EE360: Multiuser Wireless Systems and Networks

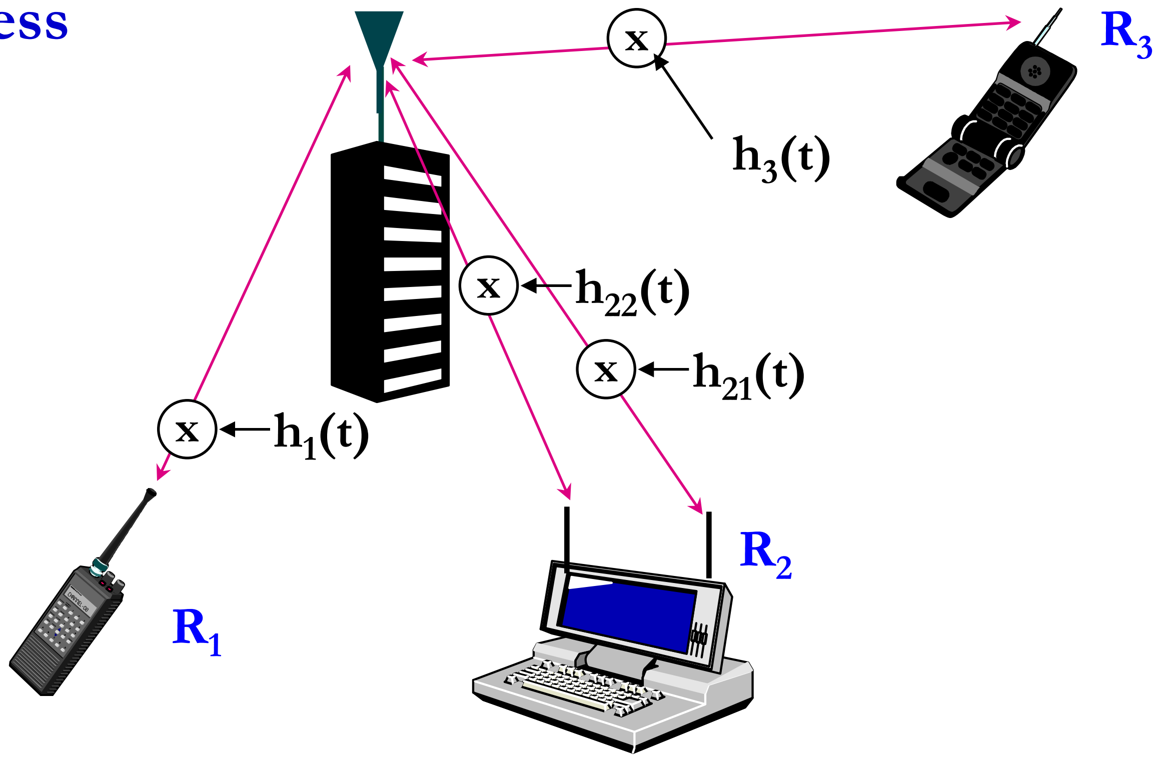
Lecture 2 Outline

- Announcements
  - HW 0 due today
  - Makeup lecture for first class (sorry for confusion):
    - Thurs eve or Friday lunch (w/ food)? Feb 2/3 or Feb 9/10?
- Bandwidth Sharing in Multiuser Channels
  - FD, TD, CD, SD, Hybrid
- Overview of Multiuser Channel Capacity
- Capacity of Broadcast Channels
  - AWGN, Fading, and ISI
- Capacity of MAC Channels
- MIMO Channels

Review of Last Lecture: Uplink and Downlink

Uplink (Multiple Access Channel or MAC):
  Many Transmitters to One Receiver.

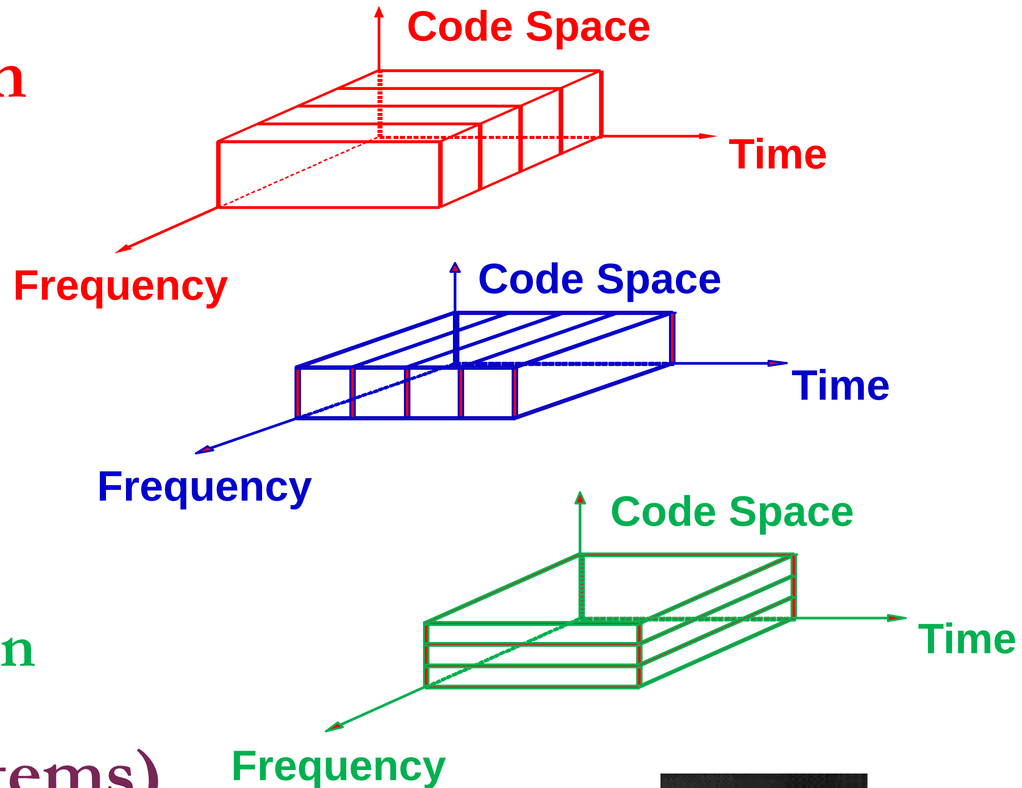
Downlink (Broadcast Channel or BC):
  One Transmitter to Many Receivers.



Uplink and Downlink typically duplexed in time or frequency

Bandwidth Sharing

- Frequency Division
- Time Division
- Code Division
  - Multiuser Detection
- Space (MIMO Systems)
- Hybrid Schemes

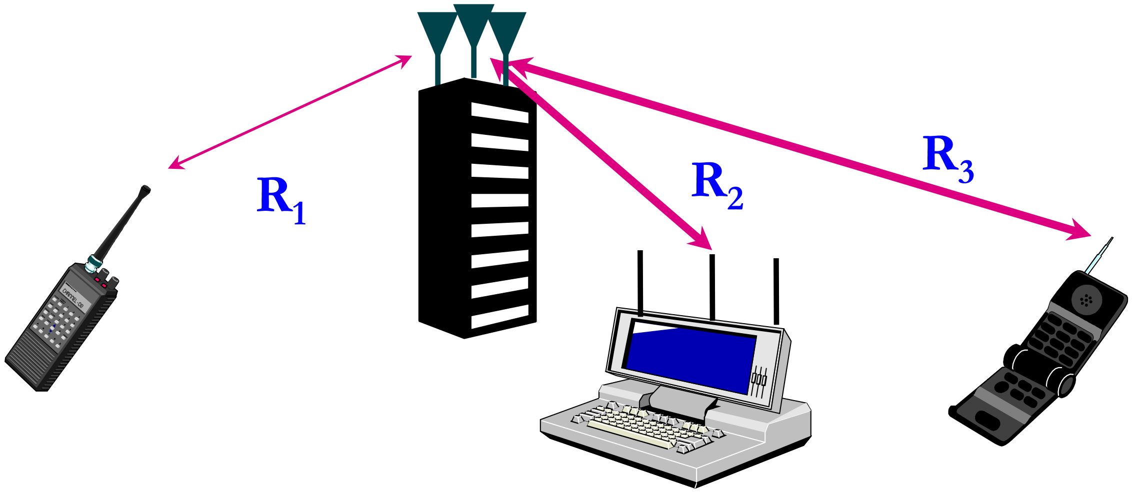


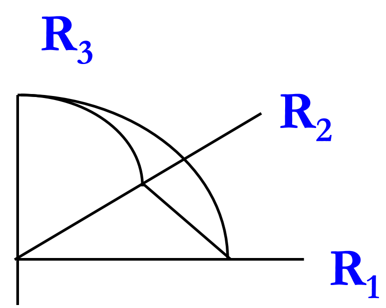
What is optimal? Look to Shannon.

Multiuser Shannon Capacity

Fundamental Limit on Data Rates

Capacity: The set of simultaneously achievable rates $\{R_1,\ldots,R_n\}$ with arbitrarily small probability of error





- Main drivers of channel capacity
  - Bandwidth and received SINR
  - Channel model (fading, ISI)
  - Channel knowledge and how it is used
  - Number of antennas at TX and RX
- Duality connects capacity regions of uplink and downlink

Broadcast Channel Capacity Region in AWGN

- Model
  - One transmitter, two receivers with spectral noise density n_1, n_2: $n_1 < n_2$.
  - Transmitter has average power Pand total bandwidth B.

- Single User Capacity:
  - Maximum achievable rate with asymptotically small P_e

$$C_i = B \log\left[1 + \frac{P}{n_i B}\right]$$

- Set of achievable rates includes $(C_1, 0)$ and $(0, C_2)$, obtained by allocating all resources to one user.

Rate Region: Time Division

- Time Division (Constant Power)
  - Fraction of time τ allocated to each user is varied

$$\{\boldsymbol{U}(\boldsymbol{R}_1 = \tau \boldsymbol{C}_1, \boldsymbol{R}_2 = (1-\tau)\boldsymbol{C}_2); 0 \le \tau \le 1\}$$

- Time Division (Variable Power)
  - Fraction of time τ and power $\boldsymbol{\sigma_i}$ allocated to each user is varied

$$\left\{ \begin{array}{c} \mathbf{U}\left(R_1 = \tau B \log\left[1 + \frac{\sigma_1}{n_1 B}\right], R_2 = (1-\tau) B \log\left[1 + \frac{\sigma_2}{n_2 B}\right]\right); \\ \tau\sigma_1 + (1-\tau)\sigma_2 = P, \qquad 0 \le \tau \le 1. \end{array} \right\}$$

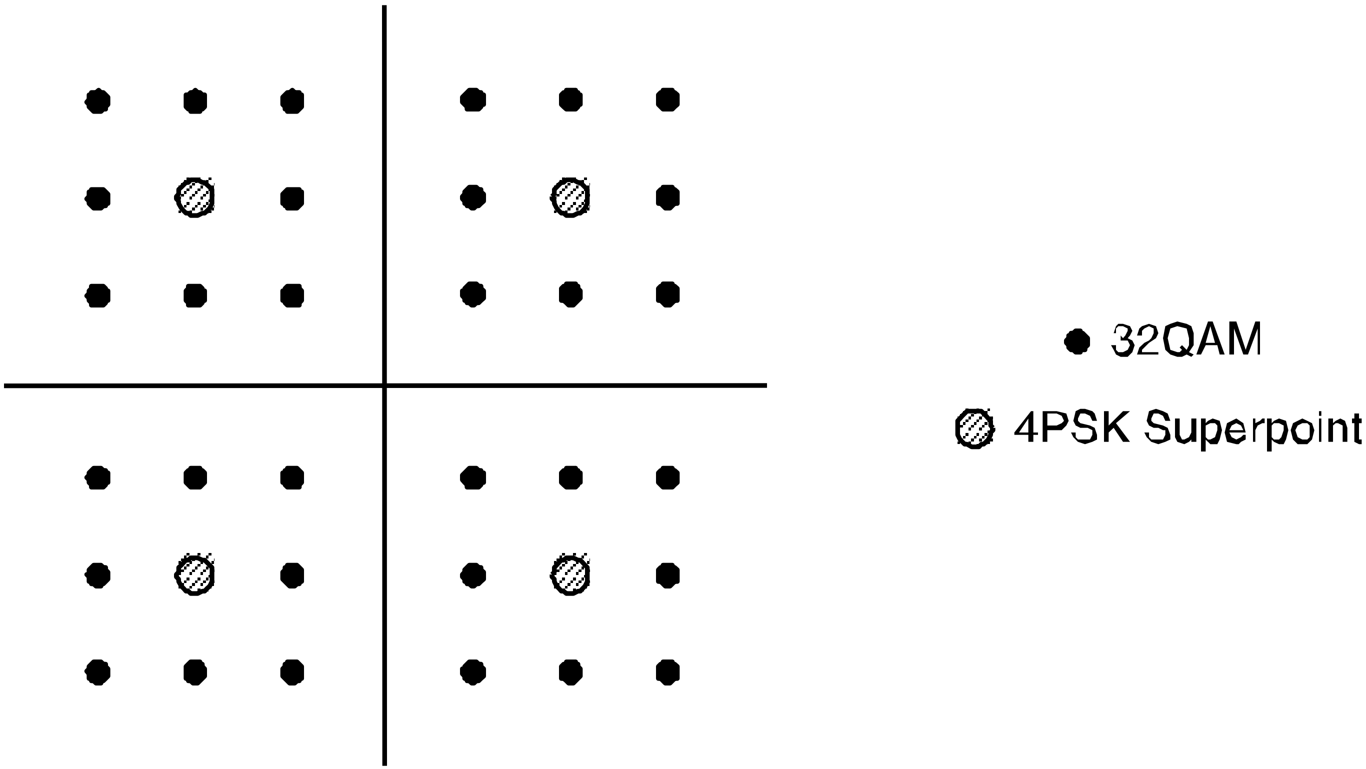
Rate Region: Frequency Division

- **Frequency Division**
  - **Bandwidth B_i and power S_i allocated to each user is varied.**

$$\left\{ \mathbf{U}\left(R_1 = B_1 \log\left[1 + \frac{P_1}{n_1 B_1}\right], R_2 = B_2 \log\left[1 + \frac{P_2}{n_2 B_2}\right]\right); \quad P_1 + P_2 = P, B_1 + B_2 = B \right\}$$

Equivalent to TD for $B_i = \tau_i B$ and $P_i = \tau_i \sigma_i$.

