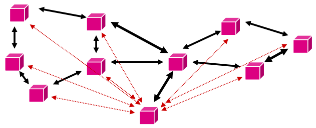
EE360: Lecture 10 Outline
Capacity of Ad Hoc Nets

- Announcements
  - Revised proposals due tomorrow
  - HW 1 posted, due Feb. 24 at 5pm
- Definition of ad hoc network capacity
- Capacity regions
- Scaling laws and extensions
- Achievable rate regions
- Capacity under cooperation
- Interference alignment
- Cross layer design

Ad-Hoc Network Capacity



- Fundamental limits on the maximum possible rates between all possible node pairs with vanishing probability of error
- Independent of transmission and reception strategies (modulation, coding, routing, etc.)
- Dependent on propagation, node capabilities (e.g. MIMO), transmit power, noise, etc

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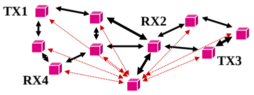
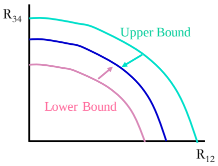
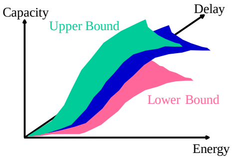
Network Capacity:
What is it?

- n(n-1)-dimensional region
  - Rates between all node pairs
  - Upper/lower bounds
    - Lower bounds achievable
    - Upper bounds hard



- Other possible axes
  - Energy and delay





Fundamental Network Capacity
The Shangri-La of Information Theory

- Much progress in finding the capacity limits of wireless single and multiuser channels
- Limited understanding about the capacity limits of wireless networks, even for simple models
- System assumptions such as constrained energy and delay may require new capacity definitions
- Is this elusive goal the right thing to pursue?

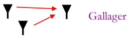
Shangri-La is synonymous with any earthly paradise;
a permanently happy land, isolated from the outside world

Some capacity questions

- How to parameterize the region
  - Power/bandwidth
  - Channel models and CSI
  - Outage probability
  - Security/robustness
- Defining capacity in terms of asymptotically small error and infinite delay has been highly enabling
  - Has also been limiting
    - Cause of unconsummated union in networks and IT
  - What is the alternative?

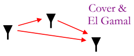
Network Capacity Results

- Multiple access channel (MAC)

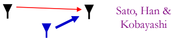


- Broadcast channel

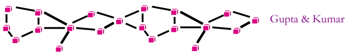
- Relay channel upper/lower bounds



- Strong interference channel



- Scaling laws



- Achievable rates for small networks

Capacity for Large Networks
(Gupta/Kumar'00)

- Make some simplifications and ask for less
  - Each node has only a single destination
  - All nodes create traffic for their desired destination at a uniform rate λ
  - Capacity (throughput) is maximum λ that can be supported by the network (1 dimensional)
- Throughput of random networks
  - Network topology/packet destinations random.
  - Throughput λ is random: characterized by its distribution as a function of network size n.
- Find scaling laws for C(n)=λ as n →∞.

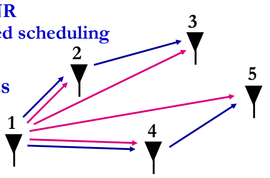
Extensions

- Fixed network topologies (Gupta/Kumar'01)
  - Similar throughput bounds as random networks
- Mobility in the network (Grossglauser/Tse'01)
  - Mobiles pass message to neighboring nodes, eventually neighbor gets close to destination and forwards message
  - Per-node throughput constant, aggregate throughput of order n, delay of order n.

