Variants of SURE-LET Approach to Image Denoising

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Abstract

Many image denoising methods assume priors due to the underdetermined nature of the image denoising problem. However, these methods might perform poorly on images whose statistics do not agree with the prior assumed. In this paper, we explored the SURE-LET approach, a Gaussian denoising approach which avoids using a specific prior. SURE-LET and PURE-LET, its extension to the Poisson noise case, offer a means to optimize the transform domain denoising methods. We implemented our version of SURE-LET and PURE-LET by using soft-threshold in discrete cosine transform domain as the denoiser. We also explored coupling SURE-LET with Anscombe transform to denoise Poisson noisy images. Our Gaussian denoisers yielded results comparable to the results from ADMM wit TV prior and NLM prior. Our Poisson denoisers outperformed Richardson-Lucy algorithm and its variant with TV prior.

1. Introduction

The process of image capturing always introduce noise to the image. For example, images captured by SLR camera will have noises from artifacts on the sensor, electronic noise sources, photon-to-electron conversion, etc. Noise in images can be categorized largely as Gaussian noise, which is additive and independent of the signal, and Poisson noise, which is dependent of the signal and is often found in images captured with low light [16]. These two kinds of noise are modeled differently. Consequently, the methods used for denoising one kind of noise are not very effective if used for the other kind of noise.

Often times, we do not have access to the ground truth. Since the image denoising problem is usually ill-posed, and there can be multiple solutions to an image denoising problem. Many existing algorithms assume priors, such as minimization of total variation (TV) norm [13] and non-local similarity [4] in order to denoise an image. However, by assuming certain statistics of the image, the denoising algorithms may not perform well on some images whose characteristics do not agree with the assumption.

Stein's unbiased risk estimator (SURE) is a statistical tool which can be used to estimate the mean squared error (MSE) of an observation corrupted by a Gaussian noise without requiring the access to the ground truth itself [14]. Blu and Luisier proposed to minimize SURE on a combination of images denoised by a set of denoisors from a class called the transform domain image denoising in order to denoise an image [2]. This framework, named SURE-LET, can be applied to any kind of transform domain denoising methods satisfying the condition that they are differentiable. Typically, in a transform domain image denoising method, a linear transformation is applied to an image, and then an operator, possibly a nonlinear operator, is applied on the transformed image; finally, the image is reverse-transformed to recover the denoised image. Many transform domain denoising methods have been proposed [11][8][6]. The question answered by SURE-LET is how to choose the best set of operators to denoise an image. Implicitly, we avoid assuming priors when denoising images using the SURE-LET approach. Hence, the SURE-LET approach is robust to varying statistics of images.

An example of a simple operator used in transform domain denoising is the soft threshold, which is a piece-wise linear point-wise function. There are studies which used soft-thresholding for image denoising, such as [7] which used soft-thresholding on the wavelet-transformed domain. For the purpose of demonstrating the use of the SURE-LET approach, we choose soft-thresholding in the discrete cosine transform (DCT) domain for its simplicity.

Even though the SURE-LET approach neither requires the knowledge of the ground truth image nor assumes statistics of the image, it requires the knowledge of the variance of the noise in the target noisy image. While statistics of the noise may not be available in some cases, there are many approaches to approximate the variance of Gaussian noise [5] as well as Poisson noise [9], and the mix of Gaussian and Poisson noise [12]. With these noise estimation approaches, we can obtain the variance of the noise required for SURE-

LET denoisers.

While the SURE-LET approach can be applied to a range of denoisers, it is only valid when applied to images corrupted by Gaussian noise. In order to denoise Poisson-noise corrupted images, we used a different MSE estimate, called the Poisson unbiased risk estimator (PURE), for estimating Poisson noise [10]. Similarly, we can apply a set of transform domain denoisers and then minimize PURE in order to denoise an image.

Another approach for denoising a Poisson noisy image is to use the Anscombe transform. The Anscombe transform approximates the Poisson random variables as the Gaussian random variables [1]. There have been studies which used the Anscombe transform to denoise Poisson noisy images [15][3]. In this work, we used the Anscombe transform to obtain the Gaussian approximation of a Poisson noisy image. Then, we used a SURE-LET denoiser and applied the inverse Anscombe transform to recover the denoised image.

