User Cooperation Diversity— System Description and Implementation Aspects and Performance Analysis

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Presented by

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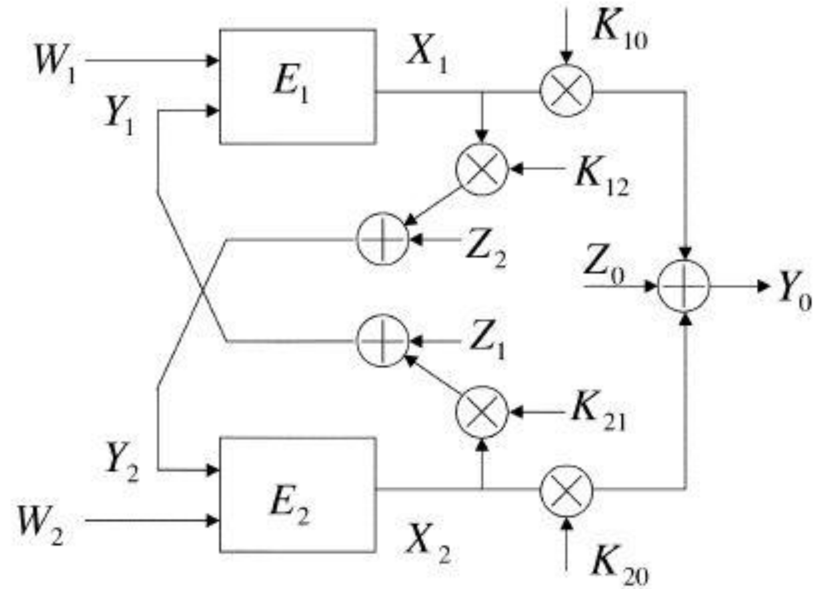
Introduction

- Proposes a method to achieve spatial diversity via cooperation of mobile users.
- Part 1 describes the user cooperation strategy i.e. the information theoretic capacity, outage and coverage analysis and proposes a cooperation scheme for a CDMA system
- Part 2 analyses capacity, outage and coverage analysis for the proposed CDMA scheme and receiver design

Channel model and problem set up

- Describes the scheme for a cellular system
- Considers a MAC (though it is applicable to other wireless systems) where two users have their own information to send and want to cooperate to send this information to the receiver at the highest rate possible.



$$Y_0(t) = K_{10}X_1(t) + K_{20}X_2(t) + Z_0(t)$$
$$Y_1(t) = K_{21}X_2(t) + Z_1(t)$$
$$Y_2(t) = K_{12}X_1(t) + Z_2(t)$$

Achievable rate region

- Cooperation strategy based on superposition block Markov encoding and backward decoding.
- Transmission is done for *B* blocks of length *n*
- Mobile divides its information into two parts: data to be sent directly to the BS and data to be sent via its partner.

$$X_1 = X_{10} + X_{12} + U_1$$

- Power allocation according to

$$P_1 = P_{10} + P_{12} + P_{U1}$$

- Achievable rates given by,

$$R_{12} < \mathrm{E}\left\{C\left(\frac{K_{12}^2 P_{12}}{K_{12}^2 P_{10} + \Xi_1}\right)\right\}$$

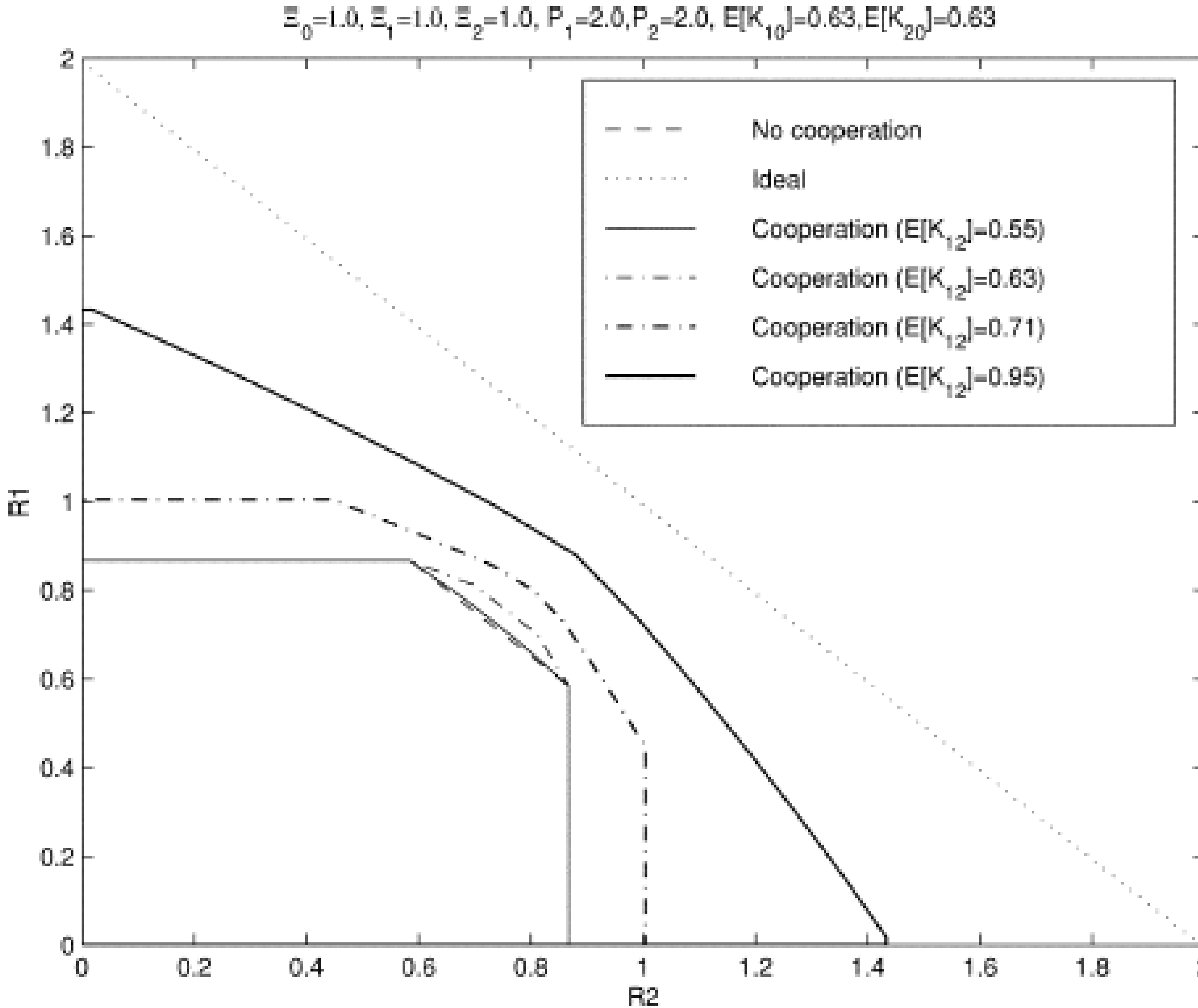
$$R_{21} < \mathrm{E}\left\{C\left(\frac{K_{21}^2 P_{21}}{K_{21}^2 P_{20} + \Xi_2}\right)\right\}$$

