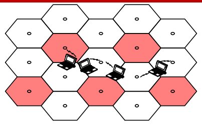
EE360: Lecture 7 Outline
Cellular System Capacity and ASE

- Announcements
  - Summary due next week
- Capacity
- Area Spectral Efficiency
- Dynamic Resource Allocation

Review of Cellular Lecture



- Design considerations:
  - Spectral sharing, reuse, cell size
- Evolution: 1G to 2G to 3G to 4G and beyond
- Multiuser Detection in cellular
- MIMO in Cellular
  - Multiuser MIMO/OFDM
  - Multiplexing/diversity/IC tradeoffs
  - Distributed antenna systems
  - Virtual MIMO

Cellular System Capacity

- Shannon Capacity
  - Shannon capacity does no incorporate reuse distance.
  - Wyner capacity: capacity of a TDMA systems with joint base station processing
- User Capacity
  - Calculates how many users can be supported for a given performance specification.
  - Results highly dependent on traffic, voice activity, and propagation models.
  - Can be improved through interference reduction techniques.
- Area Spectral Efficiency
  - Capacity per unit area

In practice, all techniques have roughly the same capacity for voice, but flexibility of OFDM/MIMO supports more heterogeneous users

Defining Cellular Capacity

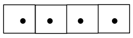
- Shannon-theoretic definition
  - Multiuser channels typically assume user coordination and joint encoding/decoding strategies
  - Can an optimal coding strategy be found, or should one be assumed (i.e. TD,FD, or CD)?
  - What base station(s) should users talk to?
  - What assumptions should be made about base station coordination?
  - Should frequency reuse be fixed or optimized?
  - Is capacity defined by uplink or downlink?
  - Capacity becomes very dependent on propagation model
- Practical capacity definitions (rates or users)
  - Typically assume a fixed set of system parameters
  - Assumptions differ for different systems: comparison hard
  - Does not provide a performance upper bound

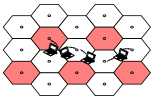
Approaches to Date

- Shannon Capacity
  - TDMA systems with joint base station processing
- Multicell Capacity
  - Rate region per unit area per cell
  - Achievable rates determined via Shannon-theoretic analysis or for practical schemes/constraints
  - Area spectral efficiency is sum of rates per cell
- User Capacity
  - Calculates how many users can be supported for a given performance specification.
  - Results highly dependent on traffic, voice activity, and propagation models.
  - Can be improved through interference reduction techniques. (Gilhousen et. al.)

Wyner Uplink Capacity

- Linear or hexagonal cells





- Received signal at base station (N total users)

$$Y_n = \sum_{k=1}^{K} X_{nk} + \alpha \sum_{n' \in A_n} \sum_{k=1}^{K} X_{n'k} + Z_n,$$

  - Propagation for out-of-cell interference captured by α
  - Average power constraint: $\mathrm{E}(X_{n,k}^2) \leq P$
- Capacity C_N defined as largest achievable rate (N users)

Linear Array

- **Theorem:** $\lim_{N\to\infty} C_N = C^*(\alpha)$

  for

$$C^*(\alpha) \triangleq \frac{1}{2K}\int_0^1 \log\left(1+\frac{(1+2\alpha\cos 2\pi\theta)^2}{\sigma_0^2}\right)d\theta,$$

$$\sigma_0^2 = \sigma^2/KP.$$

Optimal scheme uses TDMA within a cell

- Users transmit in 1/K timeslots; power KP

Treats co-channel signals as interference:

Results

- **Alternate TDMA**

$$C_{TDMA} = \frac{1}{4K}\log\left(1+\frac{2KP}{\sigma^2}\right)$$

- **CDMA w/ MMSE**

$$C_{MMSE} = -\frac{1}{K}\log\left(\int_0^1 \frac{\sigma_0^2}{(1+2\alpha\cos(2\pi\theta))^2+\sigma_0^2}\right)$$

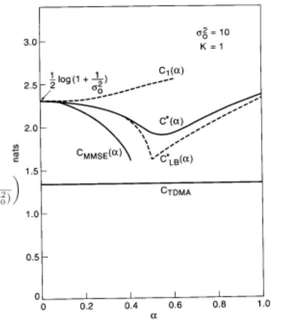


Fig. 1. Plots of $C^*(\alpha)$, $C_1(\alpha)$, C_{TDMA}, $C_{MMSE}(\alpha)$, and $C^*_{LB}(\alpha)$ versus α for $\sigma_0^2 = 0.01$, $K = 1$ (linear array). $C^*_{LB}(\alpha)$ is the lower bound of (2.5).

