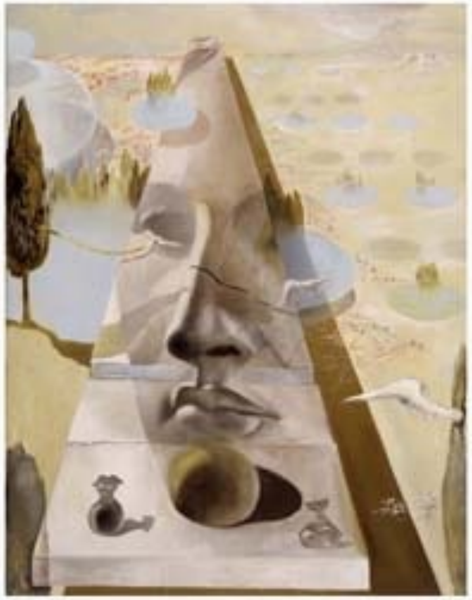
# Lecture 3

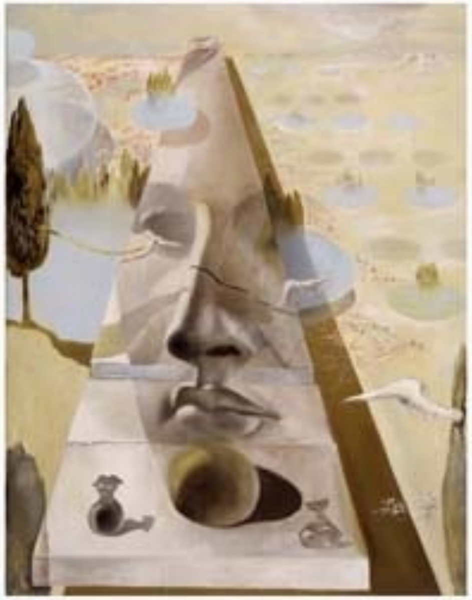
# Camera Models 2 & Camera Calibration



Professor Silvio Savarese

Stanford Vision and Learning Lab

# Lecture 3
# Camera Models 2 & Camera Calibration

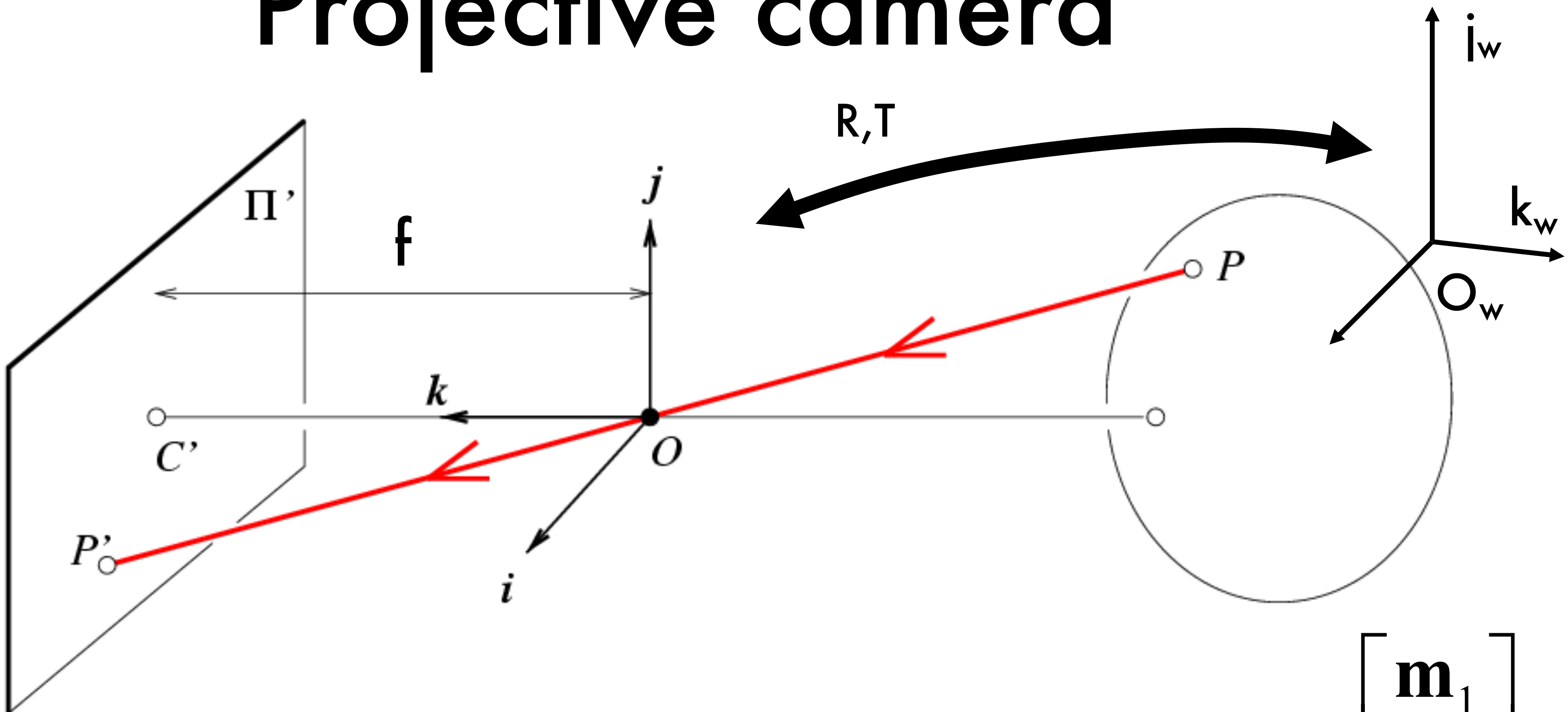


- Recap of camera models
- Camera calibration problem
- Camera calibration with radial distortion
- Example

Reading: [FP] Chapter 1 “Geometric Camera Calibration”
[HZ] Chapter 7 “Computation of Camera Matrix P”

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

# Projective camera




$$P'_{3\times1} = M\,P_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} P_{w\,4\times1} \qquad M = \begin{bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{bmatrix}$$

$$= \begin{bmatrix}\mathbf{m}_1\\ \mathbf{m}_2\\ \mathbf{m}_3\end{bmatrix} P_W = \begin{bmatrix}\mathbf{m}_1 P_W\\ \mathbf{m}_2 P_W\\ \mathbf{m}_3 P_W\end{bmatrix} \xrightarrow{\mathbb{E}} P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right)$$

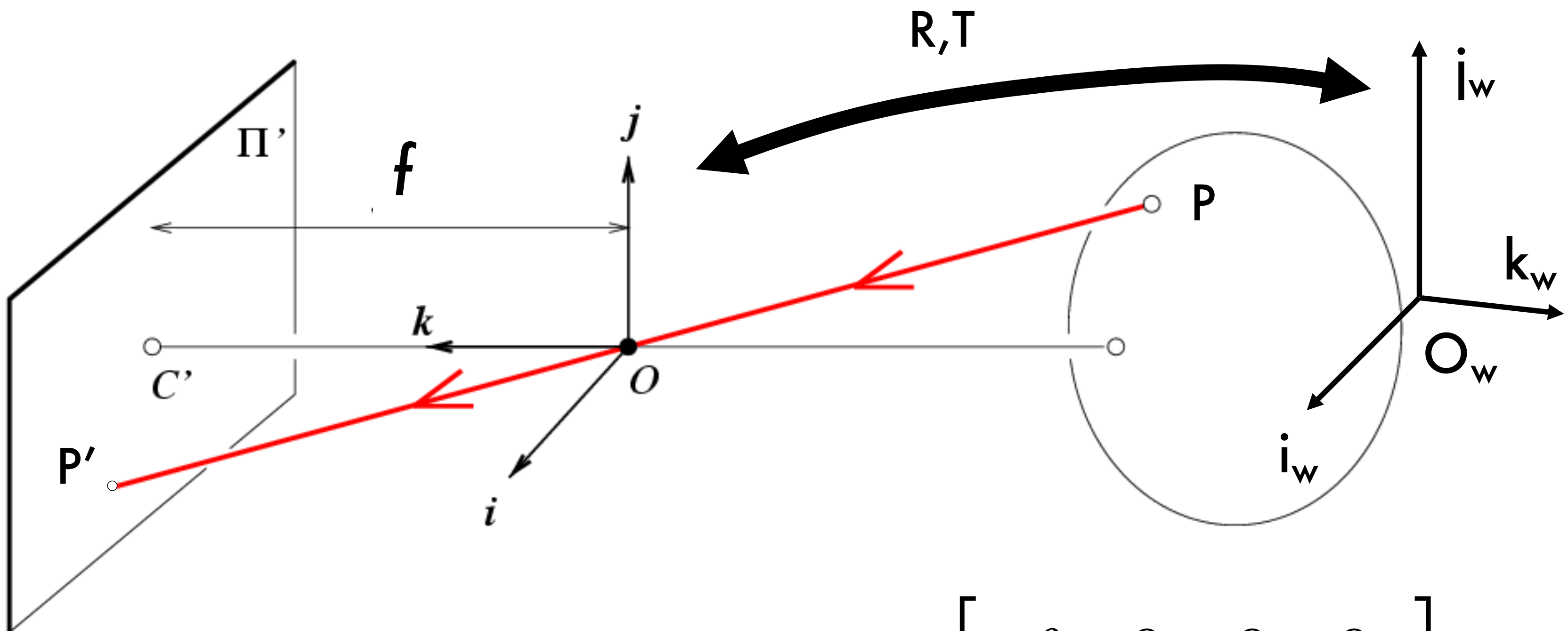
[Eq. 1]

**What's the expression of K?**

Nobody has responded yet.

Hang tight! Responses are coming in.

# Exercise!




$$M = K\begin{bmatrix} R & T \end{bmatrix} = K\begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$\rightarrow \; P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w}\right) = \left(f\frac{x_w}{z_w}, f\frac{y_w}{z_w}\right)$$

## Can we write a simplified expression for P'_E?

Nobody has responded yet.

Hang tight! Responses are coming in.

# Canonical Projective Transformation

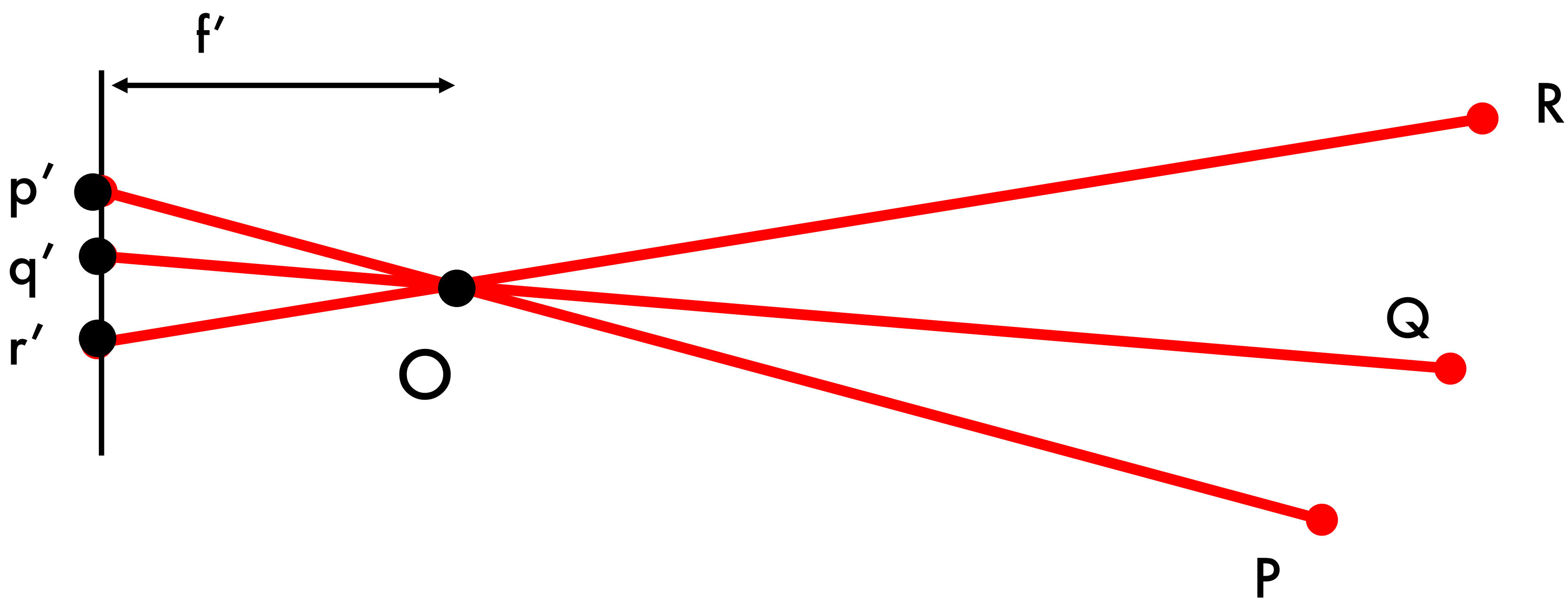
$$P' = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = M\,P$$