$$R_{10} < \mathrm{E}\left\{C\left(\frac{K_{10}^2 P_{10}}{\Xi_0}\right)\right\}$$

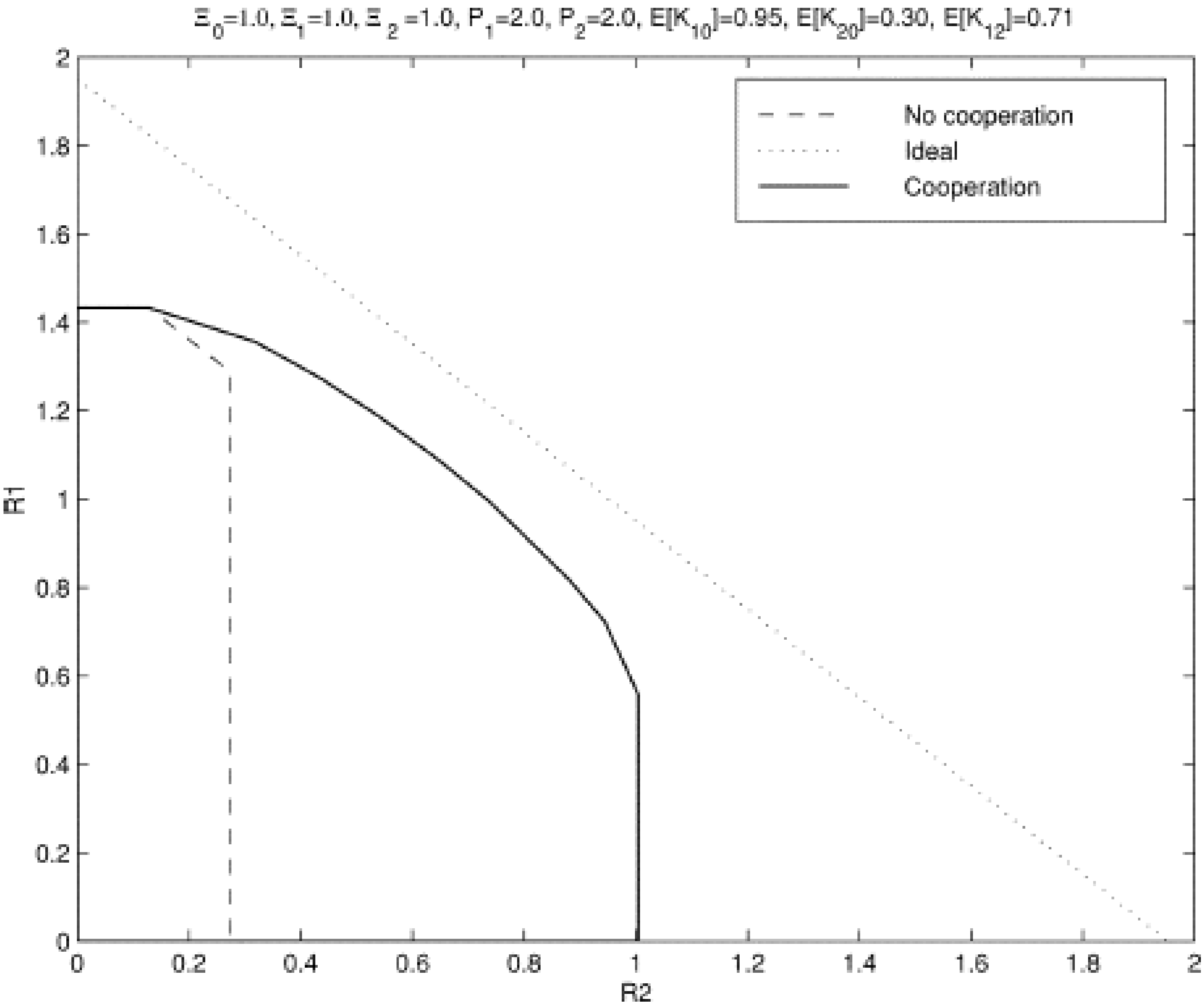
$$R_{20} < \mathrm{E}\left\{C\left(\frac{K_{20}^2 P_{20}}{\Xi_0}\right)\right\}$$