Channel Reuse in Cellular Systems

- Channel Reuse in Cellular Systems
  - Motivation: power falloff with transmission distance
  - Pro: increase system spectral efficiency
  - Con: co-channel interference (CCI)
  - “Channel”: time slot, frequency band, (semi)-orthogonal code ...

- Cellular Systems with different multiple-access techniques
  - CDMA (IS-95, CDMA2000): weak CCI, channel reuse in every cell
    - codes designed with a single and narrow autocorrelation peak
  - TDMA (GSM), FDMA (AMPS): much stronger CCI
    - a minimum reuse distance required to support target SINR

- Channel reuse: traditionally a fixed system design parameter

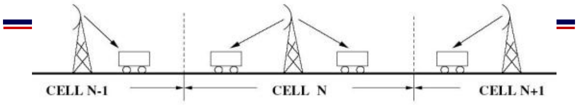
Adaptive Channel Reuse

- Tradeoff
  - Large reuse distance reduces CCI
  - Small reuse distance increases bandwidth allocation

- Related work
  - [Frodigh 92] Propagation model with path-loss only
    channel assignment based on sub-cell compatibility
  - [Horikawa 05] Adaptive guard interval control
    special case of adaptive channel reuse in TDMA systems

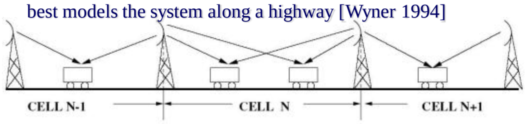
- Current work
  - Propagation models incorporating time variation of wireless channels
    static (AWGN) channel, fast fading and slow fading
  - Channel reuse in cooperative cellular systems (network MIMO)
    compare with single base station processing

System Model



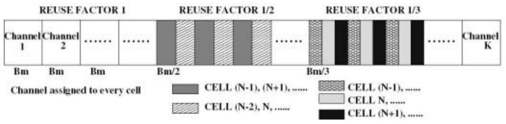
- Linear cellular array, one-dimensional, downlink, single cell processing

  best models the system along a highway [Wyner 1994]



- Full cooperation leads to fundamental performance limit
- More practical scheme: adjacent base station cooperation

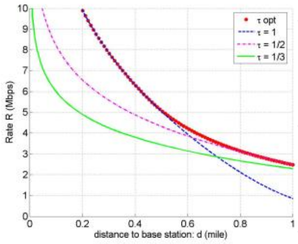
Channel Assignment



- Intra-cell FDMA, K users per cell total bandwidth in the system K·Bm

- Bandwidth allocated to each user
  - maxium bandwidth Bm, corresponding to channel reuse in each cell
  - may opt for a fraction of bandwidth, based on channel strength
    - increased reuse distance, reduced CCI & possibly higher rate

Single Base Station Transmission: AWGN



- Path loss only, receive power

$$P_r(d) = A \cdot P_t \cdot d^{-\gamma}$$

A: path loss at unit distance

γ : path-loss exponent

- Receive SINR $\rho(d,\tau) =$

$$\frac{d^{-\gamma}}{\left(\frac{2L}{\tau}+d\right)^{-\gamma}+\left(\frac{2L}{\tau}-d\right)^{-\gamma}+\frac{N_0\tau}{AP_t}}$$

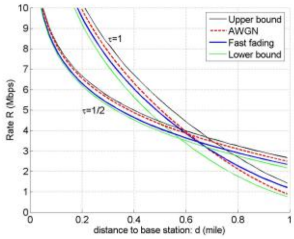
L: cell radius. N₀: noise power

- Optimal reuse factor

$$\arg\max B_m \cdot \tau \log\left[1+\rho(d,\tau)\right]$$

- Observations
  - Mobile close to base station -> strong channel, small reuse distance
  - Reuse factor changes (1 -> ½) at transition distance d_T = 0.62 mile

Rayleigh Fast Fading Channel



- Environment with rich scatters
- Applies if channel coherence time shorte than delay constraint
- Receive power $P_r = A \cdot g \cdot P_t \cdot d^{-\gamma}$

g: exponentially distributed r.v.

