Programming Abstractions

CS106B

Cynthia Bailey
Chris Gregg

pollev.com/cs106b

Today's Topics

- Recursion!
  - Functions calling functions
- Next time:
  - More recursion! It's Recursion Week!
  - Like Shark Week, but more nerdy

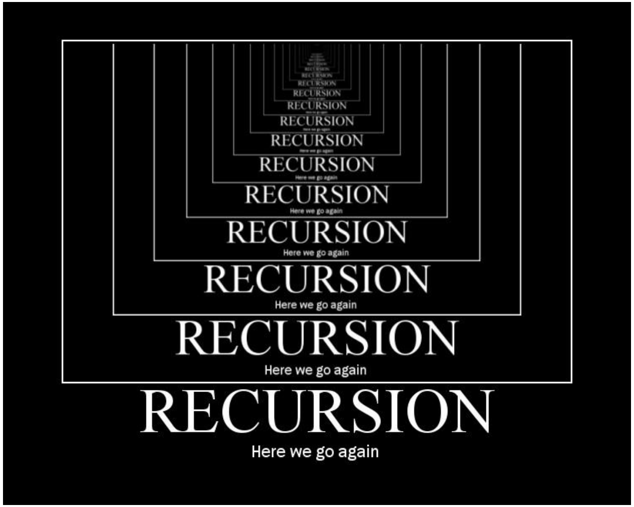
- **For important announcements, be sure to see the weekly announcements post on the Ed Q&A board! https://edstem.org**
- **Also on Ed: live lecture Q&A with Chris & Jonathan**

Quick Course Overview

- Week 1: C++
- Week 2: ADTs, *how to use them*
- Weeks 3-4: Recursion ← **YOU ARE HERE**
- Weeks 5-10: ADTs, *behind the scenes!*
  - actually implement them yourself

Recursion!

The exclamation point isn't there only because this is so exciting; it also relates to our first recursion example….



Factorial!

$$n! = n(n-1)(n-2)(n-3)(n-4)\ldots(3)(2)(1)$$

This could be a really long expression!

Recursion is a technique for tackling large or complicated problems by just "eating" one "bite" of the problem at a time.

Factorial!

$$\boldsymbol{n}! = \boldsymbol{n}(\boldsymbol{n}-\mathbf{1})(\boldsymbol{n}-\mathbf{2})(\boldsymbol{n}-\mathbf{3})(\boldsymbol{n}-\mathbf{4})\ldots(\mathbf{2})(\mathbf{1})$$

An alternate mathematical formulation:

$$n! = \begin{cases} 1 & if\ n = 1 \\ n(\boldsymbol{n}-\mathbf{1})! & otherwise \end{cases}$$

Translated to code

```
int factorial(int n) {
    if (n == 1) {
        return 1;
    } else {
        return n * factorial(n - 1);
    }
}
```

Factorial!

$$n! = n(n-1)(n-2)(n-3)(n-4)\ldots(2)(1)$$

An alternate mathematical formulation:

$$n! = \begin{cases} 1 & if\ n = 1 \\ n(n-1)! & otherwise \end{cases}$$

Translated to code

```
int factorial(int n) {
    if (n == 1) {
        return 1;
    } else {
        return n * factorial_teacher_solution(n-1);
    }
}
```

Debugging tip: when reading your own code, mentally assume the recursive call works perfectly, and reason from there.

Basic Recursive Function Design Pattern

Always two parts:

Base case:

- This problem is so tiny, it's hardly a problem anymore! Just give answer.

Recursive case:

- This problem is still a bit large, let's (1) bite off just one piece, and (2) delegate the remaining work to recursion.

The recursive function pattern

Recursive case:

- This problem is still a bit large, let's **(1) bite off just one piece**, and (2) delegate the remaining work to recursion.

```
int factorial(int n) {
    if (n == 1) {  // Easy! Return trivial answer
        return 1;
    } else {  // Not easy enough to finish yet!
        return n * factorial(n - 1);
    }
}
```

Do one of the many, many multiplications required for factorial.