$$\Re^4 \xrightarrow{H} \Re^3$$

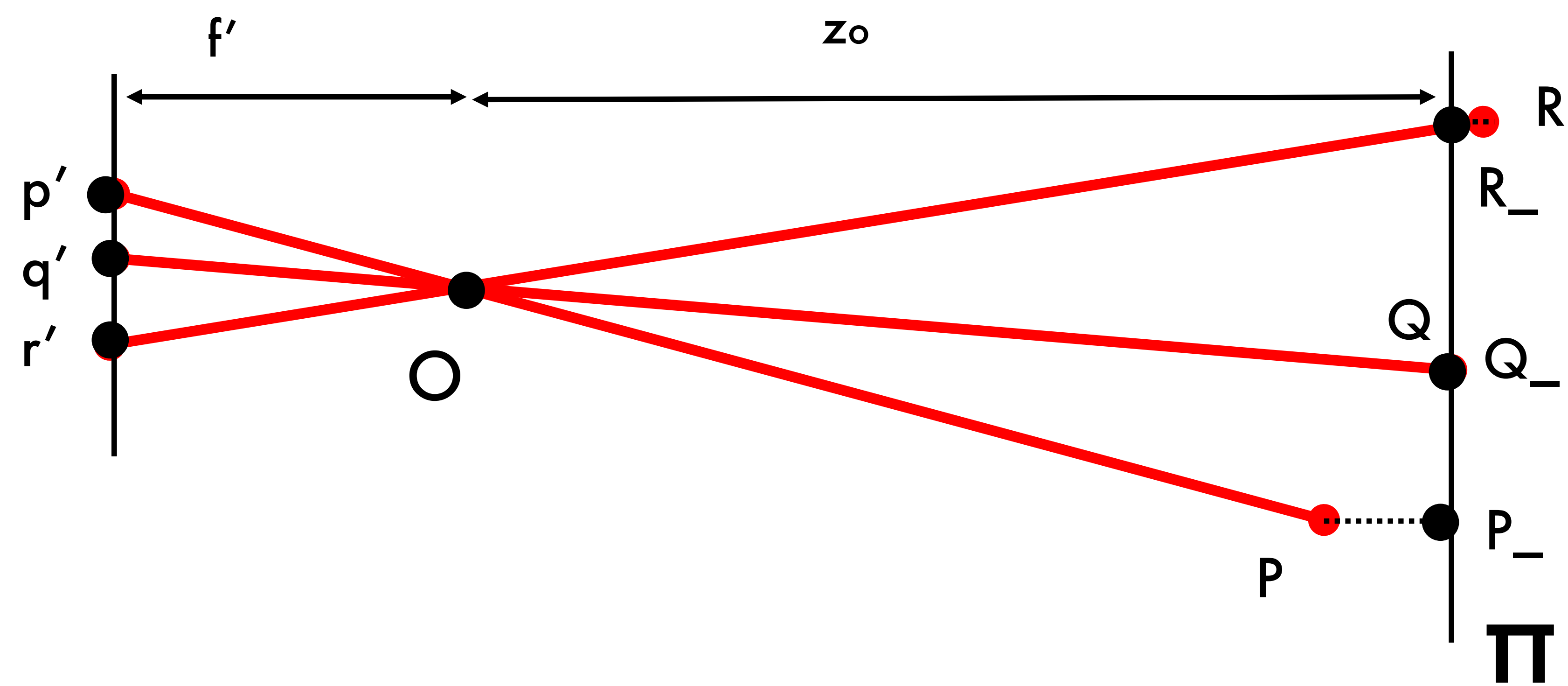
$$P_i' = \begin{bmatrix} \frac{x}{z} \\ \frac{y}{z} \end{bmatrix}$$

# Projective camera

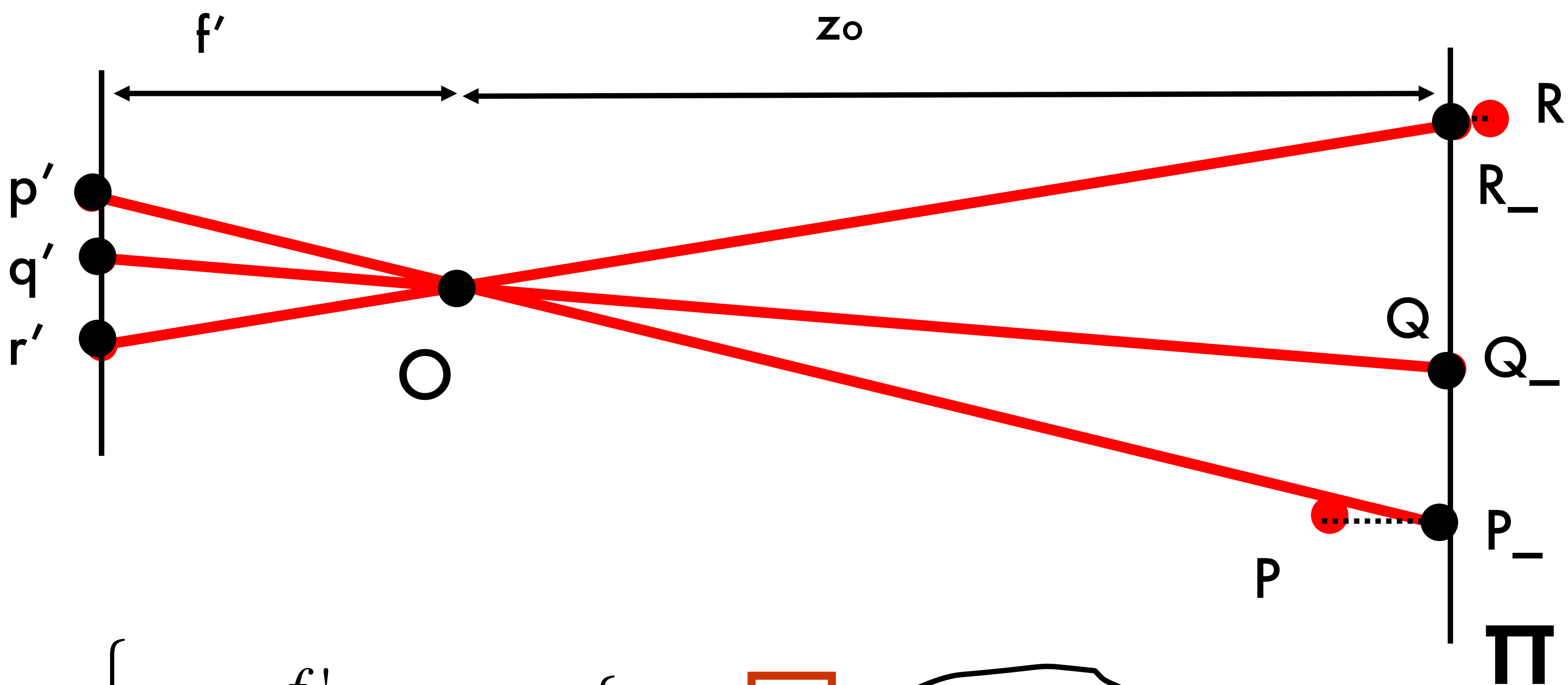


# Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



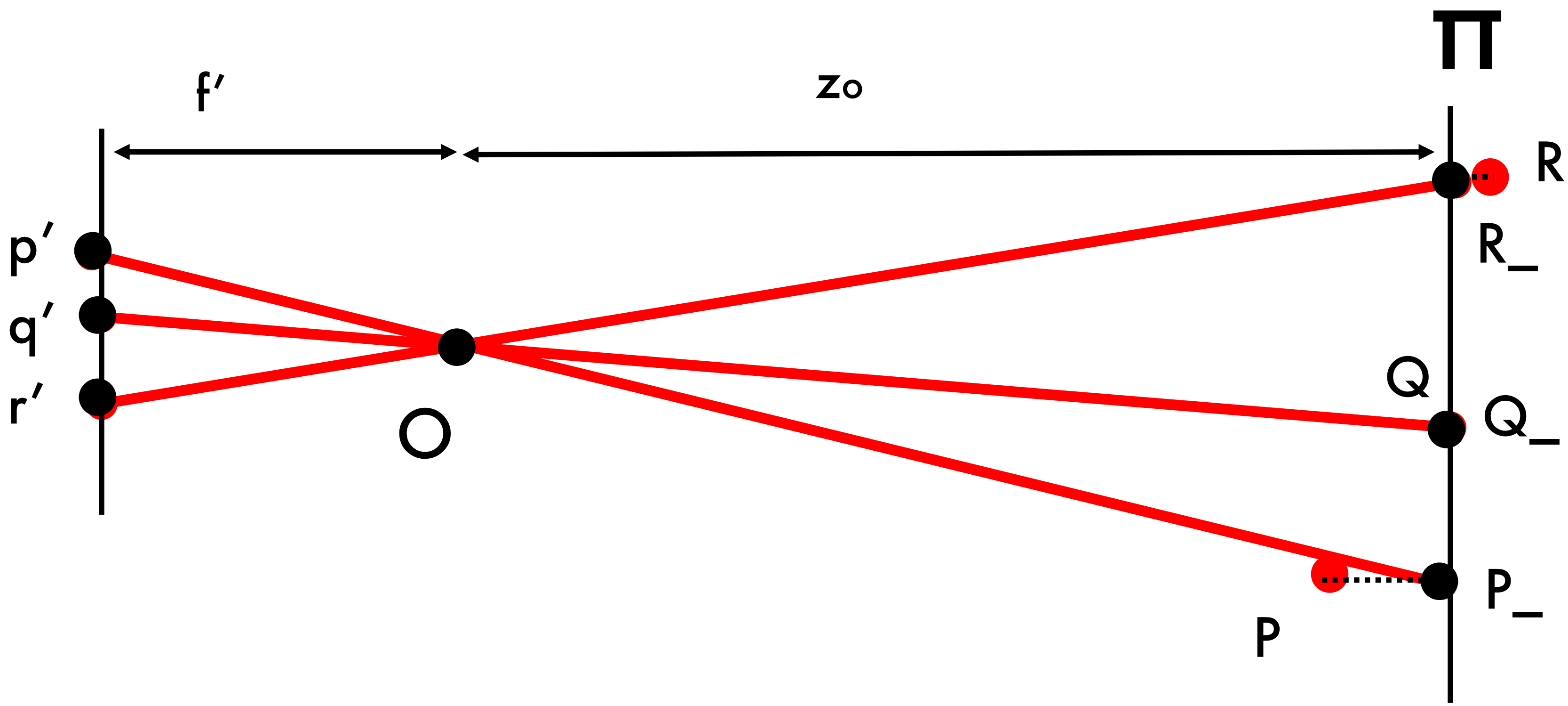
# Weak perspective projection




$$\begin{cases} x' = \dfrac{f'}{z}x \\ y' = \dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = \boxed{\dfrac{f'}{z_0}}x \\ y' = \dfrac{f'}{z_0}y \end{cases}$$

Magnification m

# Weak perspective projection




Projective (perspective)

Weak perspective

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & 1 \end{bmatrix} \rightarrow M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix}$$

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1\,P_W \\ \mathbf{m}_2\,P_W \\ \mathbf{m}_3\,P_W \end{bmatrix}$$

$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{v} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$\xrightarrow{\mathbb{E}} \left(\frac{\mathbf{m}_1\,P_w}{\mathbf{m}_3\,P_w}, \frac{\mathbf{m}_2\,P_w}{\mathbf{m}_3\,P_w}\right)$$

Perspective

---

$$P' = M\,P_w = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_W = \begin{bmatrix} \mathbf{m}_1\,P_W \\ \mathbf{m}_2\,P_W \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} = \begin{bmatrix} & \mathbf{m}_1 & & \\ & \mathbf{m}_2 & & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

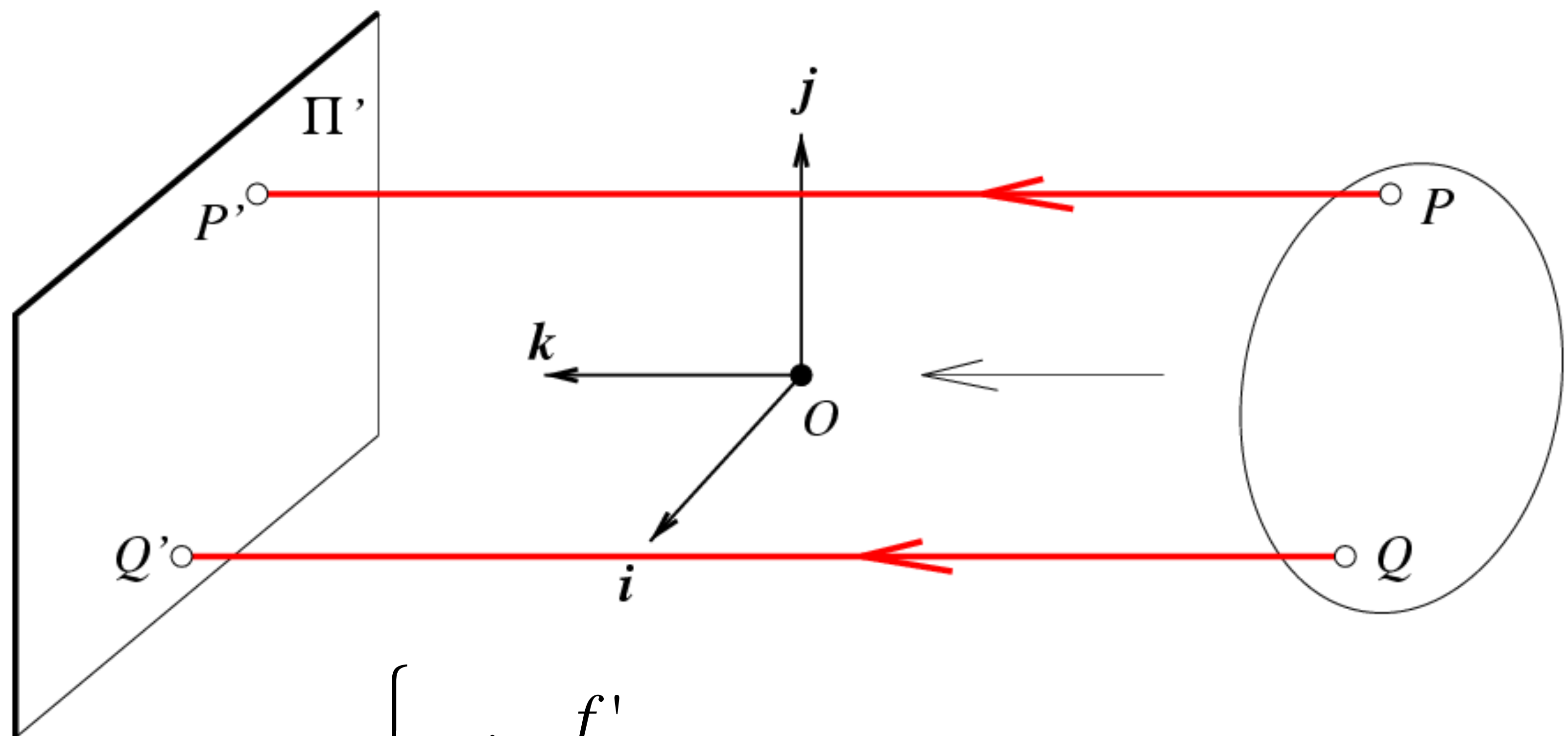
$$\xrightarrow{\mathbb{E}} (\mathbf{m}_1\,P_w, \mathbf{m}_2\,P_w)$$