In this work, we explored the SURE-LET approach implemented with soft-thresholding in DCT domain for denoising Gaussian-noise corrupted images. We used the method proposed by [5] in order to approximate the variance of the noise required for the SURE-LET approach. In the case of Poisson noise, we explored the closely related PURE-LET approach implemented with soft-thresholding in DCT domain. Finally, also in the case of Poisson noise, we explored the use of Anscombe transform coupled with the SURE-LET approach.

2. Related Work

2.1. SURE-LET

This section describes the SURE-LET approach as proposed by Blu and Luisier [2]. Following the original work, a Gaussian-noise corrupted image can be modeled as follows.

$$\mathbf{y} = \mathbf{x} + \eta \tag{1}$$

where $\mathbf{y} \in \mathbb{R}^N$ is the noisy image, $\mathbf{x} \in \mathbb{R}^N$ is the ground truth image, and $\eta \in \mathbb{R}^N$ is the additive Gaussian noise. N is the number of pixels in the image.

Let $\mathbf{F}$ be a denoising function and $\mathbf{F}(\mathbf{y}) = \hat{\mathbf{x}}$ be the corresponding denoised image, or an estimation of the ground truth. The goal is to minimize the MSE:

$$MSE = \frac{1}{N}||\hat{\mathbf{x}} - \mathbf{x}||^2 \tag{2}$$

However, we cannot calculate MSE without having access to the ground truth image. Blu and Lusier formulated SURE in the context of Gaussian image denoising as follows.

$$\text{SURE}(\mathbf{F}) = \frac{1}{N}||\mathbf{F}(\mathbf{y}) - \mathbf{y}||^2 + \frac{2\sigma^2}{N}\text{div}\{\mathbf{F}(\mathbf{y})\} - \sigma^2 \tag{3}$$

where σ^2 is the variance of the Gaussian noise.

In SURE-LET approach, the denoising function is expressed as a linear combination of elementary denoising functions

$$\mathbf{F}(\mathbf{y}) = \sum_{k=1}^{K} a_k \mathbf{F}_k(\mathbf{y}) \tag{4}$$

where K is the number of elementary denoising functions chosen. This is the linear expansion of threshold (LET) principle. Given that any $\mathbf{F}_k(\mathbf{y})$ is differentiable, we can minimize SURE over the weights of elementary denoising functions. By taking the parial derivative of SURE with respect to each weight a_k, we obtain a set of linear equations:

$$\sum_{l=1}^{K} \mathbf{F}_k(\mathbf{y})^T \mathbf{F}_l(\mathbf{y}) a_l = \mathbf{F}_k(\mathbf{y})^T \mathbf{y} - \sigma^2 \text{div}\{\mathbf{F}_k(\mathbf{y})\} \tag{5}$$

for $k = 1, ..., K$. Typically, this system of linear equations is underdetermined – there can be many solutions to this problem. In this work, we followed the solution suggested by Blu and Luisier, which is the least-norm solution obtained by applying the pseudoinverse of $[M]_{k,l} = \mathbf{F}_k(\mathbf{y})^T \mathbf{F}_l(\mathbf{y})$ to the vector $[c]_k = \mathbf{F}_k(\mathbf{y})^T \mathbf{y} - \sigma^2 \text{div}\{\mathbf{F}_k(\mathbf{y})\}$.

Blu and Luisier also showed how the SURE-LET approach can be applied for any transform domain denoising by expressing the denoising function as follows.

$$\mathbf{F}(\mathbf{y}) = \sum_{k=1}^{K} a_k \mathbf{R}\Theta_k(\mathbf{D}\mathbf{y}) \tag{6}$$

As discussed in the Theory section of this paper, we followed this formulation in the implementation of SURE-LET approach with soft-thresholding in DCT domain.

