EE102B Spring 2018-19 Pauly
Signal Processing and Linear Systems II

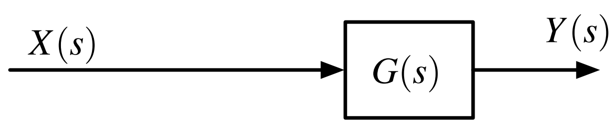
Homework 6

Due May 17, 2019

This week there are more problems than usual this week, there is no lab.

1. *Feedback Stabilization*

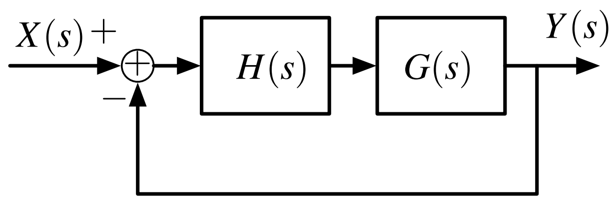
   We have a system we would like to control



   The impulse response of the plant we are trying to control is

$$g(t) = e^t \sin(2t).$$

   This is unstable. To stabilize this system we add a controller to the forward system, and unity feedback,

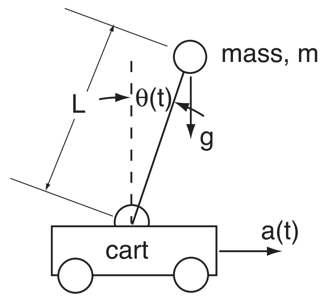


   Our goal is to produce a closed loop transfer function that is critically damped with poles at $s = -2$.

   a) Find the transfer function of the plant $G(s)$ from the impulse response $g(t)$ given above.
   b) Find the closed loop transfer function $T(s)$ in terms of $H(s)$.
   c) Choose $H(s)$ to produce a critically damped closed loop transfer function, with poles at $s = -2$.
   d) Find the impulse response of the closed loop system (call it $\tau(t)$ to avoid confusion).

2. *Balancing an Inverted Pendulum*

   We want to balance a stick with a mass at the end, mounted on a cart. We can control the acceleration of the cart $a(t)$. The stick is attached to the cart with a hinge that pivots in the direction of cart motion. The angle $\theta(t)$ is measured from vertical, as illustrated below



If we assume the stick to be massless, and equate forces perpendicular to the axis of the stick, we get

$$mL\theta''(t) = mg\sin\theta(t) - ma(t)\cos\theta(t).$$

If we assume that $\theta(t)$ is small, $\sin\theta(t) \approx \theta(t)$ and $\cos\theta(t) \approx 1$. Making these approximations, and dividing through by m,

$$L\theta''(t) = g\theta(t) - a(t).$$

If $\theta(0) \neq 0$or $\theta'(0) \neq 0$, and we do nothing the stick falls over. We want to find a method to automatically determine the cart acceleration $a(t)$ so that the stick is balanced, and $\theta(t)$ goes to zero. We will do this by using an input $a(t)$ that is a function of the output $\theta(t)$ that we are trying to control. Let $L = 1$ m (the length of the stick) and assume the gravitational constant g is $10\ \mathrm{m/s^2}$ for convenience.

Let the initial conditions be $\theta(0) = 0$, and $\theta'(0) = 1$. This means that the stick is vertical, but rotating.

(a) *Proportional Control*

For the first attempt, we let

$$a(t) = k_0\theta(t).$$

The acceleration is proportional to the angle we are trying to control.

i. Find a differential equation for $\theta(t)$.

ii. Let $k_0 = 9$, and solve for $\theta(t)$. Is this system stable? (does $\theta(t)$ go to zero?)

iii. Let $k_0 = 11$ solve for $\theta(t)$. Is this system stable?

(b) *Proportional Plus Derivative Control*

For the next attempt, we let

$$a(t) = k_1\theta'(t) + k_0\theta(t).$$

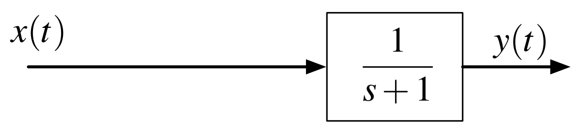
i. Find a differential equation for $\theta(t)$.

ii. Find values of k_0 and k_1 so the system is critically damped, with a time constant of 1 s.

iii. Solve for $\theta(t)$. Is this system stable?

3. *Feedback to Improve System Response*

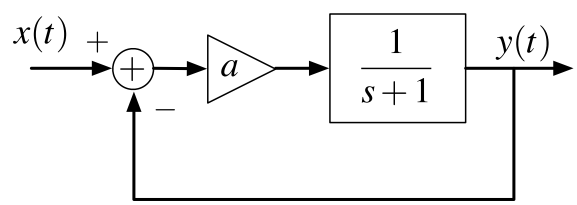
The response to a remote manipulator can be modeled by this system



$x(t)$ is the position we request, and $y(t)$ is the position of the manipulator. This has an impulse response that has a time constant of 1 s (i.e. the impulse response of the system is $e^{-t/T}$ where $T = 1$ s). This is too slow to be practically usable. In order for a manipulator to feel immediate and interactive, we would like the response time to be no more than 100 ms.

(a) Find the step response of the system, and plot it.