↑ ↑ magnification

Weak perspective

# Orthographic (affine) projection

Distance from center of projection to image plane is infinite

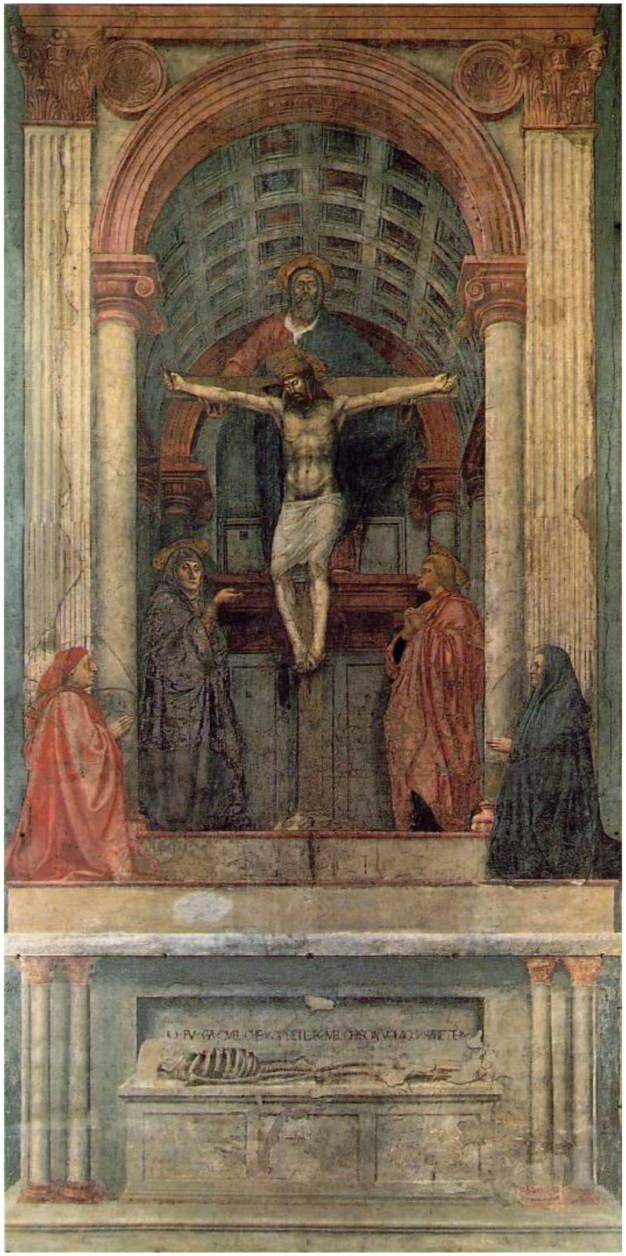



$$\begin{cases} x' = \dfrac{f'}{z} x \\ y' = \dfrac{f'}{z} y \end{cases} \rightarrow \begin{cases} x' = x \\ y' = y \end{cases}$$

# Pros and Cons of These Models

- Weak perspective results in much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.
- Pinhole perspective is much more accurate for modeling the 3D-to-2D mapping.
  - Used in structure from motion or SLAM.

# One-point perspective



Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

il Canaletto The Piazzetta, Venice,

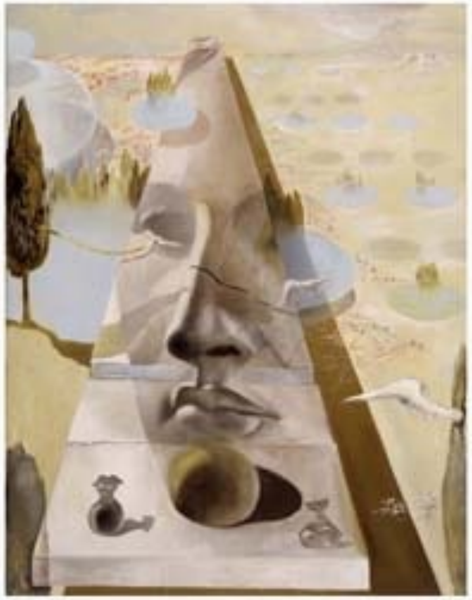
# Weak perspective projection

*The Kangxi Emperor's Southern Inspection Tour (1691-1698)*By Wang Hui

# Weak perspective projection

*The Kangxi Emperor's Southern Inspection Tour (1691-1698)*By Wang Hui

# Lecture 3
# Camera Calibration



- Recap of camera models
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# Projective camera