- Optimal reuse factor

$$\arg\max B_m \cdot \tau \, \mathsf{E}_g \log\left[1+\rho(d,\tau,g)\right]$$

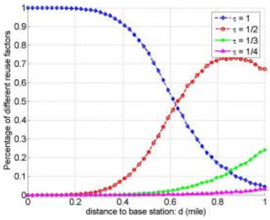
- Lower bound: random signal

Upper bound: random interference

- Observations
  - AWGN and fast fading yield similar performance

    reuse factor changes (1 -> ½) at transition distance d_T = 0.65 mile
  - Both “sandwiched” by same upper/lower bounds (small gap in between)

Rayleigh Slow Fading Channel

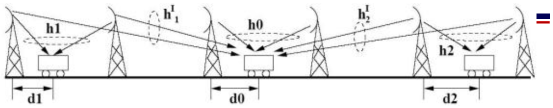


- Stringent delay constraint, entire codeword falls in one fading state
- Optimal reuse factor

$$\arg\max B_m \cdot \tau \log\left[1+\rho(d,\tau,g)\right]$$

- Compare with AWGN/slow fading: optimal reuse factor only depends on distance between mobile and base station
- Observations
  - Optimal reuse factor random at each distance, also depends on fading
  - Larger reuse distance ($1/\tau > 2$) needed when mobiles close to cell edge

Base Station Cooperation: AWGN

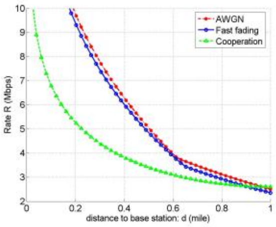


- Adjacent base station cooperation, effectively 2×1 MISO system
- Channel gain vectors: signal interference

$$h_0 = \begin{bmatrix} d_0^{-\gamma/2} \\ (2L-d_0)^{-\gamma/2} \end{bmatrix} \qquad h^I_{1,2} = \begin{bmatrix} \left(\frac{2L}{\tau} \pm d_0\right)^{-\gamma/2} \\ \left(\frac{2L}{\tau} \mp 2L \pm d_0\right)^{-\gamma/2} \end{bmatrix}$$

- Transmitter beamforming $\boldsymbol{w} = \left[w_{(j)}\right] = \left[h_{(j)} / \left|h_{(j)}\right|\right]$
  - optimal for isolated MISO system with per-base power constraint
  - suboptimal when interference present
  - an initial choice to gain insight into system design

Performance Comparison

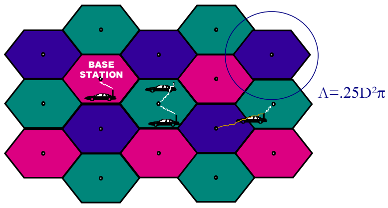


Observations

- no reuse channel in adjacent cell: to avoid base station serving user and interferer at the same time
- reuse factor ½ optimal at all d: suppressing CCI without overly shrinking the bandwidth allocation
- bandwidth reduction (1-> ½) over-shadows benefit from cooperation
- Advantage of cooperation over single cell transmission: only prominent when users share the channel; limited with intra-cell TD/FD [Liang 06]
- Remedy: allow more base stations to cooperate

  in the extreme case of full cooperation, channel reuse in every cell

Area Spectral Efficiency



- **S/I increases with reuse distance.**
- **For BER fixed, tradeoff between reuse distance and link spectral efficiency (bps/Hz).**
- **Area Spectral Efficiency: $A_e = \Sigma R_i / (.25D^2\pi)$ bps/Hz/Km².**

ASE with Adaptive Modulation

- Users adapt their rates (and powers) relative to S/I variation.
- S/I distribution for each user based on propagation and interference models.

$$\gamma_d = S_d / \sum S_i$$

  - Computed for extreme interference conditions.
  - Simulated for average interference conditions.
- The maximum rate R_i for each user in a cell is computed from its S/I distribution.
  - For narrowband system use adaptive MQAM analysis

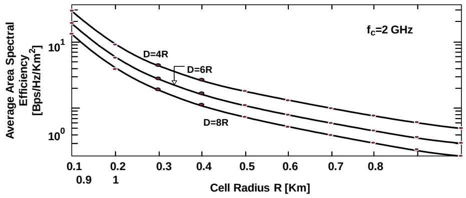
Propagation Model

- Two-slope path loss model:

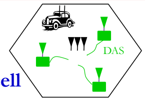
$$\overline{S}_r(d) = \frac{K}{d^a(1+d/g)^b}\overline{S}_t,$$

- Slow fading model: log-normal shadowing
- Fast fading model: Nakagami-m
  - Models Rayleigh and approximates Ricean.
- *ASE maximized with reuse distance of one!*
  - Adaptive modulation compensate for interference

ASE vs. Cell Radius



Distributed Antennas (DAS) in Cellular



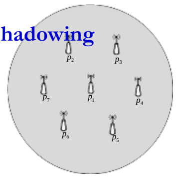
- Basic Premise:
  - Distribute BS antennas throughout cell
    - Rather than just at the center
  - Antennas connect to BS through wireless/wireline links
- Performance benefits
  - Capacity
  - Coverage
  - Power consumption

Average Ergodic Rate

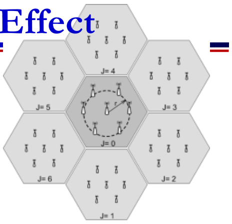
- Assume full CSIT at BS of gains for all antenna ports
- Downlink is a MIMO broadcast channel with full CSIR
- Expected rate is

$$C_{csit}(P) = E_u E_{sh}\left[\log_2\left(1+\overline{S}\left(\sum_{I=1}^{N}\sqrt{\frac{f_i}{D(p_i,u)^\alpha}}\right)^2\right)\right]$$

  - Average over user location and shadowing
- DAS optimization
  - Where to place antennas
  - Goal: maximize ergodic rate



Interference Effect



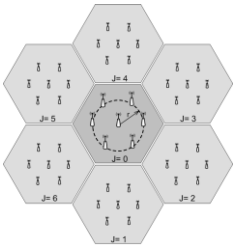
- Impact of intercell interference

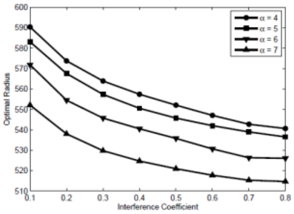
$$SINR = \frac{\sum_{i=1}^{N}\frac{f_i}{D(p_i,u)^\alpha}}{\sum_{j=1}^{6}\sum_{i=1}^{N}\gamma_j\frac{f_i}{D(p_i^j,u)^\alpha}+\sigma^2}$$

γ_j

- is the interference coefficient from cell *j*
  - Autocorrelation of neighboring cell codes for CDMA systems
  - Set to 1 for LTE(OFDM) systems with frequency reuse of one.

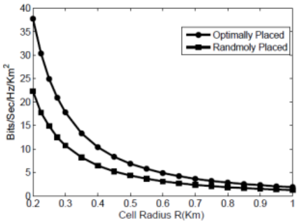
Interference Effect





Area Spectral Efficiency

- Average user rate/unit bandwidth/unit area (bps/Hz/Km^2)
  - Captures effect of cell size on spectral efficiency and interference

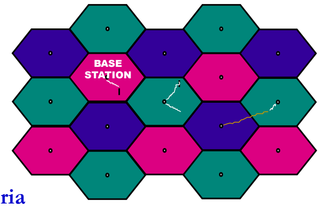


Summary

- Wireless data/multimedia are main drivers for future generations of cellular systems.
  - Killer application unknown; how will cellular users access the Internet; will cellular or WLANs prevail.
- Efficient systems are interference-limited
  - Interference reduction key to high system capacity
- Adaptive techniques in cellular can improve significantly performance and capacity
- MIMO a powerful technique, but impact on out-of-cell interference and implementation unknown.

