Flutter Shutter Deconvolution under Gaussian and Poisson Noise

Ifueko Igbinedion
Department of Electrical Engineering, Stanford University

Motivation

In a conventionally captured photograph, motion blur destroys high frequency spatial details of moving objects. The flutter shutter camera [5] preserves these high spatial frequencies by fluttering the camera's shutter open and closed during the exposure time. The flutter changes the typical box filter used in image formation to a broad band filter, preserving a wider range of spatial frequencies.

As noise degrades an image, image frequencies are further aliased. Naive deconvolution algorithms depend on this information being intact for accurate reconstruction. Utilizing regularization helps improve the results of this deconvolution, especially under the presence of noise. This project assesses the performance of flutter shutter based deblurring algorithms which utilize total variation regularization under varying Gaussian, Poisson, and mixed Gaussian-Poisson noise.

Related Work

Total variation is a commonly used regularizer to improve results of many ill-posed inverse problems solved with convex optimization. Boyd et. al. argue that the alternating direction method of multipliers (ADMM) algorithm with total-variation (TV) regularization is a well suited algorithm for many of these inverse problems. Kamilov et. al. explore ADMM with an isotropic regularizer and successfully apply it to phase unwrapping within images. Lou et. al. extend this even further by using a weighted difference of anisotropic and isotropic regularizers within their optimization scheme. This project implements an anisotropic variant of ADMM as well as the ADMM algorithm as Lou et. al., which utilizes the difference of l1 and l2 norms of the gradient function. The formulations of both these variants are discussed in the next section.

Techniques and Algorithms

The Gaussian noise term follows a zero mean i.i.d. Gaussian distribution, $\eta = N(0, \sigma^2)$, where σ^2 is the variance of the noise. The Poisson term follows a Poisson distribution, $Pois(\lambda)$, where λ is both the mean and variance. The mixed noise model approximates the Poisson noise term by a signal dependent gaussian distribution $N(\mathbf{Ax}, \mathbf{Ax})$ (Foi et. al.), which can now be considered additive noise such that the total noise term is $\eta = N(\mathbf{Ax},\mathbf{Ax}) + N(0,\sigma^2) = N(\mathbf{Ax}, \mathbf{Ax} + \sigma^2)$. Before deconvolution, noise following these distributions is added to the simulated flutter shutter image.

Two iterative ADMM algorithms for deconvolution of the fluttered image are implemented. The general formulation for image reconstruction is:

$$\text{minimize}_{\{x\}} \ \tfrac{1}{2} \|Cx - b\|_2^2 + \Gamma(x)$$
$$\text{Subject to } \mathbf{Dx} - \mathbf{z} = 0$$

Where C is the convolution matrix, b is the blurred image, $\Gamma(x)$ is the regularization term. In this project, two regularization terms are utilized: A weighted l1 norm, and the weighted difference of l1 and l2 norms:

$$\Gamma_1(x) = \lambda \|\mathbf{Dx}\|_1 \qquad \text{and} \qquad \Gamma_2(x) = \|\mathbf{Dx}\|_1 - \lambda \|\mathbf{Dx}\|_2$$

Γ_1 is the anisotropic regularization term TV while Γ_2 is isotropic. For ADMM in general, the Augmented lagrangian is formed as

$$L_\rho(x, z, y) = f(x) + g(z) + y^T(\mathbf{Dx} - \mathbf{z}) + \rho/2 \ \|\mathbf{Dx} - \mathbf{z}\|_2^2$$

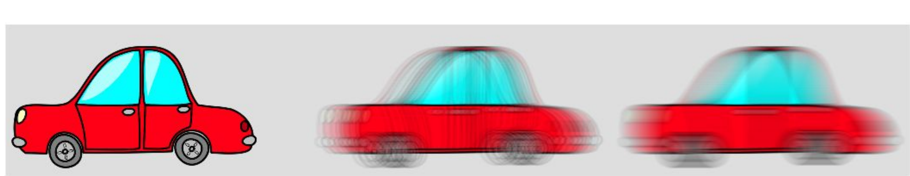
Now the update rules for both regularization schemes can be derived. For the anisotropic case, the update rules derived by Boyd et. al. defines 2 auxiliary variables, z and u:

$$\mathbf{x} \leftarrow \text{prox}_{f,\rho}(v) = \text{argmin}_{\{x\}} L_\rho(x,z,y) = \text{argmin}_{\{x\}} f(\mathbf{x}) + \rho/2 \ \|\mathbf{Dx}\text{-}\mathbf{v}\|_2^2, \ \mathbf{v} = \mathbf{z} - \mathbf{u}$$
$$\mathbf{z} \leftarrow \text{prox}_{g,\rho}(v) = \text{argmin}_{\{z\}} L_\rho(x,z,y) = \text{argmin}_{\{z\}} g(\mathbf{z}) + \rho/2 \ \|\mathbf{v}\text{-}\mathbf{u}\|_2^2, \ \mathbf{v} = \mathbf{Dx} + \mathbf{u}$$
$$\mathbf{u} \leftarrow \mathbf{u} + \mathbf{Dx} + \mathbf{z}$$

For the isotropic case, we follow the method utilized by Lou et. al, based on the difference of convex algorithm. Another auxiliary variable, q, is introduced to linearize the isotropic term. This only changes the z and q updates:

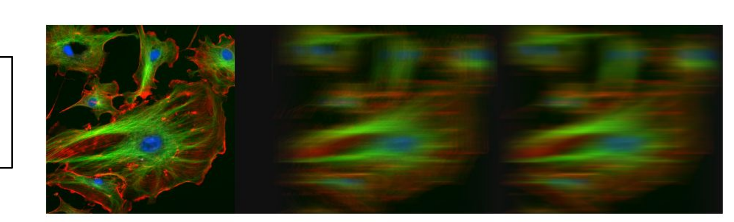
$$\mathbf{z} \leftarrow \text{prox}_{g,\rho}(v) = \text{argmin}_{\{z\}} L_\rho(x,z,y) = \text{argmin}_{\{z\}} g(\mathbf{z}) + \rho/2 \ \|\mathbf{v}\text{-}\mathbf{u}\|_2^2, \ \mathbf{v} = \mathbf{Dx} + \mathbf{u} + \lambda\mathbf{q}$$
$$\mathbf{q} \leftarrow \mathbf{Dx} / \sqrt{|\mathbf{Dx}|^2}$$