$$R_{10} + R_{20} < \mathrm{E}\left\{C\left(\frac{K_{10}^2 P_{10} + K_{20}^2 P_{20}}{\Xi_0}\right)\right\}$$

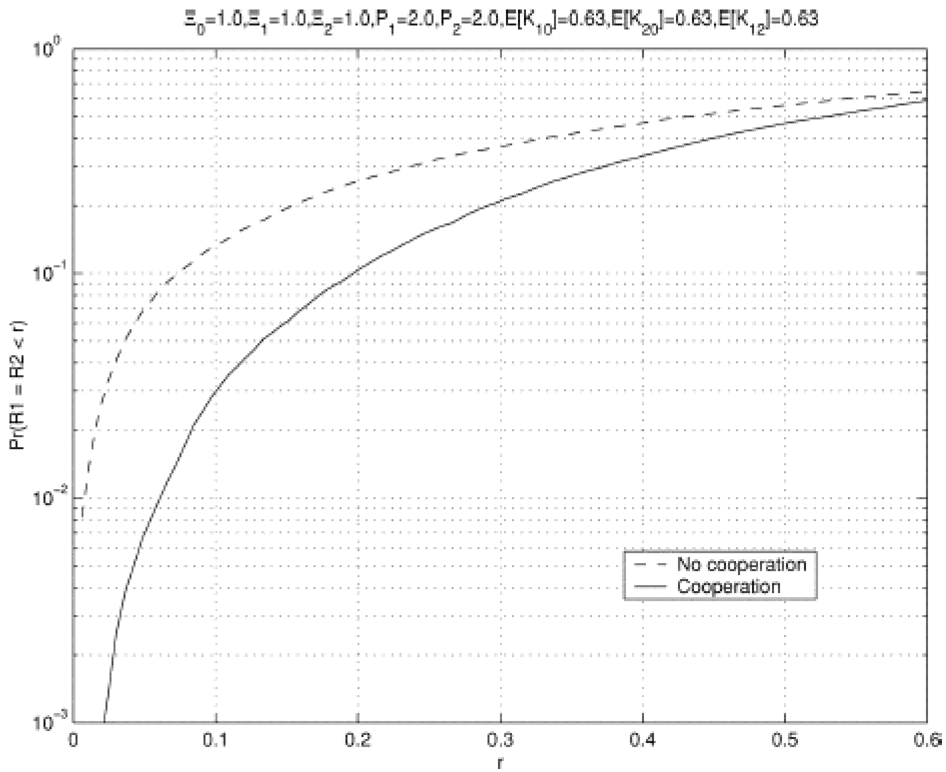
$$R_{10} + R_{20} + R_{12} + R_{21} < \mathrm{E}\left\{C\left(\frac{K_{10}^2 P_1 + K_{20}^2 P_2 + 2K_{10}K_{20}\sqrt{P_{U1}P_{U2}}}{\Xi_0}\right)\right\}$$



Ξ0=1.0, Ξ1=1.0, Ξ2=1.0, P1=2.0,P2=2.0, E[K10]=0.63,E[K20]=0.63
No cooperation
Ideal
Cooperation (E[K12]=0.55)
Cooperation (E[K12]=0.63)
Cooperation (E[K12]=0.71)
Cooperation (E[K12]=0.95)
R1
R2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Ξ_0=1.0, Ξ_1=1.0, Ξ_2=1.0, P_1=2.0, P_2=2.0, E[K_{10}]=0.95, E[K_{20}]=0.30, E[K_{12}]=0.71
No cooperation
Ideal
Cooperation
R1
R2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

Probability of Outage



Probability of outage v/s service sustainability rate, *r*, for the equal rate point where users have statistically similar channels toward the BS.

