Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks

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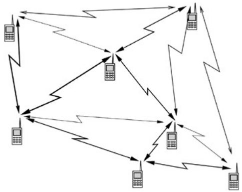
What are we trying to solve?

- Consider n source-destination pairs located randomly.
- Signals transmitted from one user to another at distance r are subject to:
  - power loss $r^{-\alpha}$, where $\alpha \in [2, 6]$,
  - a random phase



- How does information capacity scale as n grows?

What do we mean by “scaling laws”?

- Assume that each node wants to communicate to a random node at a rate $R(n)$ bits/sec.

Definition (Total throughput)

$$T(n) = nR(n)$$

- What is: $\max_{all\ schemes} T(n)$ as n grows ?

Dense vs Extended Networks

Definition (Dense networks)

Area is fixed and the density of nodes increases.

- Interference limited.
- Example: Cellular networks in urban areas.

Definition (Extended networks)

Density is fixed and the area increases.

- coverage limited.
- Example: Cellular networks in rural areas.
- Power limitation come to play.

Why is this problem important?

- Theoretical curiosity
  - FlashForward:
    - In a dense network capacity scales linearly with n. !!
- Broad design directions for the engineers
  - FlashForward:
    - distributed MIMO communication
    - Node Cooperation
    - Hierarchical and Digital Architecture
    - Many long-range communications.

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Previous Work: Dense Networks

A seminal paper by Gupta and Kumar [2] initiated this field.

- Critical Assumption:
  Signals received from other nodes (except one) are regarded as noise.
  - nearest-neighbor multihop scheme $\rightarrow$ many retransmissions $\rightarrow$ $\rightarrow$ scaling no better than $O(\sqrt{n})$. :-(
- Franceschetti et al. [4] proved that this bound is achievable.
  Thus, Scaling law: $\Theta(\sqrt{n})$

Is this scaling law a consequence of the physical-layer technology or can we do better?

Yes we can!

Let me tell you how in a few slides!

Previous Work: Extended Networks

Xie and Kumar [3] addressed the question on the extended networks.

- If $\alpha > 6$ then nearest neighbor multihop scheme is optimal.

Many subsequent works that relaxed the condition down to $\alpha > 4$.

- What about $\alpha \in [2, 4]$? Is nearest neighbor multihop scheme is optimal?

No!

Intuition: For $\alpha < 4$, the network is interference limited $\rightarrow$ like a dense network...

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Problem Formulation

- n nodes uniformly and i.d. in a square of unit area.
- Communication over flat channels
- No multipath effects and Line of sight type environment
- The channel gains are known to all the nodes.
- Far-Field Assumptions
- Path loss and random phase.

Problem Formulation

Upper bound

$T(n) = O(n \log(n))$

Main idea of the proof:

- The rate $R(n)$ from any source node s is bounded by the capacity of the SIMO channel.

Achievable Rate

$T(n) \geq K_\epsilon n^{1-\epsilon}, \forall \epsilon > 0.$

Main idea of the proof:

- Construct clusters and perform long-range MIMO transmissions between clusters.

Achievable Scheme

Divide the network in clusters of size M. Take at random a pair (s, d). Assume nodes s, d belong to clusters S and D respectively. Assume node s needs to transmit M bits to node d.

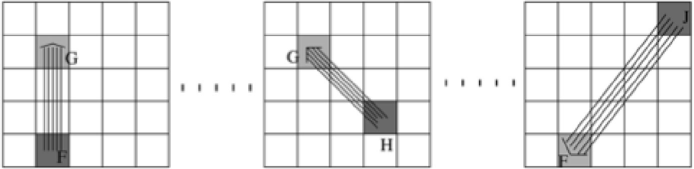
- Phase 1: Setting up Transmit Cooperation
  - Node s distributes locally the M bits to the nodes of the current cluster
- Phase 2: MIMO Transmissions
  - The nodes of the cluster S cooperate and perform long-range transmission to all the nodes of the cluster D.
- Phase 3: Cooperate to Decode
  - Nodes in D cooperate to decode the message and send it locally to d.

Phase 1: Setting up Transmit Cooperation

- Clusters work in parallel.
- Inside each cluster, each node s needs to distribute M bits to the rest $M-1$ nodes of the cluster. $\rightarrow M^2$ bits.
- Assume we have a transmission scheme that achieves M^b bits/slot, where $0 \leq b < 1$.
- Therefore, we need $\frac{M^2}{M^b} = M^{2-b}$ time slots for phase 1.

Phase 2: MIMO Transmissions

- There are n (s,d) pairs in all the network.
- The long-distance MIMO transmissions between the clusters are performed one at a time.
- We need n time slots for phase 2.



Phase 3: Cooperate to Decode

- Clusters work in parallel.
- M destination nodes in each cluster. $\rightarrow$
  Each cluster received M transmissions in phase 2. $\rightarrow$
  Each node in the cluster received M observations.
- Each node quantize each observation into Q bits. $\rightarrow$ QM^2 bits need to be locally flooded inside the cluster.
- Therefore, we need $\frac{QM^2}{M^b} = QM^{2-b}$ time slots for phase 3.

Aggregate Throughput

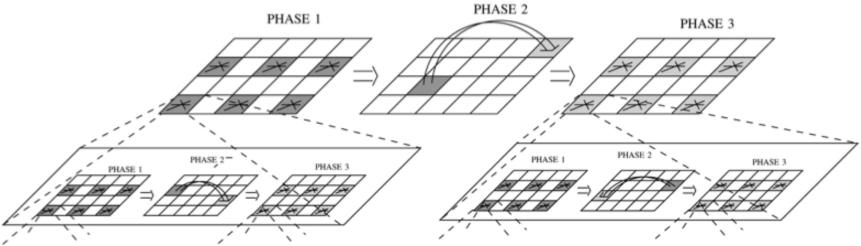
Aggregate Throughput

$$T(n) = \frac{nM}{M^{2-b}+n+QM^{2-b}} = \frac{1}{2+Q} n^{\frac{1}{2-b}}$$

- Note that $\frac{1}{2-b} > b, \quad \forall\ 0 \leq b < 1.$
- We started from a scheme with $T(n) = n^b$
- We have a new scheme that achieves $T(n) = n^{\frac{1}{2-b}} > n^b$
- By repeating this procedure we get:

$$T(n) = K_\epsilon n^{1-\epsilon}$$

Graphical Representation



Extended Networks

Main Result

The same scheme achieves $T(n) \geq K \cdot n^{2-\frac{\alpha}{2-\epsilon}}$ for $2 \leq \alpha < 3$ (better than just multihop.)

"Bursty" modification of the hierarchical scheme:

- Density is fixed, area is $\sqrt{n}$ x $\sqrt{n}$ square. $\rightarrow$
- All distances increase by $\sqrt{n} \rightarrow$
- Received powers are all decreased by $n^{\frac{\alpha}{2}}$.
- Power contraint is $\frac{P}{n^{\frac{\alpha}{2}}}$
- Run the scheme a fraction $\frac{1}{n^{\alpha/2-1}}$ with power $\frac{P}{n}$.

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Conclusions

- We achieved an optimal throughput performance for a dense network!
- We used this scheme for the extended networks to fill in the gap for $\alpha \in [2, 4]$.

Main points:

- Node cooperation
- MIMO transmissions
- Hierarchical Cooperation
- Many long-range communications.

Questions ...

References

- Ayfer Ozgur, Olivier Leveque and David N. C. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad Hoc Networks," *IEEE Trans. on Inf. Theory*, vo. 53, 2007
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