EE102B Spring 2018-19 Pauly
Signal Processing and Linear Systems II

Homework 8

Due May 31

1. Find the inverse z-transform for each of the following. Assume the signals are right-sided.

   (a) $X(z) = \dfrac{(1-4z^{-1})}{1-5z^{-1}+6z^{-2}}$

   (b) $X(z) = \dfrac{z^{-1}(1-4z^{-1})}{1-5z^{-1}+6z^{-2}}$

   (c) $X(z) = \dfrac{z^{-1}(-5+22z^{-1})}{(1+z^{-1})(1-2z^{-1})^2}$

2. A causal LTI system is described by the following difference equation:

$$y[n] - \frac{1}{2}y[n-1] + \frac{1}{4}y[n-2] = x[n]$$

   (a) Find the z transform of the impulse response.

   (b) Determine $y[n]$ when $x[n] = (\frac{1}{2})^n u[n]$. Assume $y[-1] = 0$ and $y[-2] = 0$.

3. Adam borrowed \$20,000 from a loan shark. He must pay this back over a 15-year term at 18% interest per year (1.5% per month), with equal monthly payments of m dollars.

   (a) Write down the difference equation relating $p[n+1]$ to $p[n]$, where $p[n]$ is the outstanding principal on the loan after the n^{th} month. Note that $p[0] = 20,000$, and $p[180] = 0$.

   (b) Use the unilateral z-transform to determine m, Adam's monthly payment.

   (c) What profit (*i.e*, the amount above \$20,000) has the loan shark made?

4. A system with the difference equation

$$y[n] - \frac{1}{9}y[n-2] = x[n-1]$$

   has initial conditions $y[-1] = 1$, and $y[-2] = 0$. If this system has an input

$$x[n] = 2u[n]$$

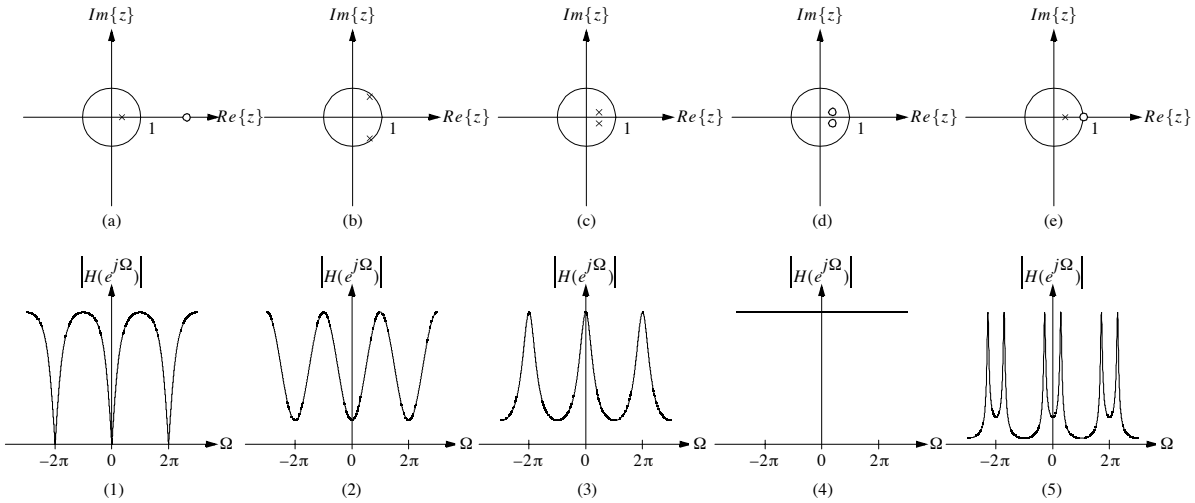
   find the

   a) Zero state response $y_{zs}[n]$,

   b) Zero input response $y_{zi}[n]$, and the

   c) Total response of the system, $y[n] = y_{zs}[n] + y_{zi}[n]$.

5. Match the pole-zero plots (a)-(e) with the corresponding magnitude responses (1)-(5).



6. *IIR vs FIR Filters*

   An Infinite Impulse Response (IIR) filter can be described by this difference equation

$$y[n] - \alpha y[n-1] = x[n] - \frac{3}{4}x[n-1] + \frac{1}{8}x[n-2]$$

   In general the impulse response of this system goes on forever (IIR), but in a few special cases it terminates, and we get an FIR filter. Your job is to find those solutions, and compute the impulse responses.

   (a) Find the transfer function $H(z)$ for this difference equation.

   (b) What values of α would result in an FIR filter.
   Hint: This is would be a filter with no poles. There are three solutions

   (c) Find the impulse response of these FIR filters.

7. *Discrete Time Systems*

   A discrete-time system with input $x[n]$ and output $y[n]$ has a transfer function

$$H(z) = \frac{1}{1 - z^{-7} - z^{-8}}$$

   while the input is

$$x[n] = n^2 u[n].$$

   Assume $y[n] = 0$ for $n < 0$. Find $y[5]$.

   Hint: The difference equation is the place to start. Convolution and/or the z-transform won't help.