Cellular Coverage

- Increase the power in the non-cooperative strategy to achieve the same sum capacity as the cooperative strategy.
- If $R^{c}_{sum}(P) = \beta R^{n}_{sum}(P')$, find P' such that

$$R^{n}_{sum}(P') = \beta R^{n}_{sum}(P)$$

$$\text{Increase in area coverage} \approx \left(\frac{2.62^{\beta} - 1}{1.62}\right)^{2/3.38}$$

$\mathrm{E}[K_{12}]$	Increase in sum capacity	Increase in coverage area (analysis)	Increase in coverage area (simulation)
0.71	11.9%	11.9%	12.2%
0.95	21.3%	21.3%	22.0%

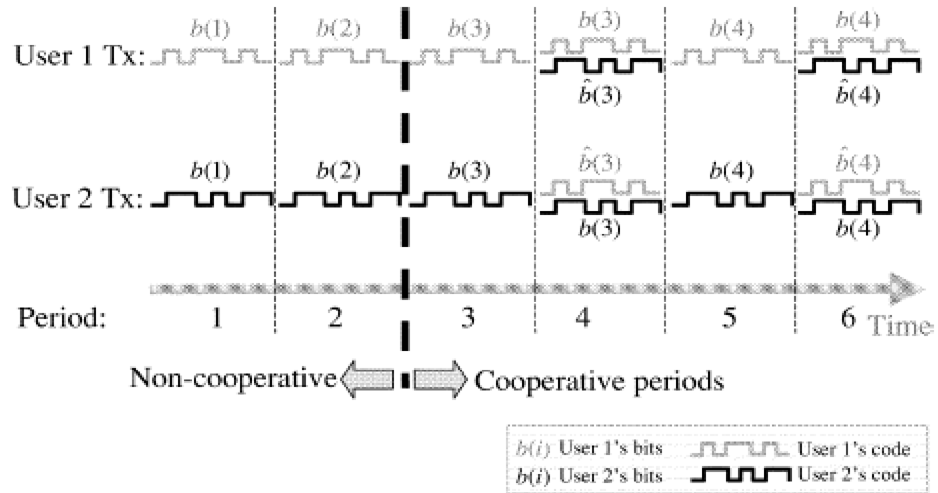
CDMA implementation

- A CDMA system in which each user has one spreading code and modulates one bit onto it. Assumes that the users' codes are orthogonal and that the coherence time of the channel is *L* symbol periods.
- For L= 3, under the cooperative scheme, the users would transmit

$$X_1(t) = a_{11} b_1^{(1)} c_1(t),\ a_{12} b_1^{(2)} c_1(t),\ a_{13} b_1^{(2)} c_1(t) + a_{14} \hat{b}_2^{(2)} c_2(t)$$

$$X_2(t) = \underbrace{a_{21} b_2^{(1)} c_2(t),}_{\text{Period 1}}\ \underbrace{a_{22} b_2^{(2)} c_2(t),}_{\text{Period 2}}\ \underbrace{a_{23} \hat{b}_1^{(2)} c_1(t) + a_{24} b_2^{(2)} c_2(t)}_{\text{Period 3}}$$

- For arbitrary L, in each L symbol period, $2L_c$ periods are used for cooperation and the remaining for non-cooperative information where L_c is some integer between 0 and L/2.



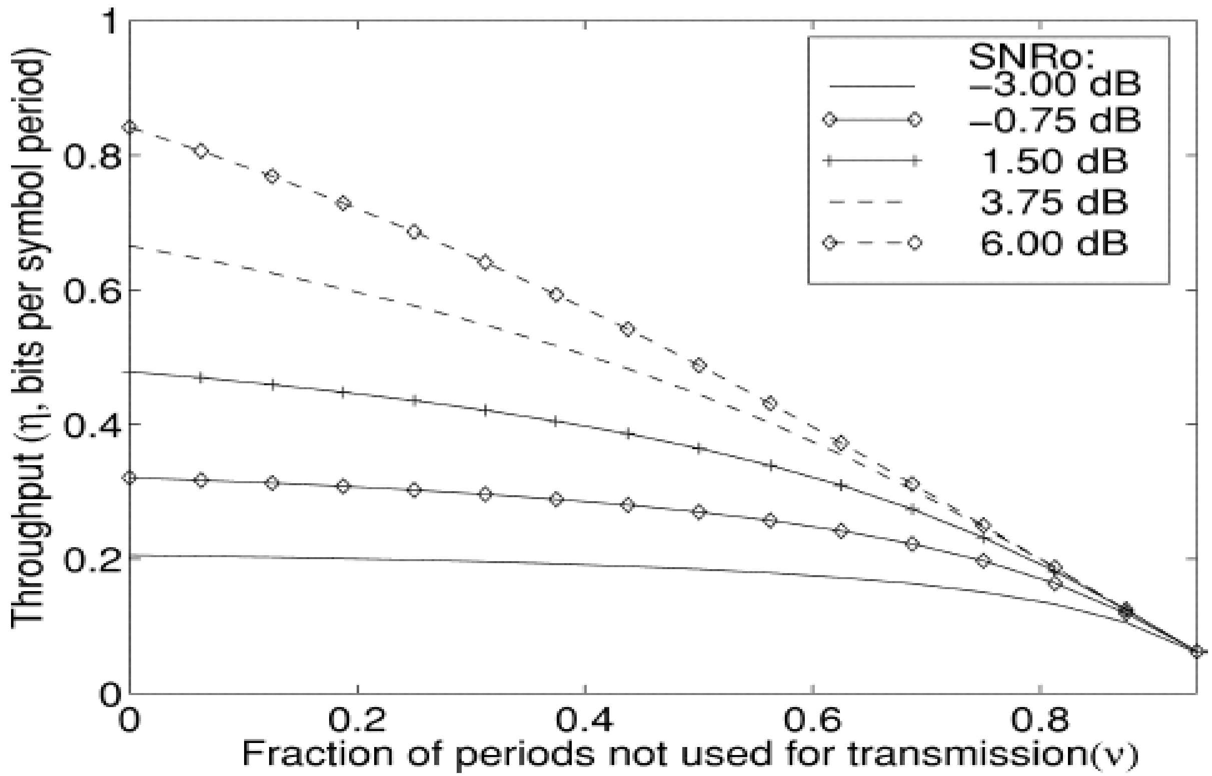
$$X_1(t) = \begin{cases} a_{11} b_1^{(i)} c_1(t), & i = 1, 2, \ldots, L_n \\ a_{12} b_1^{((L_n+1+i)/2)} c_1(t), & i = L_n+1, L_n+3, \ldots, L-1 \\ a_{13} b_1^{(L_n+i)/2} c_1(t) + a_{14} \hat{b}_2^{((L_n+i)/2)} c_2(t), & i = L_n+2, L_n+4, \ldots, L \end{cases}$$