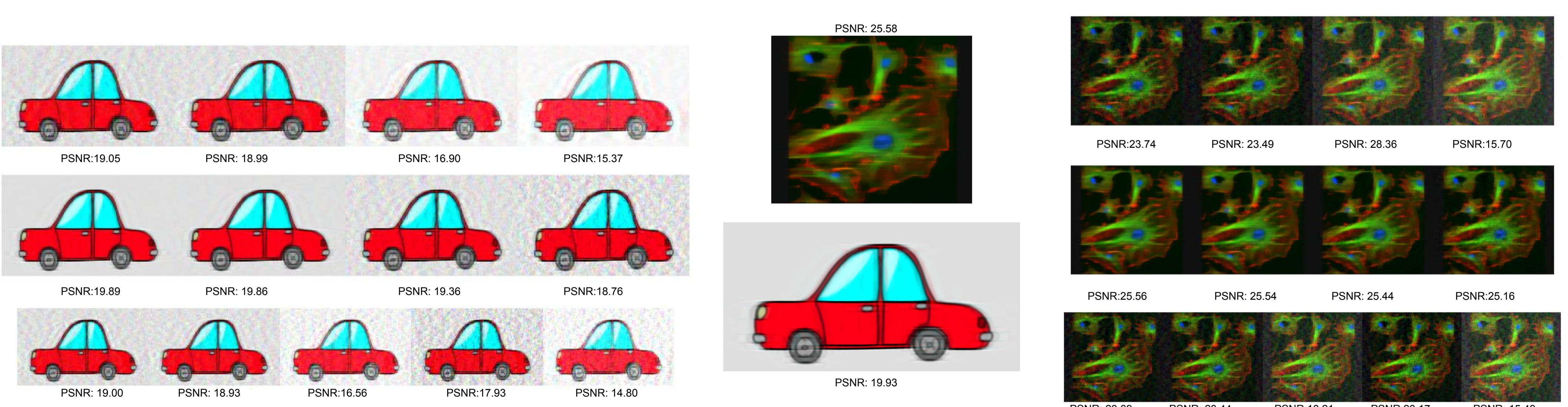
Experimental Results



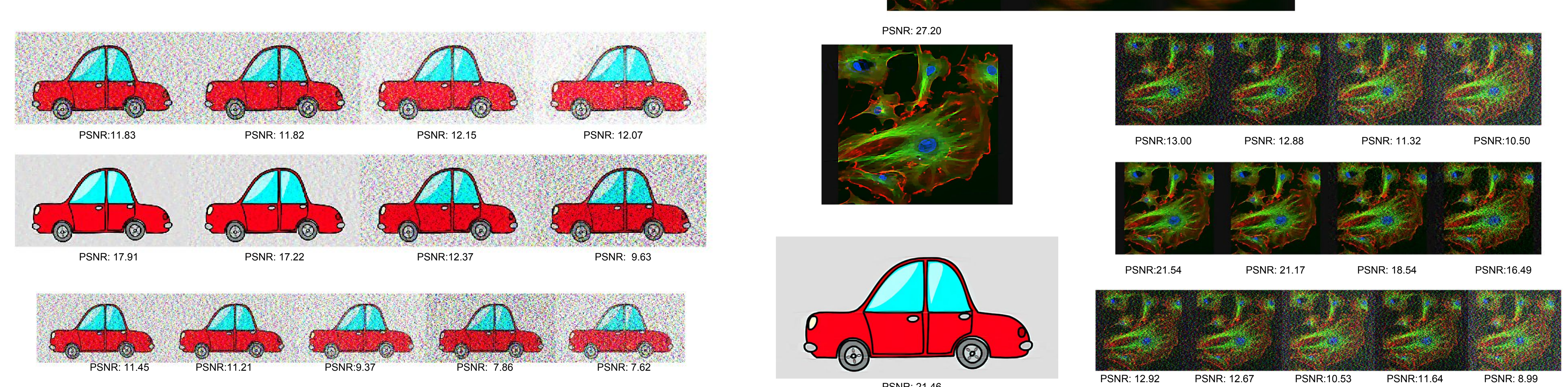
To the left: Stationary image, flutter shutter blurred image, and motion blurred image for the car test image.

To the right: Stationary image, flutter shutter blurred image, and motion blurred image for the cells test image.





Results of ADMM using Γ_1 regularizer for the car. Outer images: Top row: gaussian noise with σ=0.001, 0.01, .1, .15; Middle row: poisson noise with λ=1000,500,50; Bottom row: Mixed gaussian-poisson noise, λ=1000, σ=.001; λ=500, σ=.01; λ=100, σ=.001; λ=50, σ=.001; λ=50, σ=.15; Center images: images reconstructed with no noise.



Results of ADMM using Γ_2 regularizer for the car. Outer images: Top row: gaussian noise with σ=0.001, 0.01, .1, .15; Middle row: poisson noise with λ=1000,500,50; Bottom row: Mixed gaussian-poisson noise, λ=1000, σ=.001; λ=500, σ=.01; λ=100, σ=.001; λ=50, σ=.001; λ=50, σ=.15; Center images: images reconstructed with no noise.

References
[1] Boyd, Stephen, et al. "Distributed optimization and statistical learning via the alternating direction method of multipliers." *Foundations and Trends® in Machine Learning* 3.1 (2011): 1-122. [2] Foi, Alessandro, et al. "Practical Poissonian-Gaussian noise modeling and fitting for single-image raw-data." *Image Processing, IEEE Transactions on* 17.10 (2008): 1737-1754. [3] Kamilov, Ulugbek S., et al. "Isotropic inverse-problem approach for two-dimensional phase unwrapping." *JOSA A* 32.6 (2015): 1092-1100.
[4] Lou, Yifei, et al. "A weighted difference of anisotropic and isotropic total variation model for image processing." *SIAM Journal on Imaging Sciences* 8.3 (2015): 1798-1823. [5] Raskar, Ramesh, Amit Agrawal, and Jack Tumblin. "Coded exposure photography: motion deblurring using fluttered shutter." *ACM Transactions on Graphics (TOG)* 25.3 (2006): 795-804.