- Throughput/delay tradeoffs
  - Piecewise linear model for throughput-delay tradeoff (ElGamal et. al'04, Toumpis/Goldsmith'04)
  - Finite delay requires throughput penalty.
- Achievable rates with multiuser coding/decoding (GK'03)
  - Per-node throughput (bit-meters/sec) constant, aggregate infinite.
  - Rajiv will provide more details

Ad Hoc Network Achievable Rate Regions

- All achievable rate vectors between nodes
  - Lower bounds Shannon capacity
- An n(n-1) dimensional convex polyhedron
  - Each dimension defines (net) rate from one node to each of the others
  - Time-division strategy
  - Link rates adapt to link SINR
  - Optimal MAC via centralized scheduling
  - Optimal routing
- Yields performance bounds
  - Evaluate existing protocols
  - Develop new protocols



Achievable Rates

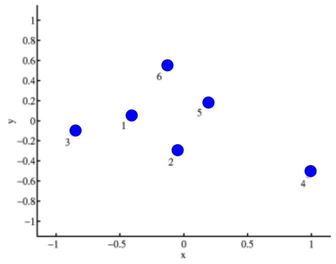
Achievable rate vectors achieved by time division ➡ Capacity region is convex hull of all rate matrices

- A matrix R belongs to the capacity region if there are rate matrices R_1, R_2, R_3 ,…, R_n such that

$$R=\sum_{i=1}^{n}\alpha_i R_i;\quad \sum_{i=1}^{n}\alpha_i\le 1;\ \alpha_i>0$$

- Linear programming problem:
  - Need clever techniques to reduce complexity
  - Power control, fading, etc., easily incorporated
  - Region boundary achieved with optimal routing

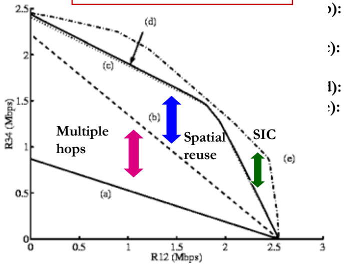
Example: Six Node Network



Capacity region is 30-dimensional

Capacity Region Slice
(*6 Node Network)*

$$\boxed{R_{ij}=0,\quad ij\neq 12{,}34,\quad i\neq j}$$

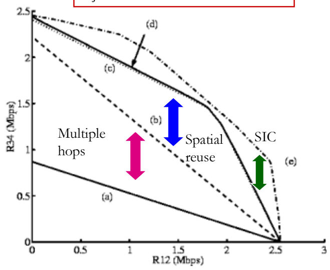


(a): Single hop, no simultaneous transmissions.
): Multihop, no simultaneous transmissions.
): Multihop, simultaneous transmissions.
): Adding power control
): Successive interference cancellation, no power control.

Extensions:
- Capacity vs. network size
- Capacity vs. topology
- Fading and mobility
- Multihop cellular

Achievable Region Slice (*6 Node Network*)

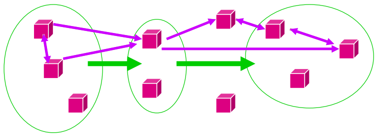
$R_{ij} = 0, \quad ij \neq 12{,}34, \quad i \neq j$



(a): Single hop, no simultaneous transmissions.
(b): Multihop, no simultaneous transmissions.
(c): Multihop, simultaneous transmissions.
(d): Adding power control
(e): Successive IC, no power control.

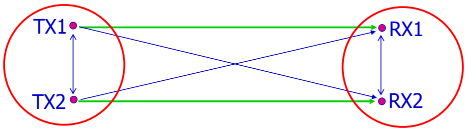
Joint work with S. Toumpis

Cooperation in Wireless Networks



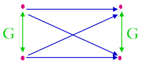
- Routing is a simple form of cooperation
- Many more complex ways to cooperate:
  - Virtual MIMO , generalized relaying, interference forwarding, and one-shot/iterative conferencing
- Many theoretical and practice issues:
  - Overhead, forming groups, dynamics, synch, …

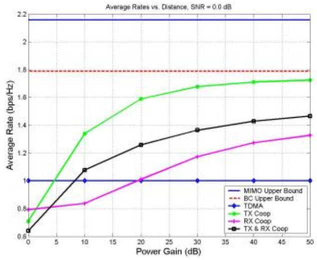
Virtual MIMO



- TX1 sends to RX1, TX2 sends to RX2
- TX1 and TX2 cooperation leads to a MIMO BC
- RX1 and RX2 cooperation leads to a MIMO MAC
- TX and RX cooperation leads to a MIMO channel
- Power and bandwidth spent for cooperation