Dynamic Resource Allocation

Allocate resources as user and network conditions change



- Resources:
  - Channels
  - Bandwidth
  - Power
  - Rate
  - Base stations
  - Access
- Optimization criteria
  - Minimize blocking (voice only systems)
  - Maximize number of users (multiple classes)
  - Maximize "revenue": utility function
    - Subject to some minimum performance for each user

Dynamic Channel Allocation

- Fixed channel assignments are inefficient
  - Channels in unpopulated cells underutilized
  - Handoff calls frequently dropped
- Channel Borrowing
  - A cell may borrow free channels from neighboring cells
  - Changes frequency reuse plan
- Channel Reservations
  - Each cell reserves some channels for handoff calls
  - Increases blocking of new calls, but fewer dropped calls
- Dynamic Channel Allocation
  - Rearrange calls to pack in as many users as possible without violating reuse constraints
  - Very high complexity

"DCA is a 2G/4G problem"

Variable Rate and Power

- Narrowband systems
  - Vary rate and power (and coding)
  - Optimal power control not obvious
- CDMA systems
  - Vary rate and power (and coding)
    - Multiple methods to vary rate (VBR, MC, VC)
  - Optimal power control not obvious
- Optimization criteria
  - Maximize throughput/capacity
  - Meet different user requirements (rate, SIR, delay, etc.)
  - Maximize revenue

Multicarrier CDMA

- Multicarrier CDMA combines OFDM and CDMA
- Idea is to use DSSS to spread a narrowband signal and then send each chip over a different subcarrier
  - DSSS time operations converted to frequency domain
- Greatly reduces complexity of SS system
  - FFT/IFFT replace synchronization and despreading
- More spectrally efficient than CDMA due to the overlapped subcarriers in OFDM
- Multiple users assigned different spreading codes
  - Similar interference properties as in CDMA

Rate and Power Control in CDMA*

- Optimize power and rate adaptation in a CDMA system
  - Goal is to minimize transmit power
- Each user has a required QoS
  - Required effective data rate

*Simultaneous Rate and Power Control in Multirate Multimedia CDMA Systems," S. Kandukuri and S. Boyd

System Model: General

- Single cell CDMA
- Uplink multiple access channel
- Different channel gains
- System supports multiple rates

System Model: Parameters

- Parameters

  N = number of mobiles
  P$_i$ = power transmitted by mobile i
  R$_i$ = raw data rate of mobile i
  W = spread bandwidth
- QoS requirement of mobile i, □$_i$, is the effective data rate

$$\gamma_i = R_i(1 - P_{ei})$$

System Model: Interference

- Interference between users represented by cross correlations between codes, C_{ij}
- Gain of path between mobile i and base station, L_i
- Total interfering effect of mobile j on mobile i, G_{ij} is $G_{ij} = L_i C_{ij}$

SIR Model (neglect noise)

$$SIR_i = \frac{G_{ii} P_i}{\sum_{j \neq i} G_{ij} P_j + \eta}$$

$$\beta_i = \left(\frac{E_b}{I_o}\right)_i = \frac{SIR_i W}{R_i}$$

QoS Formula

- Probability of error is a function of □$_I$
  - Formula depends on the modulation scheme
- Simplified P_e expression

$$P_{ei} = \frac{1}{c\beta_i}$$

- QoS formula

$$\gamma_i = R_i\left(1 - P_e\left(\frac{SIR_i W}{R_i}\right)\right)$$

Solution

- Objective: Minimize sum of mobile powers subject to QoS requirements of all mobiles
- Technique: Geometric programming
  - A non-convex optimization problem is cast as a convex optimization problem
- Convex optimization
  - Objective and constraints are all convex
  - Can obtain a global optimum or a proof that the set of specifications is infeasible
  - Efficient implementation

Problem Formulation

Minimize 1^TP (sum of powers)

Subject to

$$R_i\left(1 - P_e\left(\frac{SIR_i W}{R_i}\right)\right) \geq \gamma_i$$

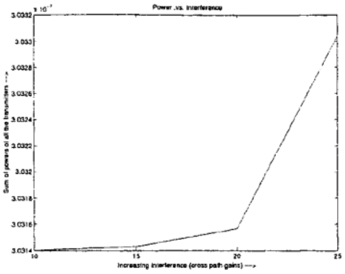
$$R_i \leq R_{thresh}$$

$$P \succ 0$$

Can also add constraints such as

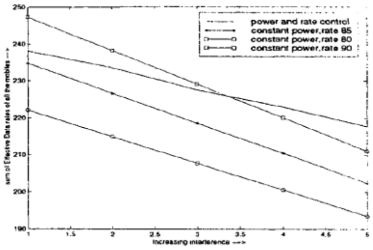
$$P_i \geq P_{\min} \qquad P_i \leq P_{\max}$$

Results



Sum of powers transmitted vs interference

Results



QoS vs. interference