2.2. PURE-LET

This section describes the PURE-LET approach as proposed by Li, Luisier, and Blu [10]. Following the original work, a Poisson noisy image can be modeled as follows:

$$\mathbf{y} = \alpha \mathcal{P}(\frac{\mathbf{H}\mathbf{x}}{\alpha}) \tag{7}$$

where $\mathbf{H} \in \mathbb{R}^{N \times N}$ is a convolution kernel, which is the identity $\mathbf{I}_N$ when the noisy image is simply the ground truth with some Poisson noise. $\mathcal{P}\{\cdot\}$ applies Poisson noise to its argument, and $\alpha \in \mathbb{R}$ is the scaling factor representing the stregth of the noise. We can think of $\frac{1}{\alpha}$ as the number of photons on each pixel of the image, or the variance of the Poisson random variable. The larger the value of α, the smaller the number of photons, and hence the stronger the Poisson noise.

The estimation of MSE in this case is PURE, which is derived in [10]. It has the following form.

$$\text{PURE}(\mathbf{F}) = \frac{1}{N}||\mathbf{F}(\mathbf{y})||^2 - \frac{2}{N}\mathbf{y}^T \mathbf{H}^{-T} \mathbf{F}^{-}(\mathbf{y}) + \mathcal{E}_{\mathcal{P}} \tag{8}$$

where $\mathbf{F}^-(\mathbf{y}) = [f_n(y_n - \alpha e_n)]_{n=1,...,N}$; e_n is the nth canonical basis (vector of all zeros except one at the nth entry). $\mathcal{E}_{\mathcal{P}} = \mathbf{y}^T\mathbf{H}^{-T}\mathbf{H}^{-1}\mathbf{y} - \alpha\mathbf{1}^T\mathbf{H}^{-T}\mathbf{H}^{-1}\mathbf{y}$ is a term which is independent of $\mathbf{F}$, and will disappear when we take the derivative of PURE with respect to coefficients of the elementary denoising functions.

By using the LET principle and taking the partial derivative of PURE with respect to weights of the elementary denoisers, we obtain a system of linear equations similar to that in the SURE-LET approach:

$$\sum_{l=1}^{K} \mathbf{F}_k(\mathbf{y})^T\mathbf{F}_l(\mathbf{y})a_l = \mathbf{y}^T\mathbf{H}^{-T}(\mathbf{F}_k(\mathbf{y}) - \alpha\partial\mathbf{F}_k(\mathbf{y})) \quad (9)$$

for $k = 1, ..., K$. Li, Luisier, and Blu noted that the matrix $[M]_{k,l} = \mathbf{F}_k(\mathbf{y})^T\mathbf{F}_l(\mathbf{y})$ may be singular, and hence a regularization term is recommended with the regularization parameter $\mu = 5 \times 10^{-4}y_{mean}^2$. Thus, a solution to this problem is $\mathbf{a} = (\mathbf{M} + \mu\mathbf{I})^{-1}\mathbf{c}$, where $[c]_k = \mathbf{y}^T\mathbf{H}^{-T}(\mathbf{F}_k(\mathbf{y}) - \alpha\partial\mathbf{F}_k(\mathbf{y}))$.

2.3. Anscombe transform

Anscombe proposed a transformation which can be used to obtain a Gaussian equivalent of a Poisson random variable [1]. In the work by Thanh and Thanh [15], the Anscbome transform is applied point-wise to the image:

$$x' = 2\sqrt{x + \frac{3}{8}} \quad (10)$$

After applying a denoiser to the Anscombe transform domain, Thanh and Thanh used the inverse Anscombe transform:

$$x = (\frac{x'}{2})^2 - \frac{3}{8} \quad (11)$$

in order to recover the image.