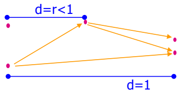
Capacity Gain with Cooperation (2x2)

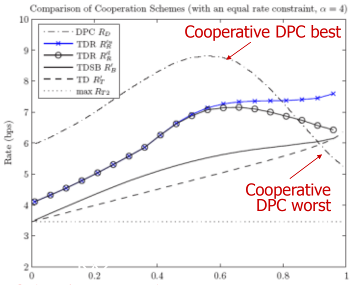




- TX cooperation needs large cooperative channel gain to approach broadcast channel bound
- MIMO bound unapproachable

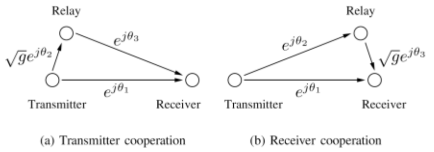
Capacity Gain vs Network Topology





Optimal cooperation coupled with access and routing

Relative Benefits of TX and RX Cooperation



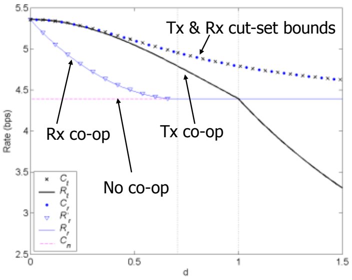
(a) Transmitter cooperation (b) Receiver cooperation

- Two possible CSI models:
  - Each node has full CSI (synchronization between Tx and relay).
  - Receiver phase CSI only (no TX-relay synchronization).
- Two possible power allocation models:
  - Optimal power allocation: Tx has power constraint *aP*, and relay (1-*a*)*P* ; 0≤*a*≤1 needs to be optimized.
  - Equal power allocation (*a* = ½).

Joint work with C. Ng

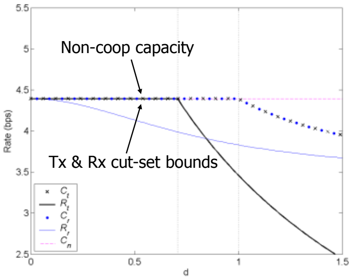
Example 1: Optimal power allocation with full CSI

- **Cut-set bounds are equal.**
- **Tx co-op rate is close to the bounds.**
- **Transmitter cooperation is preferable.**



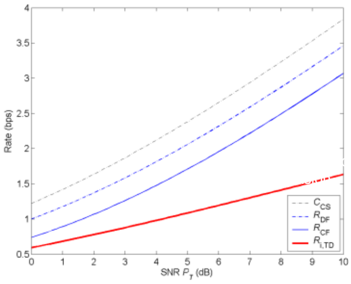
Example 2: Equal power allocation with RX phase CSI

- **Non-cooperative capacity meets the cut-set bounds of Tx and Rx co-op.**
- **Cooperation offers no capacity gain.**



Capacity: Non-orthogonal Relay Channel

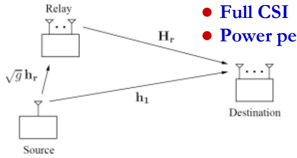
- **Compare rates to a full-duplex relay channel.**
- **Realize conference links via time-division.**
- **Orthogonal scheme suffers a considerable performance loss, which is aggravated as SNR increases.**



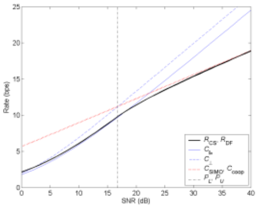
Transmitter vs. Receiver Cooperation

- **Capacity gain only realized with the right cooperation strategy**
- **With full CSI, Tx co-op is superior.**
- **With optimal power allocation and receiver phase CSI, Rx co-op is superior.**
- **With equal power allocation and Rx phase CSI, cooperation offers no capacity gain.**
- **Similar observations in Rayleigh fading channels.**