The recursive function pattern

Recursive case:

- This problem is still a bit large, let's (1) bite off just one piece, and **(2) delegate the remaining work to recursion**.

```
int factorial(int n) {
    if (n == 1) {  // Easy! Return trivial answer
        return 1;
    } else {  // Not easy enough to finish yet!
        return n * factorial(n - 1);
    }
}
```

Do one of the many, many multiplications required for factorial.

Delegate all the other multiplications to the recursive call.

Digging deeper in the recursion

Looking at how recursion works “under the hood”

Factorial!

```
int factorial(int n) {
    cout << n << endl;  // **Added for this question**
    if (n == 1) {  // Easy! Return trivial answer
        return 1;
    } else {  // Not easy enough to finish yet!
        return n * factorial(n - 1);
    }
}
```

What is the **third** thing **printed** when we call `factorial(4)`?

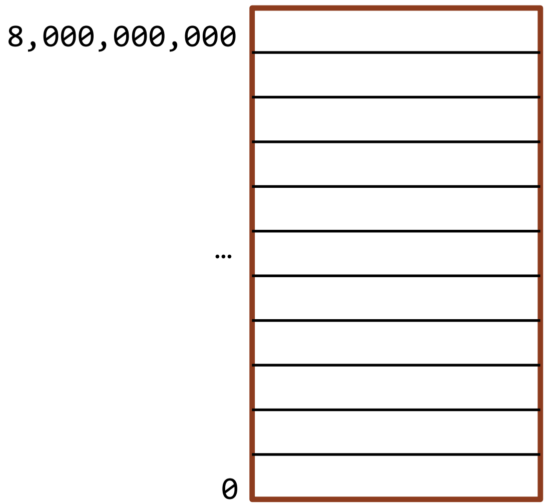
A. 1
B. 2
C. 3
D. 4
E. Other/none/more

pollev.com/cs106b

Stanford University

How does this look in memory?
A little background…

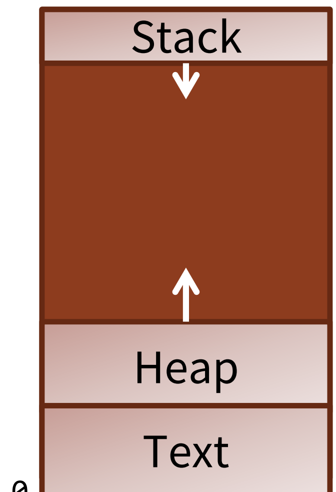
- A computer's memory is like a giant Vector/array, and like a Vector, we start counting at index `0`.
- We typically draw memory vertically (rather than horizontally like a Vector), with index `0` at the bottom.
- A typical laptop's memory has billions of these indexed slots (one byte each)



* Take CS107 to learn much more!!

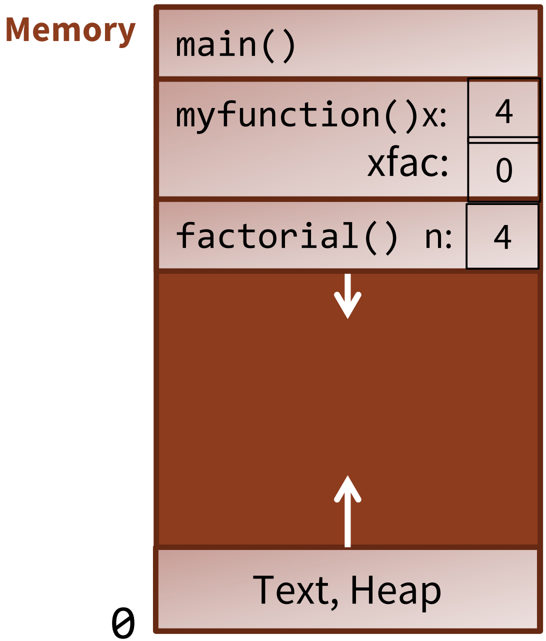
How does this look in memory?
A little background…

- Broadly speaking, we divide memory into regions:
  - **Text:** the program's own code (needs to be in memory so it can run!)
  - **Heap:** we'll learn about this later in CS106B!
  - **Stack:** this is where local variables for each function are stored.