Superposition Coding



Best user decodes fine points

Worse user decodes coarse points

Code Division

- Superposition Coding
  - Coding strategy allows better user to cancel out interference from worse user.

$$\left\{\bigcup\left(R_1=B\log\left[1+\frac{P_1}{n_1B}\right],R_2=B\log\left[1+\frac{P_2}{n_2B+S_1}\right]\right);P_1+P_2=P\right\}$$

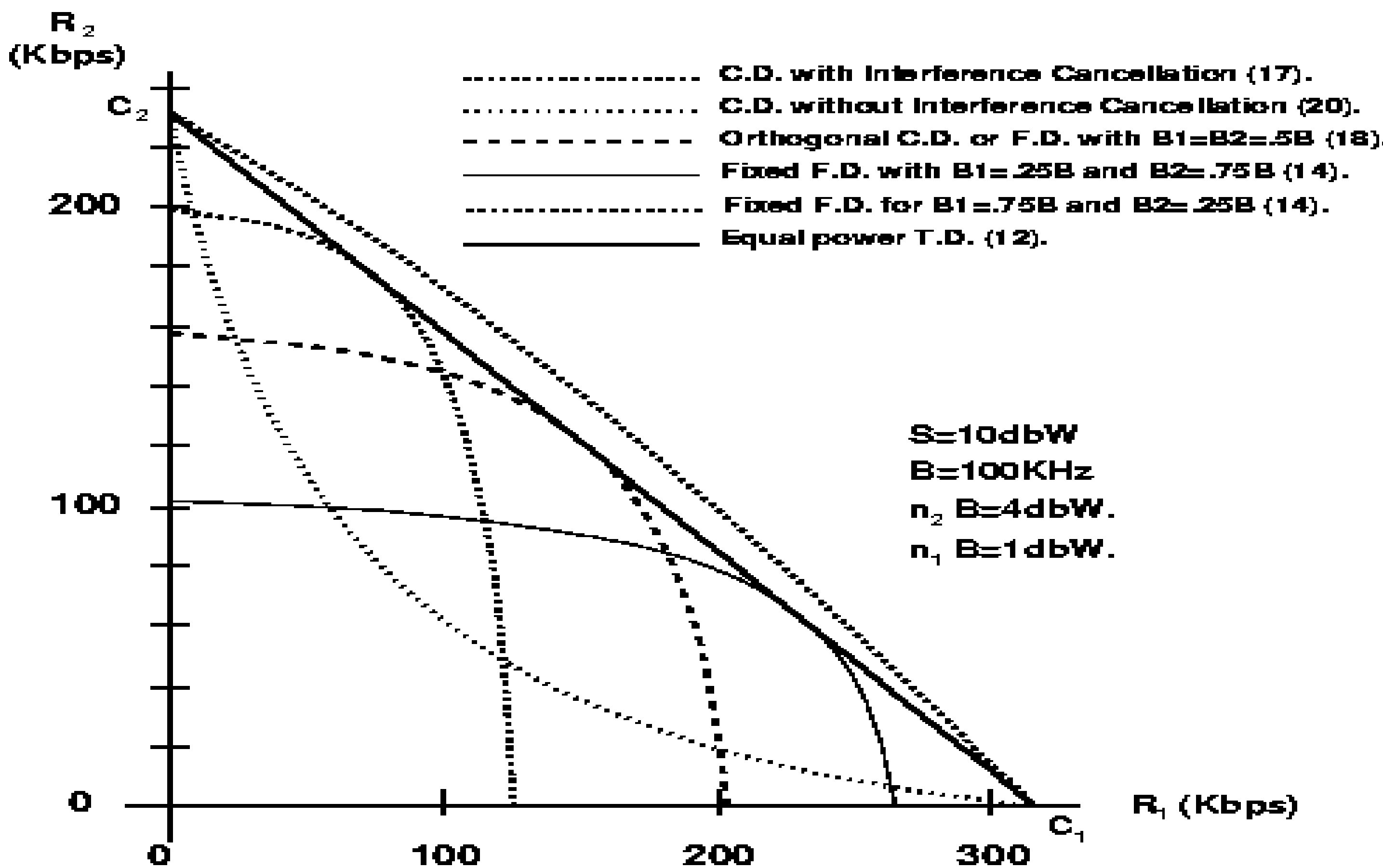
- DS spread spectrum with spreading gain *G* and cross correlation $\rho_{12}=\rho_{21}=G$:

$$\left\{\bigcup\left(R_1=\frac{B}{G}\log\left[1+\frac{P_1}{n_1B/G}\right],R_2=\frac{B}{G}\log\left[1+\frac{P_2}{n_2B/G+S_1/G}\right]\right);P_1+P_2=P\right\}$$

  - By concavity of the log function, G=1 maximizes the rate region.

- DS without interference cancellation

$$\left\{\bigcup\left(R_1=\frac{B}{G}\log\left[1+\frac{P_1}{n_1B/G+P_2/G}\right],R_2=\frac{B}{G}\log\left[1+\frac{P_2}{n_2B/G+P_1/G}\right]\right);P_1+P_2=P\right\}$$

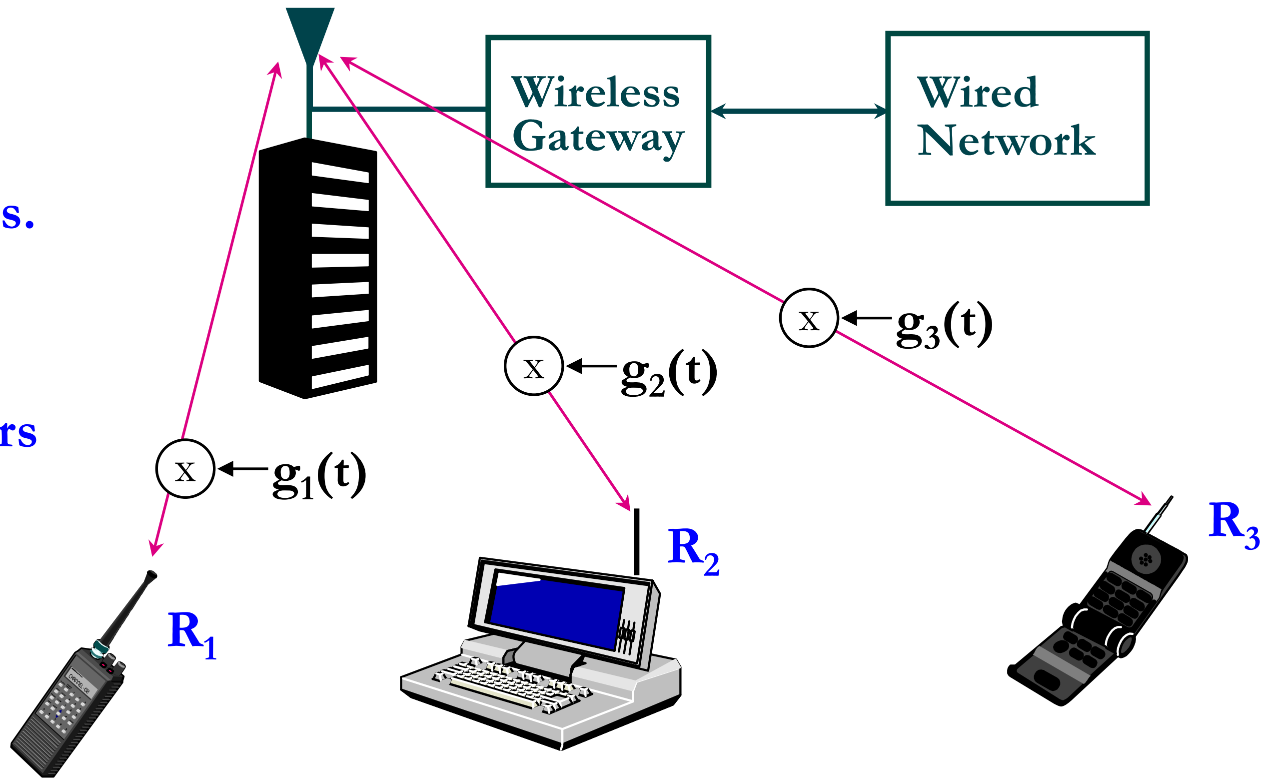


R 2
(Kbps)
C 2
200
100
0
0
100
200
300
R1 (Kbps)
C1
C.D. with Interference Cancellation (17).
C.D. without Interference Cancellation (20).
Orthogonal C.D. or F.D. with B1=B2=.5B (18).
Fixed F.D. with B1=.25B and B2=.75B (14).
Fixed F.D. for B1=.75B and B2=.25B (14).
Equal power T.D. (12).
S=10dbW
B=100KHz
n2 B=4dbW.
n1 B=1dbW.

Broadcast and MAC Fading Channels

Broadcast:
One Transmitter
to Many Receivers.

Multiple Access:
Many Transmitters
to One Receiver.



Goal: Maximize the rate region $\{R_1,\ldots,R_n\}$, subject to some minimum rate constraints, by dynamic allocation of power, rate, and coding/decoding.

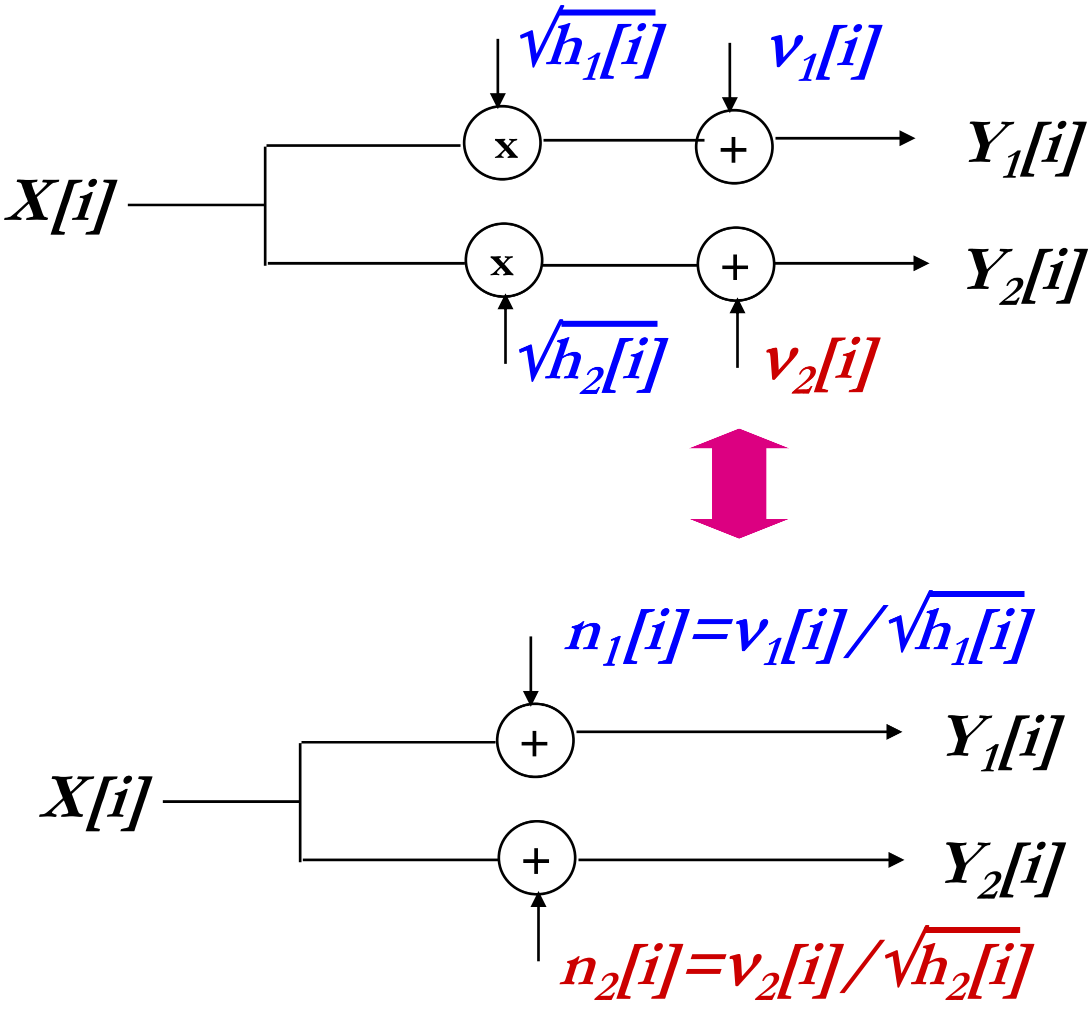
Assume transmit power constraint and perfect TX and RX CSI

Fading Capacity Definitions

- Ergodic (Shannon) capacity: maximum long-term rates averaged over the fading process.
  - Shannon capacity applied directly to fading channels.
  - Delay depends on channel variations.
  - Transmission rate varies with channel quality.
- Zero-outage (delay-limited*) capacity: maximum rate that can be maintained in all fading states.
  - Delay independent of channel variations.
  - Constant transmission rate – much power needed for deep fading.
- Outage capacity: maximum rate that can be maintained in all nonoutage fading states.
  - Constant transmission rate during nonoutage
  - Outage avoids power penalty in deep fades

*Hanly/Tse, IT, 11/98

Two-User Fading Broadcast Channel



At each time i:
$n=\{n_1[i],n_2[i]\}$

Ergodic Capacity Region*

- Capacity region: $C_{ergodic}(\overline{P}) = \bigcup_{\mathcal{P}\in\mathcal{F}} C(\mathcal{P})$, where

$$C(\mathcal{P}) = \left\{ R_j \le E_n \left[B \log\left(1 + \frac{P_j(n)}{n_j B + \sum_{i=1}^{M} P_i(n) 1[n_j > n_i]} \right) \right], \quad 1 \le j \le M \right\}$$

  - The power constraint implies $E_n \sum_{j=1}^{M} P_j(n) = \overline{P}$
- Superposition coding and successive decoding achieve capacity
  - Best user in each state decoded last
  - Power and rate adapted using multiuser water-filling: power allocated based on noise levels and user priorities

*Li/Goldsmith, IT, 3/01

Zero-Outage Capacity Region*

- The set of rate vectors that can be maintained for all channel states under power constraint $\overline{P}$

$$C_{zero}(\overline{P}) = \bigcup_{\mathcal{P}\in\mathcal{F}} \bigcap_{n\in N} C(\mathcal{P})$$

$$C(\mathcal{P}) = \left\{ R_j \le B\log\left(1 + \frac{P_j(n)}{n_j B + \sum_{i=1}^{M} P_i(n) 1[n_j > n_i]}\right), \quad 1 \le j \le M \right\}$$

- Capacity region defined implicitly relative to power:
  - For a given rate vector R and fading state n we find the minimum power $P^{min}(R,n)$ that supports R.
  - $R \in C_{zero}(\overline{P})$ if $E_n[P^{min}(R,n)] \le \overline{P}$

*Li and Goldsmith, IT, 3/01

Outage Capacity Region

- Two different assumptions about outage:
  - All users turned off simultaneously (common outage Pr)
  - Users turned off independently (outage probability vector Pr)
- Outage capacity region implicitly defined from the minimum outage probability associated with a given rate
- Common outage: given (R,n), use threshold policy
  - If $P^{min}(R,n) > s^*$ declare an outage, otherwise assign this power to state n.
  - Power constraint dictates s^* : $\bar{P} = E_{n:P^{\min}(R,n)\le s^*}\left[P^{\min}(R,n)\right]$
  - Outage probability: $Pr = \int_{n:P^{\min}(R,n) > s^*} p(n)$

Independent Outage

- With independent outage cannot use the threshold approach:
  - Any subset of users can be active in each fading state.
- Power allocation must determine how much power to allocate to each state and which users are on in that state.
- Optimal power allocation maximizes the reward for transmitting to a given subset of users for each fading state
  - Reward based on user priorities and outage probabilities.
  - An iterative technique is used to maximize this reward.
  - Solution is a generalized threshold-decision rule.

Minimum-Rate Capacity Region

- Combines ergodic and zero-outage capacity:
  - Minimum rate vector maintained in all fading states.
  - Average rate in excess of the minimum is maximized.
- Delay-constrained data transmitted at the minimum rate at all times.
- Channel variation exploited by transmitting other data at the maximum excess average rate.

