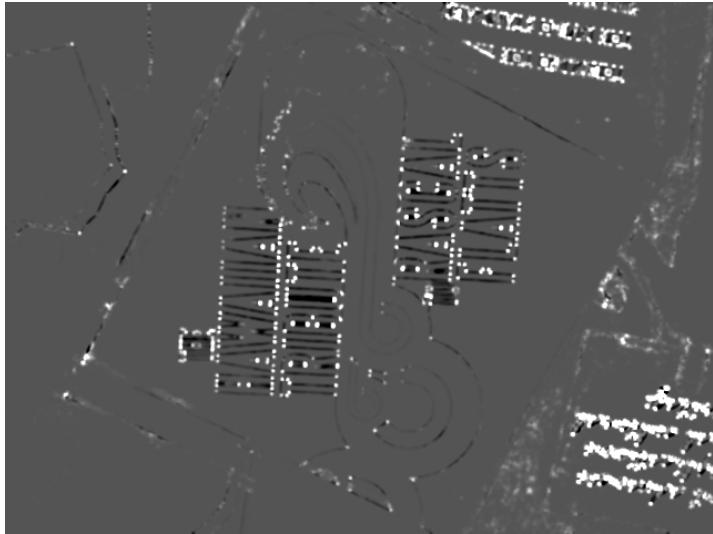
$k = 0.05$



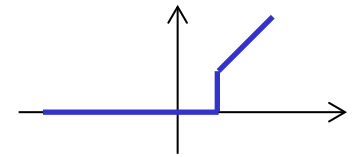
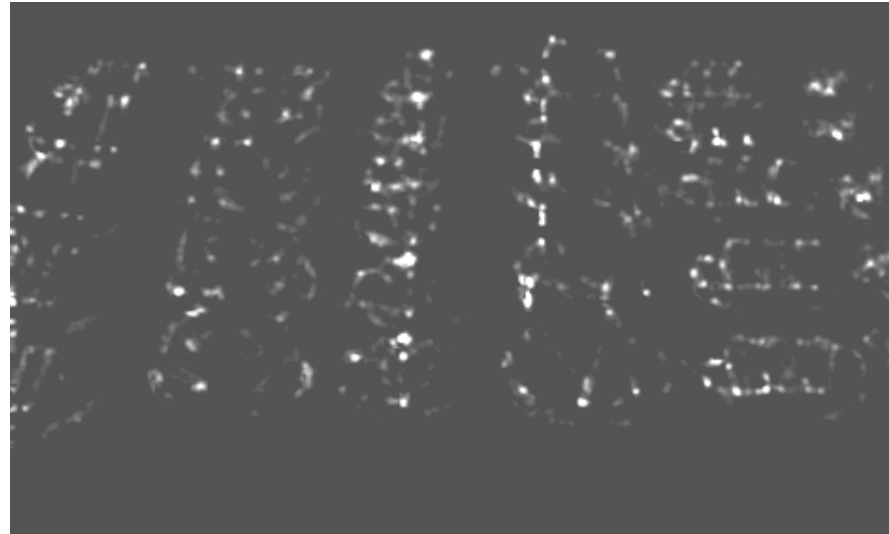
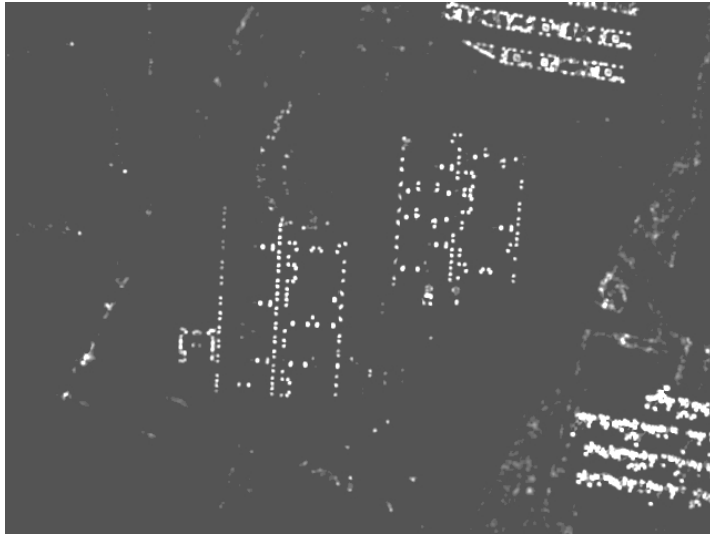
Input images