$$X_2(t) = \begin{cases} a_{21} b_2^{(i)} c_2(t), & i = 1, 2, \ldots, L_n \\ a_{22} b_2^{((L_n+1+i)/2)} c_2(t), & i = L_n+1, L_n+3, \ldots, L-1 \\ a_{23} \hat{b}_1^{((L_n+i)/2)} c_1(t) + a_{24} b_2^{((L_n+i)/2)} c_2(t), & i = L_n+2, L_n+4, \ldots, L \end{cases}$$

Subject to the power constraint

$$\frac{1}{L}\left(L_n a_{11}^2 + L_c\left(a_{12}^2 + a_{13}^2 + a_{14}^2\right)\right) = P_1$$
$$\frac{1}{L}\left(L_n a_{21}^2 + L_c\left(a_{22}^2 + a_{23}^2 + a_{24}^2\right)\right) = P_2.$$

Throughput v/s fraction of periods not used for transmission



$$\eta = (1-\nu) C_{\mathrm{BSC}} \left(Q \left(\sqrt{\frac{\mathrm{SNR}_0}{1-\nu}} \right) \right)$$

Probability of error

- Non-cooperative period:

Receiver's detector: $\hat{b}_1 = \text{sign}\left(\frac{1}{N_c}\mathbf{c}_1^T\mathbf{Y}_0\right) = \text{sign}\left(K_{10}a_{11}b_1 + n_0\right)$

Probability of error: $P_{e_1} = Q\left(K_{10}a_{11}\frac{\sqrt{N_c}}{\sigma_0}\right).$

- Cooperative period:

Optimal detector:

$$(1-P_{e_{12}})\Lambda^{-1}e^{\mathbf{v}_1^T\mathbf{y}} + P_{e_{12}}\Lambda e^{\mathbf{v}_2^T\mathbf{y}} \underset{-1}{\overset{1}{\gtrless}} (1-P_{e_{12}})\Lambda^{-1}e^{-\mathbf{v}_1^T\mathbf{y}} + P_{e_{12}}\Lambda e^{-\mathbf{v}_2^T\mathbf{y}}$$