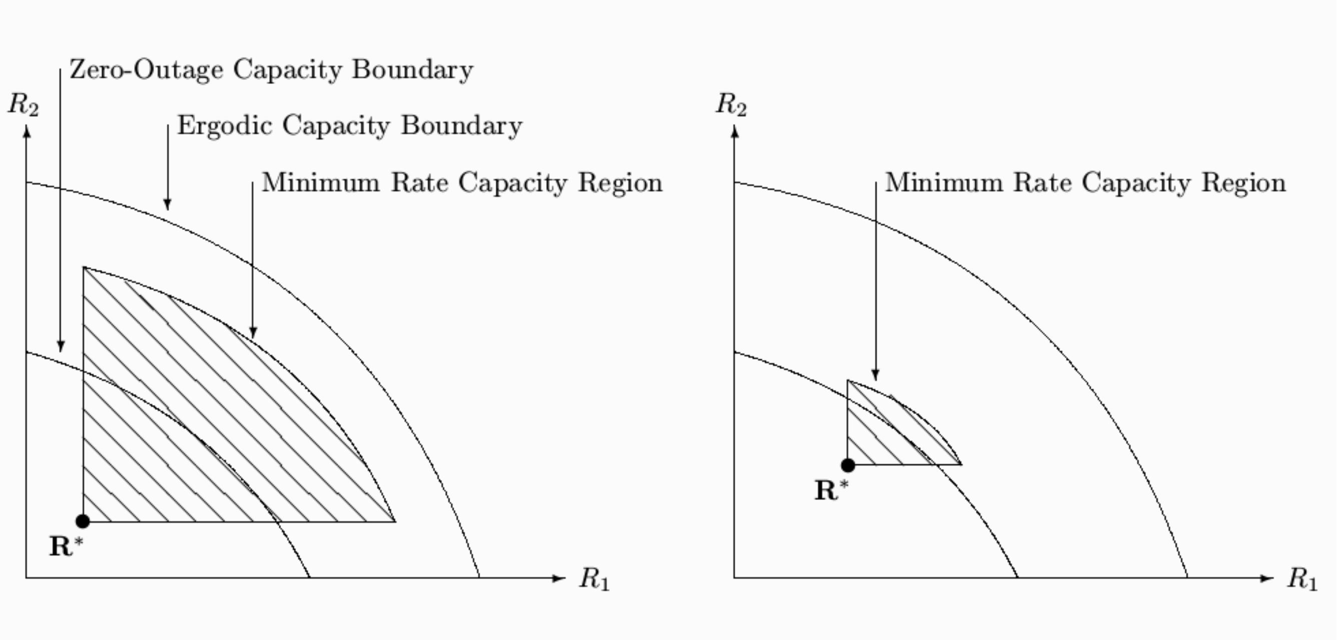
Minimum Rate Constraints

- Define minimum rates $R^* = (R^*_1,\ldots,R^*_M)$:
  - These rates must be maintained in all fading states.
- For a given channel state n:

$$R_j(n) \le B\log\left(1+\frac{P_j(n)}{n_jB+\sum_{i=1}^{M}P_i(n)1[n_j>n_i]}\right), \quad R_j(n)\ge R_j^*\ \forall n$$

- R^* must be in zero-outage capacity region
  - Allocate excess power to maximize excess ergodic rate
  - The smaller R^*, the bigger the min-rate capacity region

Comparison of Capacity Regions



- For R^* far from C_{zero} boundary, $C_{min\text{-}rate} \approx C_{ergodic}$
- For R^* close to C_{zero} boundary, $C_{min\text{-}rate} \approx C_{zero} \cap R^*$

Optimal Coding and Power Allocation

- Superposition coding with SIC in usual order (best user decoded last) is optimal.

- Power allocation broken down into two steps:
  - First allocate the minimum power needed to achieve the minimum rates in all fading states.
  - Then optimally allocate the excess power to maximize the excess ergodic rate.
  - Power allocation between users: insights
    - Excess power given to better user impacts interference of worse user but not vice versa
    - Excess power given to better user results in a higher rate increase
    - Power allocation depends on channel state and user priorities

Minimum Rates for Single-User Channels

- Maximize excess ergodic rate:

$$\max E[\log(1+\tfrac{P(n)}{n})] \quad s.t.\ E[P(n)] \le P,\ \ R(n) \ge R^* \ \forall n$$

- Power required to achieve R^* in state n:

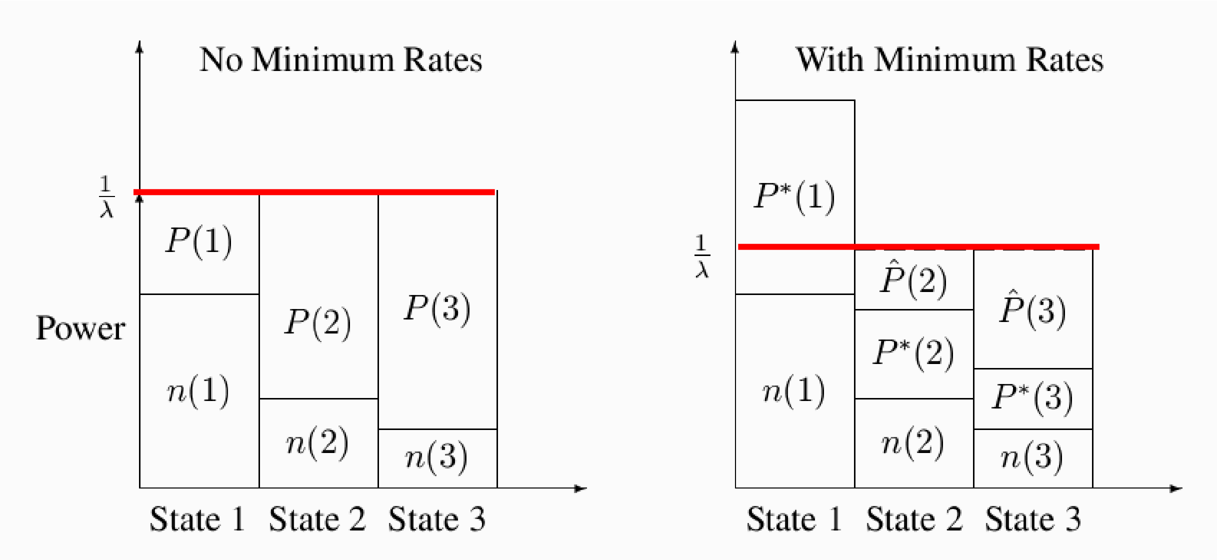
$$\boldsymbol{P}^*(\boldsymbol{n}) = \boldsymbol{n}(\boldsymbol{e}^{\boldsymbol{R}^*} - 1)$$

- Optimal excess power allocation: $P(n)=P^*(n)+\hat{P}(n)$

$$\hat{\boldsymbol{P}}(\boldsymbol{n}) = \begin{cases} \frac{1}{\lambda} - (\boldsymbol{n} + \boldsymbol{P}^*(\boldsymbol{n})) & \boldsymbol{n} + \boldsymbol{P}^*(\boldsymbol{n}) \le \frac{1}{\lambda} \\ 0 & \boldsymbol{else} \end{cases}$$

Waterfilling to modified noise

Water-filling to Modified Noise for SU Channel



- Without no minimum rate all 3 states are allocated power.
- With a minimum rate the noise level in state i increases by $P^*(i)$
  - Only the two best states are allocated excess power.

Two-User Broadcast Channel with Minimum Rates

- Min-rate capacity region boundary defined by:

$$\max_{P(n)} E_n[\mu_1 R_1(n) + \mu_2 R_2(n)] \quad s.t.$$

$$E_n[P_1(n) + P_2(n)] \le P, \quad R_i(n) \ge R_i^* \ \forall n$$

- Minimum power required in state n ($n_2 > n_1$):

$$P_1^* = n_1(e^{R_1^*} - 1), \quad P_2^* = (P_1^* + n_2)(e^{R_2^*} - 1)$$

- Total excess power to allocate over all states

$$\hat{P} = P - E_n[P_1^*(n) + P_2^*(n)]$$

Modified Problem

- Optimize relative to excess power ($n_2 > n_1$):

$$\max_{P(n)} E_n\left[\mu_1 \log\left(1+\frac{\hat{P}_1(n)+P_1^*(n)}{n_1}\right)+\mu_2 \log\left(1+\frac{\hat{P}(n)-\hat{P}_1(n)+P_2^*(n)}{n_2+\hat{P}_1(n)+P_1^*(n)}\right)\right] \quad s.t.$$

$$E_n[\hat{P}(n)] \le \hat{P}, \quad 0 \le \hat{P}_1(n) \le \hat{P}(n) e^{-R_2^*} \qquad \forall n$$

- Excess power allocation:
  - Optimize excess power $\hat{P}(n)$ allocated to state n
  - Divide $\hat{P}(n)=\hat{P}_1(n)+\hat{P}_2(n)$ between the two users
  - Solved via two dimensional Lagrangian or greedy algorithm

Total Excess Power Allocation

- Optimal allocation of *excess* power to state n is a multilevel water-filling:

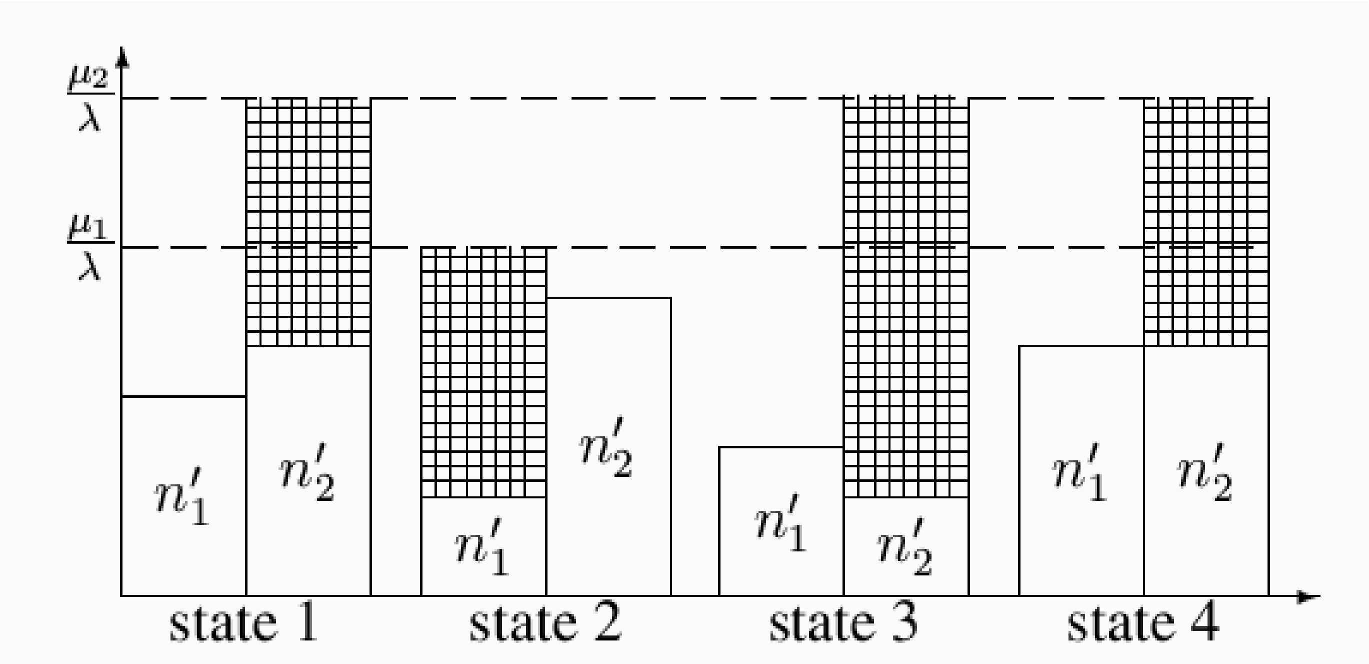
$$\hat{P}(\mathbf{n}) = \max\left(\frac{\mu_1}{\lambda} - n_1',\ \frac{\mu_2}{\lambda} - n_2',\ 0\right)$$

where n_1' and n_2' are effective noises:

$$\begin{cases} n_1' = (P_1^*(\mathbf{n}) + n_1)e^{R_2^*},\ n_2' = (P_1^*(\mathbf{n}) + n_2)e^{R_2^*} & n_1 < n_2 \\ n_1' = (P_2^*(\mathbf{n}) + n_1)e^{R_1^*},\ n_2' = (P_2^*(\mathbf{n}) + n_2)e^{R_1^*} & n_1 \geq n_2 \end{cases}$$

and the water-level λ satisfies the power constraint

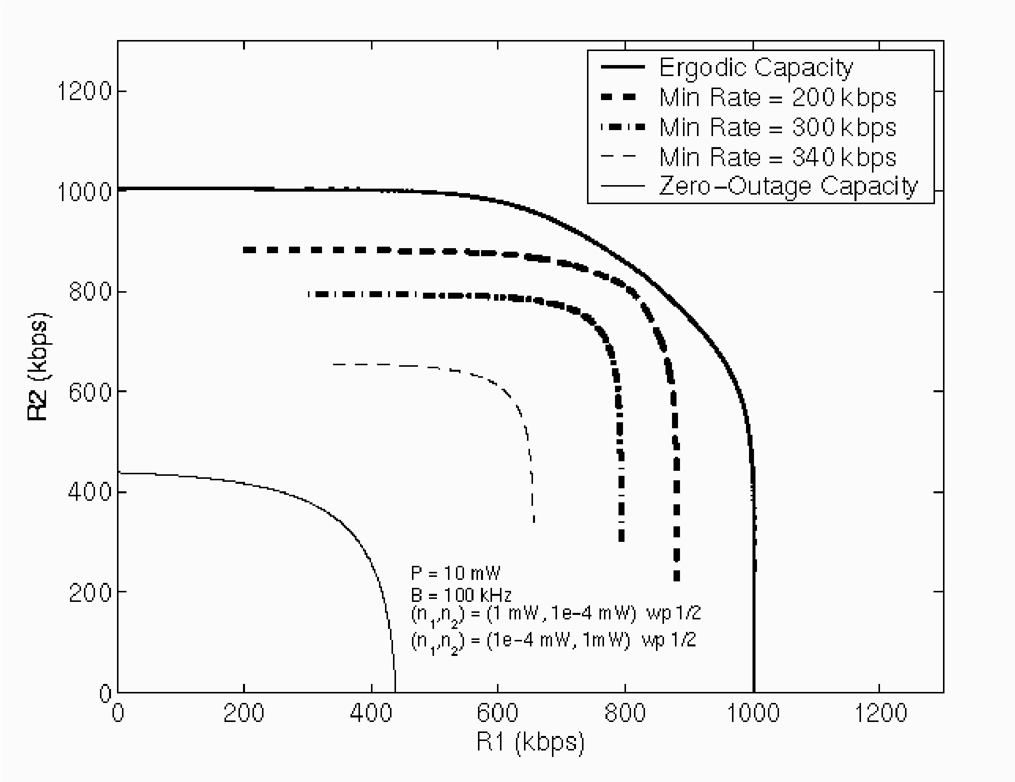
Multi-User Water-filling



- Identical to the optimal power allocation scheme for ergodic capacity with modified noise and power constraint.
- Once $\hat{P}(n)$ known, division between users straightforward.
  - Depends on user priorities and effective noises

Min-Rate Capacity Region: Large Deviation in User Channels

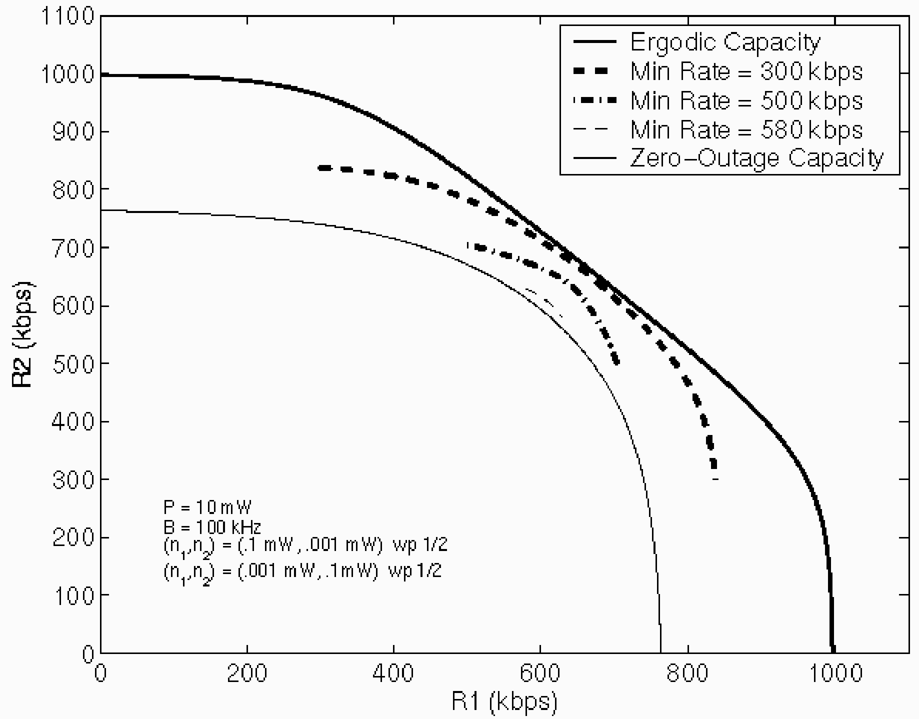
P = 10 mW,
B = 100 KHz



Symmetric channel with 40 dB difference in noises in each fading state (user 1 is 40 dB stronger in 1 state, and vice versa).

