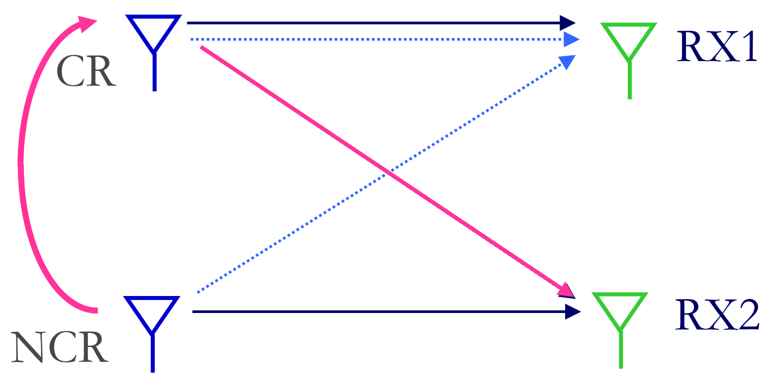
EE360: Lecture 13 Outline
Capacity of Cognitive Radios

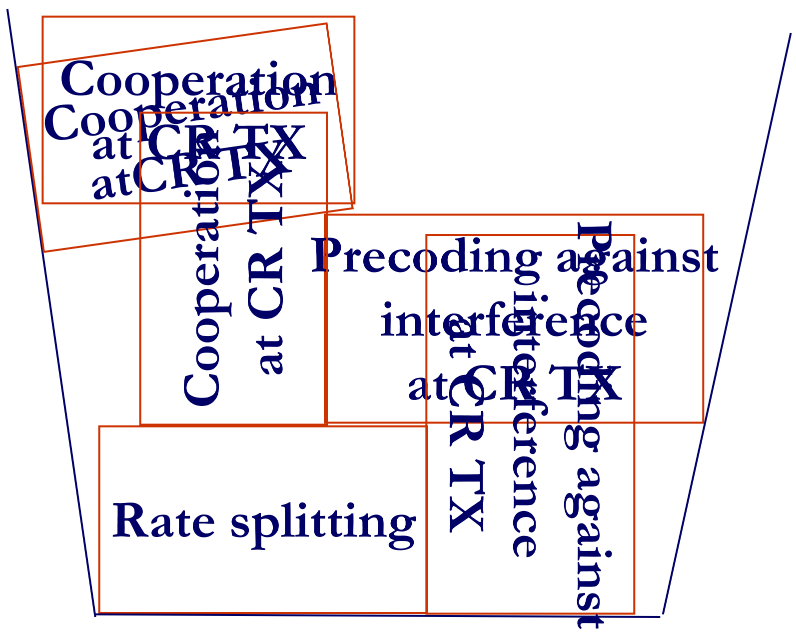
- Announcements
  - Progress reports due Feb. 29 at midnight
- Overview
- Achievable rates in Cognitive Radios
- Better achievable scheme and upper bounds
- MIMO cognitive radio capacity
- BC with cognitive relays
- Summary

Overlay Systems

- Cognitive user has knowledge of other user's message and/or encoding strategy
  - Used to help noncognitive transmission
  - Used to presubtract noncognitive interference



Transmission Strategy "Pieces"



To allow each receiver to decode part of the other node's message
⇨ reduces interference

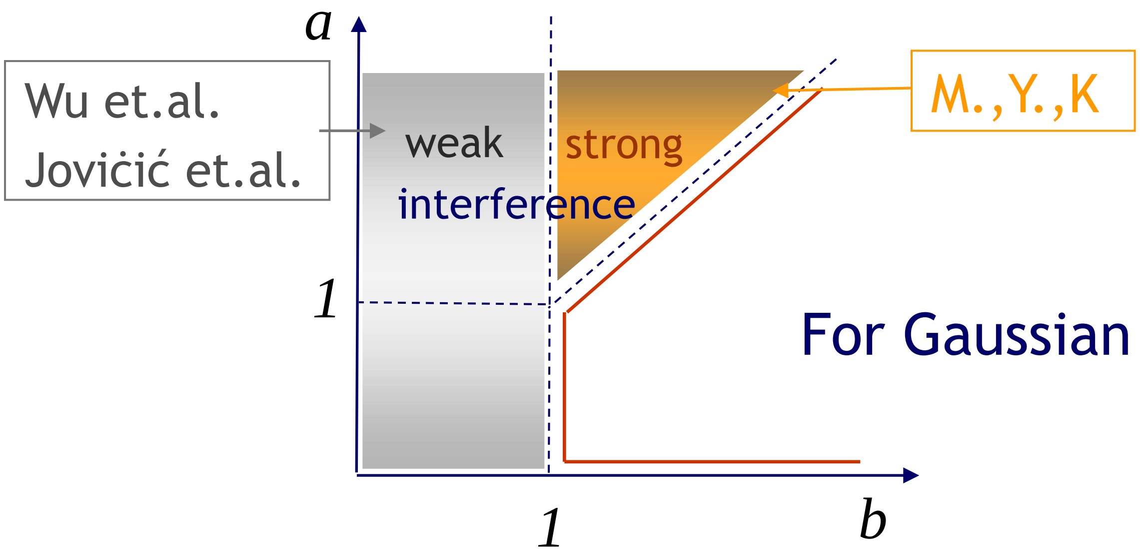
Removes the NCR interference at the CR RX