3. Theory

In this section, we discuss the use of SURE-LET and PURE-LET for our specific case of denoiser – soft-thresholding in DCT domain. The soft-threshold function has the following form.

$$\theta_T(x) = \begin{cases} x - T & x > T \\ 0 & |x| \leq T \\ x + T & x < -T \end{cases} \quad (12)$$

The SURE-LET and PURE-LET approaches also require the derivative of the denoising functions. Hence, we show the derivative of the soft-threshold function here.

$$\theta'_T(x) = \begin{cases} 1 & |x| > T \\ 0 & |x| \leq T \end{cases} \quad (13)$$

In practice, we only have to apply the soft-threshold function to a target transformed image and check each pixel whether its value is zero in order to determine θ'_T. To summarize, our denoising function has the following form.

$$\mathbf{F}^-(\mathbf{y}) = \sum_{k=1}^{K} \text{IDCT}(\theta_T(\text{DCT}(\mathbf{y}))) \quad (14)$$

We used this expression as the denoiser in our implementation of SURE-LET and PURE-LET denoisers.

3.1. SoftSURE

For SURE-LET with soft-thresholding in DCT domain, we substitute the expression of our denoiser in equation (14) and the derivative of the soft threshold function (13) to equation (5). We obtain a system of linear equations

$$\mathbf{Ma} = \mathbf{c} \quad ; \quad \mathbf{M} \in \mathbb{R}^{K\times K} \quad (15)$$

where

$$\begin{aligned} [M]_{k,l} &= [\text{IDCT}(\theta_{T_k}(\text{DCT}(\mathbf{y})))]^T[\text{IDCT}(\theta_{T_l}(\text{DCT}(\mathbf{y})))] \\ [c]_k &= [\text{IDCT}(\theta_{T_k}(\text{DCT}(\mathbf{y})))]^T\mathbf{y} - \sigma^2(\gamma^T\theta'_{T_k}(\text{DCT}(\mathbf{y}))) \\ \gamma &= \text{diag}\{(\text{IDCT})(\text{DCT})\} \end{aligned}$$

By solving this system of equations using pseudoinverse, we obtain the weight for each of the denoising results from elementary denoisers. Hence, we can reconstruct the denoised image by simply taking the linear combination of these elementary denoised images as in equation (6).

In order to estimate the variance of the Gaussian noise required for solving equation (15), we used the algorithm described in [5] which requires only the target noisy image.

3.2. SoftPURE

In the case of PURE-LET with soft-thresholding in DCT Domain, by using our denoiser in equation (14) and the derivative in equation (13), we obtain the following form of (9).

$$\mathbf{Ma} = \mathbf{c} \quad ; \quad \mathbf{M} \in \mathbb{R}^{K\times K} \quad (16)$$

where

$$\begin{aligned} [M]_{k,l} &= [\text{IDCT}(\theta_{T_k}(\text{DCT}(\mathbf{y})))]^T[\text{IDCT}(\theta_{T_l}(\text{DCT}(\mathbf{y})))] \\ [c]_k &= \mathbf{y}^T(\text{IDCT}(\theta_{T_k}(\text{DCT}(\mathbf{y}))) - \alpha\text{IDCT}(\theta'_{T_k}(\text{DCT}(\mathbf{y})))) \end{aligned}$$

which can be solved by using the regularized inverse as described in the PURE-LET subsection.

In this work, we did not implement the Poisson noise variance estimator. When denoising Poisson noisy images with unknown variance using PURE-LET, we assumed a set of values for the variance and compared the results.

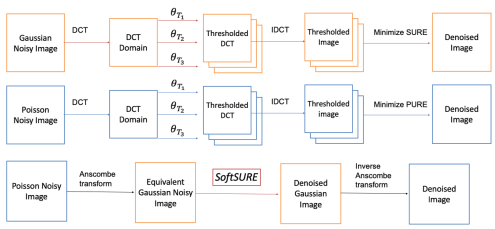


Figure 1: Block diagrams of the three denoising approaches implemented in this paper. (top) *SoftSURE*: SURE-LET with soft-thresholding in DCT domain (middle) *SoftPURE*: PURE-LET with Soft-Thresholding in DCT Domain (bottom) *ASURE*: *SoftSURE* applied to Anscombe transformed image.

3.3. ASURE

In order to denoise a Poisson noisy image with the SURE-LET approach, we first apply the Anscombe transform to the image. Note that the variance of the resulting Gaussian noisy image equivalent is approximately 1 [12]. Hence, there is no need to use any noise estimation algorithm in this case.