Multiple-Antenna Relay Channel



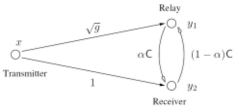
- **Full CSI**
- **Power per transmit antenna: *P/M*.**



- Single-antenna source and relay
- Two-antenna destination
  - SNR < P_L: MIMO Gain
  - SNR > P_U: No multiplexing gain; can't exceed SIMO channel capacity (Host-Madsen'05)

Joint work with C. Ng and N. Laneman

Conferencing Relay Channel

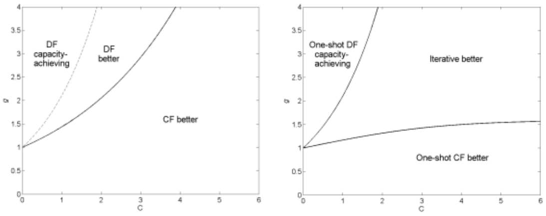


$$y_1 = \sqrt{g}\,x + n_1$$

$$y_2 = x + n_2$$

- **Willems introduced conferencing for MAC (1983)**
  - **Transmitters conference before sending message**
- **We consider a relay channel with conferencing between the relay and destination**
- **The conferencing link has total capacity C which can be allocated between the two directions**

Iterative vs. One-shot Conferencing

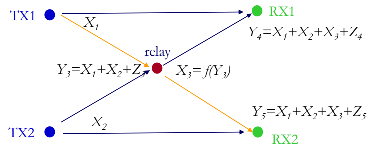


- Weak relay channel: the iterative scheme is disadvantageous.
- Strong relay channel: iterative outperforms one-shot conferencing for large C.

Lessons Learned

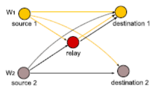
- Orthogonalization has considerable capacity loss
  - Applicable for clusters, since cooperation band can be reused spatially.
- DF vs. CF
  - DF: nearly optimal when transmitter and relay are close
  - CF: nearly optimal when transmitter and relay far
  - CF: not sensitive to compression scheme, but poor spectral efficiency as transmitter and relay do not joint-encode.
- The role of SNR
  - High SNR: rate requirement on cooperation messages increases.
  - MIMO-gain region: cooperative system performs as well as MIMO system with isotropic inputs.

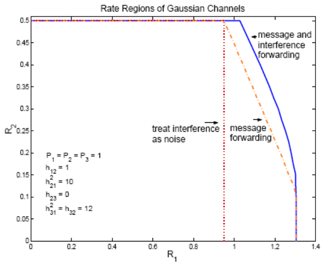
Generalized Relaying



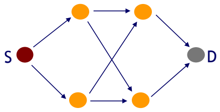
- Can forward message and/or interference
  - Relay can forward all or part of the messages
    - Much room for innovation
  - Relay can forward interference
    - To help subtract it out

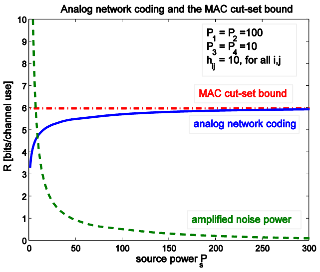
Beneficial to forward both interference and message





In fact, it can achieve capacity





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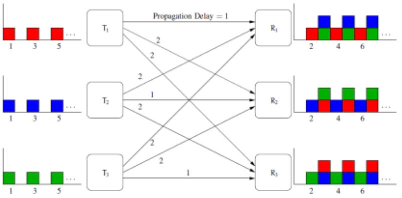
Interference Alignment

- Addresses the number of interference-free signaling dimensions in an interference channel
- Based on our orthogonal analysis earlier, it would appear that resources need to be divided evenly, so only 2BT/N dimensions available
- Jafar and Cadambe showed that by aligning interference, 2BT/2 dimensions are available
- Everyone gets half the cake!