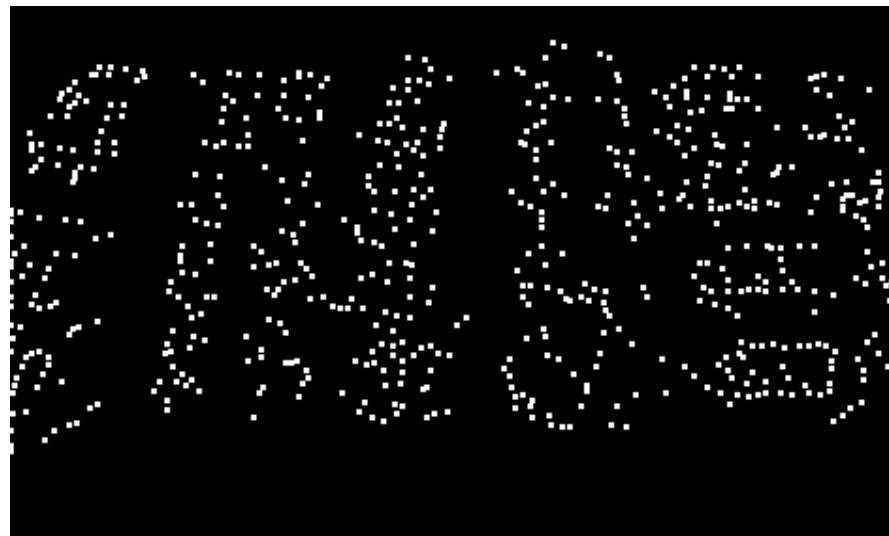
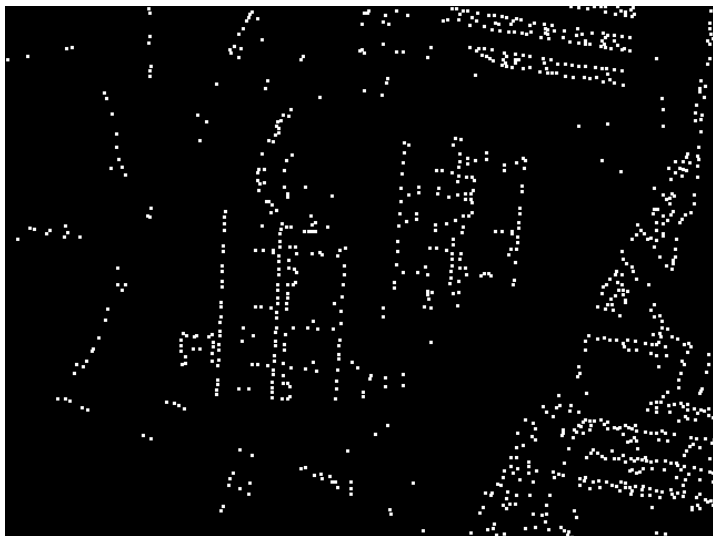
Harris cornerness