4. Results and Discussion

4.1. Threshold Values

We experimented with a range of threshold values for *SoftSURE* denoiser. According to Blu and Luisier, the performance of SURE-LET denoisers does not change significantly as the number of elementary denoisers increases from 2 onward. In figure 2, we chose a set of 3 threshold values and compare the resulting MSEs. While the performance of the denoiser for a specific set of thresholds may depend on the variance of the noise, we chose a single set of threshold values because we wanted to demonstrate the flexibility of SURE-LET denoisers when performing against different sets of noisy images. For all other results in this paper, we chose the set of threshold values (0, 0.1, 0.2, 0.5).

4.2. Comparison with Other Denoising Methods

We applied our denoising methods as well as some other methods to simulated noisy images. In Figure 3, *SoftSURE*, ADMM with the total variation (TV) prior, and ADMM with the non-local means (NLM) prior are applied to the Cameraman image with injected Gaussian noise of standard deviation $\sigma = 0.1$; *SoftPURE*, *ASURE*, Richardson-Lucy (RL) algorithm, and Richardson-Lucy algorithm with TV prior are applied to a the Cameraman image simulated to have Poisson noise with variance $\sigma^2 = 100$. We described Poisson noise with variance since it reflects the number of

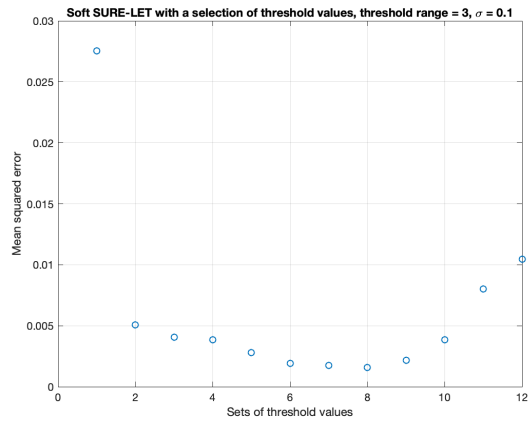


Figure 2: Plot of MSEs from denoising an image corrupted by Gaussian noise with standard deviation $\sigma = 0.1$ using the *SoftSURE* denoiser and different sets of threshold values. From left to right: (0, 0.0001, 0.0005), (0.0005, 0.001, 0.005), (0.001, 0.005, 0.01), (0.005, 0.01, 0.05), (0.01, 0.05, 0.1), (0.05, 0.1, 0.5), (0.1, 0.5, 1), (0.5, 1, 5), (1, 5, 10), (5, 10, 50), (10, 50, 100)

photons detected by the sensor ($\sigma = \sqrt{\text{number of photons}}$). Note that we estimate σ of the Gaussian noise using the noise estimator from [5] and not the oracle value $\sigma = 0.1$.

For the Gaussian noise case, when $\sigma = 0.1$, *SoftSURE* yields the result which is worse than the result from ADMM with TV prior, but better than the result from ADMM with NLM prior in terms of the peak signal-to-noise ratio (PSNR). Qualitatively, the result from *SoftSURE* is not as sharp as the result from ADMM with TV prior. The blur in *SoftSURE* image might be the consequence of suppressing high-frequency components in the image when performing thresholding in DCT domain. However, when the noise level is low, *SoftSURE* outperforms ADMM denoisers since they tend to oversmooth the image.

PSNR of results \ σ	**0.001**	**0.01**	**0.1**
noisy image	59.95	39.87	19.89
SoftSURE	41.98	37.98	24.13
ADMM+TV	29.48	29.47	27.29
ADMM+NLM	22.93	22.93	22.98