Min-Rate Capacity Region: Smaller Deviation

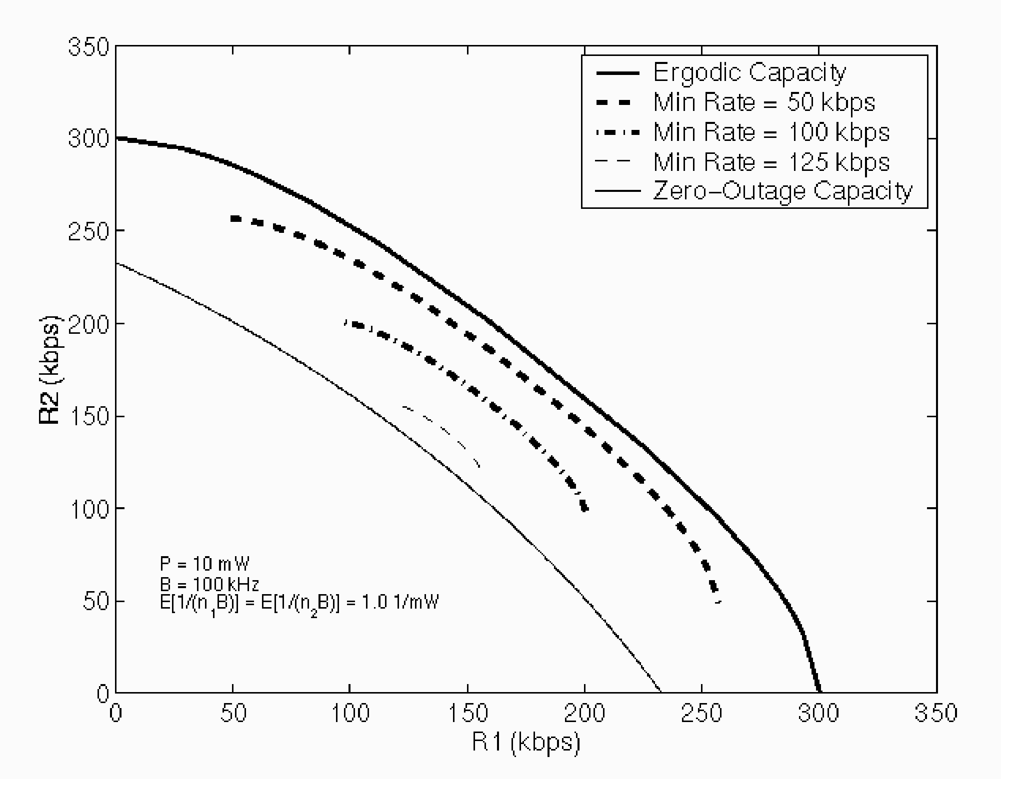
P = 10 mW,
B = 100 KHz



Symmetric channel with 20 dB difference in noises in each fading state (user 1 is 20 dB stronger in 1 state, and vice versa).

Min-Rate Capacity Region: Severe Rician Fading

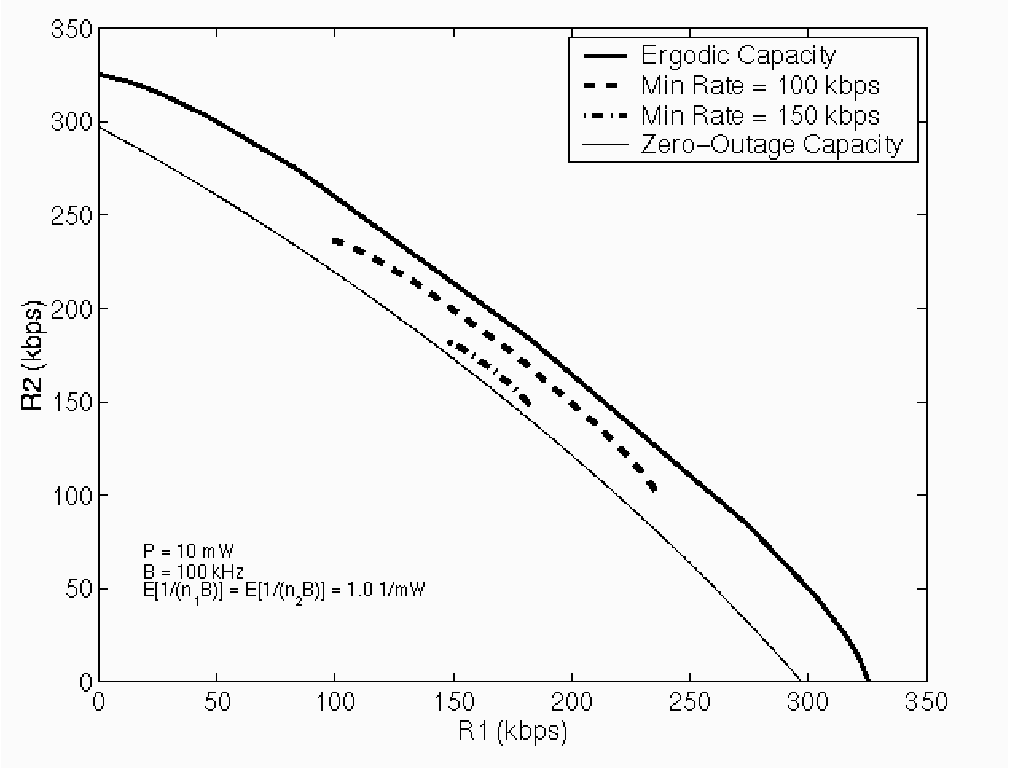
P = 10 mW,
B = 100 KHz



Independent Rician fading with K=1 for both users (severe fading, but not as bad as Rayleigh).

Min-Rate Capacity Region: Mild Rician Fading

P = 10 mW,
B = 100 KHz

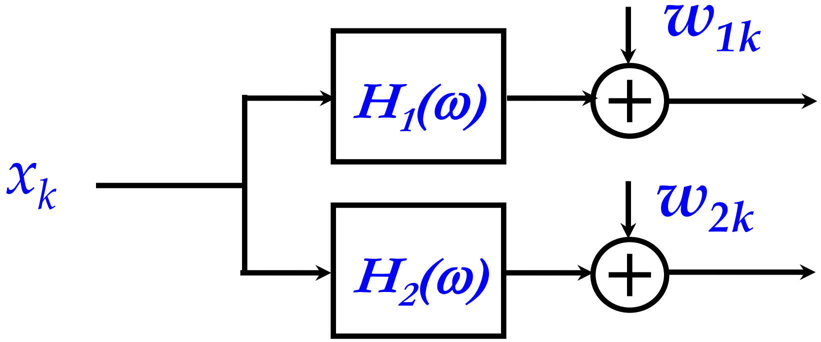


Independent Rician fading with K=5 for both users.

Broadcast Channels with ISI

- ISI introduces memory into the channel
- The optimal coding strategy decomposes the channel into parallel broadcast channels
  - Superposition coding is applied to each subchannel.
- Power must be optimized across subchannels and between users in each subchannel.

Broadcast Channel Model



$$y_{1k} = \sum_{i=1}^{m} h_{1i} x_{k-i} + w_{1k}$$

$$y_{2k} = \sum_{i=1}^{m} h_{2i} x_{k-i} + w_{2k}$$

- **Both H_1 and H_2 are finite IR filters of length m.**
- **The w_{1k} and w_{2k} are correlated noise samples.**
- **For $1<k\leq n$, we call this channel the n-block discrete Gaussian broadcast channel (n-DGBC).**
- **The channel capacity region is C=(R_1,R_2).**

Circular Channel Model

- **Define the zero padded filters as:**

$$\{\tilde{h}_i\}_{i=1}^{n} = (h_1, \ldots, h_m, 0, \ldots, 0)$$

- **The n-Block Circular Gaussian Broadcast Channel (n-CGBC) is defined based on circular convolution:**

$$\tilde{y}_{1k} = \sum_{i=0}^{n-1} \tilde{h}_{1i} x_{((k-i))} + w_{1k} = x_i \otimes h_{1i} + w_{1k}$$

$0 \le k < n$

$$\tilde{y}_{2k} = \sum_{i=0}^{n-1} \tilde{h}_{2i} x_{((k-i))} + w_{2k} = x_i \otimes h_{2i} + w_{2k}$$

where $((\cdot))$ denotes addition modulo n.

Equivalent Channel Model

- **Taking DFTs of both sides yields**

$$\tilde{Y}_{1j} = \tilde{H}_{1j} X_j + W_{1j}$$

$$\tilde{Y}_{2j} = \tilde{H}_{2j} X_j + W_{2j}$$

0≤j<n

- **Dividing by $\tilde{H}$ and using additional properties of the DFT yields**

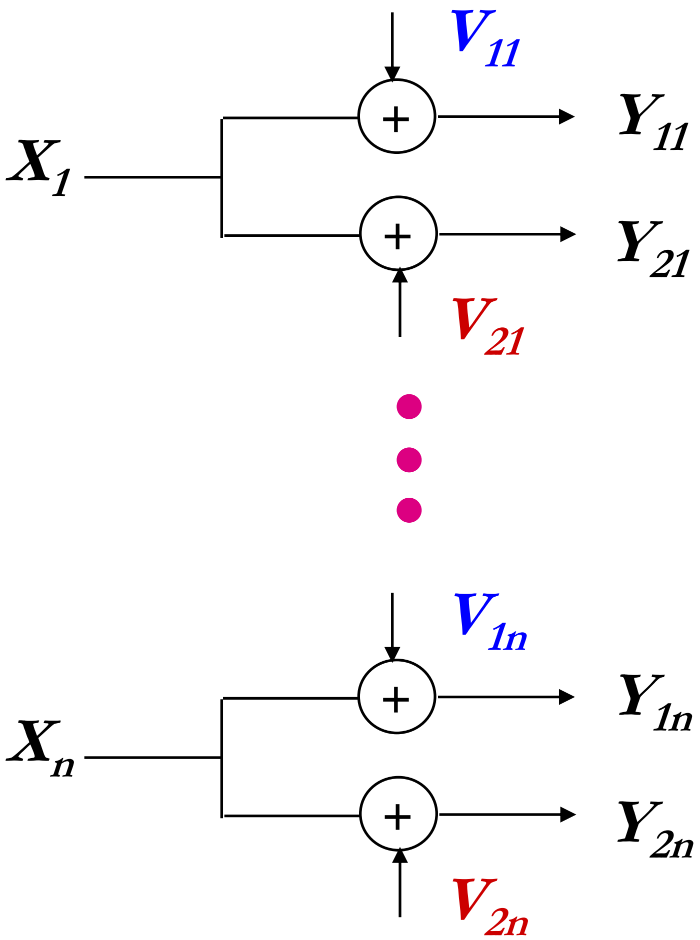
$$Y'_{1j} = X'_j + V'_{1j}$$

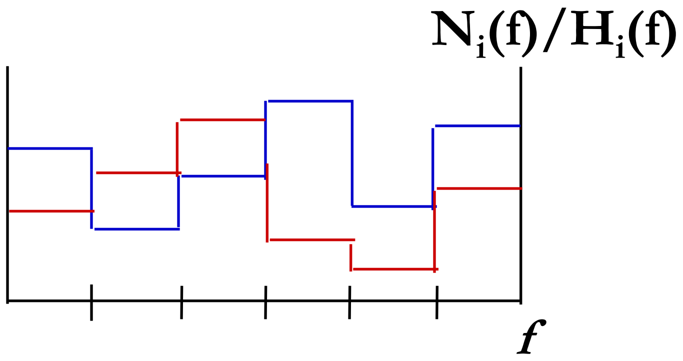
$$Y'_{2j} = X'_j + V'_{2j}$$

0≤j<n

where {V'_{1j}} and {V'_{2j}} are independent zero-mean Gaussian random variables with $\sigma_{lj}^2 = n(N_l(2\pi j/n)/|\tilde{H}_{lj}|^2, l = 1,2.$

Parallel Channel Model





Channel Decomposition

- **The n-CGBC thus decomposes to a set of n parallel discrete memoryless degraded broadcast channels with AWGN.**
  - **Can show that as n goes to infinity, the circular and original channel have the same capacity region**
- **The capacity region of parallel degraded broadcast channels was obtained by El-Gamal (1980)**
  - **Optimal power allocation obtained by Hughes-Hartogs('75).**
- **The power constraint** $\sum_{i=0}^{n-1} E[x_i^2] \leq nP$ **on the original channel is converted by Parseval's theorem to** $\sum_{i=0}^{n-1} E[(X_i')^2] \leq n^2 P$ **on the equivalent channel.**

Capacity Region of Parallel Set

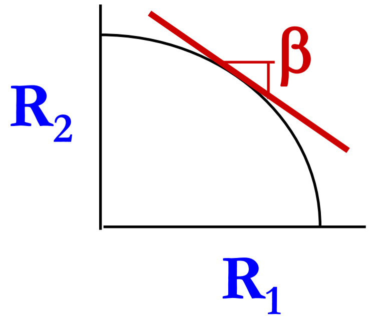
- **Achievable Rates (no common information)**

$$\Big\{R_1 \le .5 \sum_{j:\sigma_{1j}<\sigma_{2j}} \log\left(1+\frac{\alpha_j P_j}{\sigma_{1j}}\right) + .5 \sum_{j:\sigma_{1j}\ge\sigma_{2j}} \log\left(1+\frac{\alpha_j P_j}{(1-\alpha_j)P_j+\sigma_{1j}}\right),$$

$$R_2 \le .5 \sum_{j:\sigma_{1j}<\sigma_{2j}} \log\left(1+\frac{(1-\alpha_j)P_j}{\alpha_j P_j+\sigma_{2j}}\right) + .5 \sum_{j:\sigma_{1j}\ge\sigma_{2j}} \log\left(1+\frac{(1-\alpha_j)P_j}{\sigma_{2j}}\right),$$

$$0 \le \alpha_j \le 1, \sum P_j \le n^2 P\Big\}$$

- **Capacity Region**
  - For $0<\beta\le\infty$ find $\{\alpha_j\}$, $\{P_j\}$ to maximize $R_1+\beta R_2+\lambda \sum P_j$.
  - Let $(R_1^*,R_2^*)_{n,\beta}$ denote the corresponding rate pair.
  - $\mathcal{C}_n=\{(R_1^*,R_2^*)_{n,\beta} : 0<\beta \le\infty \}$, $\mathcal{C}=\text{liminf}_n \frac{1}{n}\, \mathcal{C}_n$.