Basic Premise

- For any number of TXs and RXs, each TX can transmit half the time and be received without any interference
  - Assume different delay for each transmitter-receiver pair
  - Delay odd when message from TX *i* desired by RX *j*; even otherwise.
  - Each TX transmits during odd time slots and is silent at other times.
  - All interference is aligned in even time slots.



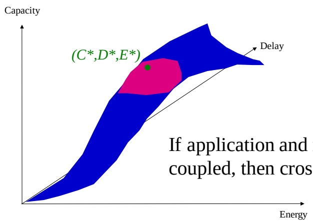
Extensions

- Multipath channels
- Fading channels
- MIMO channels
- Cellular systems
- Imperfect channel knowledge
- …

Is a capacity region all we need to design networks?

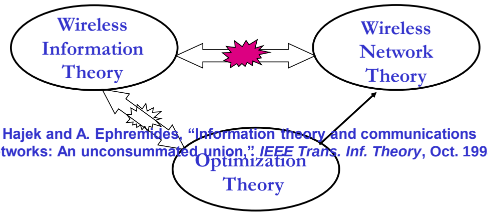
Yes, if the application and network design can be decoupled

Application metric: *f(C,D,E)*: *(C*,D*,E*)=arg max f(C,D,E)*



If application and network design are coupled, then cross-layer design needed

Limitations in theory of ad hoc networks today



B. Hajek and A. Ephremides, "Information theory and communications networks: An unconsummated union," *IEEE Trans. Inf. Theory*, Oct. 1998.

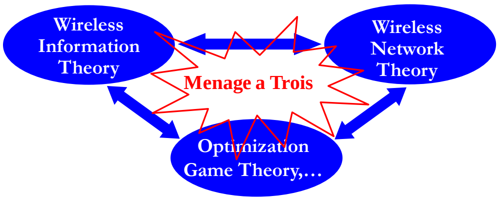
- Shannon capacity pessimistic for wireless channels and intractable for large networks

– Large body of wireless (and wired) network theory that is ad-hoc, lacks a basis in fundamentals, and lacks an objective success criteria.

– Little cross-disciplinary work spanning these fields

– Optimization techniques applied to given network models, which rarely take into account fundamental network capacity or dynamics

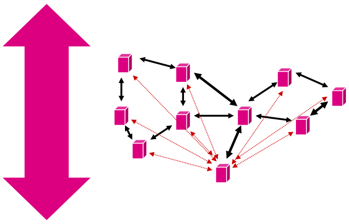
Consummating Unions



- When capacity is not the only metric, a new theory is needed to deal with nonasymptopia (i.e. delay, random traffic) and application requirements
  - Shannon theory generally breaks down when delay, error, or user/traffic dynamics must be considered
- Fundamental limits are needed outside asymptotic regimes
- Optimization, game theory, and other techniques provide the missing link

Crosslayer Design in Ad-Hoc Wireless Networks

- Application
- Network
- Access
- Link
- Hardware

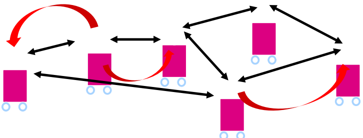


Substantial gains in throughput, efficiency, and end-to-end performance from cross-layer design

Why a crosslayer design?

- The technical challenges of future mobile networks cannot be met with a layered design approach.
- QoS cannot be provided unless it is supported across all layers of the network.
  - The application must adapt to the underlying channel and network characteristics.
  - The network and link must adapt to the application requirements
- Interactions across network layers must be understood and exploited.