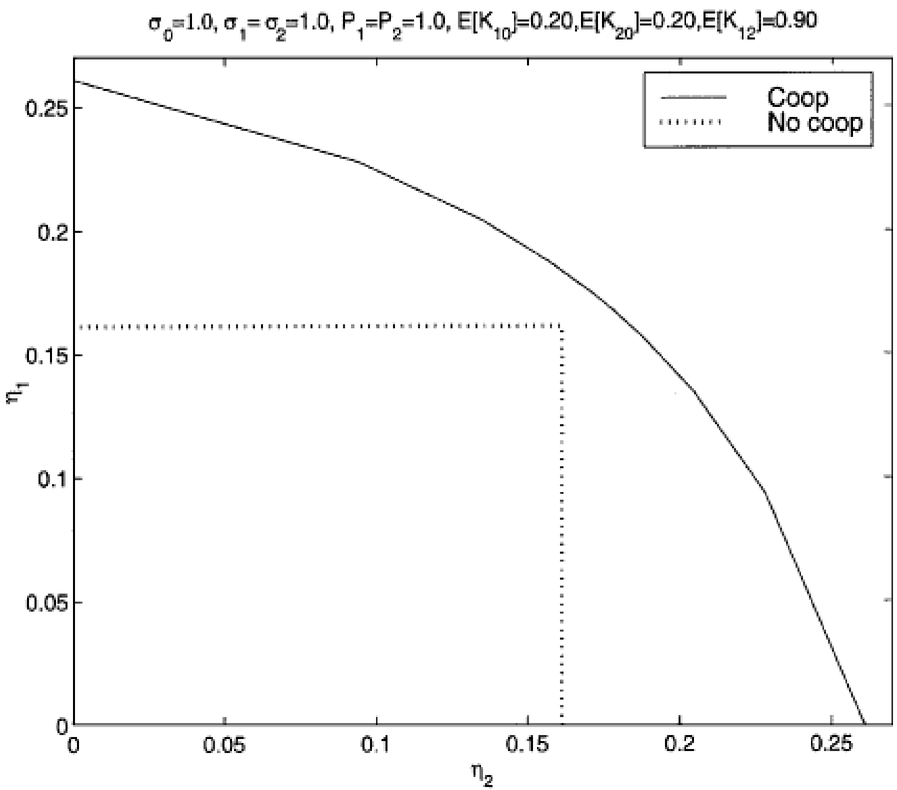
where

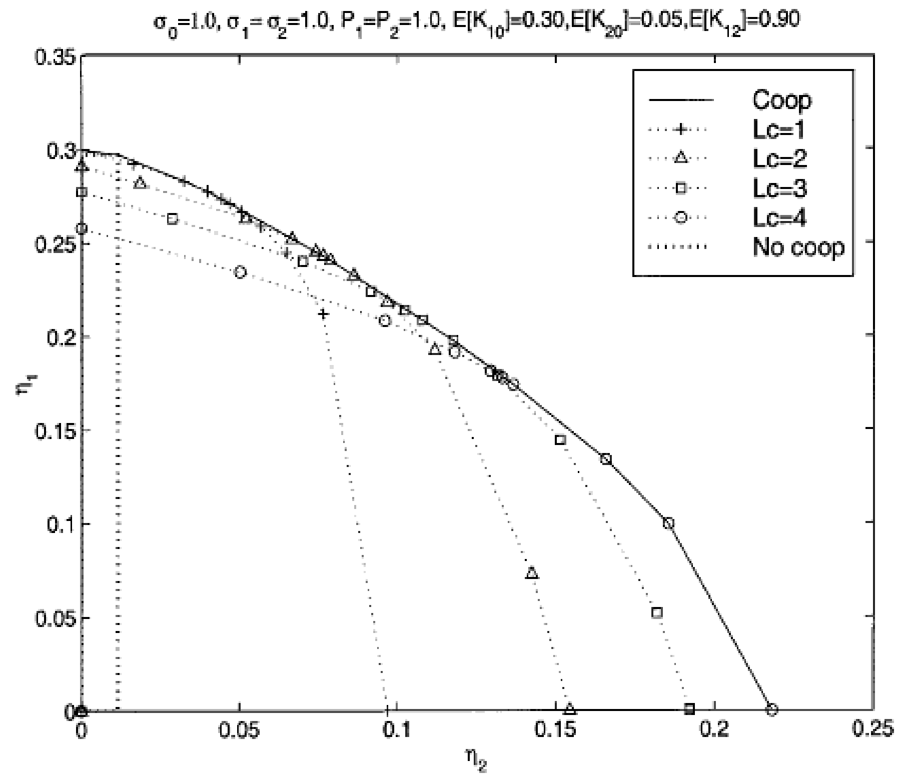
$$P_{e_{12}} = Q\left(K_{12}a_{12}\frac{\sqrt{N_c}}{\sigma_1}\right)$$

Suboptimal detector: $\hat{b}_1 = \text{sign}\left(\left[K_{10}a_{12} \quad \lambda(K_{10}a_{13} + K_{20}a_{23})\right]\mathbf{y}\right)$

Probability of error: $P_{e_2} = (1-P_{e_{12}})Q\left(\frac{\mathbf{v}_\lambda^T\mathbf{v}_1}{\sqrt{\mathbf{v}_\lambda^T\mathbf{v}_\lambda}}\right) + P_{e_{12}}Q\left(\frac{\mathbf{v}_\lambda^T\mathbf{v}_2}{\sqrt{\mathbf{v}_\lambda^T\mathbf{v}_\lambda}}\right)$

System Throughput





$$\eta_1(L_c, \{a_{ij}\}) = E_{\{K_{ij}\}}\left[\eta_1(L_c\{a_{ij}\}, \{K_{ij}\})\right].$$

where $$\eta_1(L_c, \{a_{ij}\}, \{K_{ij}\}) = \frac{1}{L}\left[L_n(1 - \Pi(P_{e_1})) + L_c(1 - \Pi(P_{e_2}))\right]$$

