Convex Optimization

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12. Conclusions

Modeling

mathematical optimization

- problems in engineering design, data analysis and statistics, economics, management, . . . , can often be expressed as mathematical optimization problems
- techniques exist to take into account multiple objectives or uncertainty in the data

tractability

- roughly speaking, tractability in optimization requires convexity
- algorithms for nonconvex optimization find local (suboptimal) solutions, or are very expensive
- surprisingly many applications can be formulated as convex problems

Theoretical consequences of convexity

- local optima are global
- extensive duality theory
  - systematic way of deriving lower bounds on optimal value
  - necessary and sufficient optimality conditions
  - certificates of infeasibility
  - sensitivity analysis
- solution methods with polynomial worst-case complexity theory (with self-concordance)

Practical consequences of convexity

(most) **convex problems can be solved globally and efficiently**

- interior-point methods require $20 - 80$ steps in practice
- basic algorithms (*e.g.*, Newton, barrier method, ...) are easy to implement and work well for small and medium size problems (larger problems if structure is exploited)
- high-quality solvers (some open-source) are available
- high level modeling tools like `CVXPY` ease modeling and problem specification

How to use convex optimization

to use convex optimization in some applied context

- use rapid prototyping, approximate modeling
  - start with simple models, small problem instances, inefficient solution methods
  - if you don't like the results, no need to expend further effort on more accurate models or efficient algorithms
- work out, simplify, and interpret optimality conditions and dual
- even if the problem is quite nonconvex, you can use convex optimization
  - in subproblems, *e.g.*, to find search direction
  - by repeatedly forming and solving a convex approximation at the current point

Further topics

some topics we didn't cover:

- methods for very large scale problems
- subgradient calculus, convex analysis
- localization, subgradient, proximal and related methods
- distributed convex optimization
- applications that build on or use convex optimization

these are all covered in EE364b.

Related classes

- EE364b — convex optimization II (Pilanci)
- EE364m — mathematics of convexity (Duchi)
- CS261, CME334, MSE213 — theory and algorithm analysis (Sidford)
- AA222 — algorithms for nonconvex optimization (Kochenderfer)
- CME307 — linear and conic optimization (Ye)