$$P' = M\,P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

# Goal of calibration

$$P' = M\,P_w = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

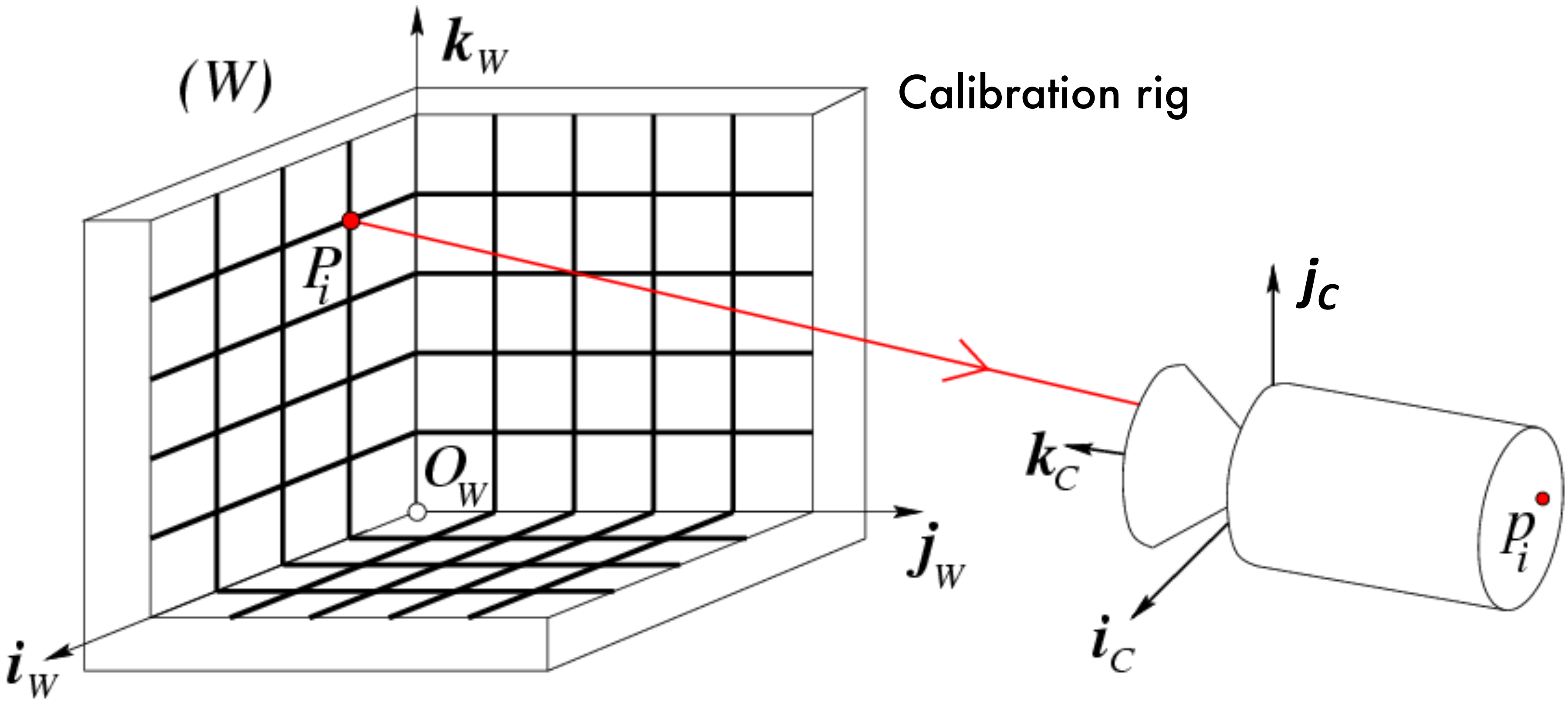
External parameters

Estimate intrinsic and extrinsic parameters from 1 or multiple images

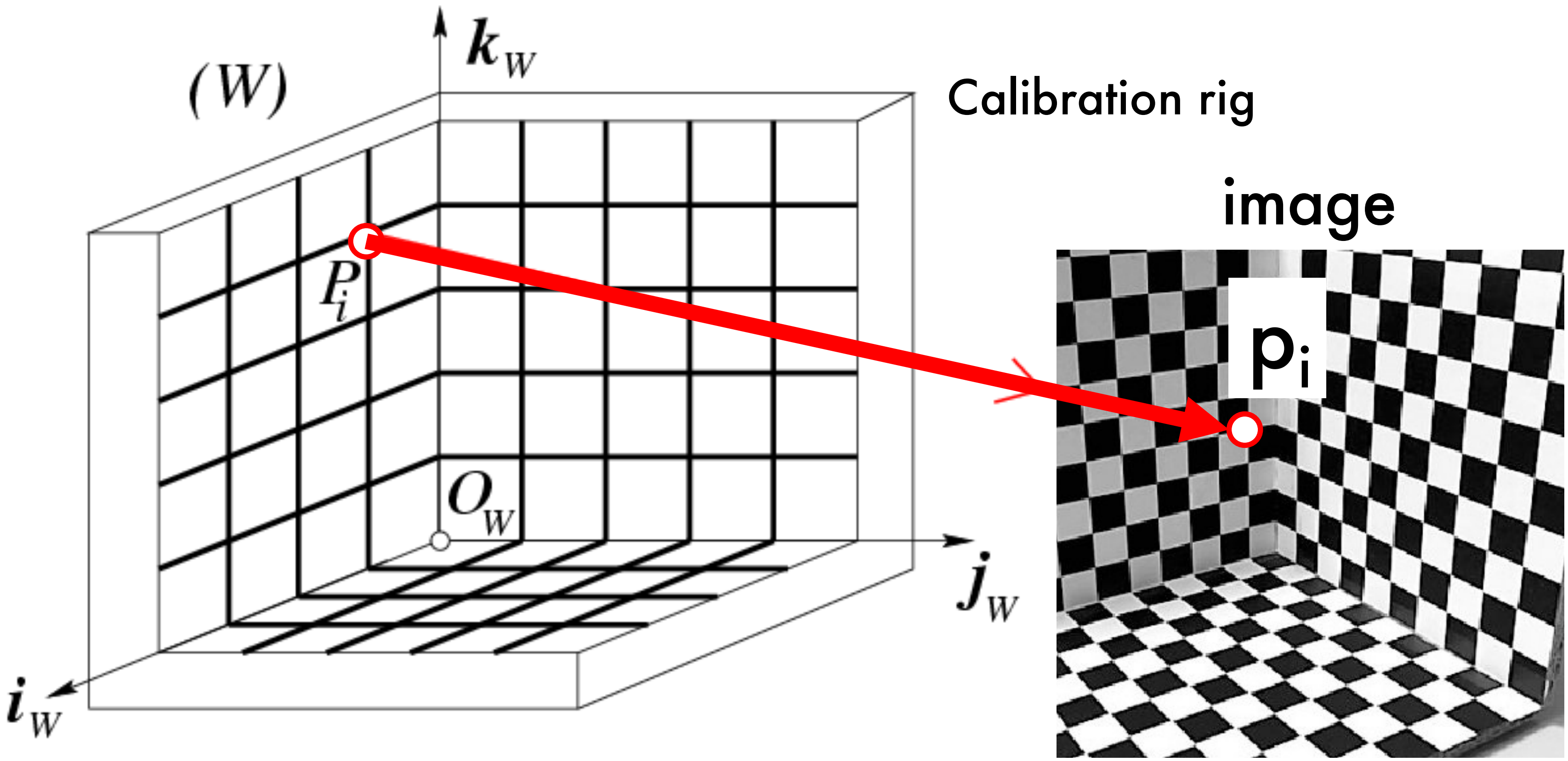
Change notation:
$P = P_w$

$p = P'$

# Calibration Problem



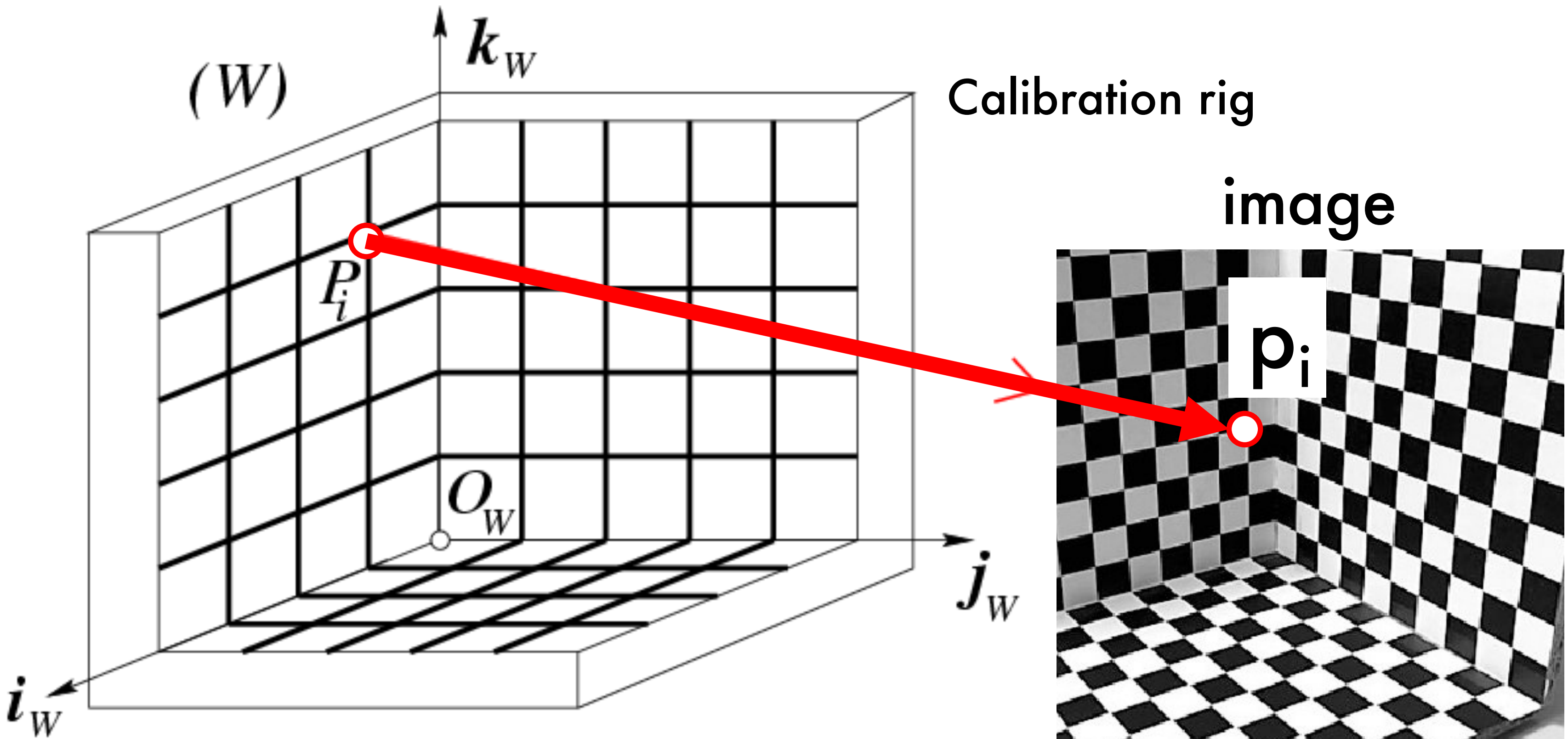
# Calibration Problem




- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

# Calibration Problem




# How many correspondences do we need?

- M has 11 unknowns • We need 11 equations

## How many of such correspondences would we need to compute both intrinsics and extrinsics?

A 11 — 0%

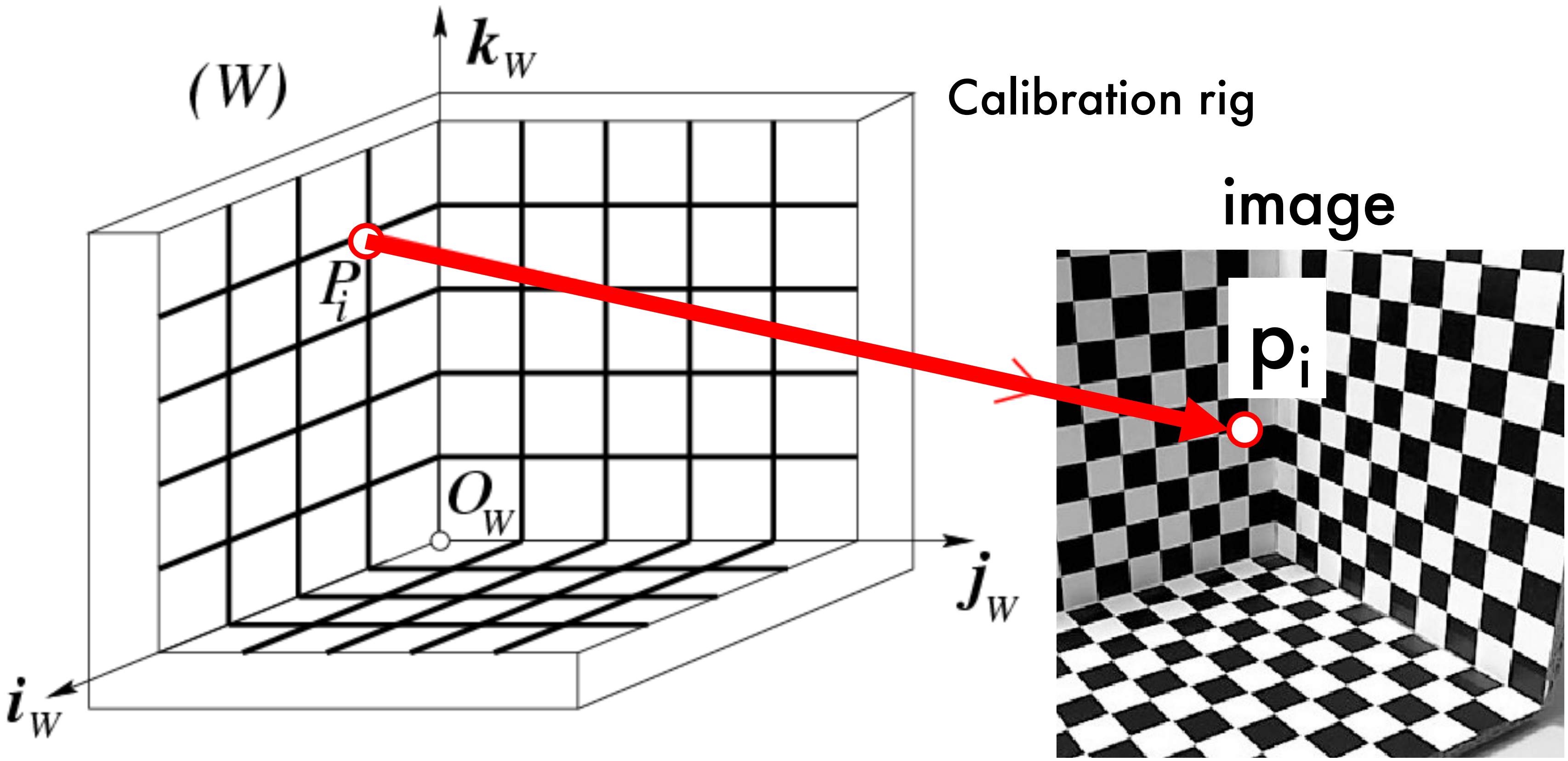
B 17 — 0%

C 5 — 0%

D 6 — 0%

None of the above — 0%

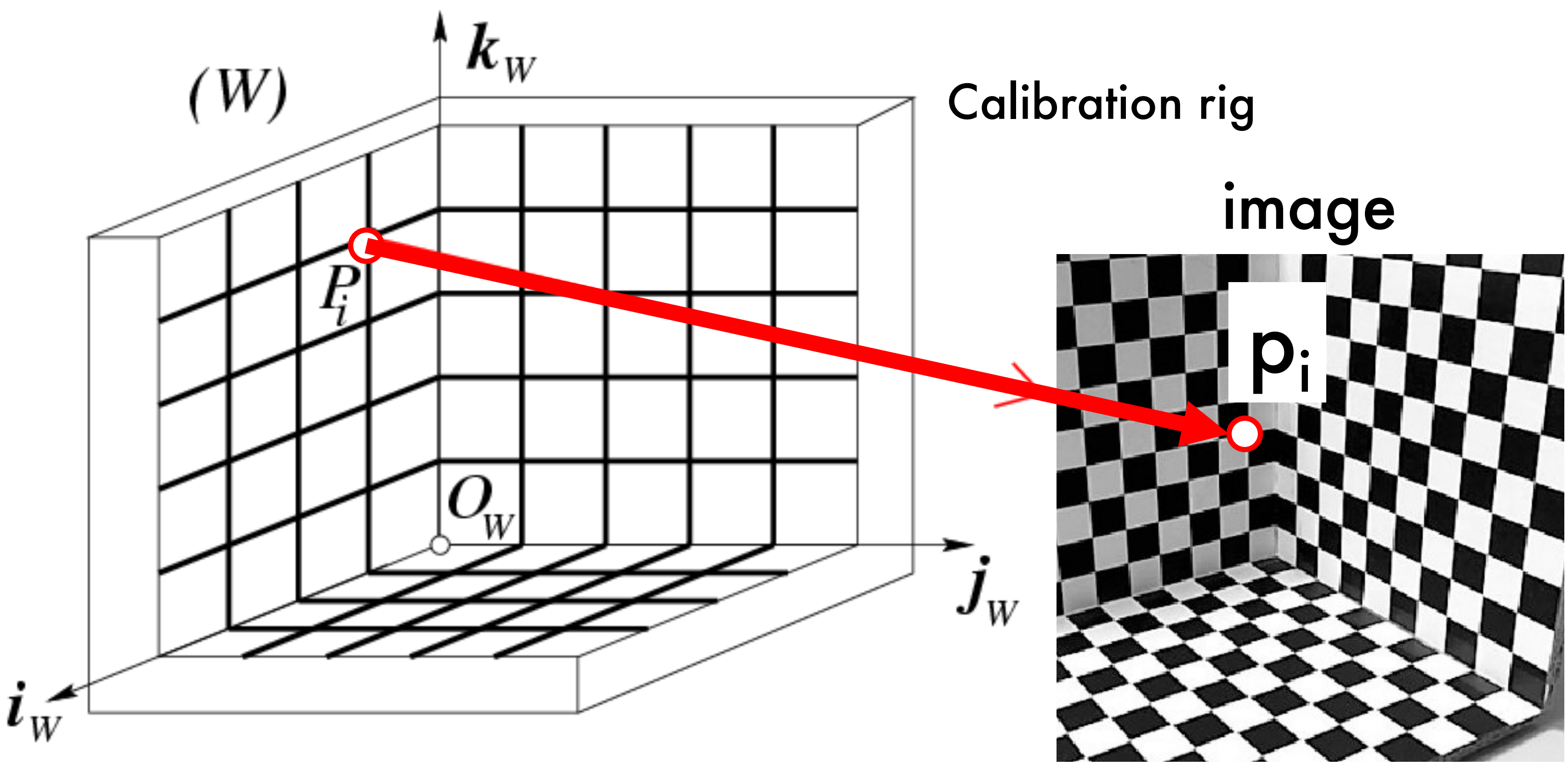
# Calibration Problem




# How many correspondences do we need?

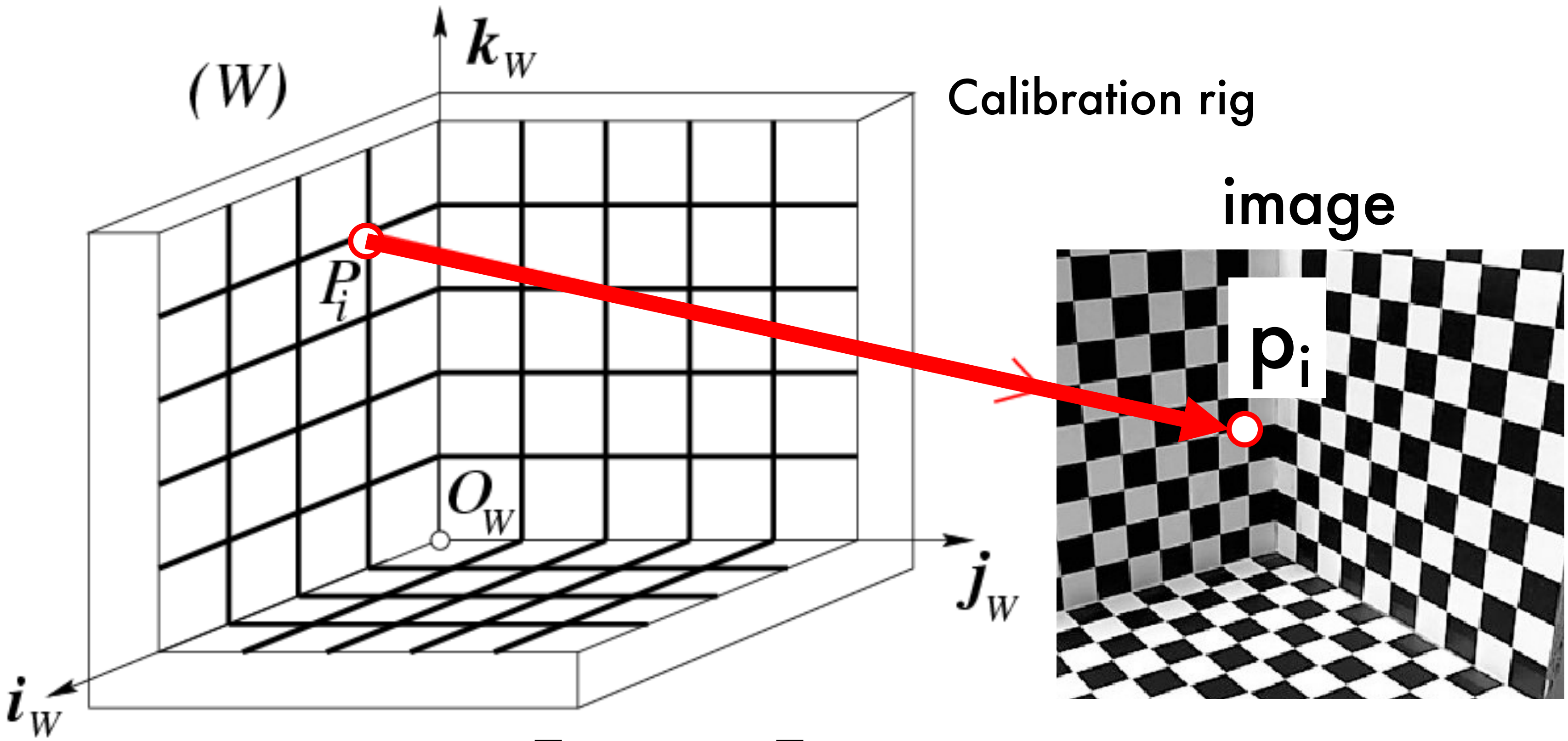
• M has 11 unknowns • We need 11 equations • 6 correspondences would do it