* Take CS107 to learn much more!!

How does this look in memory?

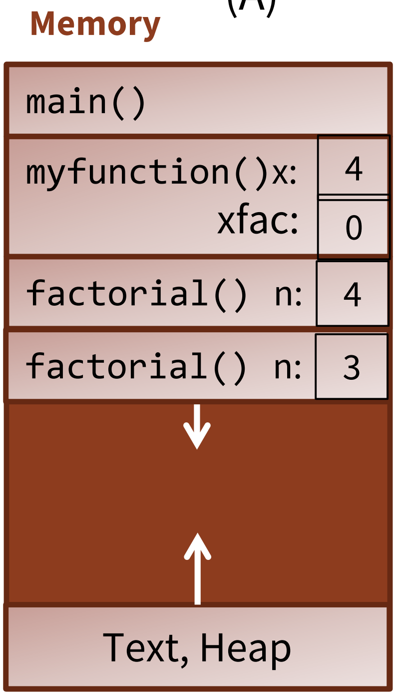


Recursive code

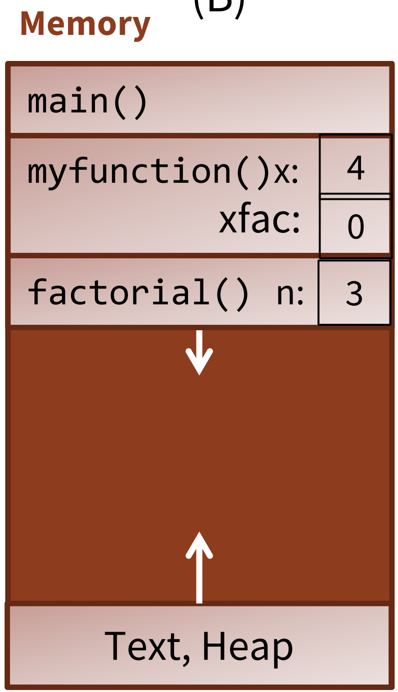
```
int factorial(int n) {
     cout << n << endl;
     if (n == 1) return 1;
     else return n * factorial(n - 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

pollev.com/cs106b

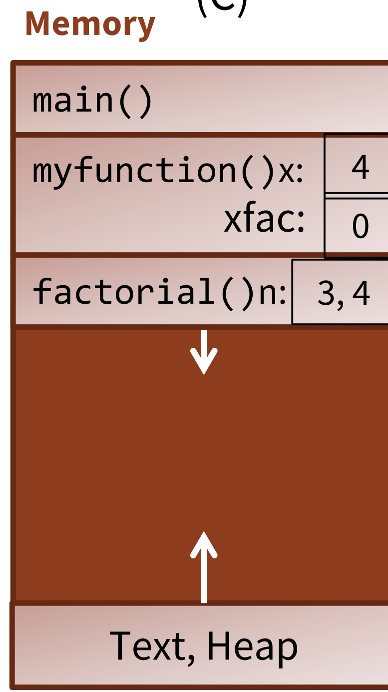
(A)



(B)



(C)



(D) Other/none of the above

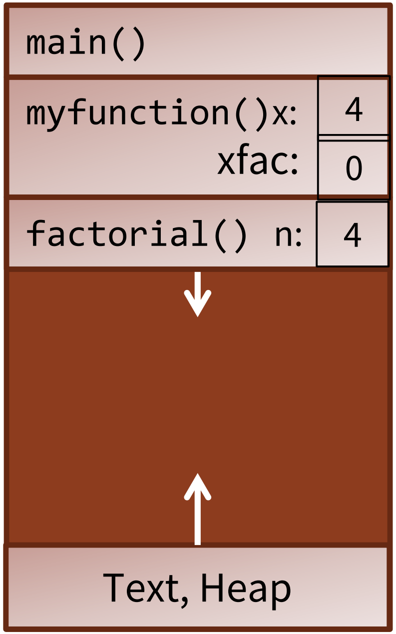
Fun fact:
The “stack” part of memory is a stack

Function **call** = **push** a stack frame

Function **return** = **pop** a stack frame

* Take CS107 to learn much more!!

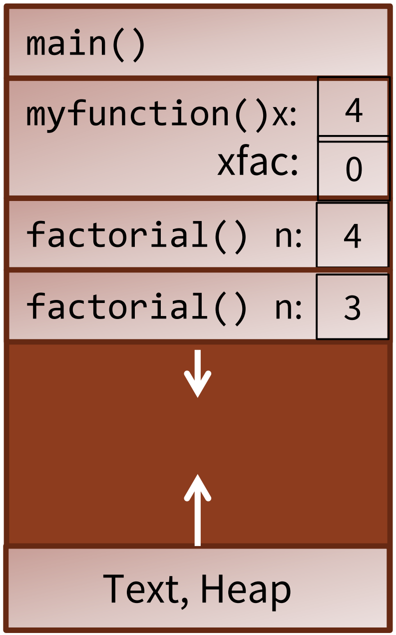
The “stack” part of memory is a stack



Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