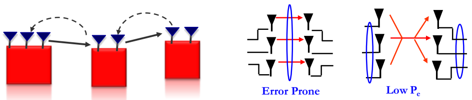
How to use Feedback in Wireless Networks



- Types of Feedback
  - Output feedback
  - CSI
  - Acknowledgements
  - Network/traffic information
  - Something else

  } Noisy/Compressed
- What is the metric to be improved by feedback
  - Capacity
  - Delay
  - Other

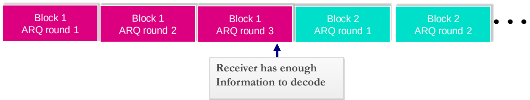
Diversity-Multiplexing-Delay Tradeoffs for MIMO Multihop Networks with ARQ



- MIMO used to increase data rate or robustness
- Multihop relays used for coverage extension
- ARQ protocol:
  - Can be viewed as 1 bit feedback, or time diversity,
  - Retransmission causes delay (can design ARQ to control delay)
- Diversity multiplexing (delay) tradeoff - DMT/DMDT
  - Tradeoff between robustness, throughput, and delay

Multihop ARQ Protocols

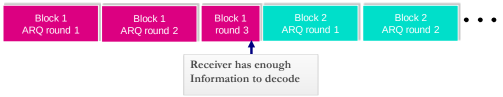
- Fixed ARQ: fixed window size
  - Maximum allowed ARQ round for ith hop L_i satisfies $\sum_{i=1}^{N} L_i \leq L$
- Adaptive ARQ: adaptive window size
  - Fixed Block Length (FBL) (block-based feedback, easy synchronization)



Block 1 ARQ round 1 | Block 1 ARQ round 2 | Block 1 ARQ round 3 | Block 2 ARQ round 1 | Block 2 ARQ round 2 • • •

Receiver has enough Information to decode

  - Variable Block Length (VBL) (real time feedback)

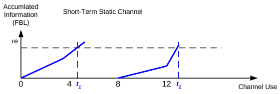


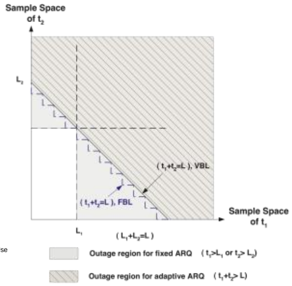
Block 1 ARQ round 1 | Block 1 ARQ round 2 | Block 1 round 3 | Block 2 ARQ round 1 | Block 2 ARQ round 2 • • •

Receiver has enough Information to decode

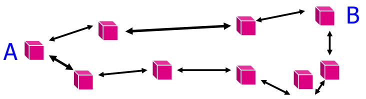
Asymptotic DMDT Optimality

- Theorem: VBL ARQ achieves optimal DMDT in MIMO multihop relay networks in long-term and short-term static channels.
- Proved by cut-set bound
- An intuitive explanation by stopping times: VBL ARQ has the smaller outage regions among multihop ARQ protocols



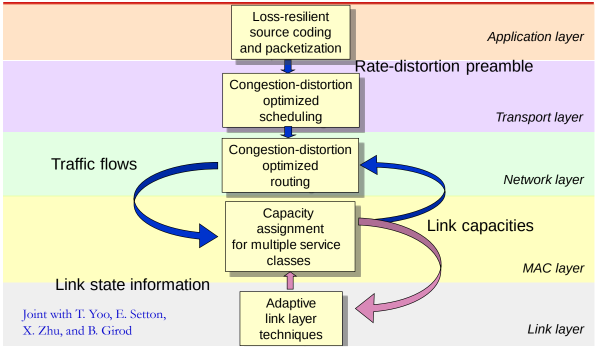


Delay/Throughput/Robustness across Multiple Layers



- Multiple routes through the network can be used for multiplexing or reduced delay/loss
- Application can use single-description or multiple description codes
- Can optimize optimal operating point for these tradeoffs to minimize distortion