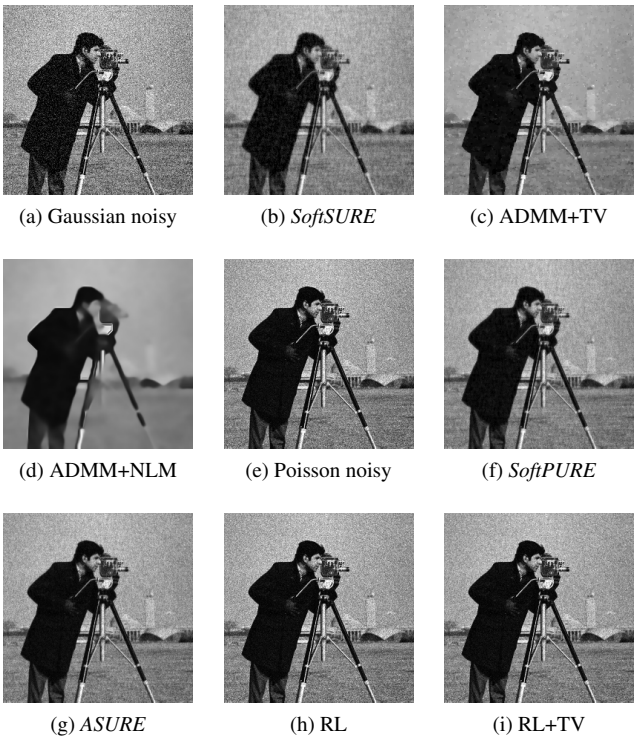
Table 1: PSNRs of the results from denoising the Cameraman image injected with Gaussian noise of known standard deviation σ.

In the case of Poisson noisy image, both *SoftPURE* and *ASURE* outperform the RL algorithm and RL algorithm with TV prior. Similar to the case of *SoftSURE*, the results from *SoftPURE* and *ASURE* appear to be less sharp than the original image due to suppression of high frequency compo-

nents when performing soft-thresholding in DCT domain. We observed that the effect of denoising decreases as σ^2 increases; this result is expected since high variance suggests that the means of the pixels values are also higher in the case of Poisson noise. In other words, higher variance is equivalent to the sensor detecting more photons, and hence less noise strength.

PSNR of results $\setminus$ σ^2	**100**	**500**	**1000**
noisy image	23.21	30.21	33.24
SoftPURE	26.21	31.31	33.84
ASURE	25.57	31.16	33.78
RL	23.22	30.21	33.24
RL+TV	24.51	31.23	32.66

Table 2: PSNRs of the results from denoising the Cameraman image with simulated Poisson noise of known variance σ^2.



(a) Gaussian noisy (b) *SoftSURE* (c) ADMM+TV

(d) ADMM+NLM (e) Poisson noisy (f) *SoftPURE*

(g) *ASURE* (h) RL (i) RL+TV

Figure 3: Simulated noisy images and results from denoising methods. (a) Gaussian noisy image ($\sigma = 0.1$), PSNR = 19.89 (b) *SoftSURE*, PSNR = 24.13 (c) ADMM with TV prior, PSNR = 27.29 (d) ADMM with NLM prior, PSNR = 22.98 (e) Poisson noisy image ($\sigma^2 = 100$), PSNR = 23.21 (f) *SoftPURE*, PSNR = 26.21 (g) *ASURE*, PSNR = 25.57 (h) Richardson-Lucy algorithm, PSNR = 23.21 (i) Richardson-Lucy algorithm with TV prior, PSNR = 24.51.

4.3. Application to Real Noisy Image

We applied the same set of denoising methods on a two-photon image of bovine endolithelial (BPAE) cells obtained from a Poisson-Gaussian fluorescent microscopy dataset [17]. Since we do not know the Poisson noise variance, for denoising methods requiring the Poisson variance value, including *SoftPURE*, RL, and RL with TV prior, we experimented on a range of values and presented the best results, which are when $\sigma^2 = 50$. The PSNRs are calculated by comparing the denoised image to the ground truth provided in the original source, which was obtained by averaging 50 images captured using the same setting [17].