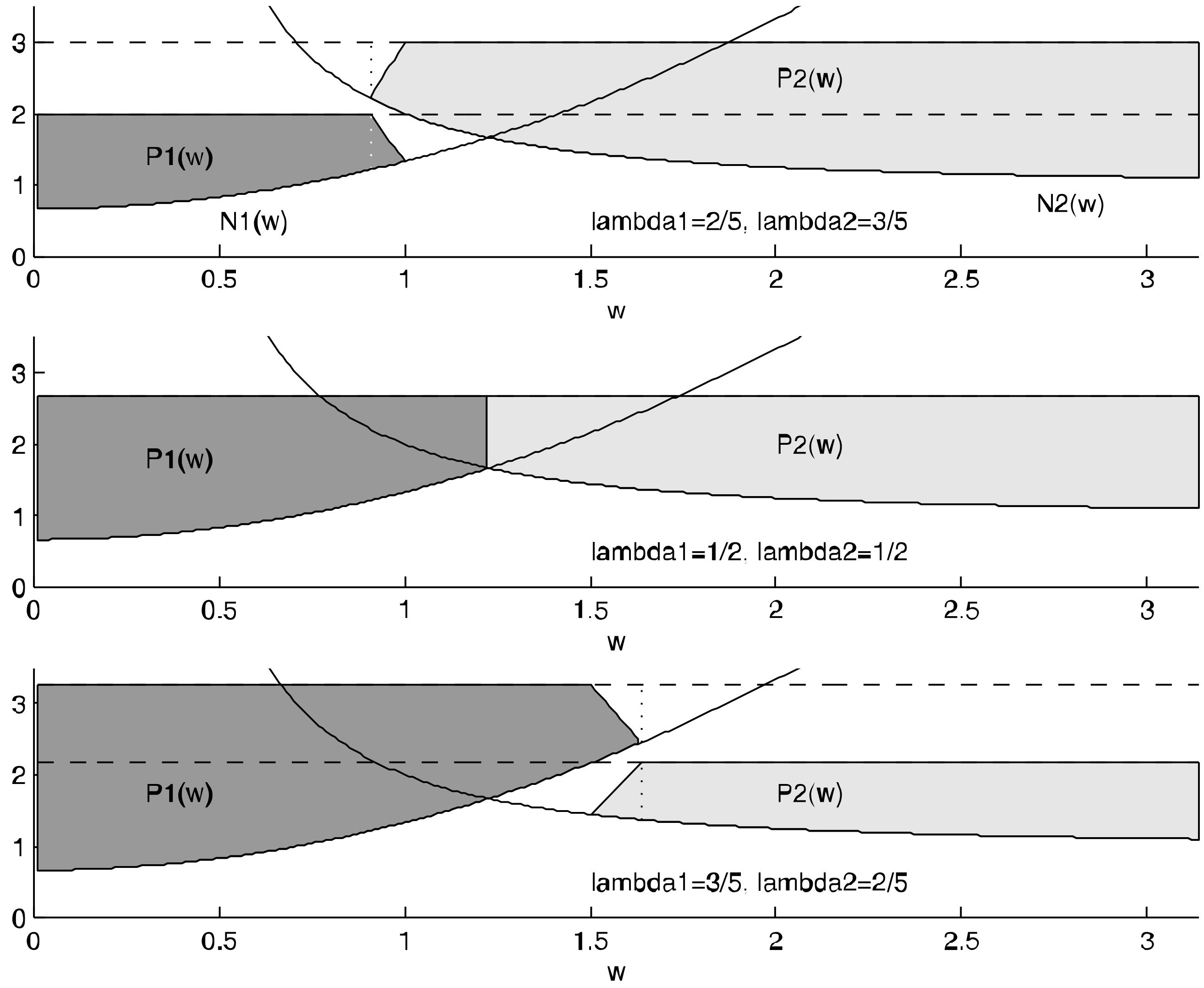
Limiting Capacity Region

$$\Big\{ \boldsymbol{R}_1 \le .5 \int_{f:\boldsymbol{H}_1(f) > \boldsymbol{H}_2(f)} \log\left(1 + \frac{\alpha(f)\boldsymbol{P}(f)\,|\,\boldsymbol{H}_1(f)\,|^2}{.5\boldsymbol{N}_0}\right) + .5 \int_{f:\boldsymbol{H}_1(f) \le \boldsymbol{H}_2(f)} \log\left(1 + \frac{\alpha_j \boldsymbol{P}_j}{(1-\alpha_j)\boldsymbol{P}_j + \sigma_{1j}}\right),$$

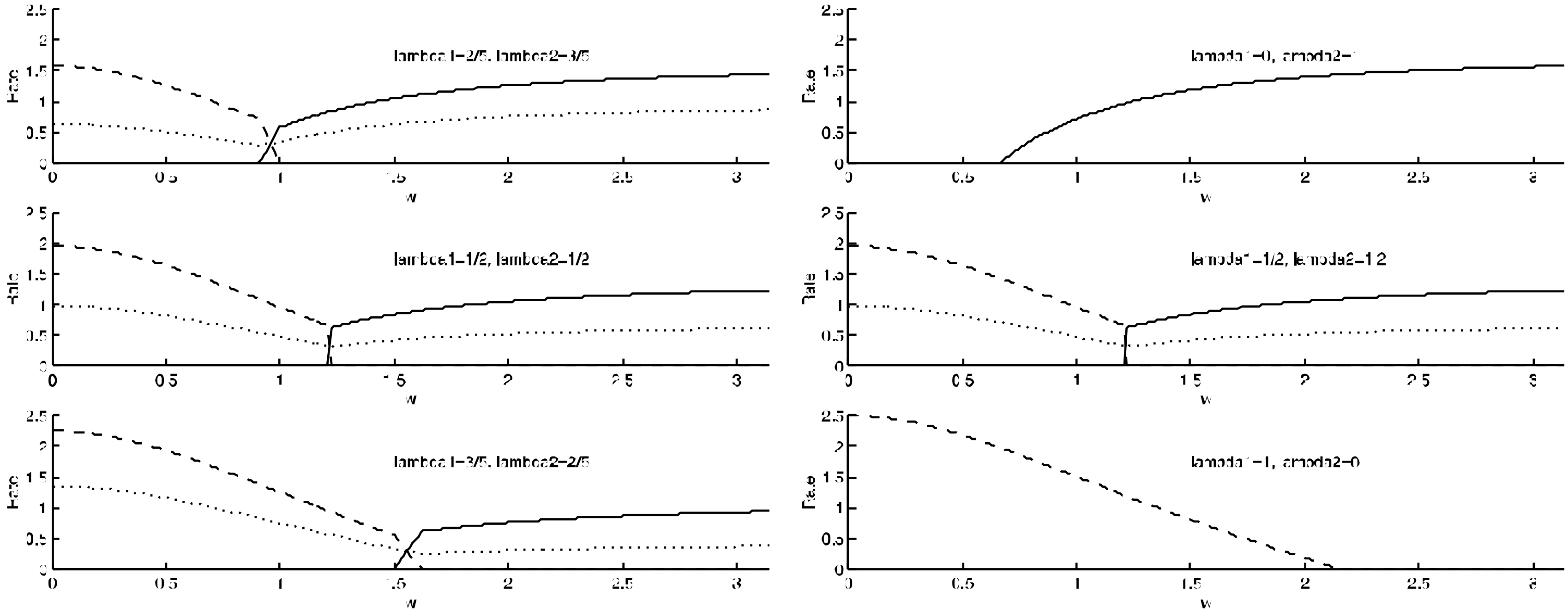
$$\boldsymbol{R}_2 \le .5 \int_{f:\boldsymbol{H}_1(f) > \boldsymbol{H}_2(f)} \log\left(1 + \frac{(1-\alpha(f))\boldsymbol{P}(f)}{\alpha(f)\boldsymbol{P}(f) + .5\boldsymbol{N}_0 / |\,\boldsymbol{H}_2(f)\,|^2}\right) + .5 \int_{f:\boldsymbol{H}_1(f) \le \boldsymbol{H}_2(f)} \log\left(1 + \frac{(1-\alpha(f))\boldsymbol{P}(f)\,|\,\boldsymbol{H}_2(f)\,|^2}{.5\boldsymbol{N}_0}\right),$$

$$0 \le \alpha(f) \le 1, \qquad \int \boldsymbol{P}(f)df \le \boldsymbol{P} \Big\}$$

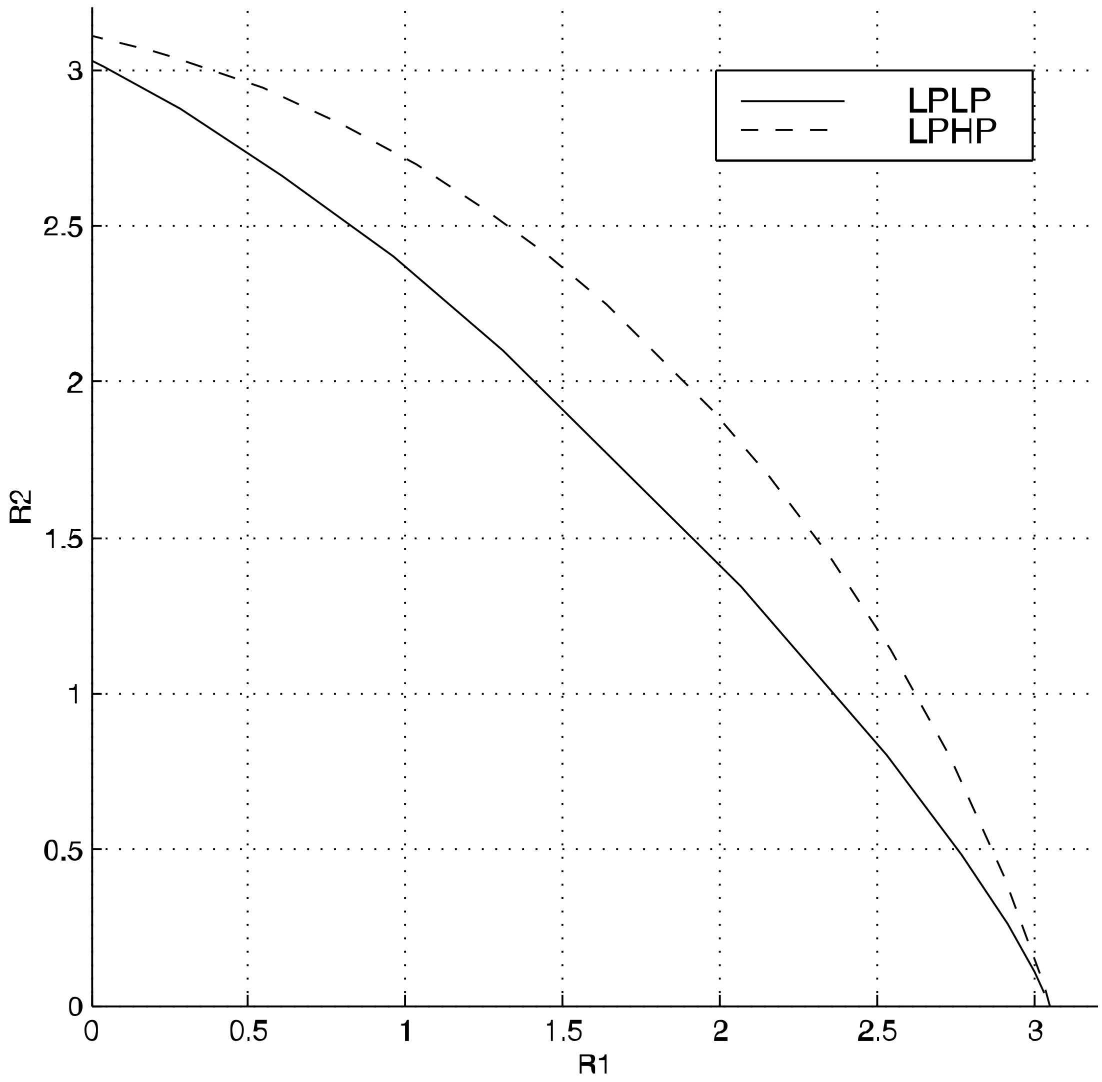
Optimal Power Allocation: Two Level Water Filling



Capacity vs. Frequency



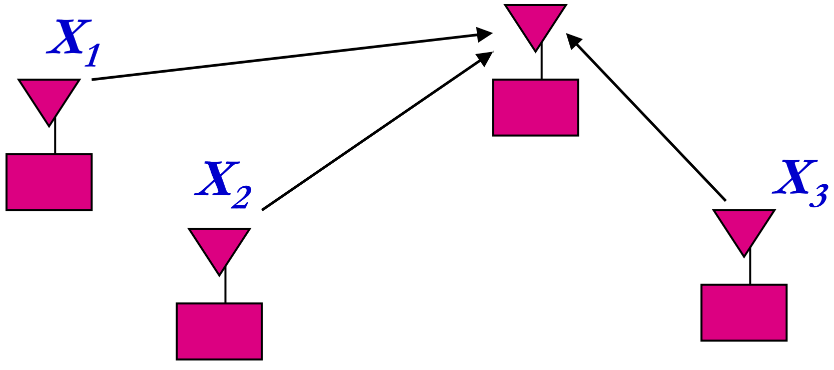
Capacity Region



Multiple Access Channel

- Multiple transmitters
  - Transmitter *i* sends signal X_i with power P_i
- Common receiver with AWGN of power N_0B
- Received signal:

$$Y = \sum_{i=1}^{M} X_i + N$$



MAC Capacity Region

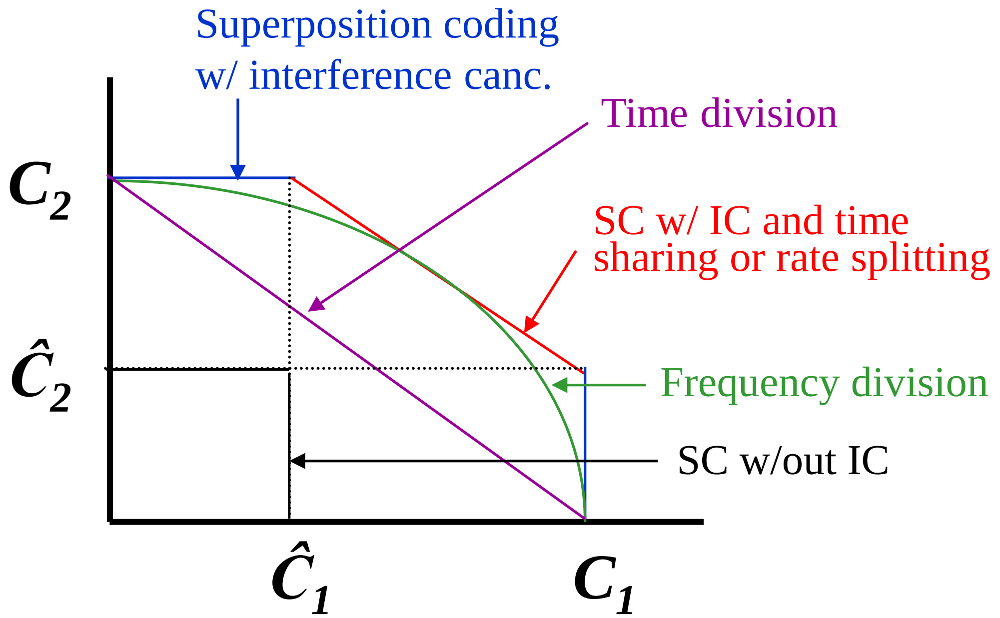
- Closed convex hull of all $(R_1,\ldots,R_M)$ s.t.