To help in sending NCR's message to its RX

Must optimally combine these approaches

"Achievable Rates in Cognitive Radios" by Devroye, Mitran, Tarokh

Results around 2007



For Gaussian channel with $P_1 = P_2$

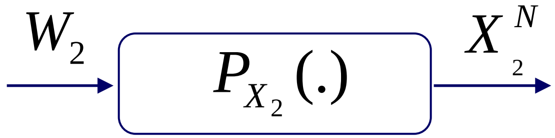
New encoding scheme uses same techniques as previous work:
rate splitting, G-P precoding against interference and cooperation

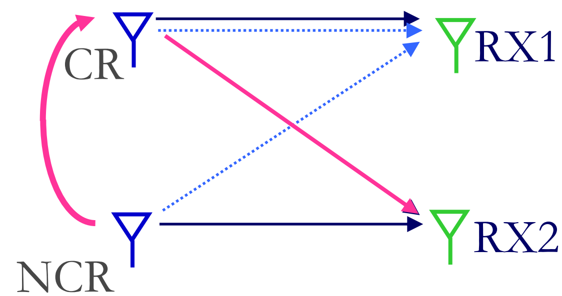
Differences:

- More general scheme than the one that suffices in weak interference
- Different binning than the one proposed by *[Devroye et.al]* and *[Jiang et.al.]*

Improved Scheme
Transmission for Achievable Rates

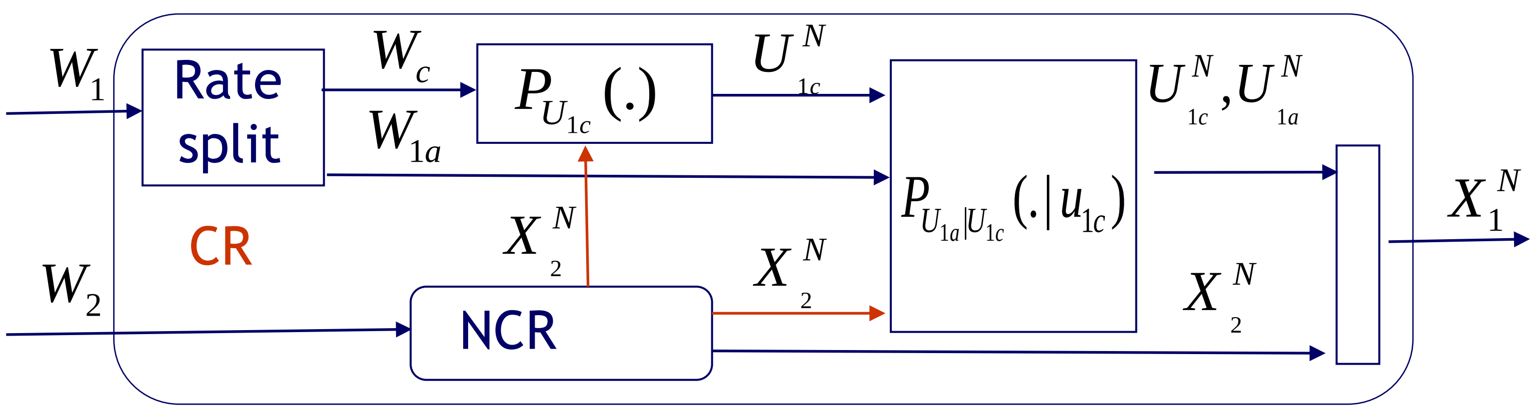
The NCR uses single-user encoder





The CR uses

- Rate-splitting to allow receiver 2 to decode part of cognitive user's message and thus reduce interference at that receiver
- Precoding while treating the codebook for user 2 as interference to improve rate to its own receiver
- Cooperation to increase rate to receiver 2



Outer Bound

The set of rate triples (R_0, R_1, R_2) satisfying

$$
\begin{aligned}
R_0 &\le I(V;Y_2) \\
R_1 &\le I(V,U_1;Y_1) \\
R_0 + R_2 &\le I(V,U_2;Y_2) \\
R_1 + R_2 &\le I(V,U_1;Y_1) + I(U_2;Y_2 | U_1,V) \\
R_0 + R_1 + R_2 &\le I(U_1;Y_1 | U_2,V) + I(V,U_2;Y_2)
\end{aligned}
$$

for input distributions that factor as

$$p(u_1)p(u_2)p(v|u_1,u_2)p(x_2|v,u_2)p(x_1|v,u_1,u_2,x_2)$$

For $R_2=0$, $U_2=\emptyset$, and by redefining R_0 as R_2:

outer bound for the IC with full cooperation

Outer Bound: Full Cooperation

The set of rate triples (R_0, R_1, R_2) satisfying

$$
\begin{aligned}
R_1 &\leq I(V,U_1;Y_1) \\
R_2 &\leq I(V,U_2;Y_2) \\
R_1 + R_2 &\leq \min\{I(V,U_1;Y_1) + I(U_2;Y_2 | U_1,V) \\
&\quad I(U_1;Y_1 | U_2,V) + I(V,U_2;Y_2)\}
\end{aligned}
$$

for input distributions that factor as

$$p(u_1)p(u_2)p(v|u_1,u_2)\underline{p(x_2|u_2)}p(x_1|u_1,u_2)$$

- **The exact same form as the Nair-El Gamal outer bound on the broadcast channel capacity**
- The difference is in the factorization of the input distribution reflecting the fact that only one-way cooperation is possible