# Calibration Problem




In practice, using more than 6 correspondences enables more robust results

# Calibration Problem




$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix} = M \, P_i$$

in pixels

[Eq. 1]

$$M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

# Calibration Problem

[Eq. 1] $$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \rightarrow u_i(\mathbf{m}_3 P_i) = \mathbf{m}_1 P_i \rightarrow u_i(\mathbf{m}_3 P_i) - \mathbf{m}_1 P_i = 0$$

$$v_i = \frac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \rightarrow v_i(\mathbf{m}_3 P_i) = \mathbf{m}_2 P_i \rightarrow v_i(\mathbf{m}_3 P_i) - \mathbf{m}_2 P_i = 0$$

[Eqs. 2]

# Calibration Problem

$$\begin{cases} u_1(\mathbf{m}_3\, P_1) - \mathbf{m}_1\, P_1 = 0 \\ v_1(\mathbf{m}_3\, P_1) - \mathbf{m}_2\, P_1 = 0 \\ \vdots \\ u_i(\mathbf{m}_3\, P_i) - \mathbf{m}_1\, P_i = 0 \\ v_i(\mathbf{m}_3\, P_i) - \mathbf{m}_2\, P_i = 0 \\ \vdots \\ u_n(\mathbf{m}_3\, P_n) - \mathbf{m}_1\, P_n = 0 \\ v_n(\mathbf{m}_3\, P_n) - \mathbf{m}_2\, P_n = 0 \end{cases}$$

[Eqs. 3]

# Block Matrix Multiplication

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

# Calibration Problem

$$\begin{cases} -u_1(\mathbf{m}_3\ P_1) + \mathbf{m}_1\ P_1 = 0 \\ -v_1(\mathbf{m}_3\ P_1) + \mathbf{m}_2\ P_1 = 0 \\ \vdots \\ -u_n(\mathbf{m}_3\ P_n) + \mathbf{m}_1\ P_n = 0 \\ -v_n(\mathbf{m}_3\ P_n) + \mathbf{m}_2\ P_n = 0 \end{cases}$$

[Eqs. 3]

$\longrightarrow$

known  unknown

$$\mathbf{P}\,\mathbf{m} = 0$$

[Eq. 4]

Homogenous linear system

$$\mathbf{P} \stackrel{\text{def}}{=} \begin{pmatrix} \mathrm{P}_1^T & \mathbf{0}^T & -u_1 \mathrm{P}_1^T \\ \mathbf{0}^T & \mathrm{P}_1^T & -v_1 \mathrm{P}_1^T \\ & \vdots & \\ \mathrm{P}_n^T & \mathbf{0}^T & -u_n \mathrm{P}_n^T \\ \mathbf{0}^T & \mathrm{P}_n^T & -v_n \mathrm{P}_n^T \end{pmatrix}_{2n \times 12}$$

1x4

$$\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^{\mathrm{T}} \\ \mathbf{m}_2^{\mathrm{T}} \\ \mathbf{m}_3^{\mathrm{T}} \end{pmatrix}_{12\times 1}$$

4x1

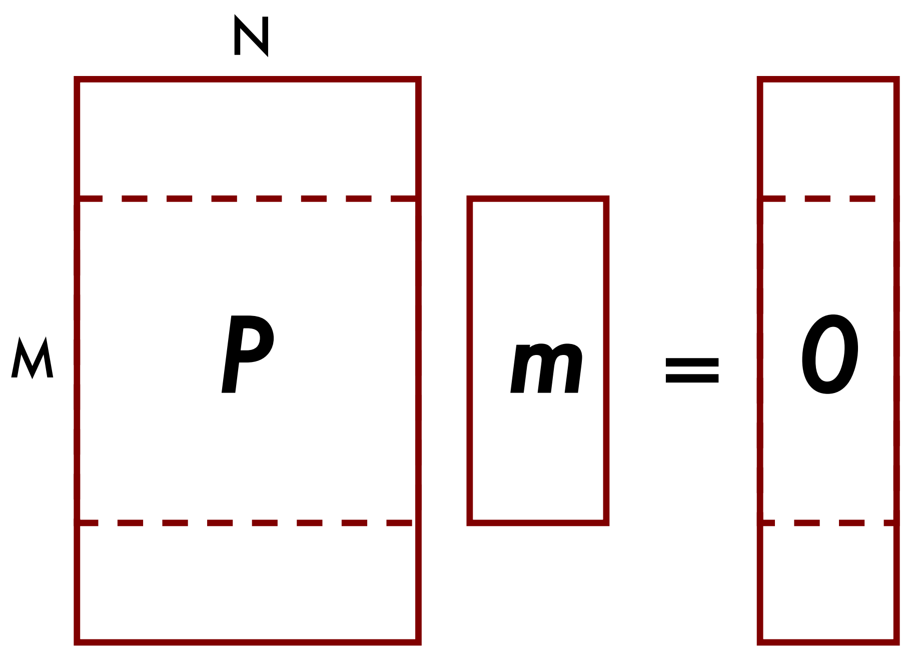
**What do we call the system if the number of correspondences is greater than 6, that is 2n>11?**

Nobody has responded yet.

Hang tight! Responses are coming in.

# Homogeneous M x N Linear Systems

M=number of equations = 2n
N=number of unknown = 11




Rectangular system (M>N)

- 0 is always a solution
- To find non-zero solution

Minimize $|\mathbf{P}\ \mathbf{m}|^2$

under the constraint $|\mathbf{m}|^2 = 1$

# Calibration Problem

$$\mathbf{P}\ \mathbf{m} = 0$$

- How do we solve this homogenous linear system?
- Via SVD decomposition!

# Calibration Problem

$$\mathbf{P}\,\mathbf{m} = 0$$

SVD decomposition of P

$$\mathrm{U}_{2n\times 12}\ \mathrm{D}_{12\times 12}\mathrm{V}^{T}_{12\times 12}$$

Last column of V gives $\boldsymbol{m}$

Why? See pag 592 of HZ

$$\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}$$

$$\boldsymbol{m} \rightarrow M$$

# Extracting camera parameters

$$M = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} \rho$$

# Extracting camera parameters

See [FP], Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}$$

$A$ $\quad$ $\mathbf{b}$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

## Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \qquad u_o = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_3)$$

$$v_o = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$

$$\cos\theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

# Extracting camera parameters

See [FP], Sec. 1.3.1

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}$$

$A$ $\qquad$ $\mathbf{b}$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

## Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin\theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin\theta$$

# Extracting camera parameters

See [FP], Sec. 1.3.1

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \frac{\beta}{\sin\theta} \boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \frac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix} = \mathrm{K}[\mathrm{R} \quad \mathrm{T}]$$

$$\underbrace{\qquad}_{A} \qquad \underbrace{\qquad}_{\mathbf{b}}$$

Box 1

$$\mathrm{A} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}} \\ \mathbf{a}_2^{\mathrm{T}} \\ \mathbf{a}_3^{\mathrm{T}} \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} \mathrm{b}_1 \\ \mathrm{b}_2 \\ \mathrm{b}_3 \end{bmatrix}$$

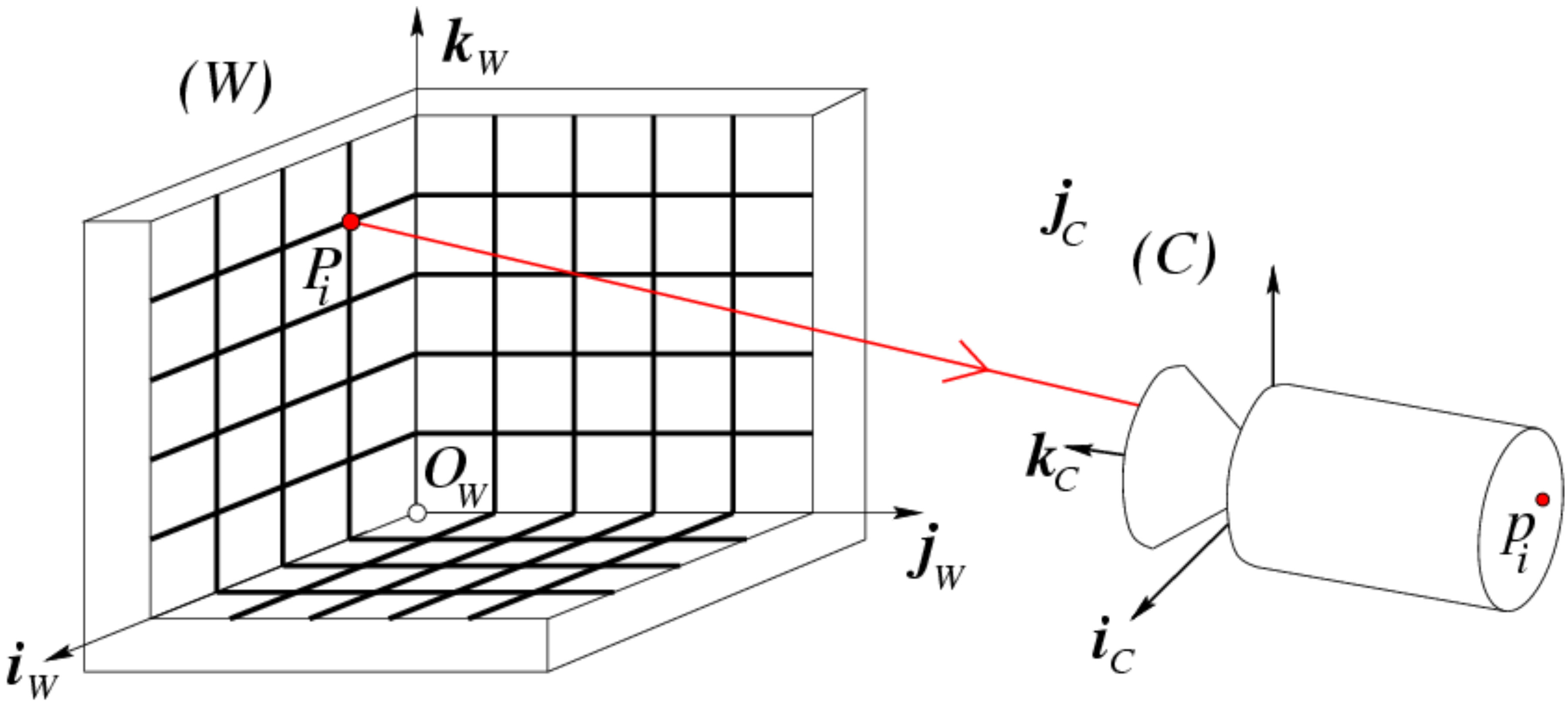
Estimated values

## Extrinsic

$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \qquad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$

$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \qquad \mathrm{T} = \rho\, \mathrm{K}^{-1} \mathbf{b}$$

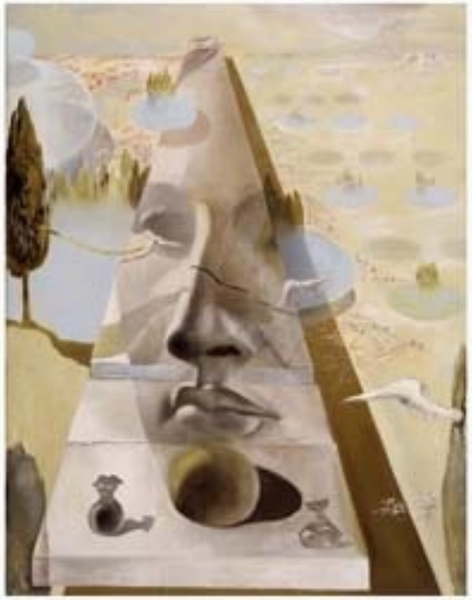
# Degenerate cases




- $P_i$'s cannot lie on the same plane!
- Points cannot lie on the intersection curve of two quadric surfaces [FP] section 1.3

# Lecture 3

# Camera Calibration



- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

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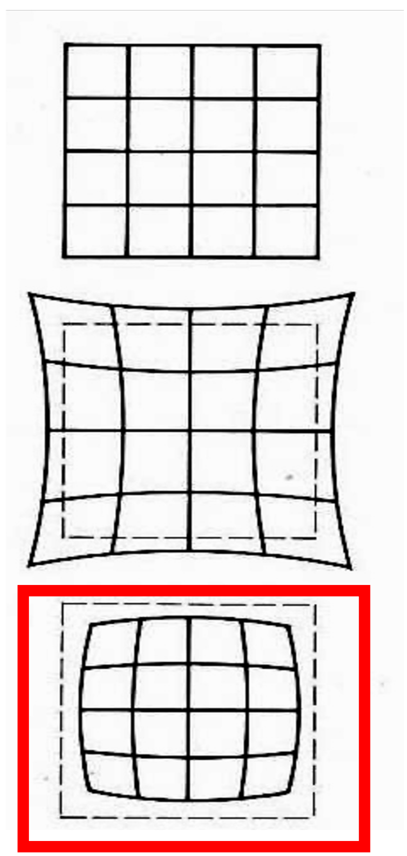
[HZ] Chapter 7 “Computation of Camera Matrix P”