$$\sum_{i\in S} R_i \le B\log\left[1+\sum_{i\in S} P_i / N_0 B\right], \quad \forall S \subseteq \{1,\ldots,M\}$$

  - For all subsets of users, rate sum equals that of 1 superuser with sum of powers from all users

- Power Allocation and Decoding Order
  - Each user has its own power (no power alloc.)
  - Decoding order depends on desired rate point

Two-User Region



$$C_i = B\log\left[1+\frac{P_i}{N_0 B}\right], i=1,2$$

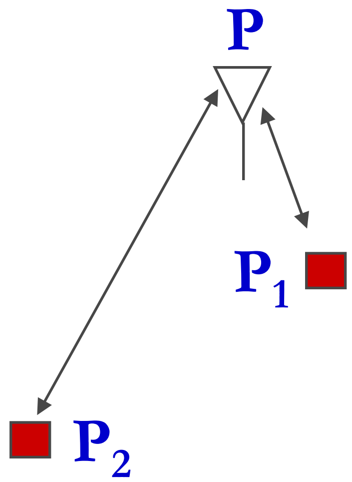
$$\hat{C}_1 = B\log\left[1+\frac{P_1}{N_0 B + P_2}\right], \qquad \hat{C}_2 = B\log\left[1+\frac{P_2}{N_0 B + P_1}\right],$$

Fading and ISI

- MAC capacity under fading and ISI determined using similar techniques as for the BC
- In fading, can define ergodic, outage, and minimum rate capacity similar as in BC case
  - Ergodic capacity obtained based on AWGN MAC given fixed fading, averaged over fading statistics
  - Outage can be declared as common, or per user
- MAC capacity with ISI obtained by converting to equivalent parallel MAC channels over frequency

Comparison of MAC and BC

- Differences:
  - Shared vs. individual power constraints
  - Near-far effect in MAC



- Similarities:
  - Optimal BC "superposition" coding is also optimal for MAC (sum of Gaussian codewords)
  - Both decoders exploit successive decoding and interference cancellation

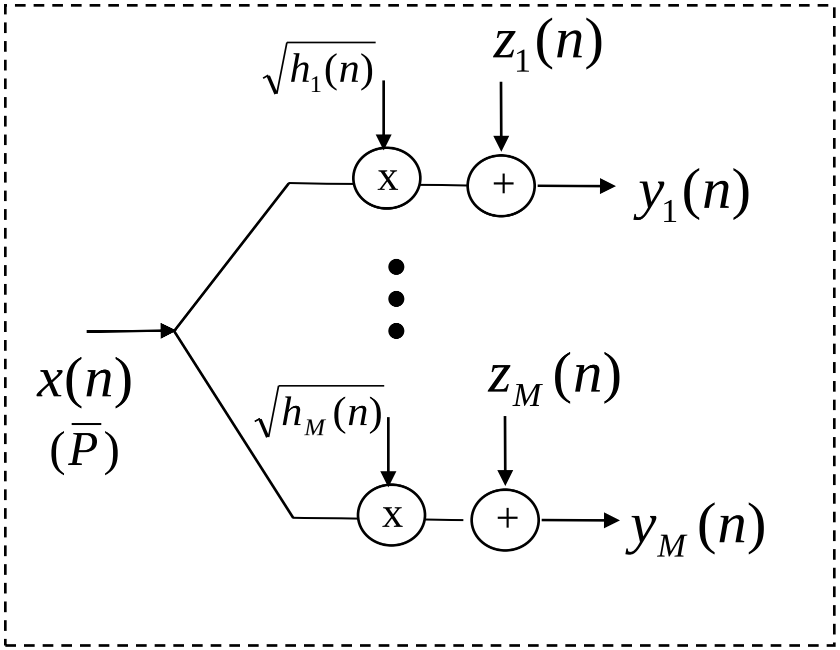
MAC-BC Capacity Regions

- MAC capacity region known for many cases
  - Convex optimization problem
- BC capacity region typically only known for (parallel) degraded channels
  - Formulas often not convex
- Can we find a connection between the BC and MAC capacity regions?

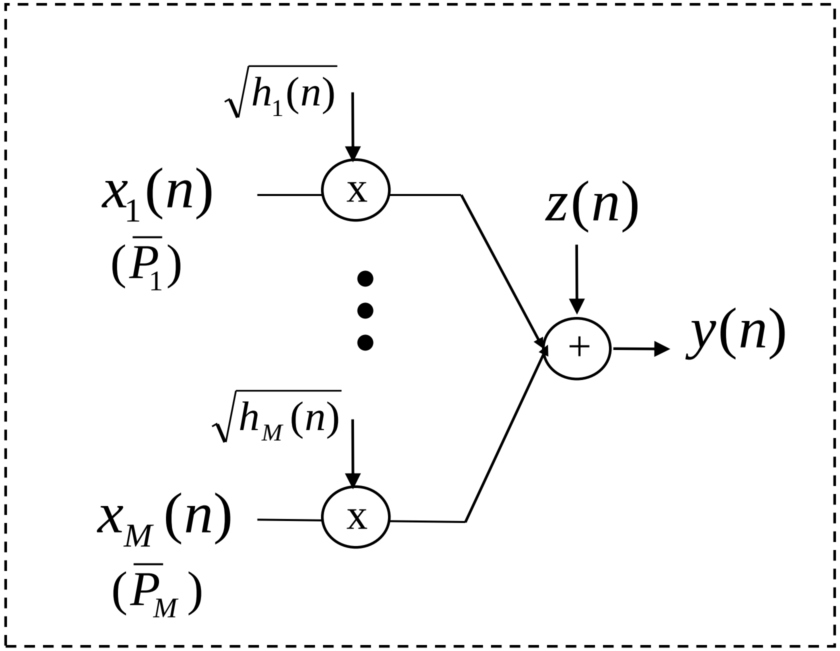
Duality

Dual Broadcast and MAC Channels

Gaussian BC and MAC with *same* channel gains and *same* noise power at each receiver



Broadcast Channel (BC)



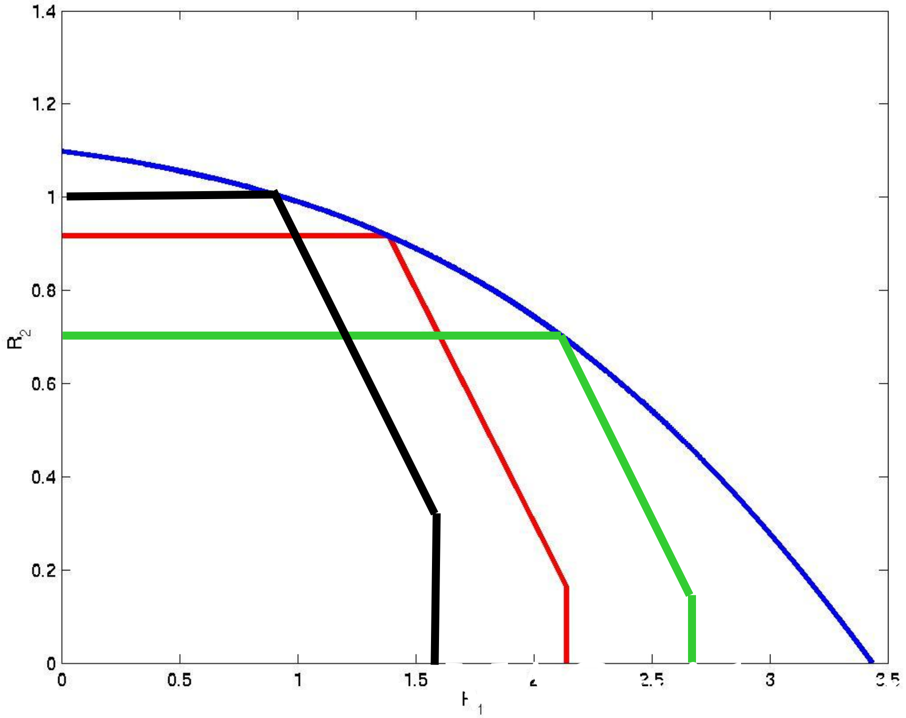
Multiple-Access Channel (MAC)

The BC from the MAC

$$C_{MAC}(P_1, P_2; h_1, h_2) \subseteq C_{BC}(P_1 + P_2; h_1, h_2)$$

$h_1 > h_2$

Blue = BC
Red = MAC



P_1=0.5, P_2=1.5

P_1=1, P_2=1

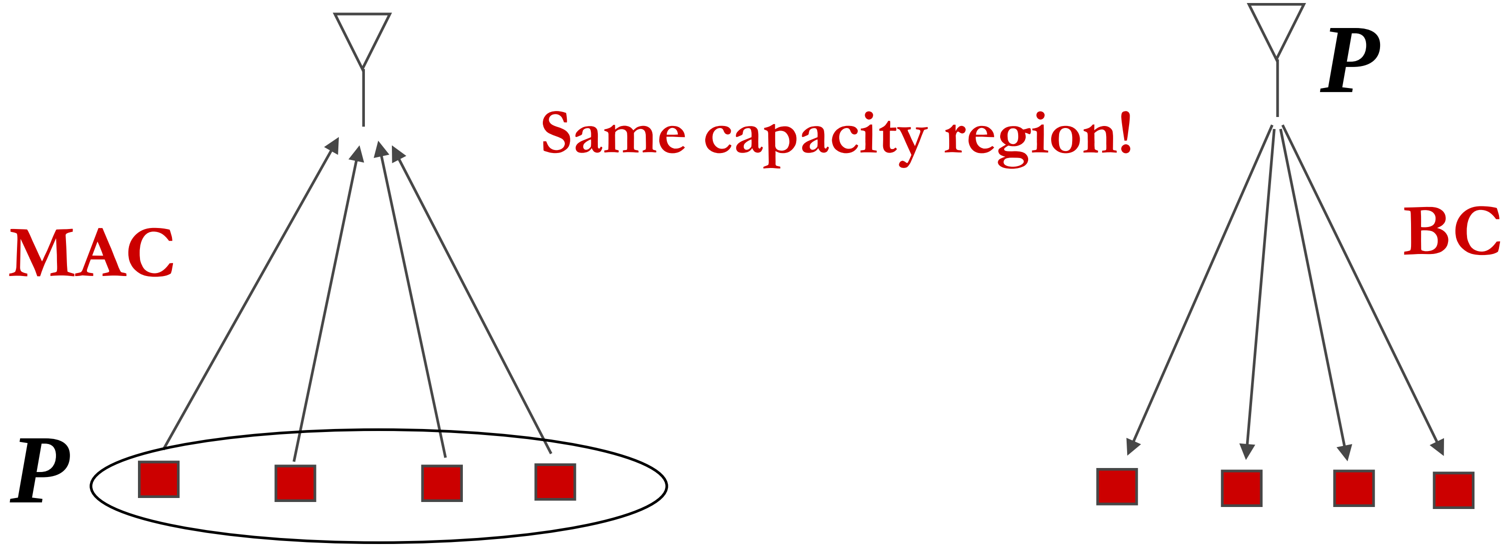
P_1=1.5, P_2=0.5

$$C_{BC}(P; h_1, h_2) = \bigcup_{0 \le P_1 \le P} C_{MAC}(P_1, P - P_1; h_1, h_2)$$

Sum-Power MAC

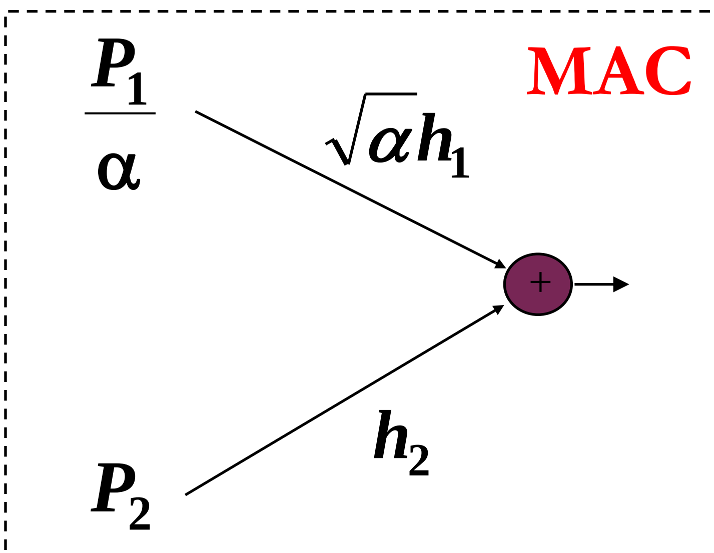
$$C_{BC}(P;h_1,h_2) = \bigcup_{0\le P_1\le P} C_{MAC}(P_1, P-P_1; h_1, h_2) \equiv C_{MAC}^{Sum}(P;h_1,h_2)$$

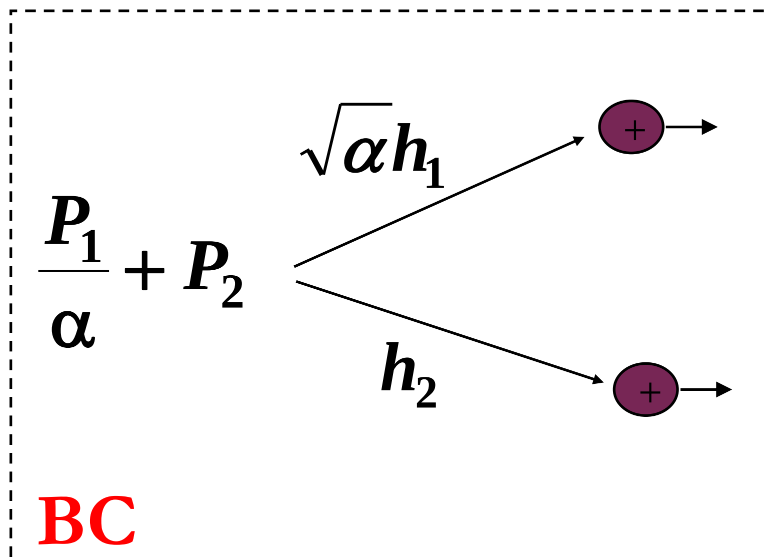
- MAC with <u>sum</u> power constraint
  - Power pooled between MAC transmitters
  - No transmitter coordination



BC to MAC: Channel Scaling

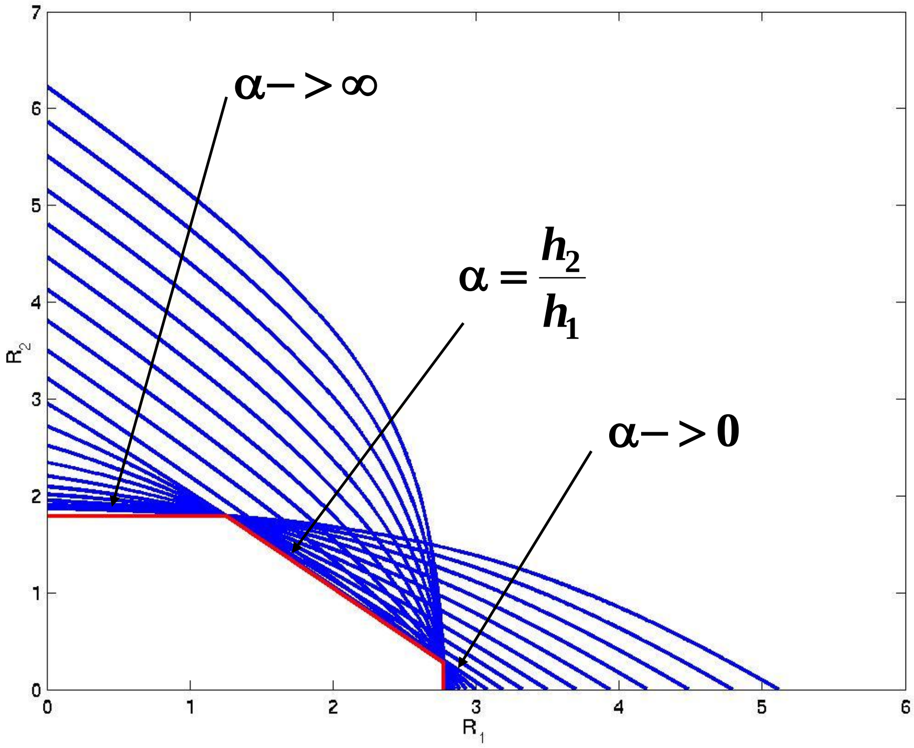
- Scale channel gain by $\sqrt{\alpha}$, power by $1/\alpha$
- MAC capacity region <u>unaffected</u> by scaling
- Scaled MAC capacity region is a subset of the scaled BC capacity region for any α
- MAC region inside scaled BC region for any scaling