Summary of new technique

Outer bound

- Follows from standard approach: invoke Fano's inequality
- Reduces to outer bound for full cooperation for $R_2=0$
- Has to be evaluated for specific channels

Achievable rates: combine

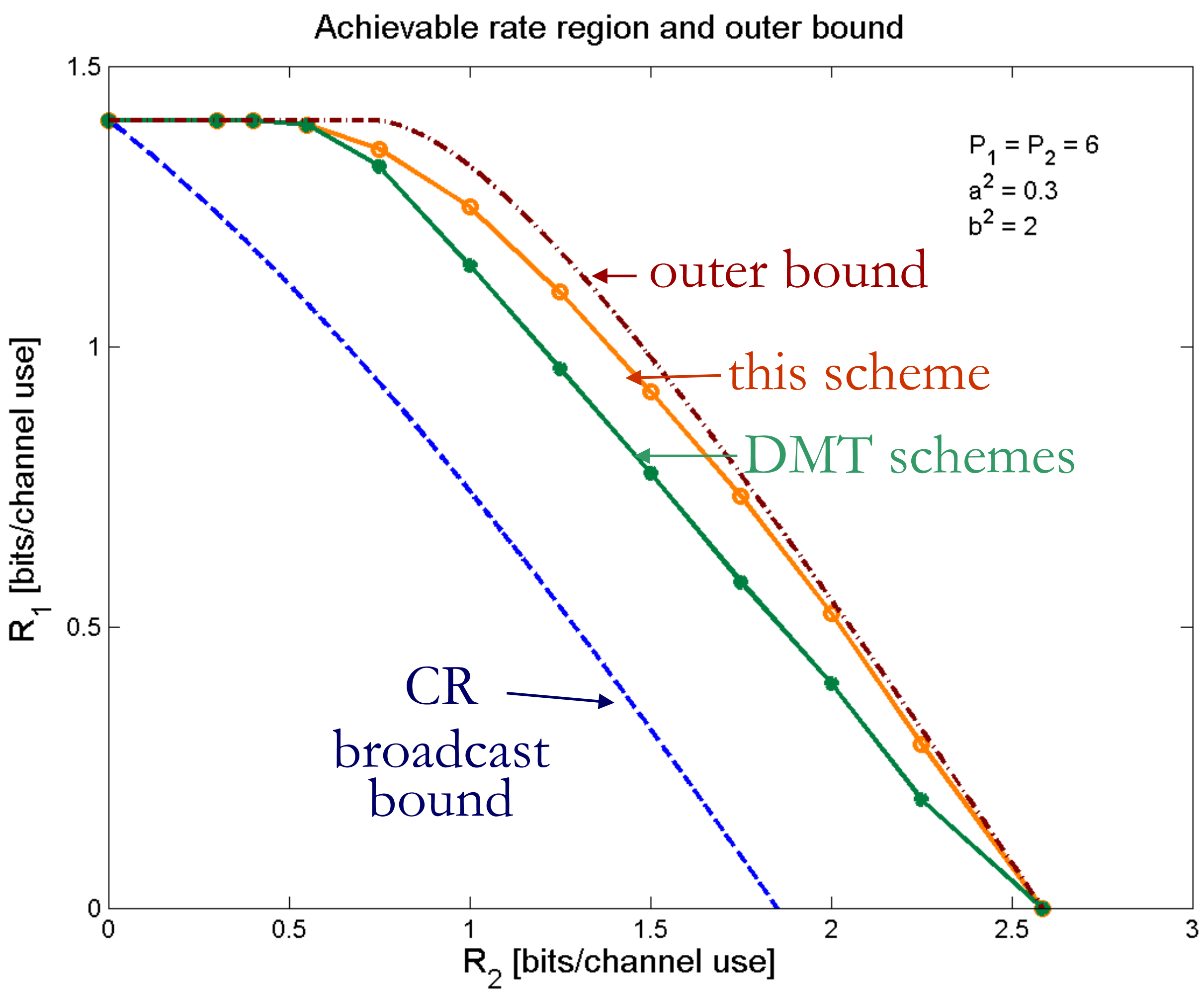
rate splitting

precoding against interference at encoder 1

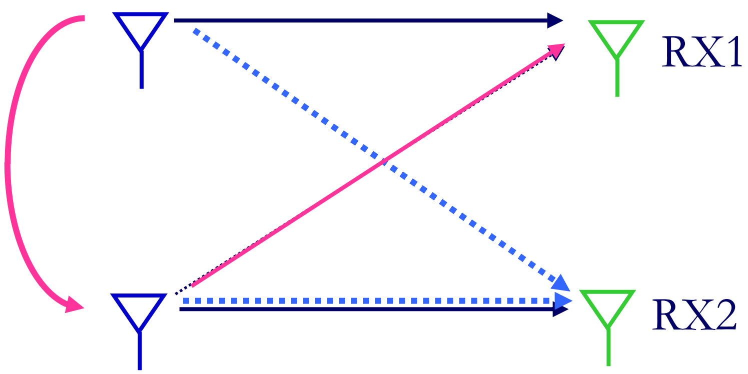
cooperation at encoder 1

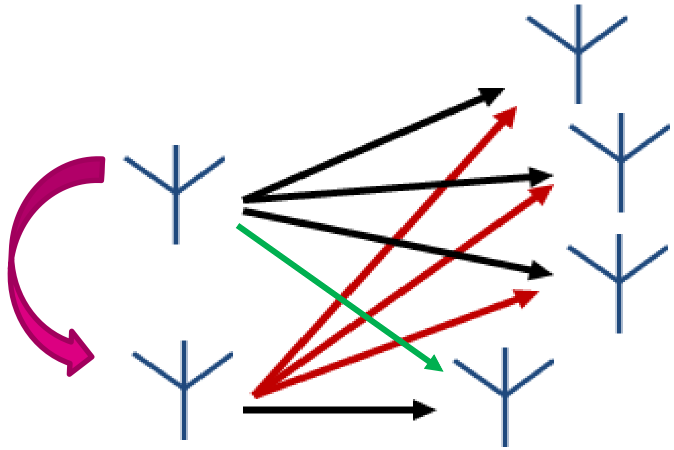
- **How far are the achievable rates from the outer bound?**
- **Capacity for other regimes?**