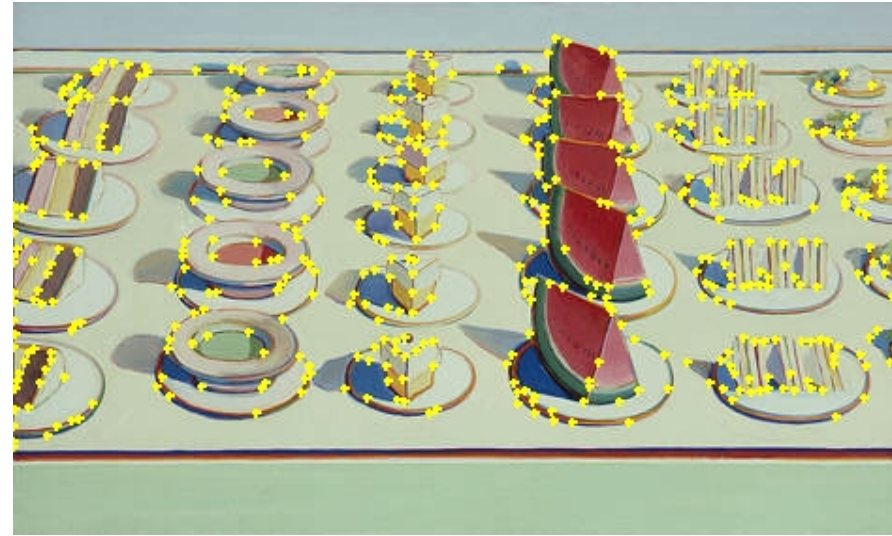
Thresholded cornerness



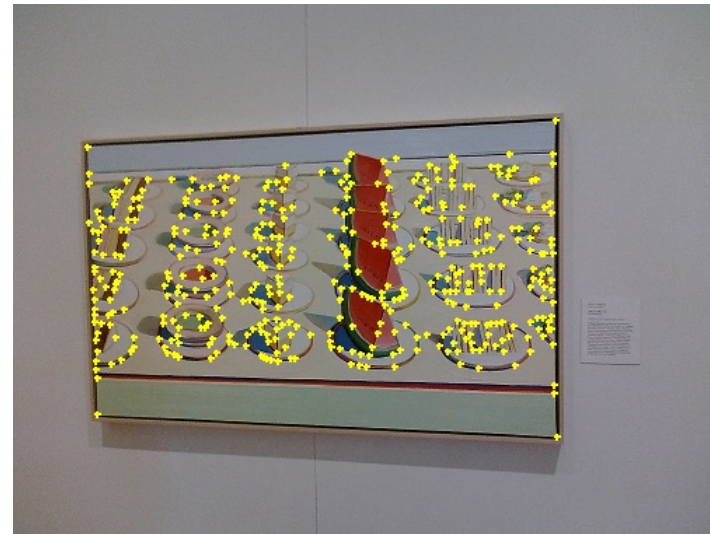
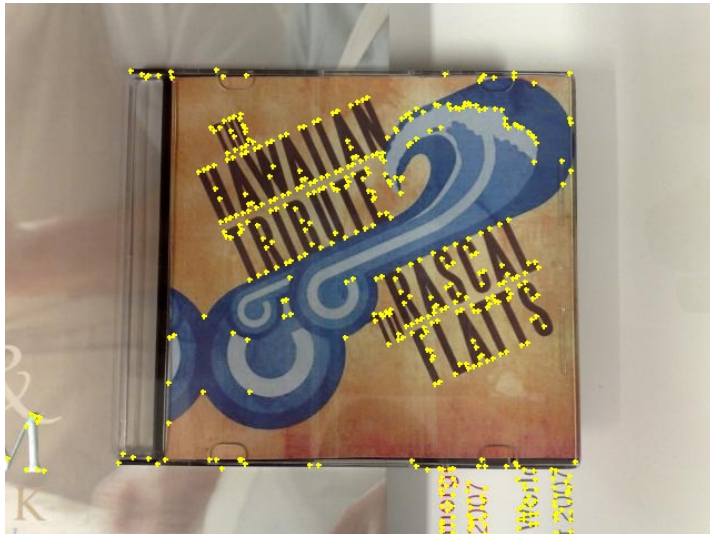
Local maxima of corneriness



Superimposed Harris keypoints



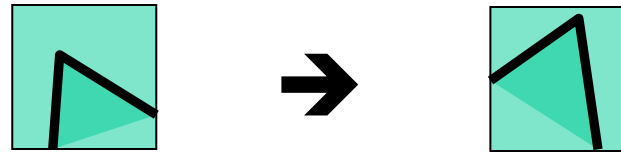
500 strongest
keypoints



Robustness of Harris detector

- Invariant to brightness offset: $f[x,y] \rightarrow f[x,y] + c$

- Invariant to shift and rotation



- Not invariant to scaling