The BC from the MAC

Blue = Scaled BC
Red = MAC

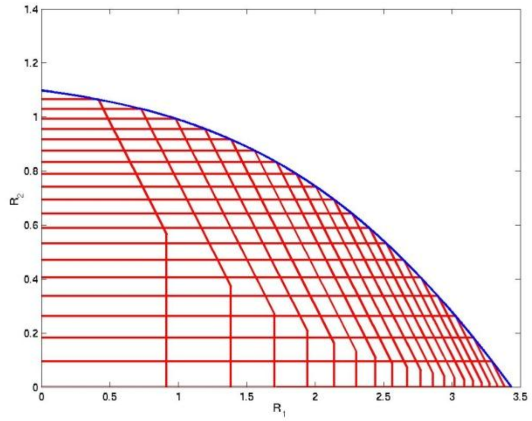


$$C_{MAC}(P_1, P_2; h_1, h_2) = \bigcap_{\alpha>0} C_{BC}\left(\frac{P_1}{\alpha} + P_2; \sqrt{\alpha}h_1, h_2\right)$$

Duality: Constant AWGN Channels

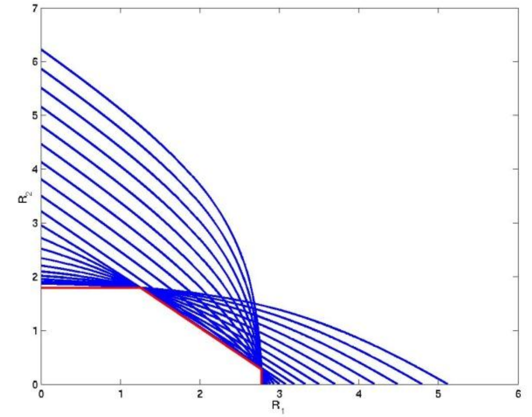
- **BC in terms of MAC**

$$C_{BC}(P;h_1,h_2)=\bigcup_{0\le P_1\le P} C_{MAC}(P_1,P-P_1;h_1,h_2)$$



- **MAC in terms of BC**

$$C_{MAC}(P_1,P_2;h_1,h_2)=\bigcap_{\alpha>0} C_{BC}\left(\frac{P_1}{\alpha}+P_2;\alpha h_1,h_2\right)$$



What is the relationship between the optimal transmission strategies?

Transmission Strategy Transformations

- Equate rates, solve for powers

$$R_1^M = \log(1+\frac{h_1^2 P_1^M}{h_2 P_2^M + \sigma^2}) = \log(1+\frac{h_1^2 P_1^B}{\sigma^2}) = R_1^B$$

$$R_2^M = \log(1+\frac{h_2^2 P_2^M}{\sigma^2}) = \log(1+\frac{h_2^2 P_2^B}{h_2^2 P_1^B + \sigma^2}) = R_2^B$$

- <u>Opposite</u> decoding order
  - Stronger user (User 1) decoded last in BC
  - Weaker user (User 2) decoded last in MAC

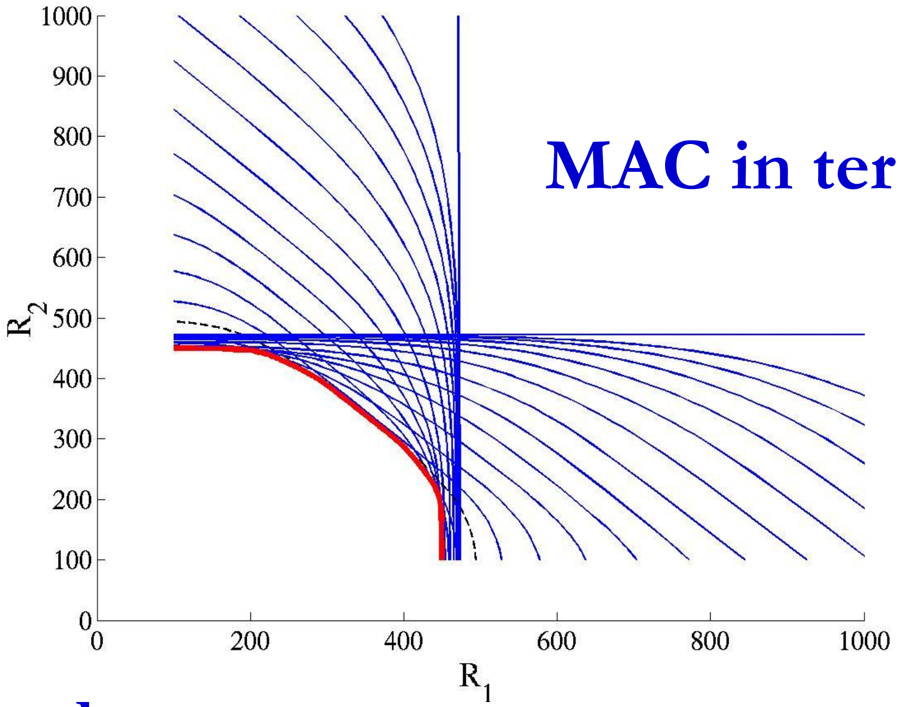
Duality Applies to Different Fading Channel Capacities

- Ergodic (Shannon) capacity: maximum rate averaged over all fading states.
- Zero-outage capacity: maximum rate that can be maintained in all fading states.
- Outage capacity: maximum rate that can be maintained in all nonoutage fading states.
- Minimum rate capacity: Minimum rate maintained in all states, maximize average rate in excess of minimum

Explicit transformations between transmission strategies

Duality: Minimum Rate Capacity

Blue = Scaled BC
Red = MAC



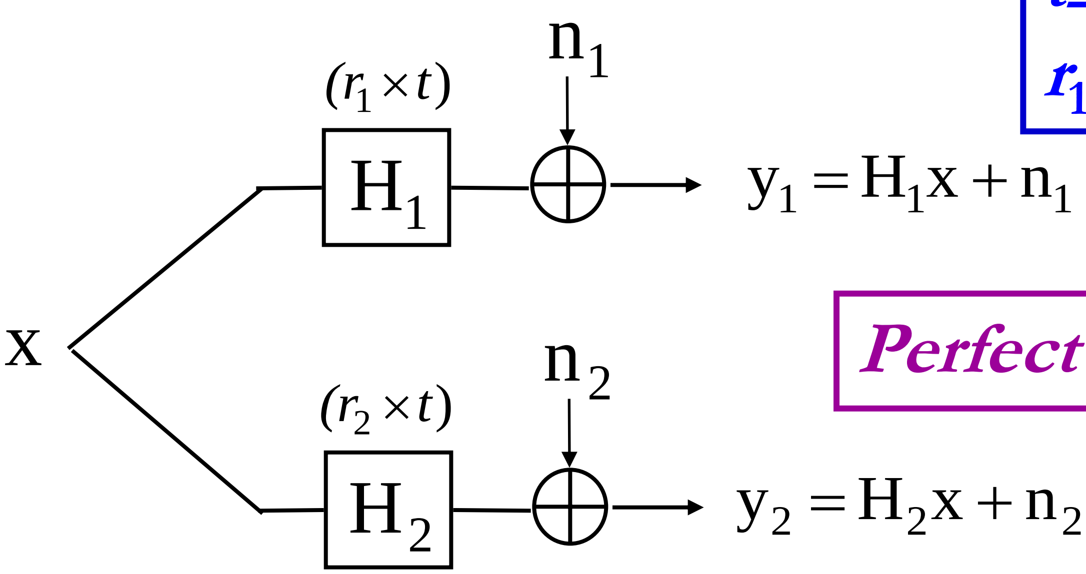
MAC in terms of BC

- BC region known
- MAC region can only be obtained by duality

What other capacity regions can be obtained by duality?

Broadcast MIMO Channels

Broadcast MIMO Channel



***t*≥*1* TX antennas**
***r*$_1$≥1, *r*$_2$≥1 RX antennas**

Perfect CSI at TX and RX

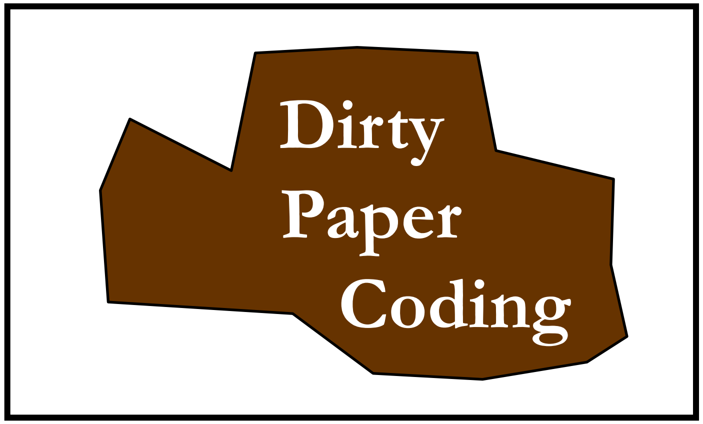
$$n_1 \sim N(0, I_{r_1}) \quad n_2 \sim N(0, I_{r_2})$$

Non-degraded broadcast channel

Dirty Paper Coding (Costa'83)

- Basic premise
  - If the interference is known, channel capacity same as if there is no interference
  - Accomplished by cleverly distributing the writing (codewords) and coloring their ink
  - Decoder must know how to read these codewords

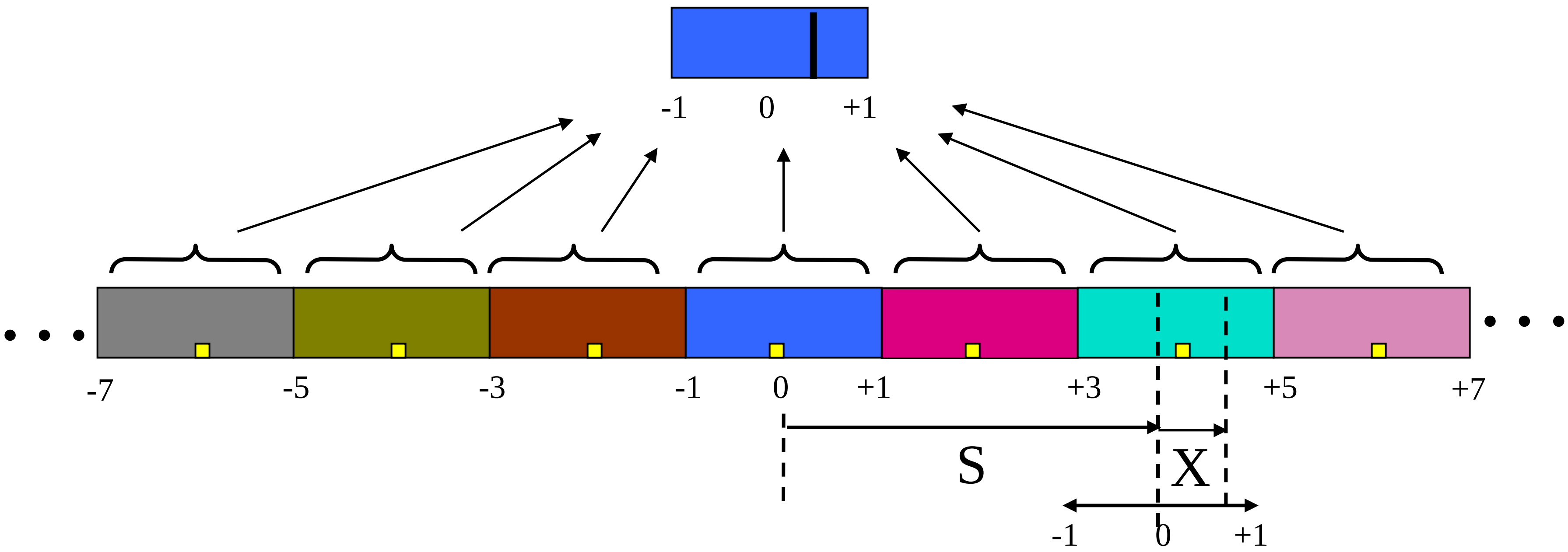
Clean Channel



Dirty Channel

Modulo Encoding/Decoding

- Received signal Y=X+S, -1≤X≤1
  - S known to transmitter, not receiver
- Modulo operation removes the interference effects
  - Set X so that $\lfloor Y \rfloor_{[-1,1]}$=desired message (e.g. 0.5)
  - Receiver demodulates modulo [-1,1]



Capacity Results

- Non-degraded broadcast channel
  - Receivers not necessarily "better" or "worse" due to multiple transmit/receive antennas
  - Capacity region for general case unknown

- Pioneering work by Caire/Shamai (Allerton'00):
  - Two TX antennas/two RXs (1 antenna each)
  - Dirty paper coding/lattice precoding (achievable rate)
    - Computationally very complex
  - MIMO version of the Sato upper bound
  - Upper bound is achievable: capacity known!

Dirty-Paper Coding (DPC) for MIMO BC

- Coding scheme:
  - Choose a codeword for user 1
  - Treat this codeword as interference to user 2
  - Pick signal for User 2 using "pre-coding"
- Receiver 2 experiences no interference:

$$\mathrm{R}_2 = \log(\det(\mathrm{I} + H_2 \Sigma_2 H_2^T))$$

- Signal for Receiver 2 interferes with Receiver 1:

$$\mathrm{R}_1 = \log\left(\frac{\det(\mathrm{I} + H_1(\Sigma_1 + \Sigma_2)H_1^T)}{\det(\mathrm{I} + H_1 \Sigma_2 H_1^T)}\right)$$

- Encoding order can be switched
- DPC optimization highly complex

Does DPC achieve capacity?