Cross-layer protocol design for real-time media



Loss-resilient source coding and packetization

Application layer

Rate-distortion preamble

Congestion-distortion optimized scheduling

Transport layer

Traffic flows

Congestion-distortion optimized routing

Network layer

Capacity assignment for multiple service classes

Link capacities

MAC layer

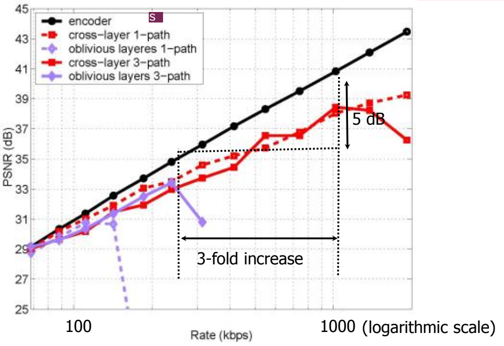
Link state information

Adaptive link layer techniques

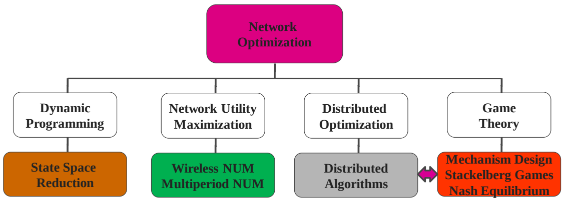
Link layer

Joint with T. Yoo, E. Setton, X. Zhu, and B. Girod

Video streaming performance



Approaches to Cross-Layer Resource Allocation*



Network Optimization

Dynamic Programming

Network Utility Maximization

Distributed Optimization

Game Theory

State Space Reduction

Wireless NUM Multiperiod NUM

Distributed Algorithms

Mechanism Design Stackelberg Games Nash Equilibrium

*Much prior work is for wired/static networks

Network Utility Maximization

- Maximizes a network utility function

flow k

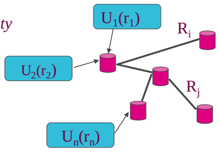
$$\max \quad \sum U_k(r_k)$$

$$s.t. \quad Ar \le R$$

routing

Fixed link capacity

- Assumes
  - Steady state
  - Reliable links
  - Fixed link capacities



- Dynamics are only in the queues

Wireless NUM

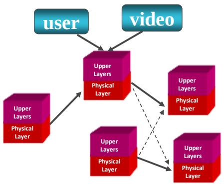
- Extends NUM to random environments
- Network operation as stochastic optimization algorithm

$$\max \quad E[\sum U(r_m(G))]$$

$$\text{st}$$

$$E[r(G)] \le E[R(S(G),G)]$$

$$E[S(G)] \le \bar{S}$$

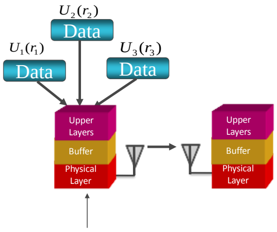


WNUM Policies

- Control network resources
- Inputs:
  - Random network channel information G^k
  - Network parameters
  - Other policies
- Outputs:
  - Control parameters
  - Optimized performance, that
  - Meet constraints
- Channel sample driven policies

Example: NUM and Adaptive Modulation

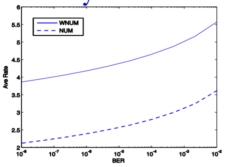
- Policies
  - Information rate
  - Tx power
  - Tx Rate
  - Tx code rate
- Policy adapts to
  - Changing channel conditions
  - Packet backlog
  - Historical power usage

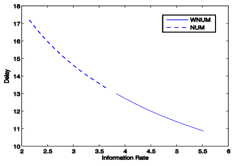


Block codes used

Rate-Delay-Reliability

- Policy Results





Game theory

- Coordinating user actions in a large ad-hoc network can be infeasible
- Distributed control difficult to derive and computationally complex
- Game theory provides a new paradigm
  - Users act to "win" game or reach an equilibrium
  - Users heterogeneous and non-cooperative
  - Local competition can yield optimal outcomes
  - Dynamics impact equilibrium and outcome
  - Adaptation via game theory

Summary

- Capacity of wireless ad hoc networks largely unknown, even for simple canonical models.
- Scaling laws, degrees of freedom (interference alignment) and other approximations promising
- Capacity not the only metric of interest
- Cross layer design requires new tools such as optimization and game theory

Presentation

- "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks" by Ayfer Ozgur, Olivier Leveque, and David N. C. Tse
- Presented by Alexandros Manolakos