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

# Radial Distortion

- Image magnification (in)decreases with distance from the optical axis
- Caused by imperfect lenses
- Deviations are most noticeable for rays that pass through the edge of the lens



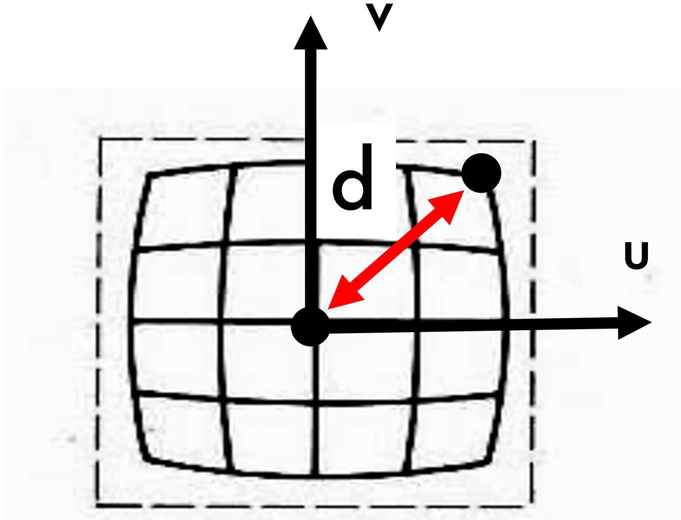
No distortion

Pin cushion

Barrel

# Radial Distortion

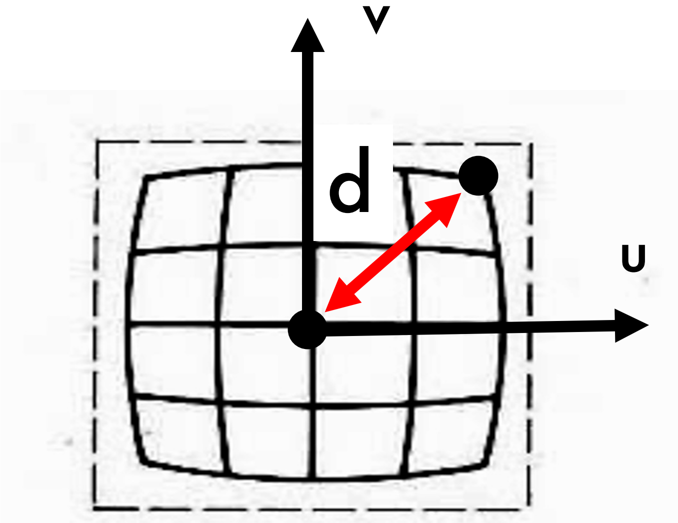
Image magnification decreases with distance from the optical center




$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S_\lambda} M\, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

# Radial Distortion

Image magnification decreases with distance from the optical center




$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{S_\lambda} M\, P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

Distortion coefficient

$$\lambda = 1 \pm \sum_{p=1}^{3} \kappa_p d^{2p}$$

[Eq. 5]

Polynomial function

$$d^2 = a\,u^2 + b\,v^2 + c\,u\,v$$

To model radial behavior

[Eq. 6]

# Radial Distortion

$$\underbrace{\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M}_{Q} P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$Q = \begin{bmatrix} \mathbf{q}_1 \\ \mathbf{q}_2 \\ \mathbf{q}_3 \end{bmatrix}$$

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{q}_1 P_i}{\mathbf{q}_3 P_i} \\ \frac{\mathbf{q}_2 P_i}{\mathbf{q}_3 P_i} \end{bmatrix} \rightarrow \begin{cases} u_i \mathbf{q}_3 P_i = \mathbf{q}_1 P_i \\ v_i \mathbf{q}_3 P_i = \mathbf{q}_2 P_i \end{cases}$$

Is this a linear system of equations?

No!

[Eqs.7]

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \ \mathrm{P_i}}{\mathbf{q}_3 \ \mathrm{P_i}} \\ \dfrac{\mathbf{q}_2 \ \mathrm{P_i}}{\mathbf{q}_3 \ \mathrm{P_i}} \end{bmatrix} \longrightarrow X = f(Q) \quad \text{[Eq .8]}$$

i=1…n

measurements ($X$), parameters ($Q$)

f( ) is the nonlinear mapping

-Newton Method

-Levenberg-Marquardt Algorithm

- Iterative, starts from initial solution
- May be slow if initial solution far from real solution
- Estimated solution may be function of the initial solution (because of local minima)
- Newton requires the computation of J, H
- Levenberg-Marquardt doesn’t require the computation of H

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \end{bmatrix} \quad \longrightarrow \quad X = f(Q)$$

[Eq .8]

i=1…n

measurements

parameters

f( ) is the nonlinear mapping

# A possible algorithm

1. Solve linear part of the system to find approximated solution
2. Use this solution as initial condition for the full system
3. Solve full system using Newton or L.M.

# General Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \\ \dfrac{\mathbf{q}_2 \, \mathrm{P}_i}{\mathbf{q}_3 \, \mathrm{P}_i} \end{bmatrix} \longrightarrow X = f(Q) \quad \text{[Eq .8]}$$

i=1…n

measurements ($X$), parameters ($Q$)

f( ) is the nonlinear mapping

Typical assumptions:

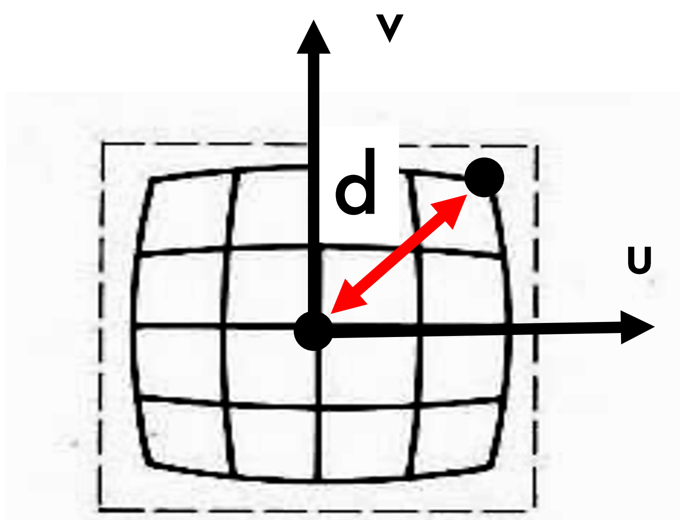
- zero-skew, square pixel
- $u_o$, $v_o$ = known center of the image

# Radial Distortion

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{q}_1 P_i}{\mathbf{q}_3 P_i} \\ \dfrac{\mathbf{q}_2 P_i}{\mathbf{q}_3 P_i} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

Can we estimate $m_1$ and $m_2$ and ignore the radial distortion?

Hint:




$$\frac{u_i}{v_i} = \text{slope}$$

# Radial Distortion Tsai [87]

## Estimating $m_1$ and $m_2$...

$$\mathrm{p}_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda}\begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix} \rightarrow \frac{u_i}{v_i} = \frac{\dfrac{(\mathbf{m}_1 \, P_i)}{(\mathbf{m}_3 \, P_i)}}{\dfrac{(\mathbf{m}_2 \, P_i)}{(\mathbf{m}_3 \, P_i)}} = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_2 \, P_i}$$

[Eq .9]

[Eq .10]

$$\begin{cases} v_1(\mathbf{m}_1 \, P_1) - u_1(\mathbf{m}_2 \, P_1) = 0 \\ v_i(\mathbf{m}_1 \, P_i) - u_i(\mathbf{m}_2 \, P_i) = 0 \\ \vdots \\ v_n(\mathbf{m}_1 \, P_n) - u_n(\mathbf{m}_2 \, P_n) = 0 \end{cases}$$

[Eq .11]

$$L \, \mathbf{n} = 0$$

$$\mathbf{L} \stackrel{\text{def}}{=} \begin{pmatrix} v_1 \mathrm{P}_1^T & -u_1 \mathrm{P}_1^T \\ v_2 \mathrm{P}_2^T & -u_2 \mathrm{P}_2^T \\ \vdots & \vdots \\ v_n \mathrm{P}_n^T & -u_n \mathrm{P}_n^T \end{pmatrix}$$

$$\mathbf{n} = \begin{bmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \end{bmatrix}$$

Get $\mathbf{m}_1$ and $\mathbf{m}_2$ by SVD

# Radial Distortion

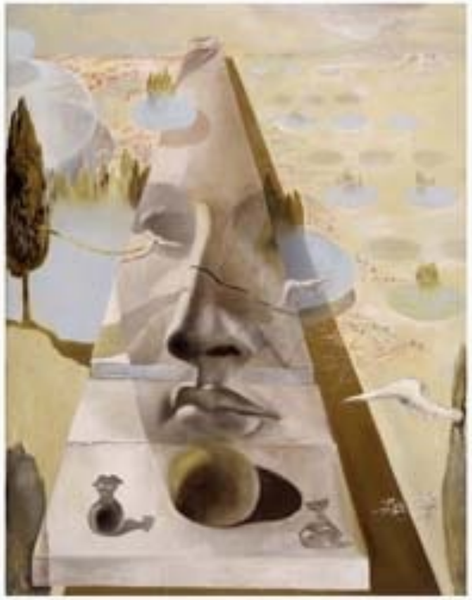
Once that $\mathbf{m}_1$ and $\mathbf{m}_2$ are estimated…

$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} \dfrac{\mathbf{m}_1 P_i}{\mathbf{m}_3 P_i} \\ \dfrac{\mathbf{m}_2 P_i}{\mathbf{m}_3 P_i} \end{bmatrix}$$

$\mathbf{m}_3$ is non linear function of $\mathbf{m}_1, \mathbf{m}_2, \lambda$

There are some degenerate configurations for which $m_1$ and $m_2$ cannot be computed

# Lecture 3
# Camera Calibration



- Recap of projective cameras
- Camera calibration problem
- Camera calibration with radial distortion
- Example

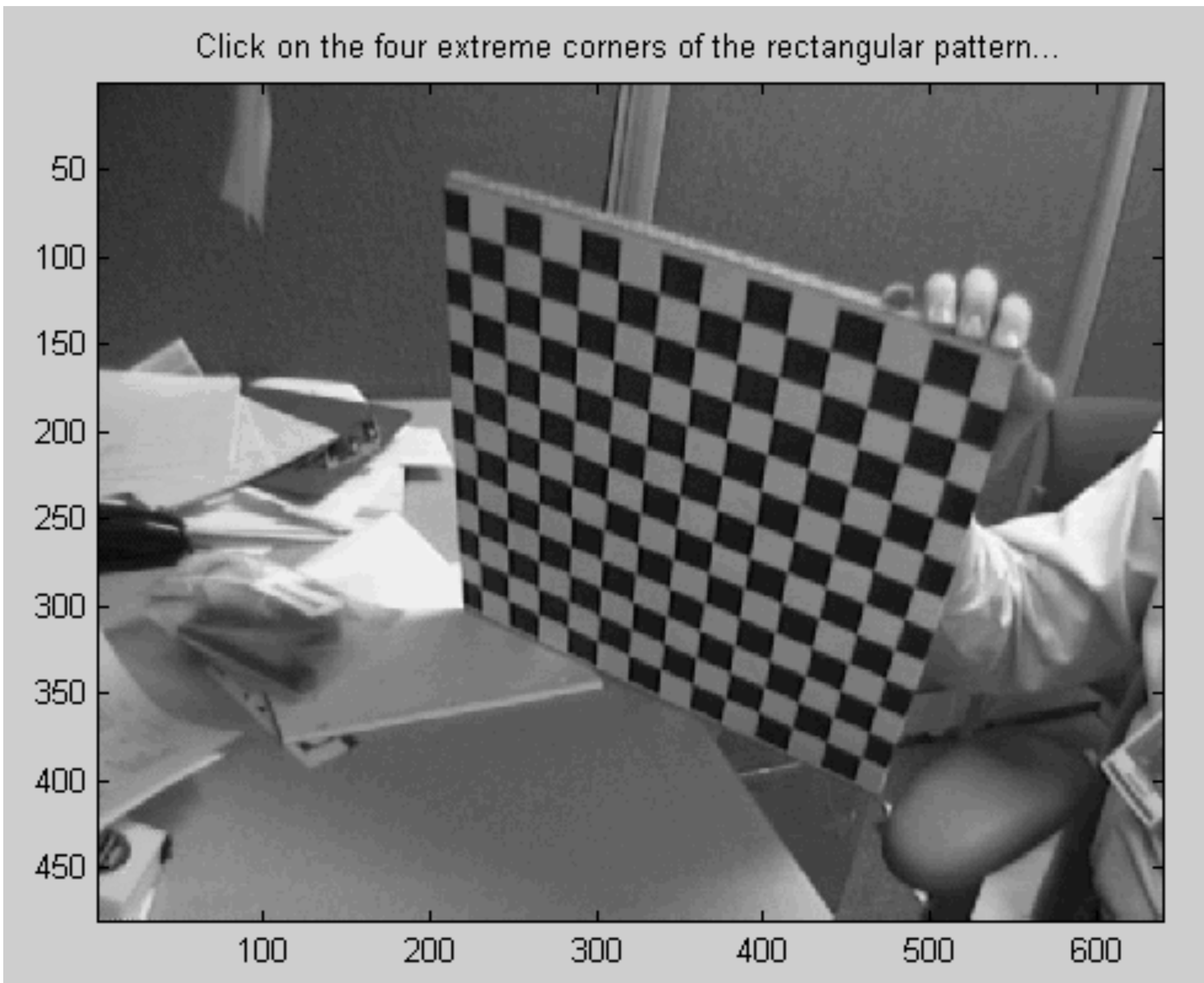
Reading: [FP] Chapter 1 "Geometric Camera Calibration"
[HZ] Chapter 7 "Computation of Camera Matrix P"