Performance Gains from Cognitive Encoding



Cognitive MIMO Networks





- Noncognitive user unaware of secondary users
- Cognitive user doesn't impact rate of noncognitive user

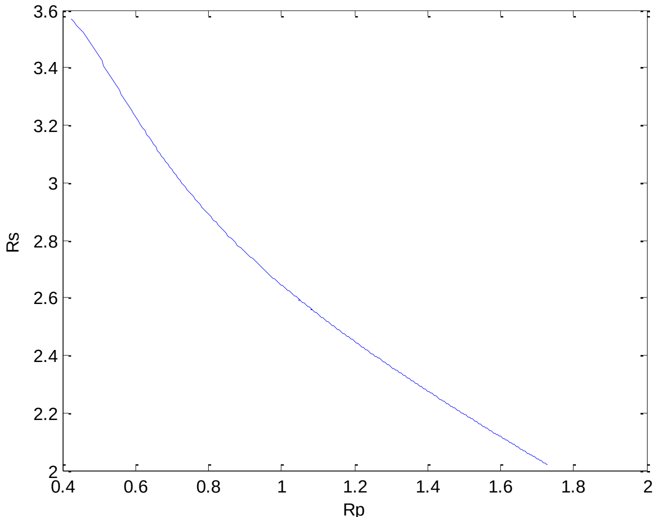
- Encoding rule for the cognitive encoder:
  - Generates codeword for primary user message
  - Generates codeword for its message using dirty paper coding
  - Two codewords superimposed to form final codeword

Achievable rates (2 users)

- **For MISO secondary users, beamforming is optimal**
- **Maximum achievable rate obtained by solving**

$$\begin{aligned} &Maximize && log_2(1+\parallel hx_{c,c} \parallel^2) \\ &Subject\ to && \parallel x_c \parallel^2 \leq P_t \\ &log_2\left(1+\frac{\parallel x_p + gx_{c,p} \parallel^2}{1+\parallel gx_{c,c} \parallel^2}\right) = log_2(1+\parallel x_p \parallel^2) \\ & && \Sigma_c \succeq 0 \end{aligned}$$

- **Closed-form relationship between primary/secondary user rates.**



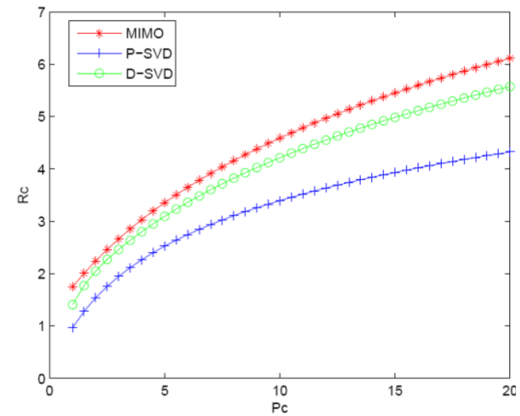
MIMO cognitive users (2 Users)

D-SVD

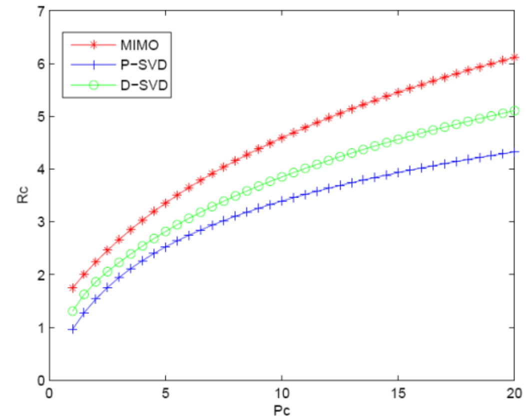
Precode based on SVD of cognitive user's channel

P-SVD

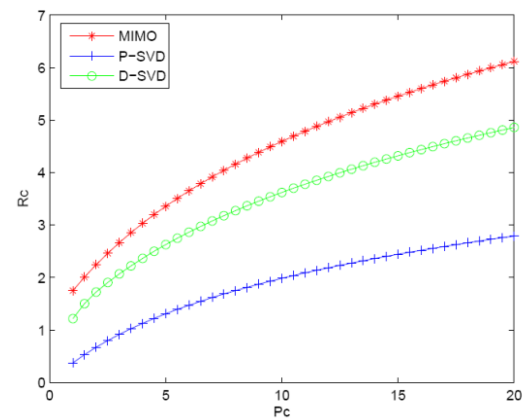
Project cognitive user's channel onto null space between C_{TX} and NC_{RX}, then perform SVD on projection



