Étude Problem

This is one of the étude problems for this winter. Please turn it in by uploading to Gradescope by 5pm on Thursday, with a regraded version by 5pm on Friday. We will not be lenient with upload times: because of the nature of the regrading, we will be posting solutions as close as possible to 5:01pm on Thursdays, so turning the étude in late will result in no credit. Please upload a placeholder to Gradescope well before the due date.

You may use any books, notes, or computer programs, but you may not discuss the étude with anyone—including online—until the solutions are posted. The only exception is that you can ask us for clarification, via the course staff email address. We’ve tried pretty hard to make these questions unambiguous and clear, so we’re unlikely to say much.

Please submit your étude via Gradescope.

We will deduct points from long, needlessly complex solutions, even if they are correct. Our solutions are not long, so if you find that your solution to a problem goes on and on for many pages, you should try to figure out a simpler one. We expect neat, legible exams from everyone, including those enrolled Cr/N.

When a problem involves computation you must give all of the following: a clear discussion and justification of exactly what you did, the source code that produces the result, and the final numerical results or plots.

Files containing problem data can be found in the following location:

http://www.stanford.edu/~jduchi/teaching/364a/data/

Please respect the honor code. Although we allow you to work on homework assignments in small groups, you cannot discuss the études with anyone.

Some études are (quite) straightforward. Others, not so much.

Be sure you are using the most recent version of CVX, CVXPY, CVXR, or Convex.jl.

Some problems involve applications. But you do not need to know anything about the problem area to solve the problem; the problem statement contains everything you need.
1. Identification of convex sets. For each of the following, state whether the indicated set is convex or not. If it is, provide a 2 sentence or less justification (which may consist of simply a citation to the book or lecture notes); if it is not, give a counterexample.

(a) The set
\[ \left\{ X \in S^2 \mid X = \begin{bmatrix} x & y \\ y & z \end{bmatrix}, X \succeq 0 \right\}. \]

(b) The set
\[ \left\{ X \in S^2 \mid X = \begin{bmatrix} x & y \\ y & z \end{bmatrix}, x = z \right\}. \]

(c) The set
\[ \left\{ X \in S^2 \mid X = \begin{bmatrix} x & y \\ y & z \end{bmatrix}, xz = 0 \right\}. \]

(d) The set
\[ \left\{ X \in S^2 \mid X = \begin{bmatrix} x & y \\ y & z \end{bmatrix}, xz \geq 0 \right\}. \]

(e) For a fixed \( b > 0 \), the set
\[ \{ x \in \mathbb{R}^n \mid \|x\|_2 \leq b \}. \]

(f) For a fixed \( b > 0 \), the set
\[ \{ x \in \mathbb{R}^n \mid \|x\|_2 \geq b \}. \]

(g) For a vector \( x \in \mathbb{R}^n \), let \( x[j] \) be the \( j \)th largest component of \( x \), i.e.
\[ x[1] \geq x[2] \geq \cdots \geq x[n]. \]

For a fixed \( k \leq n \), the set
\[ \{ (x, t) \in \mathbb{R}_+^n \times \mathbb{R}_+ \mid x[1] + \cdots + x[k] \leq t \}. \]