Some slides in this lecture are courtesy to Profs. J. Ponce, F-F Li

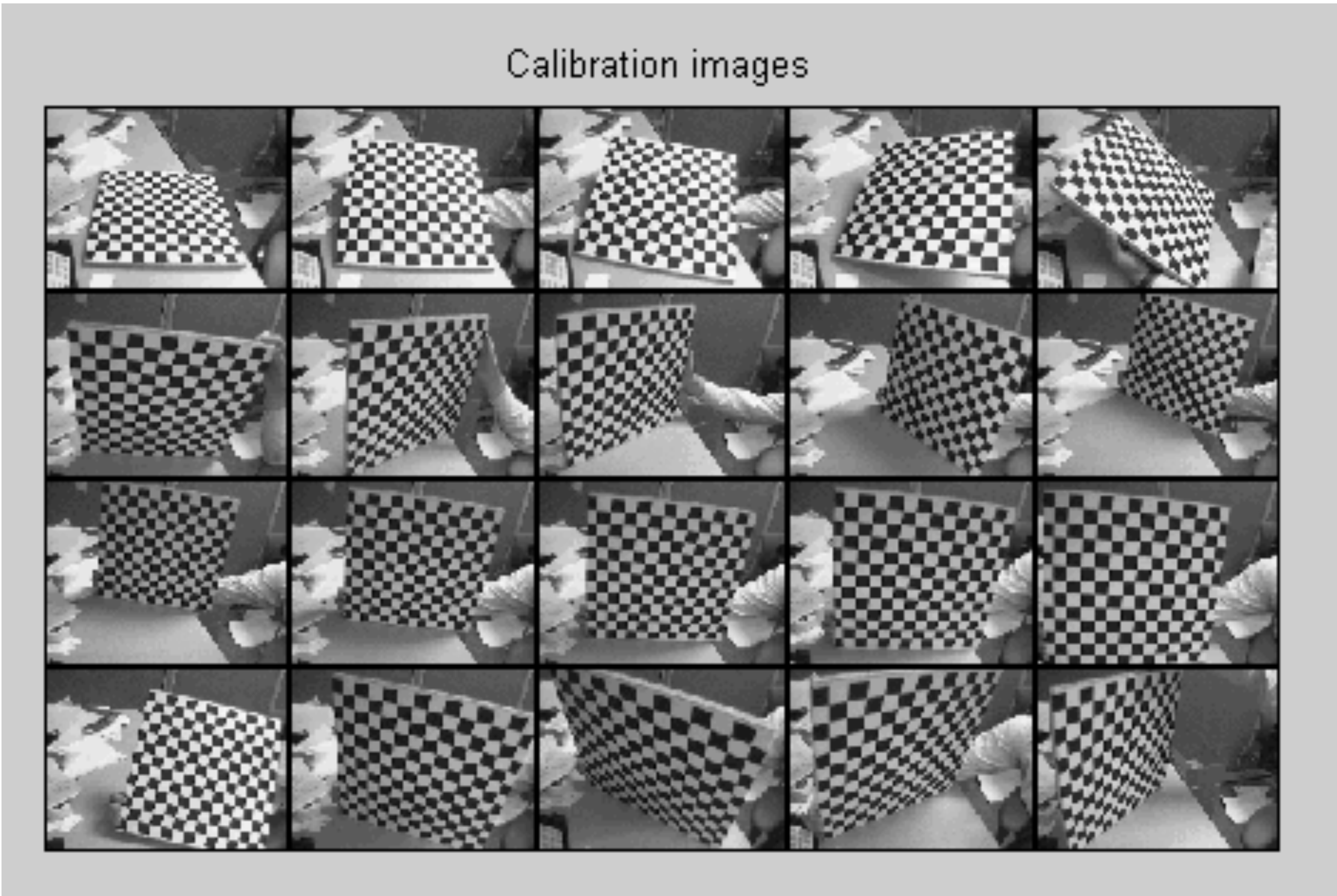
# Calibration Procedure

*Camera Calibration Toolbox for OpenCV*
*J. Bouguet* – [1998-2000]

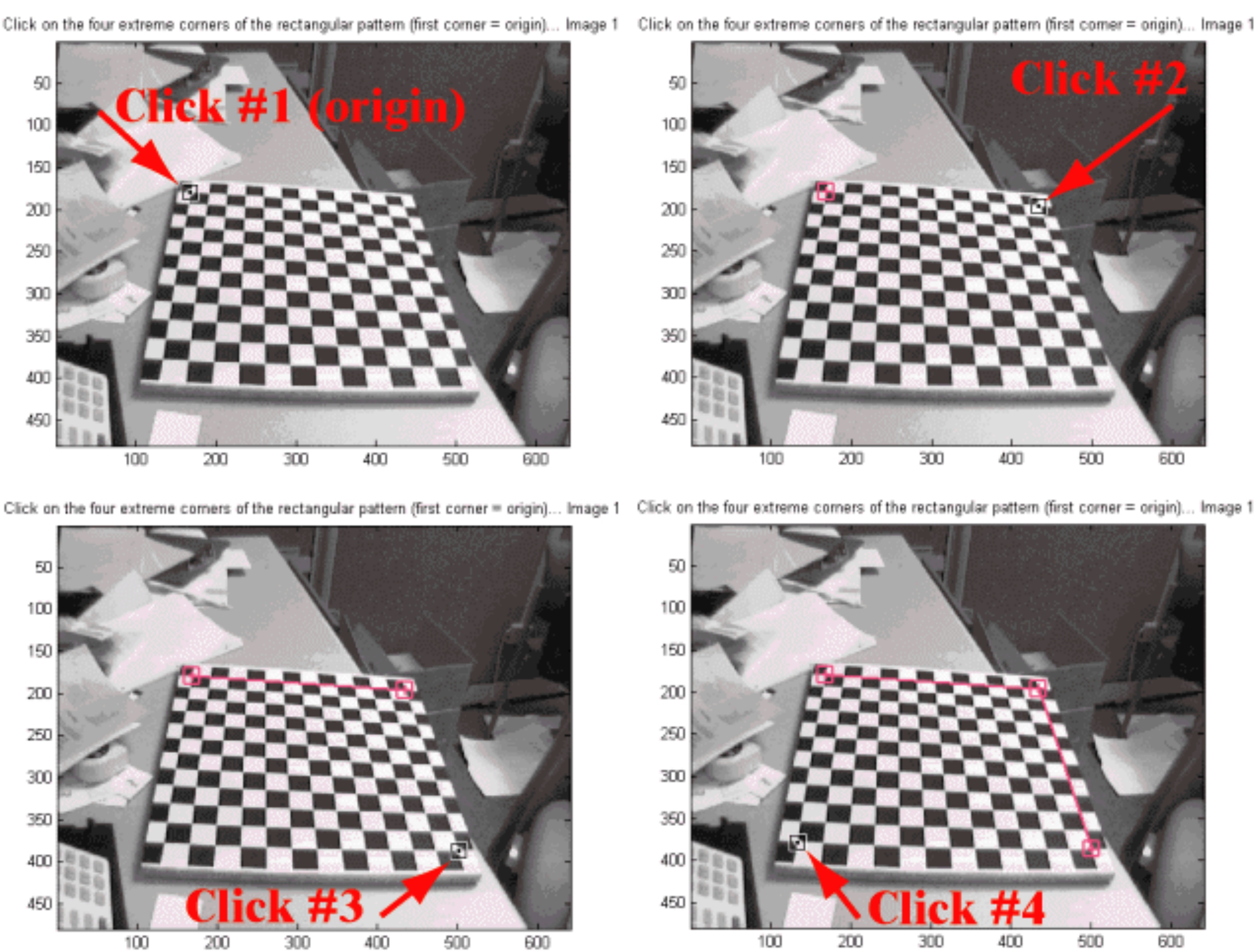
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html#examples