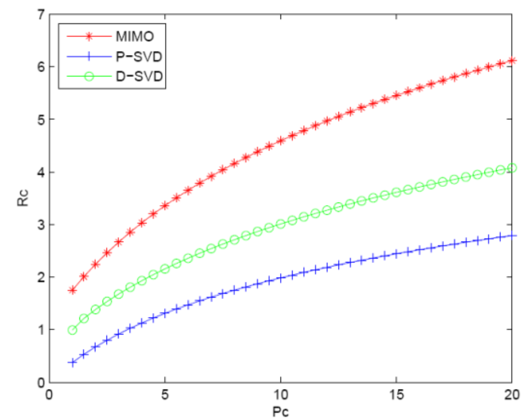
$P_p = 5, \|g\| = 0.374$



$P_p = 15, \|g\| = 0.374$



$P_p = 5, \|g\| = 0.707$

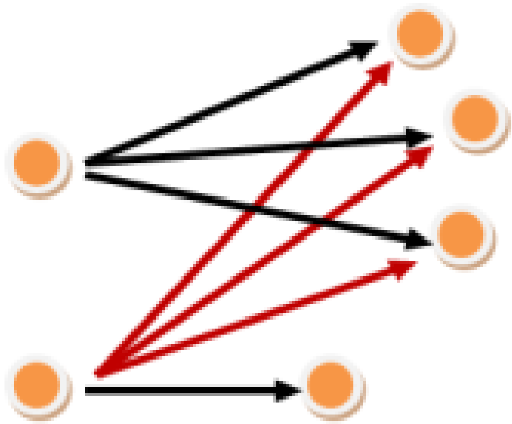


$P_p = 15, \|g\| = 0.707$

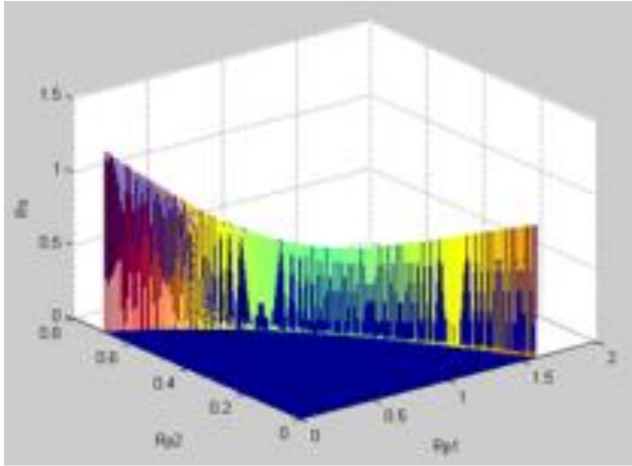
Multi-user Cognitive MIMO Networks

- Extend analysis to multiple primary users
- Assume each transmitter broadcasts to multiple users
  - Primary receivers have one antenna
  - Secondary users are MISO.

- Main Result:
  - With appropriate power allocation among primary receivers, the secondary users achieve their maximum possible rate.



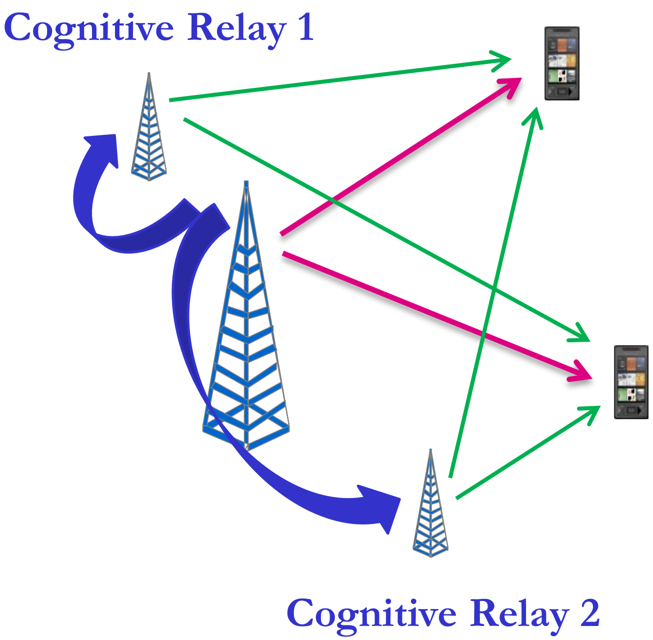
Cognitive MIMO network with multiple primary users