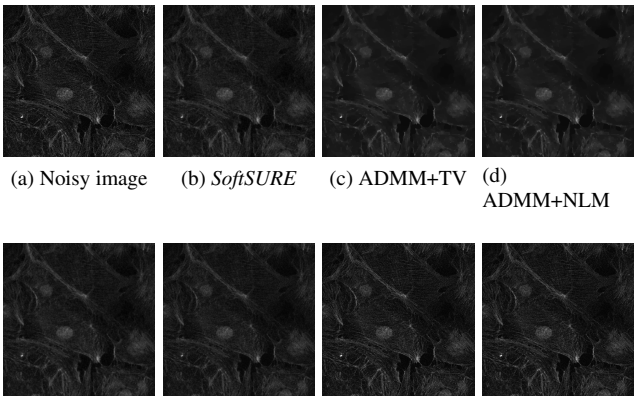
For the Gaussian denoisers, *SoftSURE* yields the result with PSNR comparable to the result from ADMM with TV prior, and outperforms ADMM with NLM prior. Qualitatively, *SoftSURE* preserves more fine details, while the ADMM methods oversmooth the image; the small fibers which are visible in the noisy image and the result from *SoftSURE* are not visible in the results from ADMM approaches. This results demonstrate how the performance of denoising methods using priors are dependent on the characteristics of specific images.

For the Poisson denoisers, *SoftPURE* and *ASURE* significantly outperform RL and RL with TV prior in terms of PSNR. However, qualitatively, the results from *SoftPURE* and *ASURE* appear to be less sharp than the noisy image, likely due to how soft-thresholding in DCT domain suppresses higher frequency components of the image. We observed that the PSNRs from *SoftPURE* and *ASURE* are higher than the PSNR of the result from *SoftSURE*; this reason is likely due to the low-light nature of the image capturing process which introduces more Poisson noise than Gaussian noise.

5. Conclusion

In the case of Gaussian noise, our version of SURE-LET denoiser, *SoftSURE*, may yield the results comparable to methods such as ADMM with TV prior and ADMM with NLM prior. When there are fine details in the image, *SoftSURE* preserve these details better than the ADMM approaches with any of the priors, since they tend to oversmooth images. In the case of Poisson noise, our version of PURE-LET denoiser, *SoftPURE*, and the SURE-LET denoiser coupled with Anscombe transform, *ASURE*, outperform Richardson-Lucy algorithm and its variant with TV prioir.

Our methods are non-iterative. Due to the availability of very efficient linear system solvers, our methods are efficient compared to iterative methods such as ADMM and RL. However, in the case of *SoftSURE*, the noise estimation algorithm needs to process a large number of small patches of the image; it is the usually the most computation-



(a) Noisy image (b) *SoftSURE* (c) ADMM+TV (d) ADMM+NLM

(e) *SoftPURE* (f) *ASURE* (g) RL (h) RL+TV

Figure 4: Noisy two-photon image of BPAE and results from denoising methods. (a) Noisy image, PSNR = 25.99 (b) *SoftSURE*, PSNR = 30.71 (c) ADMM with TV prior, PSNR = 30.87 (d) ADMM with NLM prior, PSNR = 28.34 (e) *SoftPURE*, PSNR = 30.74 (f) *ASURE*, PSNR = 29.29 (g) Richardson-Lucy algorithm, PSNR = 25.99 (h) Richardson-Lucy algorithm with TV prior, PSNR = 26.43.

ally expensive in *SoftSURE*. We can improve our *SoftSURE* approach to denoising Gaussian noisy images by assuming similar noise statistics for images captured with similar settings, or using a more efficient noise estimator.

Currently, our denoisers can be applied to the case of pure Gaussian noise or pure Poisson noise. However, extending the SURE-LET approach to the case of mixed Poisson-Gaussian noise is straightforward – we can add the contribution of Gaussian noise to PURE in order to estimate the MSE in this case. In fact, Li, Luisier, and Blu proposed this estimate of the MSE, named SPURE, which can be used for denoising or deconvolving mixed Poisson-Gaussian noisy images using the approach very similar to SURE-LET and PURE-LET [10]. Combining a mixed Poisson-Gaussian noise level estimator with SPURE and the denoised image optimization approach as used in SURE-LET and PURE-LET may provide a means to denoise a larger range of noisy images which can be modeled as corrupted by mixed Poisson-Gaussian noise.

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