Outage

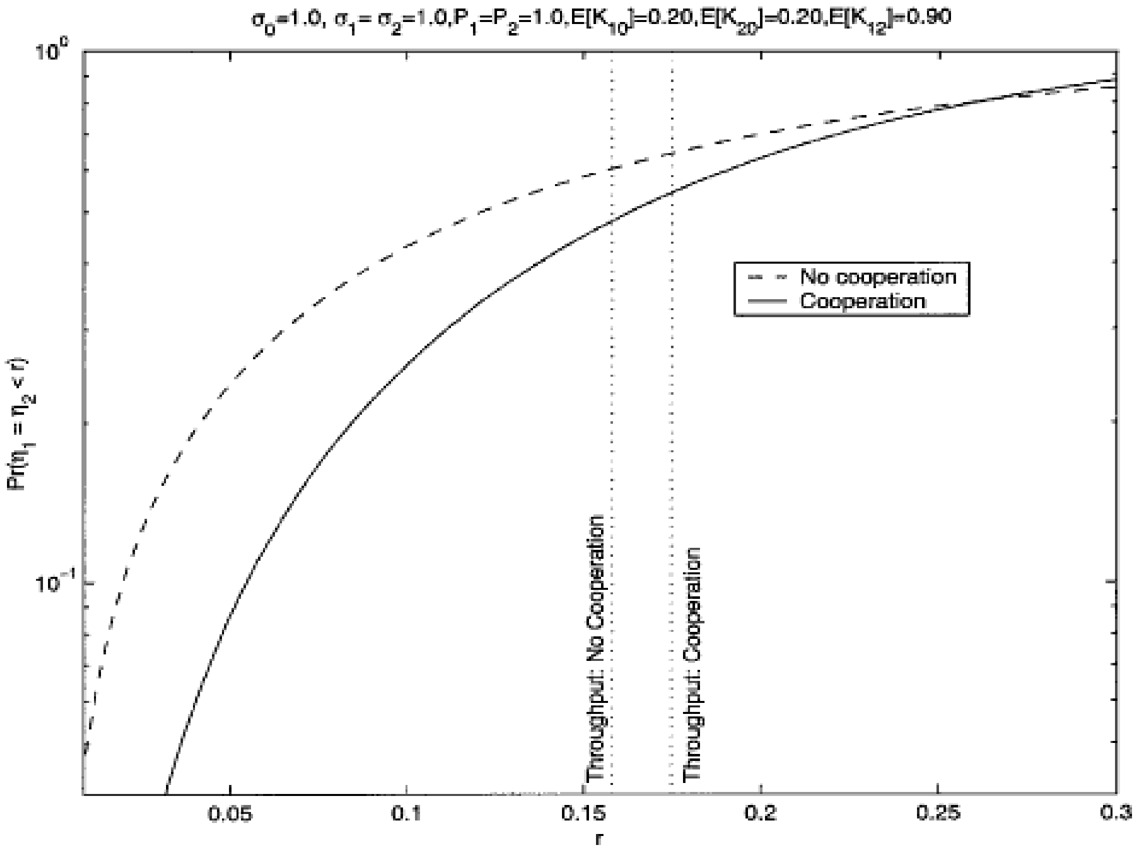


Fig. 6. Probability of outage for conventional CDMA implementation.

Coverage

- Find P’ such that

$$\eta_{\text{sum}}^{n}(P') = (1+\delta)\eta_{\text{sum}}^{n}(P).$$

- Through numerical analysis, it was found that

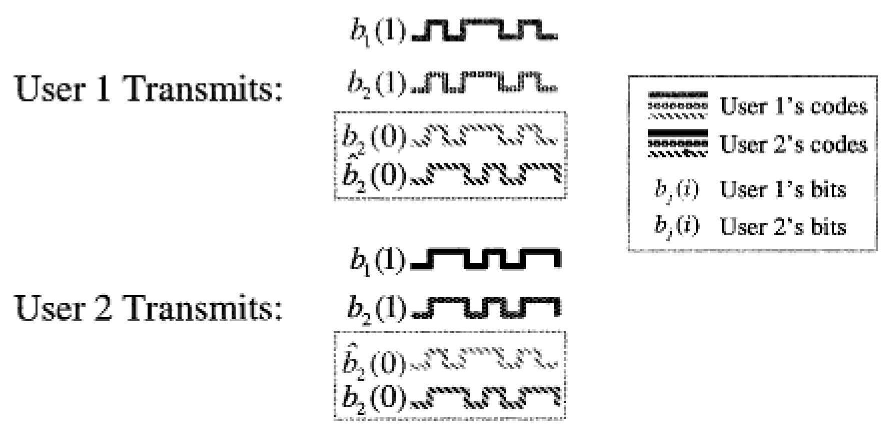
$$\text{Increase in area coverage} \approx \mu\delta$$

where μ is a parameter that depends on the channel condition.

High-rate CDMA system

- Multiple codes per user for higher data rates
- Imitates the information-theoretic cooperative scheme in code space

$$
\begin{aligned}
X_1(t) = & a_{11}b_{11}c_{11}(t) \\
& + a_{12}b_{12}c_{12}(t) \\
& + \left[a_{13}b_{12}^{(-1)}c_{13}(t) + \tilde{a}_{23}\hat{b}_{22}^{(-1)}c_{23}(t)\right] \\
X_2(t) = & \underbrace{a_{21}b_{21}c_{21}(t)}_{\substack{\text{akin to } X_{20} \\ \text{Part I, Sec. III.1}}} \\
& + \underbrace{a_{22}b_{22}c_{22}(t)}_{\substack{\text{akin to } X_{21} \\ \text{Part I, Sec. III.1}}} \\
& + \underbrace{\left[\tilde{a}_{13}\hat{b}_{12}^{(-1)}c_{13}(t) + a_{23}b_{22}^{(-1)}c_{23}(t)\right]}_{\substack{\text{akin to } U \\ \text{Part I, Sec. III.1}}}
\end{aligned}
$$



With no phase knowledge at the transmitters

- Proposed cooperative scheme

$$X_1(t) = \underbrace{a_{11} b_1^{(1)} c_1(t),}_{\text{Period 1}} \quad \underbrace{a_{12} b_1^{(2)} c_1(t),}_{\text{Period 2}} \quad \underbrace{a_{13} \hat{b}_2^{(2)} c_1(t)}_{\text{Period 3}}$$

$$X_2(t) = \underbrace{a_{21} b_2^{(1)} c_2(t),}_{\text{Period 1}} \quad \underbrace{a_{22} b_2^{(2)} c_2(t),}_{\text{Period 2}} \quad \underbrace{a_{23} \hat{b}_1^{(2)} c_2(t).}_{\text{Period 3}}$$