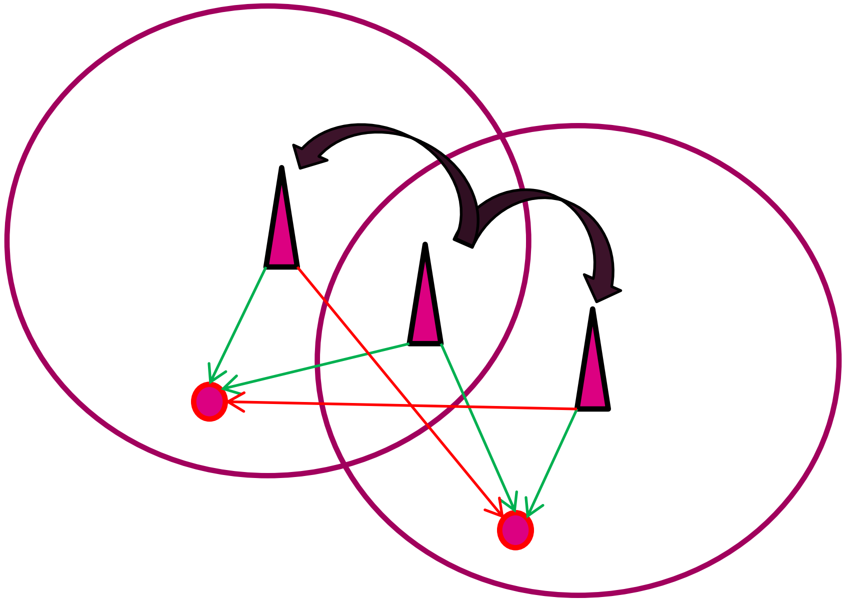


Achievable rates with two primary users

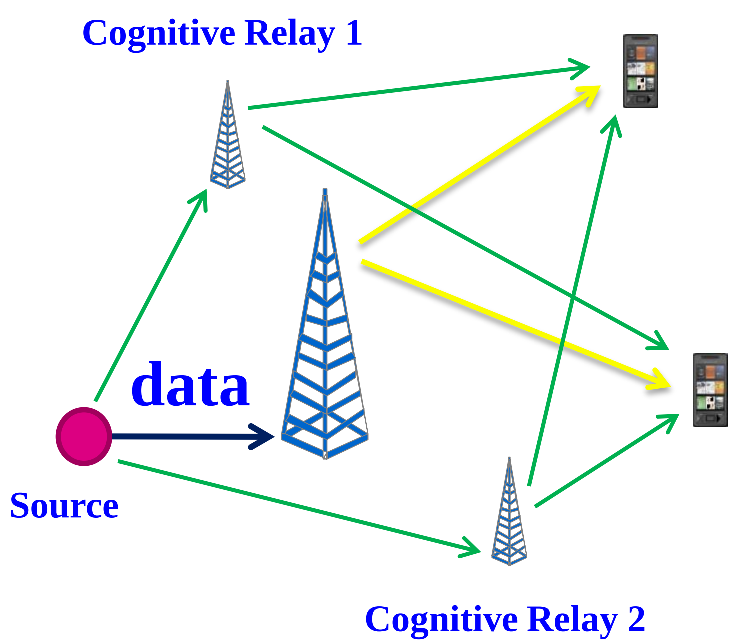
Other Overlay Systems

- Cognitive relays
- Cognitive BSs



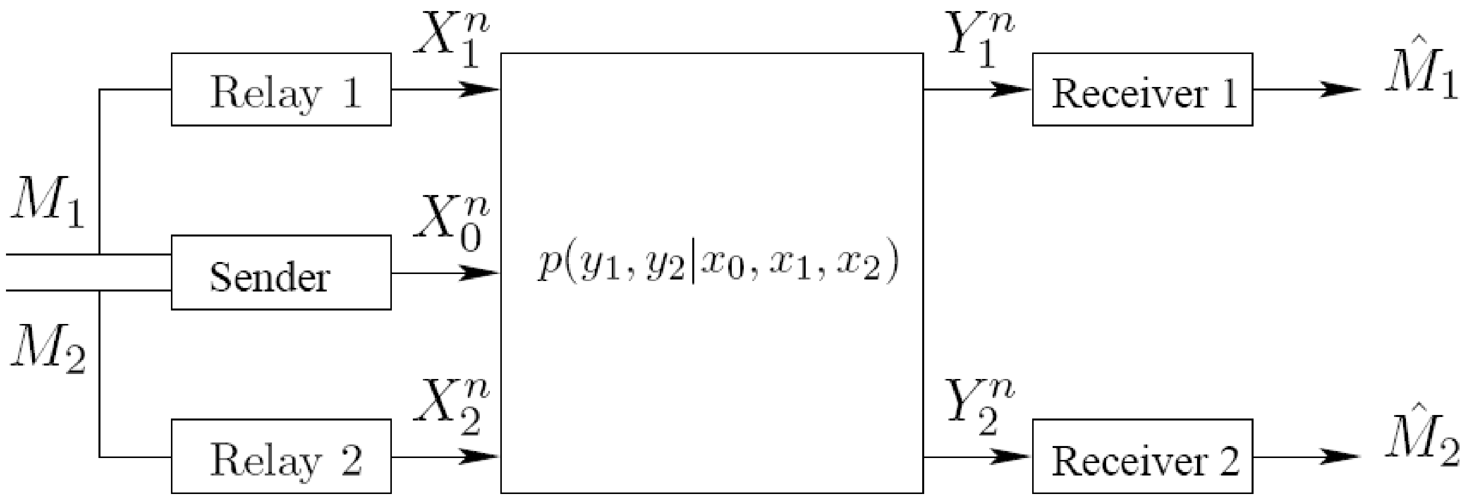


Broadcast Channel with Cognitive Relays (BCCR)