(b) To speed up the response, we add a feedback loop around the system, along with a gain stage



Find the transfer function of this system.

(c) Choose a such that the time constant is 100 ms. Solve for the step response, and plot it on the same graph as part (a).

(d) What is the steady state error between the position you request and the manipulator position?

4. *Feedback and System Dynamics*

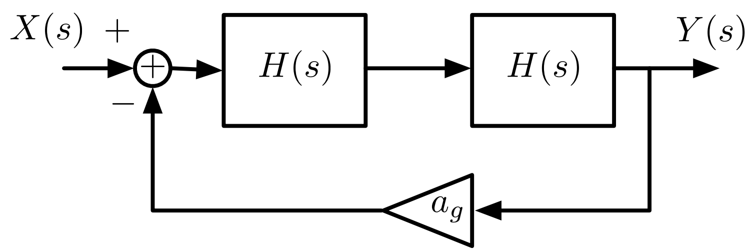
In this problem we have two amplifiers we are going to connect in series. Each amplifier has a transfer function of

$$H(s) = \frac{100}{s+1}$$

We are going to use feedback for the reasons described in class (linearization, insensitivity to the actual gain of the amplifiers).

With two amplifiers, we can use a global feedback loop around both of them, or use local feedback around each individually.

a) The case where we use a single global feedback loop is shown here:

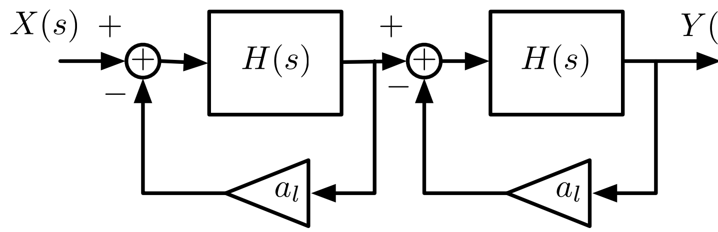


Find the constant gain a_g so that the DC gain (the transfer function at zero frequency) of the system is 100.

b) Find the transfer function $T_g(s)$ of this system.

c) Find the impulse response $\tau_g(t)$ of this system.

d) The case where we use two local feedback loops is shown here:



Assume both feedback gains are the same, and find a_l so that the DC gain of the system is again 100.

e) Find the transfer function $T_l(s)$ of this system.

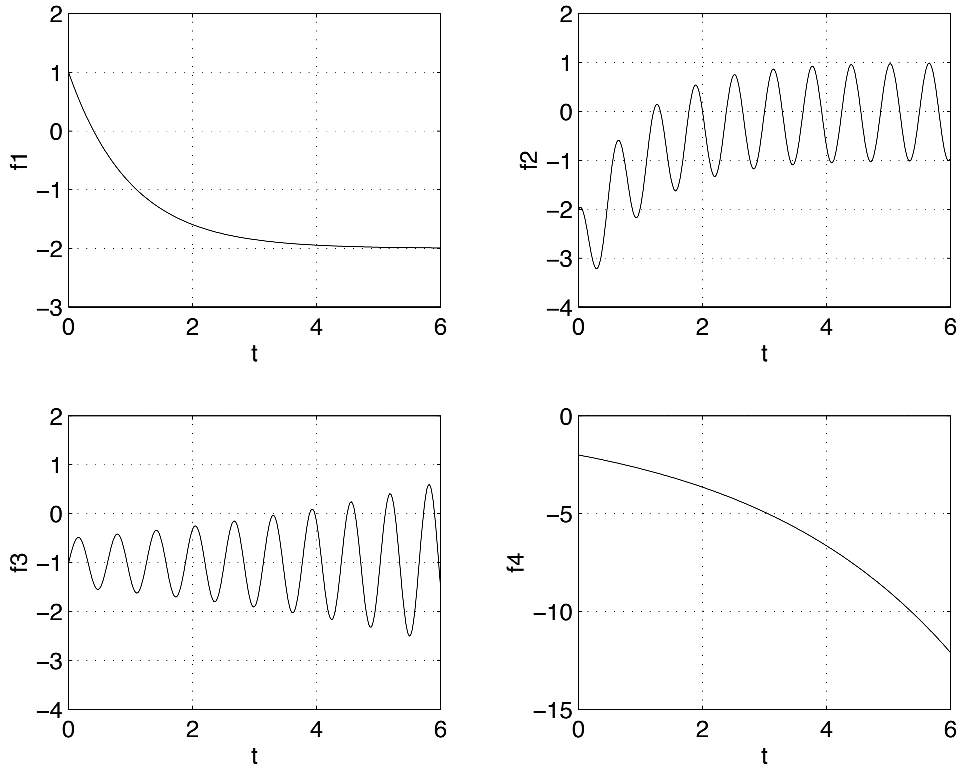
f) Find the impulse response $\tau_l(t)$ of this system.

g) Describe the solutions for the global feedback and local feedback cases. Some possible answers are of the type "critically damped", "oscillatory", "lightly damped", "overdamped", or "unstable."

h) Briefly describe which system you would recommend, and why.

5. *Identifiing poles from an impulse response*

   For each of theses signals, estimate what the poles of the Laplace transform must be.



6. *Pole-Zero Identification from a Bode Plot*

   Find the poles, zeros, and DC gain for this Bode plot.