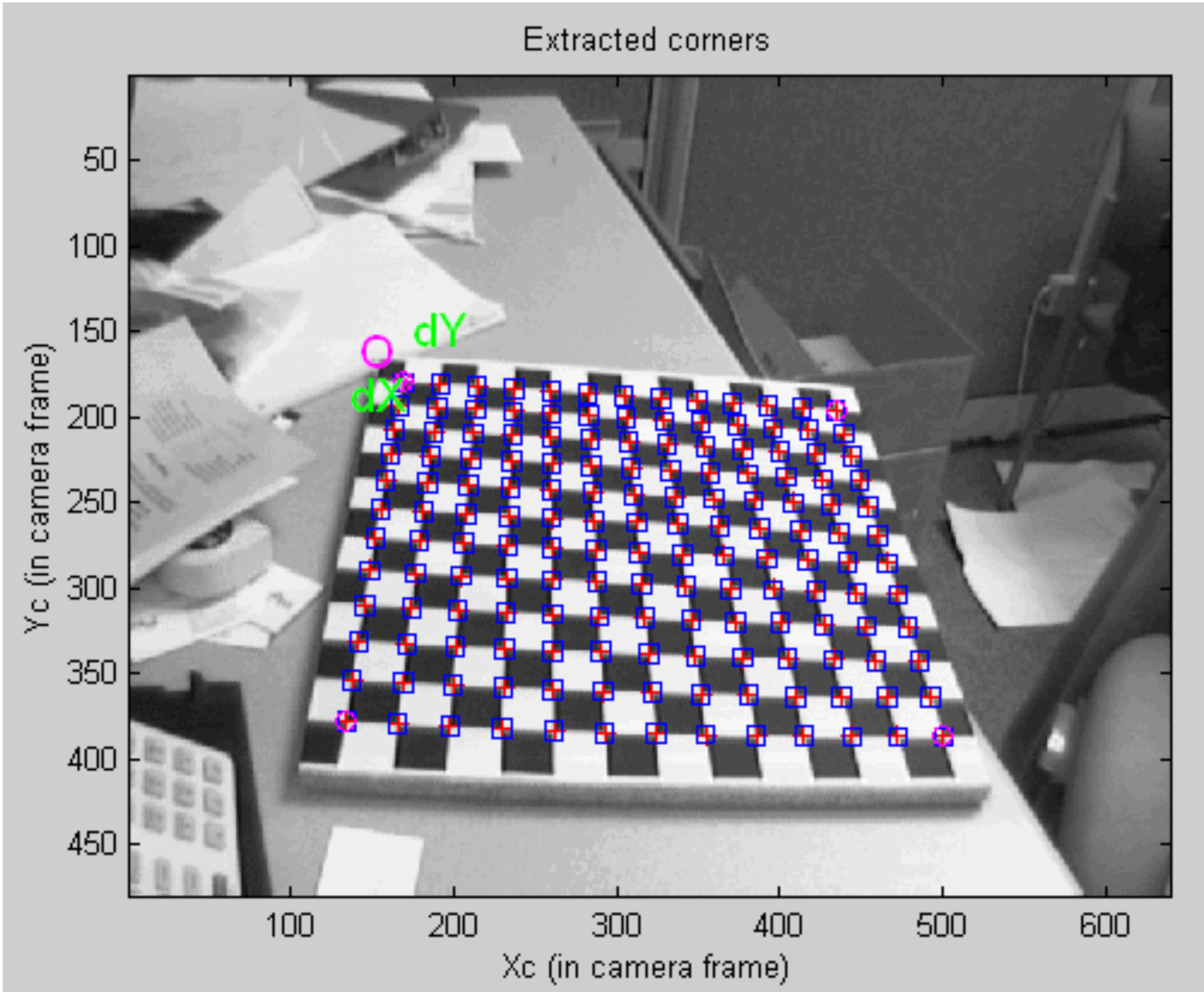
# Calibration Procedure



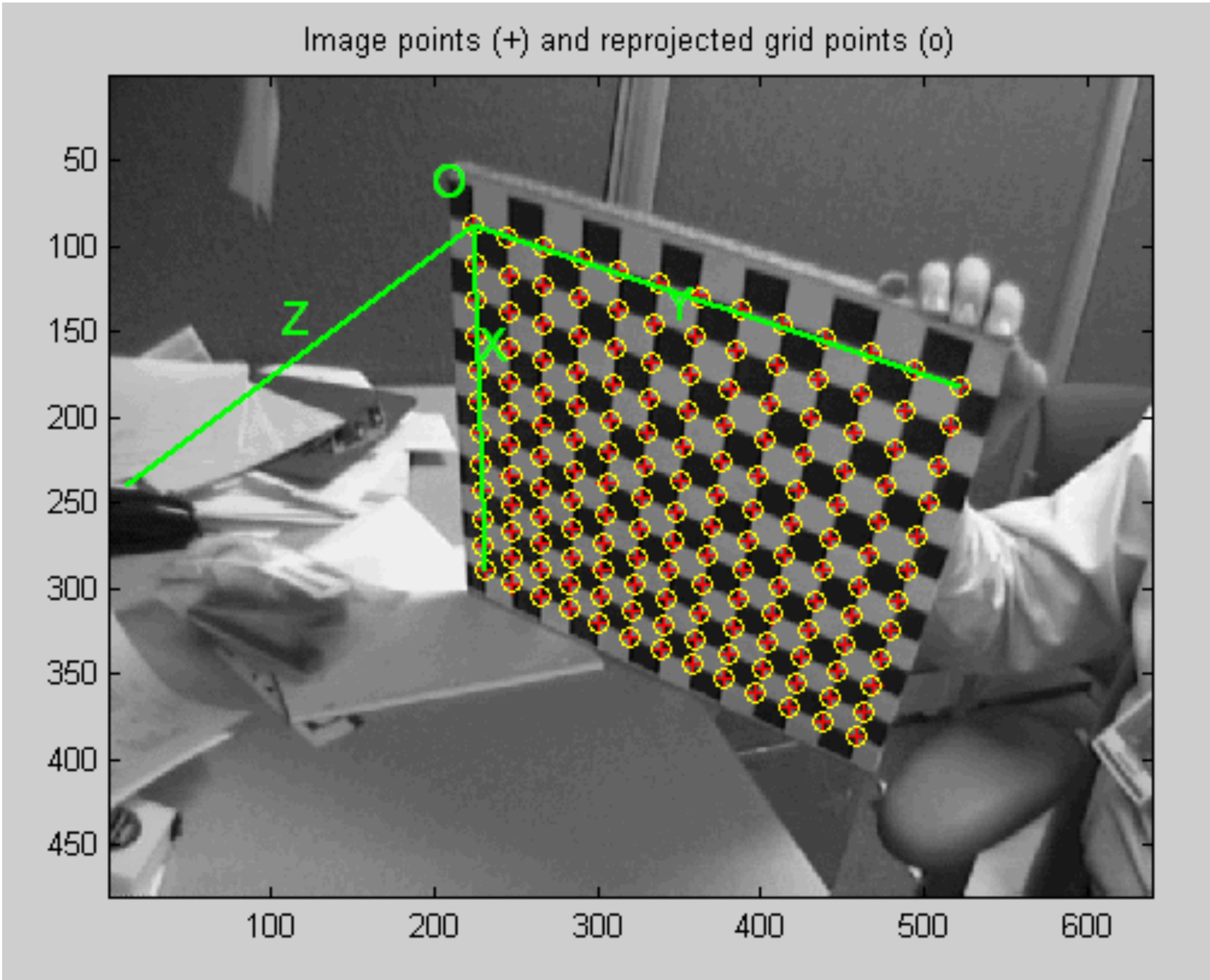
# Calibration Procedure



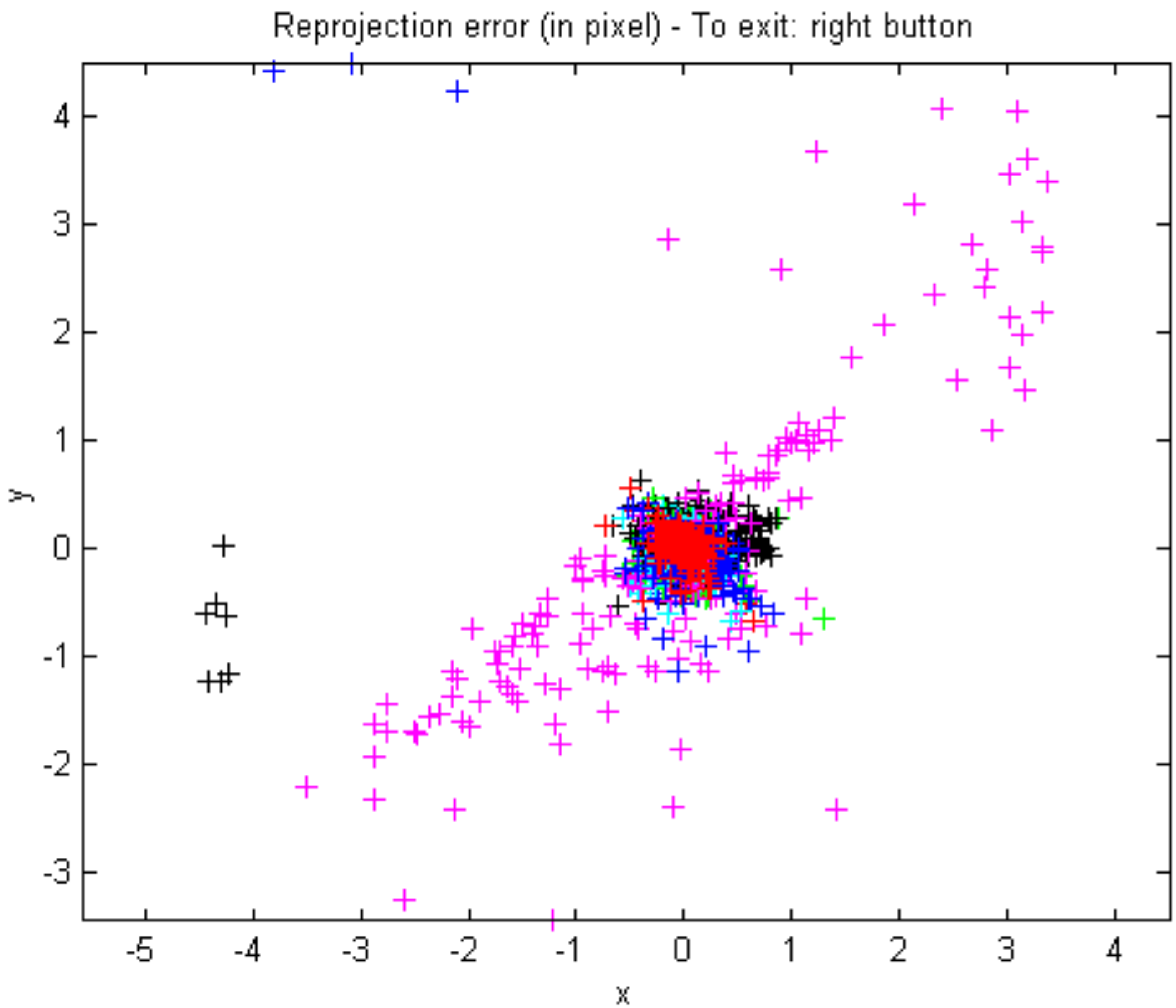
# Calibration Procedure



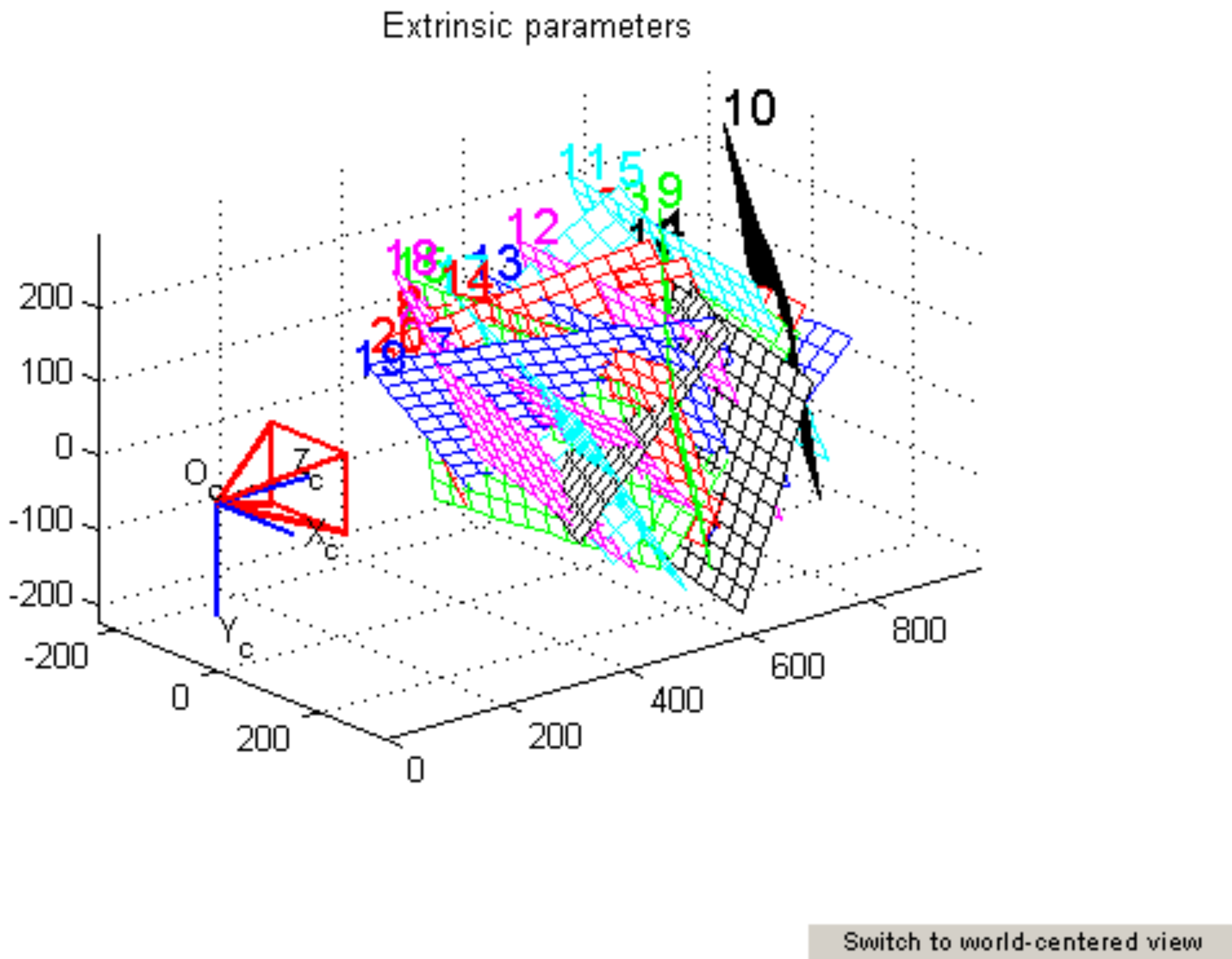
# Calibration Procedure



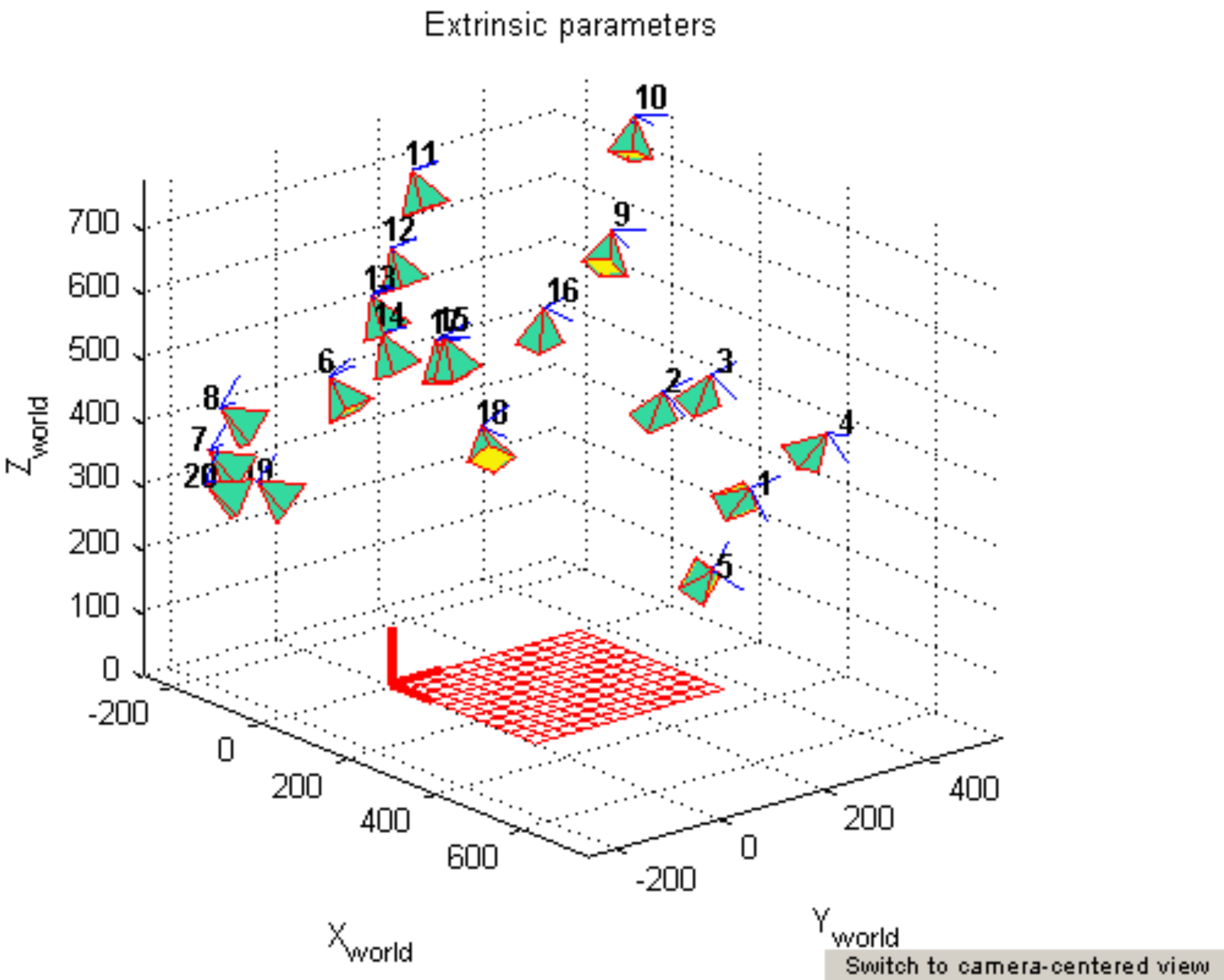
# Calibration Procedure



# Calibration Procedure



# Calibration Procedure



# Next lecture

- Single view reconstruction