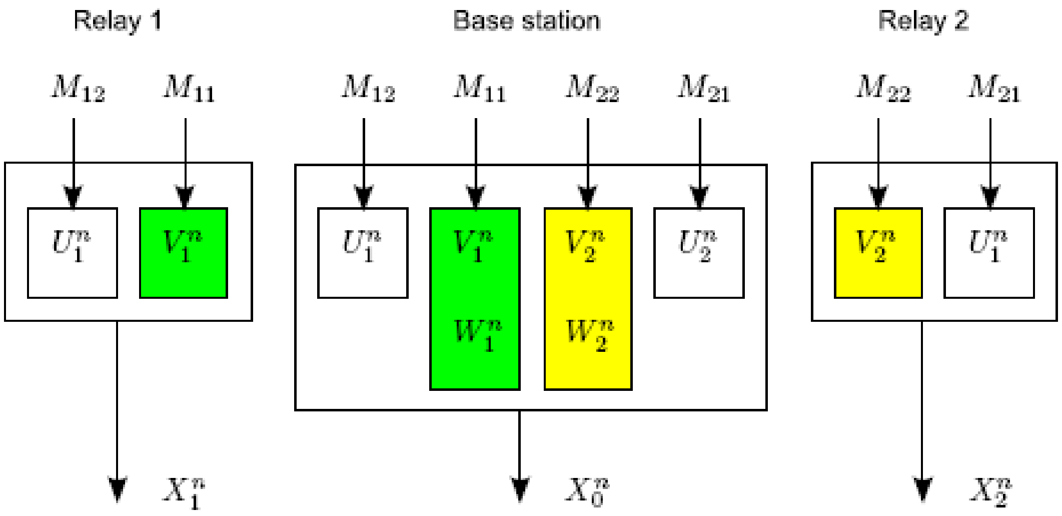
- Enhance capacity via cognitive relays
  - Cognitive relays overhear the source messages
  - Cognitive relays then cooperate with the transmitter in the transmission of the source messages

Channel Model



- **Sender (Base Station) wishes to send two independent messages to two receivers**
- **Messages uniformly generated**
- **Each cognitive relay knows only one of the messages to send**

Coding Scheme for the BCCR



- **Each message split into two parts: common and private**
- **Cognitive relays cooperate with the base station to transmit the respective common messages**
- **Each private message encoded with two layers**
  - **Inner layer exposed to the respective relay**
  - **Outer layer pre-codes for interference (GGP coding)**

Achievable Rate Region

- **Joint probability distribution**

$$p(u_1)p(u_2)p(v_1|u_1)p(v_2|u_2)p(w_1,w_2|v_1,v_2,u_1,u_2)p(x_1|v_1,u_1)p(x_2|v_2,u_2)p(x_0|w_1,w_2,v_1,v_2,u_1,u_2)$$

- **Achievable rate region: all rates ($R_{12}+R_{11}, R_{21}+R_{22}$) s.t.**

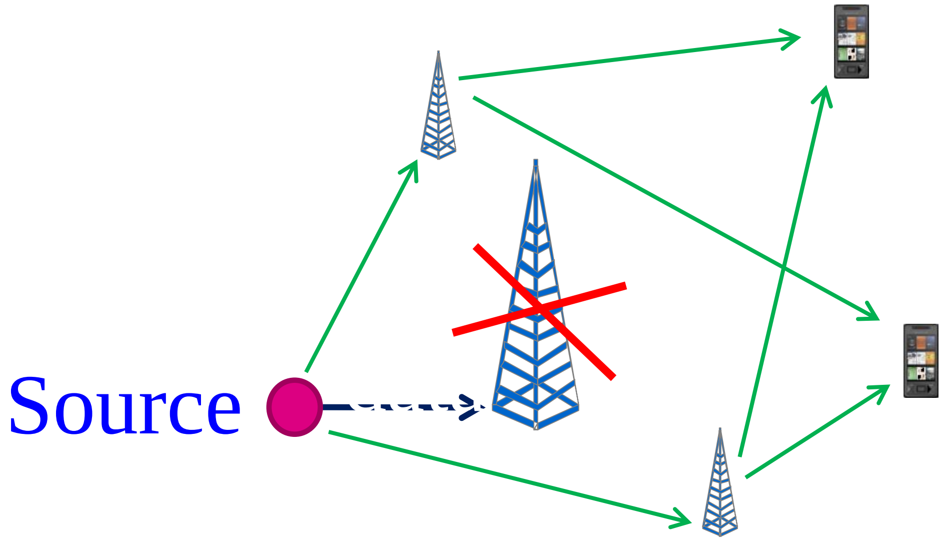
$$\begin{aligned}
&R_{11} - L_{11} \le -I(W_1; V_2|V_1,U_1,U_2),\\
&R_{22} - L_{22} \le -I(W_2; V_1|V_2,U_1,U_2),\\
&R_{11} - L_{11} + R_{22} - L_{22} \le -(I(W_1; V_2|V_1,U_1,U_2) + I(W_2; V_1|V_2,U_1,U_2) + I(W_2;W_1|V_1, V_2,U_1,U_2)),\\
&L_{11} \le I(V_1,W_1; Y_1|U_1,U_2),\\
&R_{12} + L_{11} \le I(U_1, V_1,W_1; Y_1|U_2),\\
&L_{11} + R_{21} \le I(V_1,W_1,U_2; Y_1|U_1),\\
&R_{12} + L_{11} + R_{21} \le I(U_1, V_1,W_1,U_2; Y_1),\\
&L_{22} \le I(V_2,W_2; Y_2|U_2,U_1),\\
&R_{21} + L_{22} \le I(U_2, V_2,W_2; Y_2|U_1),\\
&L_{22} + R_{12} \le I(V_2,W_2,U_1; Y_2|U_2),\\
&R_{21} + L_{22} + R_{12} \le I(U_2, V_2,W_2,U_1; Y_2).
\end{aligned}$$

Generality of the Result

- Without the cognitive relays
  - BCCR reduces to a generic BC
  - Correspondingly, rate region reduces to Marton's region for the BC (best region to date for the BCs)
- Without the base station
  - BCCR reduces to an IC
  - Correspondingly, rate region reduces to the Han-Kobayashi Region for the IC (best region for ICs)