- DPC yields MIMO BC achievable region.
  - We call this the dirty-paper region
- Is this region the capacity region?
- We use duality, dirty paper coding, and Sato's upper bound to address this question
- First we need MIMO MAC Capacity

MIMO MAC Capacity

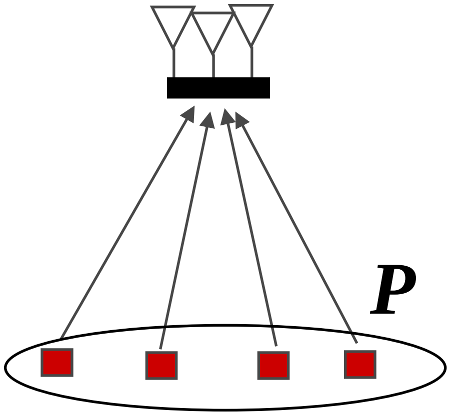
- **MIMO MAC follows from MAC capacity formula**

$$C_{MAC}(P_1,\ldots,P_k) = \bigcup \left\{ (R_1,\ldots,R_k) : \sum_{k\in S} R_k \le \log_2 \det\left[I + \sum_{k\in S} H_k Q_k H_k^H \right], \quad \forall S \subseteq \{1,\ldots,K\} \right\}$$

- **Basic idea same as single user case**
  - **Pick some subset of users**
  - **The sum of those user rates equals the capacity as if the users pooled their power**
- **Power Allocation and Decoding Order**
  - **Each user has its own power (no power alloc.)**
  - **Decoding order depends on desired rate point**

MIMO MAC with sum power

- **MAC with sum power:**
  - **Transmitters code independently**
  - **Share power**



$$C_{MAC}^{Sum}(P) = \bigcup_{0 \le P_1 \le P} C_{MAC}(P_1, P - P_1)$$

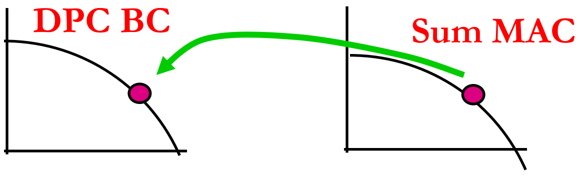
- **Theorem: Dirty-paper BC region equals the dual sum-power MAC region**

$$C_{BC}^{DPC}(P) = C_{MAC}^{Sum}(P)$$

Transformations: MAC to BC

- Show any rate achievable in sum-power MAC also achievable with DPC for BC:

$$C_{BC}^{DPC}(P) \supseteq C_{MAC}^{Sum}(P)$$

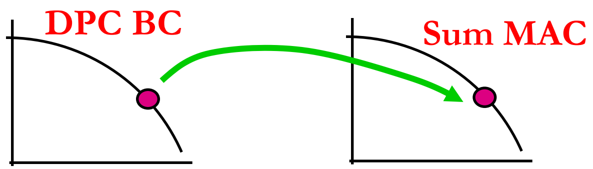


  - A sum-power MAC strategy for point (R_1,…R_N) has a given input covariance matrix and encoding order
  - We find the corresponding PSD covariance matrix and encoding order to achieve (R_1,…,R_N) with DPC on BC
    - The rank-preserving transform "flips the effective channel" and reverses the order
    - Side result: beamforming is optimal for BC with 1 Rx antenna at each mobile

Transformations: BC to MAC

- Show any rate achievable with DPC in BC also achievable in sum-power MAC:

$$C_{BC}^{DPC}(P) \subseteq C_{MAC}^{Sum}(P)$$



  - We find transformation between optimal DPC strategy and optimal sum-power MAC strategy
    - "Flip the effective channel" and reverse order

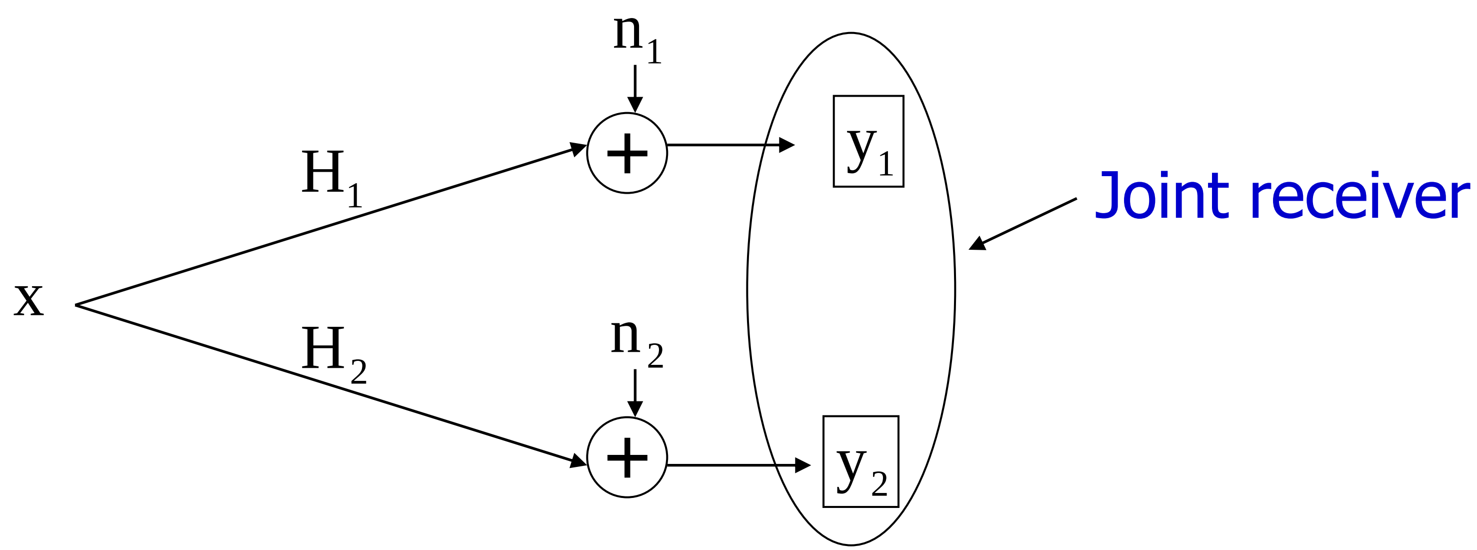
Computing the Capacity Region

$$C_{BC}^{DPC}(P) = C_{MAC}^{Sum}(P)$$

- Hard to compute DPC region (Caire/Shamai'00)
- "Easy" to compute the MIMO MAC capacity region
  - Obtain DPC region by solving for sum-power MAC and applying the theorem
  - Fast iterative algorithms have been developed
  - Greatly simplifies calculation of the DPC region and the associated transmit strategy

Sato Upper Bound on the BC Capacity Region

- **Based on receiver cooperation**



- **BC sum rate capacity ≤ Cooperative capacity**

$$C_{BC}^{\text{sumrate}}(P,H) \leq \max_{\Sigma_x} \frac{1}{2} \log |I + H\Sigma_x H^T|$$

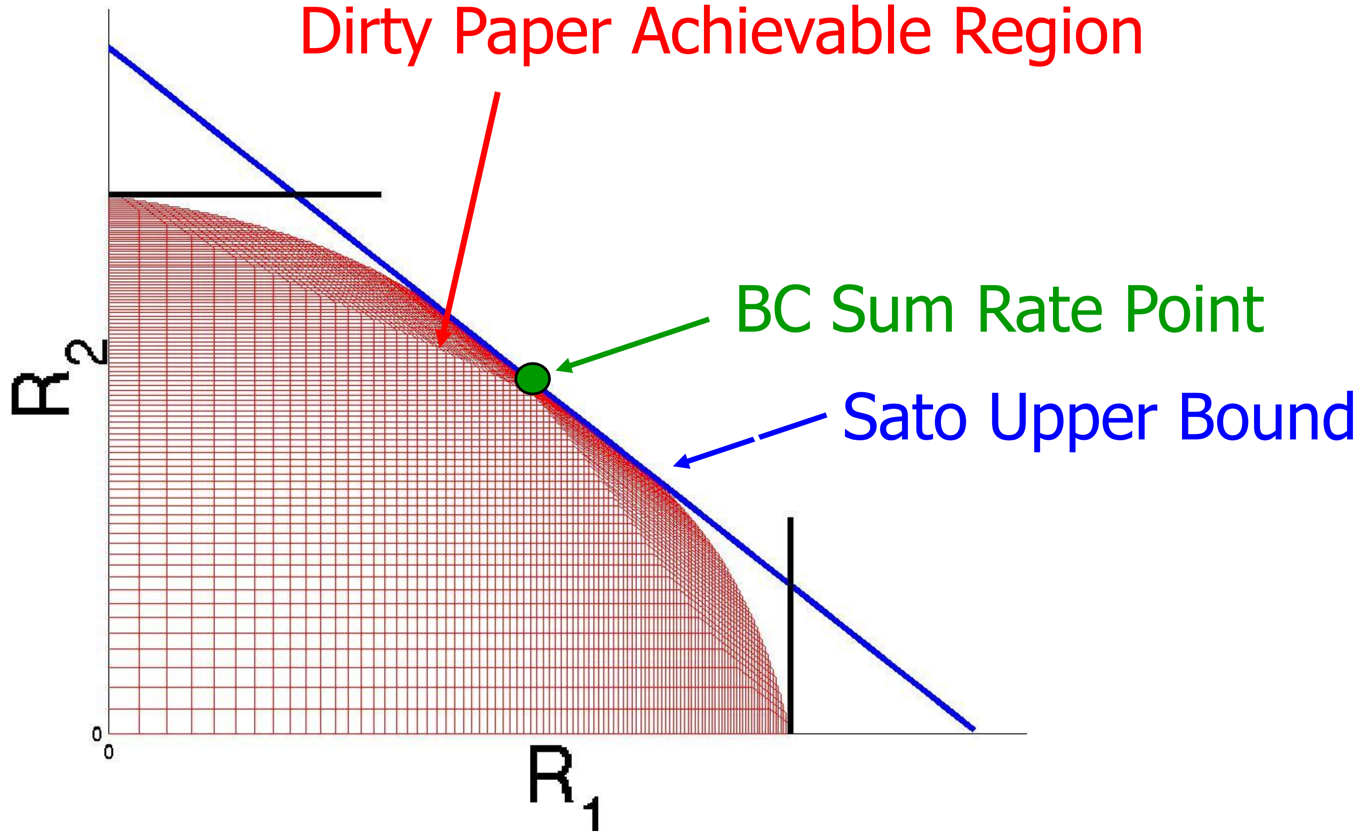
The Sato Bound for MIMO BC

- **Introduce noise correlation between receivers**
- **BC capacity region unaffected**
  - **Only depends on noise marginals**
- **Tight Bound (Caire/Shamai'00)**
  - **Cooperative capacity with <u>worst-case</u> noise correlation**

$$C_{BC}^{\text{sum rate}}(P,H) \le \inf_{\Sigma_z} \max_{\Sigma_x} \frac{1}{2} \log | I + \Sigma_z^{-1/2} H \Sigma_x H^T \Sigma_z^{-1/2} |$$

- **Explicit formula for worst-case noise covariance**
- **By Lagrangian duality, cooperative BC region equals the sum-rate capacity region of MIMO MAC**

MIMO BC Capacity Bounds



Does the DPC region equal the capacity region?

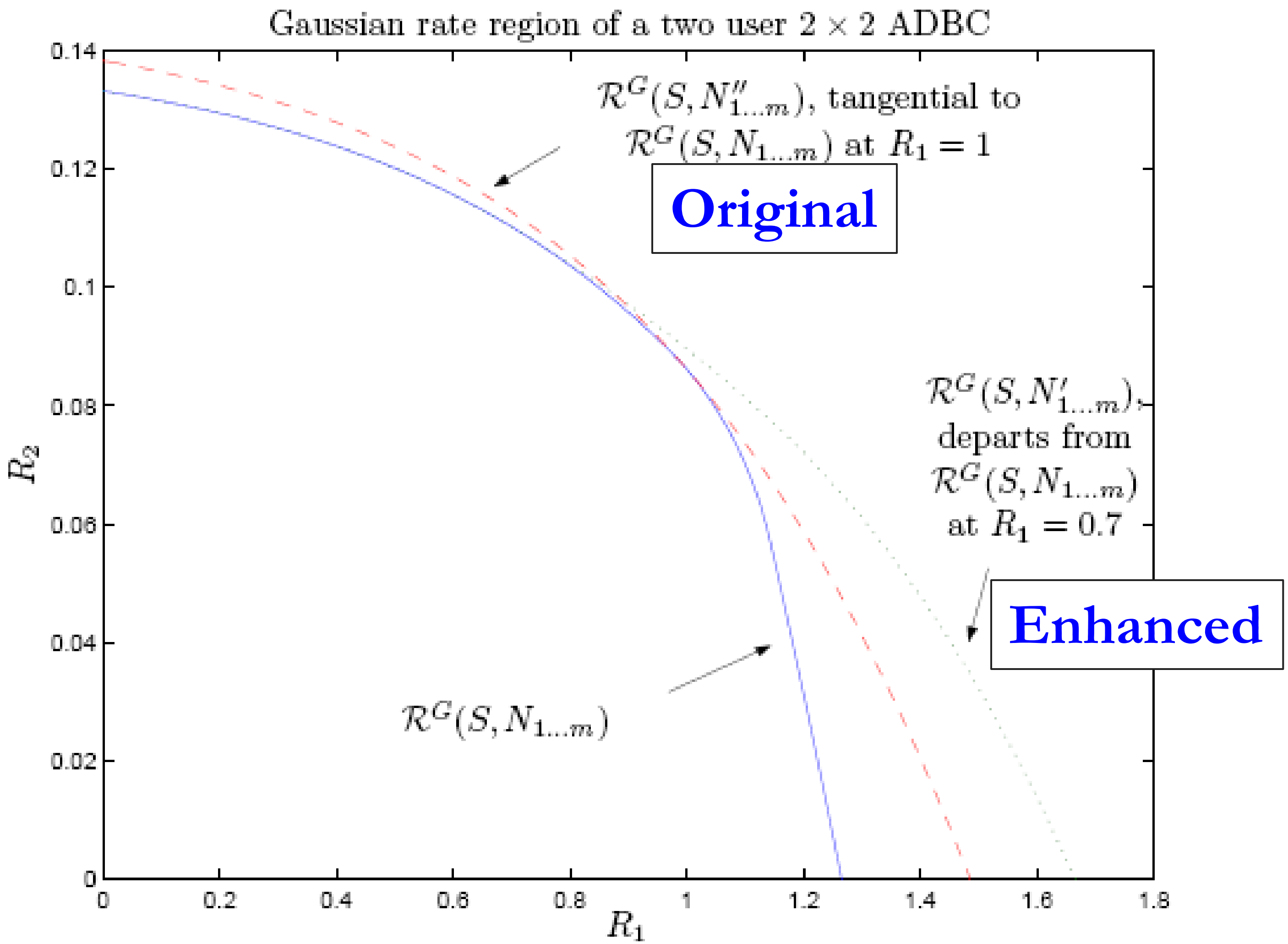
Full Capacity Region

- DPC gives us an achievable region
- Sato bound only touches at sum-rate point
- Bergman's entropy power inequality is not a tight upper bound for nondegraded broadcast channel
- A tighter bound was needed to prove DPC optimal
  - It had been shown that if Gaussian codes optimal, DPC was optimal, but proving Gaussian optimality was open.
- Breakthrough by Weingarten, Steinberg and Shamai
  - Introduce notion of enhanced channel, applied Bergman's converse to it to prove DPC optimal for MIMO BC.

Enhanced Channel Idea

- The aligned and degraded BC (AMBC)
  - Unity matrix channel, noise innovations process
  - Limit of AMBC capacity equals that of MIMO BC
  - Eigenvalues of some noise covariances go to infinity
  - Total power mapped to covariance matrix constraint
- Capacity region of AMBC achieved by Gaussian superposition coding and successive decoding
  - Uses entropy power inequality on enhanced channel
  - Enhanced channel has less noise variance than original
  - Can show that a power allocation exists whereby the enhanced channel rate is inside original capacity region
- By appropriate power alignment, capacities equal

Illustration



Main Points

- Shannon capacity gives fundamental data rate limits for multiuser wireless channels
- Fading multiuser channels optimize at each channel instance for maximum average rate
- Outage capacity has higher (fixed) rates than with no outage.
- OFDM is near optimal for broadcast channels with ISI
- Duality connects BC and MAC channels
  - Used to obtain capacity of one from the other
- Capacity of broadcast MIMO channel obtained using duality and the notion of an enhanced channel