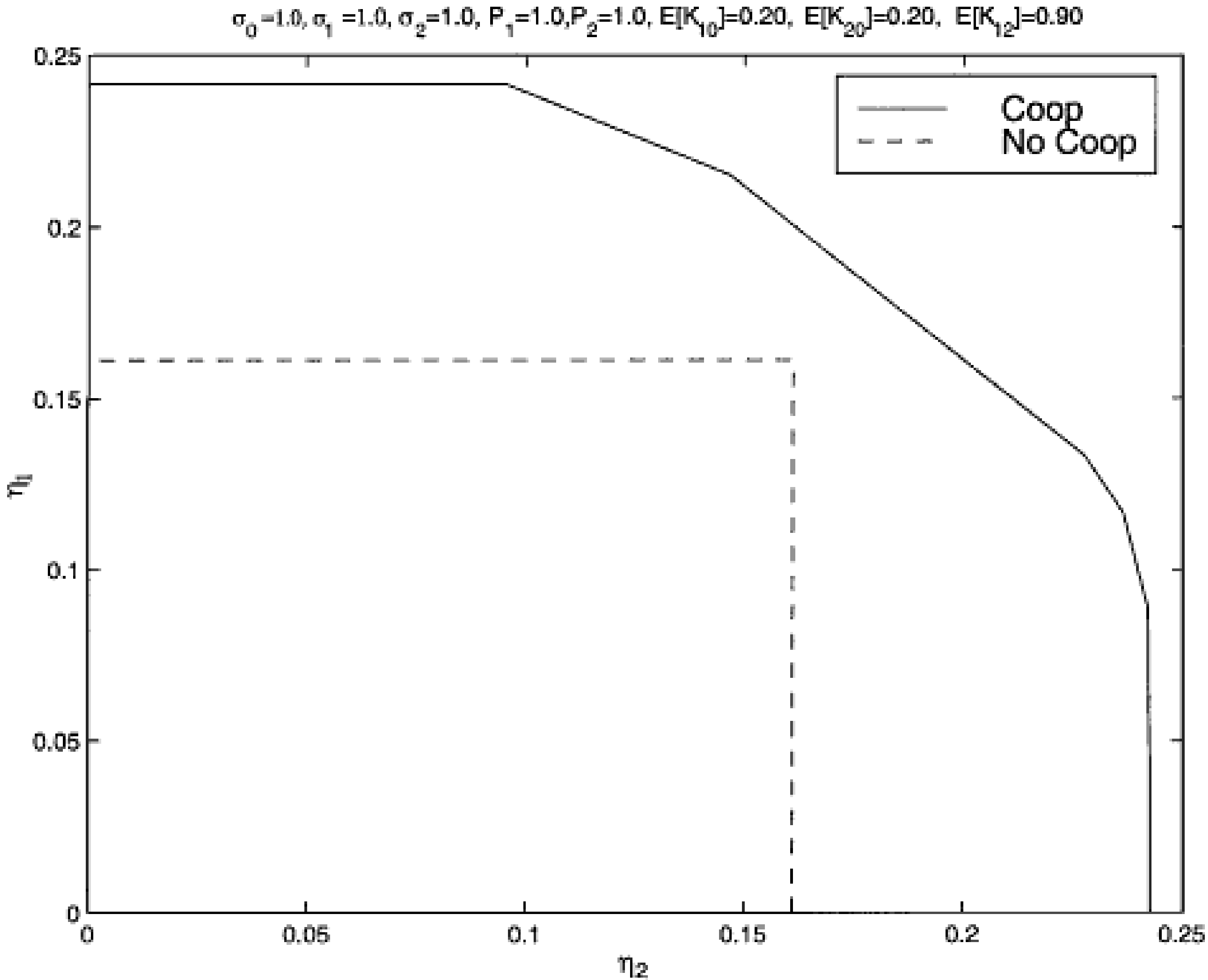
Subject to the power constraints

$$\frac{1}{L}(L_n a_{11}^2 + L_c(a_{12}^2 + a_{13}^2)) = P_1$$
$$\frac{1}{L}(L_n a_{21}^2 + L_c(a_{22}^2 + a_{23}^2)) = P_2.$$

- Throughput is given by

$$\eta_1(L_c, \{a_{ij}\}, \{k_{ij}^{\mathrm{lg}}\}) = E_{\{K_{ij}^{\mathrm{sm}}\}}\left[\eta_1(\{a_{ij}\}, \{k_{ij}^{\mathrm{lg}} K_{ij}^{\mathrm{sm}}\})\right]$$

- Adapt L_c and a_{ij} to maximize $(\alpha\eta_1 + (1-\alpha)\eta_2)$



σ0 =1.0, σ1 =1.0, σ2=1.0, P1=1.0,P2=1.0, E[K10]=0.20, E[K20]=0.20, E[K12]=0.90
Coop
No Coop
0.25
0.2
0.15
0.1
0.05
0
η1
0
0.05
0.1
0.15
0.2
0.25
η2

Conclusions

- Gains: Higher data-rate and a decreased sensitivity to channel variations
- Although complexity is increased, we get an increased and robust data rate and/or extended battery life and/or extended cell coverage and better QoS (due to lower probability of outage).
- Can be used in ad-hoc networks.
- Useful in networks with nodes that have size constraints.
- Several higher level issues, such as which mobiles must partner up, under what conditions they should partner, what point on the achievable rate region they must operate at etc., must be addressed.