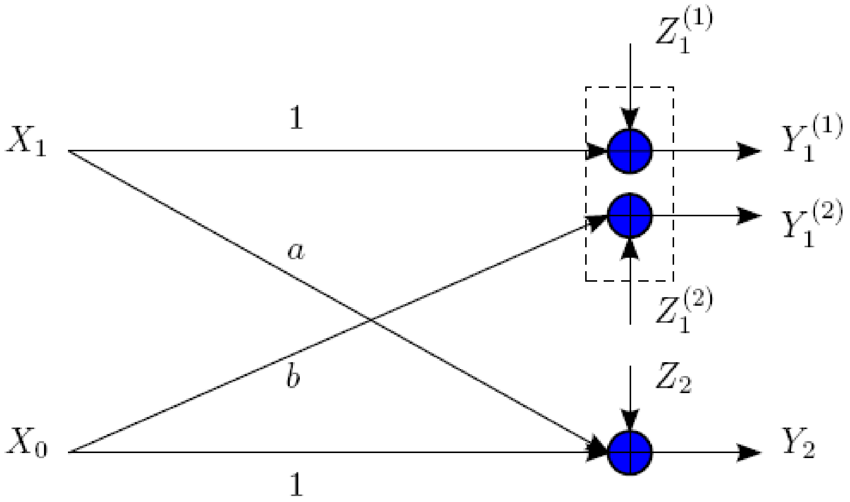
Improved Robustness

- Without cognitive relays
  - When base station is gone, the entire transmission is dead
- With cognitive relays
  - When base station is gone, cognitive relays can pick up the role of base station, and the ongoing transmission continues
  - Cognitive relays and the receivers form an interference channel



A Numerical Example

- **Special Gaussian configuration with a single cognitive relay,**



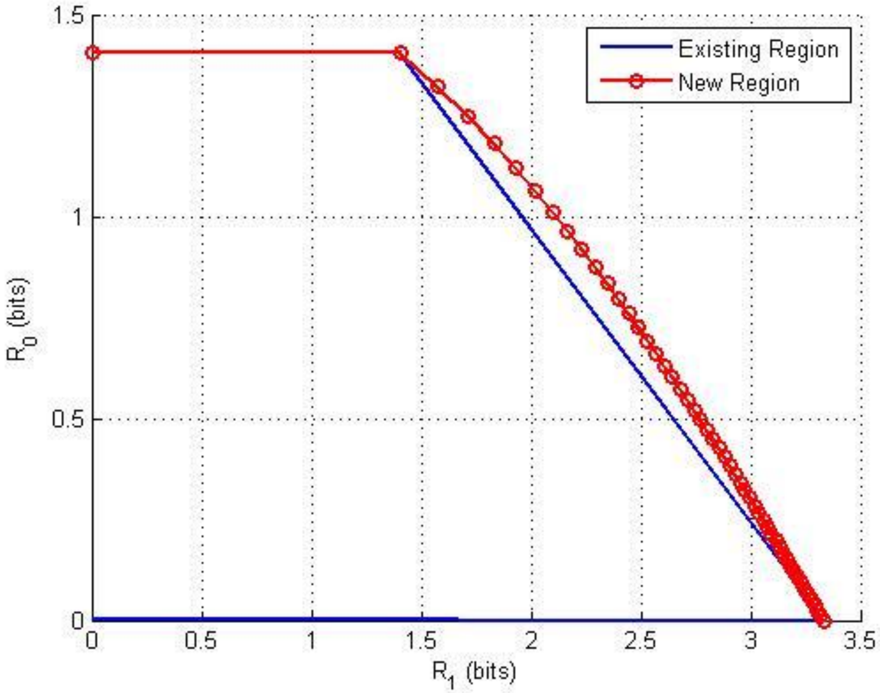
$$Y_1^{(1)} = X_1 + Z_1^{(1)}$$
$$Y_1^{(2)} = bX_0 + Z_1^{(2)} \quad (b \geq 1)$$
$$Y_2 = X_0 + aX_1 + Z_2$$

- **Rate region for this special case (obtained from region for BCCR)**

$$R_1 \leq \frac{1}{2}\log_2(1+P_1) + \frac{1}{2}\log_2(1+(1-\alpha)b^2P_0),$$
$$R_2 \leq \frac{1}{2}\log_2\left(1+\frac{\alpha P_0}{(1-\alpha)P_0+1}\right).$$

A Numerical Example

- **No existing rate region can be specialized to this region for the example except [Sridharan et al'08]**
- **This rate region also demonstrates strict improvement over one of the best known region for the cognitive radio channel**



Channel parameters:
$a = 0.5$, $b = 1.5$;
$P_1 = 6$, $P_2 = 6$;

Overlay Challenges

- Complexity of transmission and detection
- Obtaining information about channel, other user's messages, etc.
- Full-duplex vs. half duplex
- Synchronization
- And many more …

Summary

- Cognition can substantially increase capacity
  - Can be applied to many types of systems
- Capacity of cognitive channels uses all "tricks" from broadcast, MAC, interference channels
- Many idealized assumptions used in obtaining capacity
- Very interesting to reduce these ideas to practice

Decentralized Cognitive MAC for Opportunistic Spectrum Access in Ad-Hoc Networks: A POMDP Framework

- Authors: Qing Zhao, Lang Tong, Anathram Swami, and Yunxia Chen
- Presented by: Kun Yi