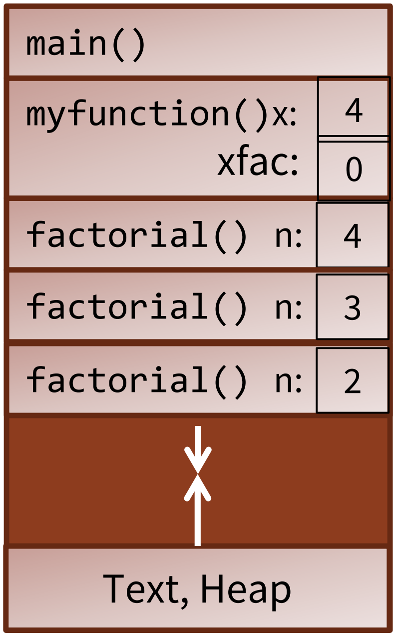
The “stack” part of memory is a stack



Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

The "stack" part of memory is a stack

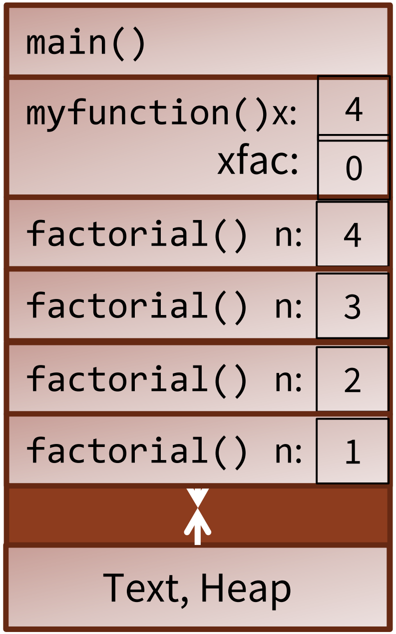


Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

Answer: 3rd thing printed is 2

The “stack” part of memory is a stack



Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

Factorial!

What is the **fourth** value ever **returned** when we call `factorial(4)`?

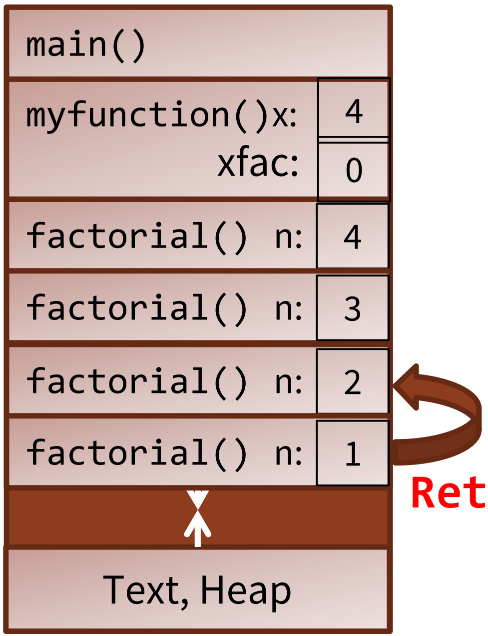
A. 4
B. 6
C. 10
D. 24
E. Other/none/more than one

Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n - 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

pollev.com/cs106b

The “stack” part of memory is a stack

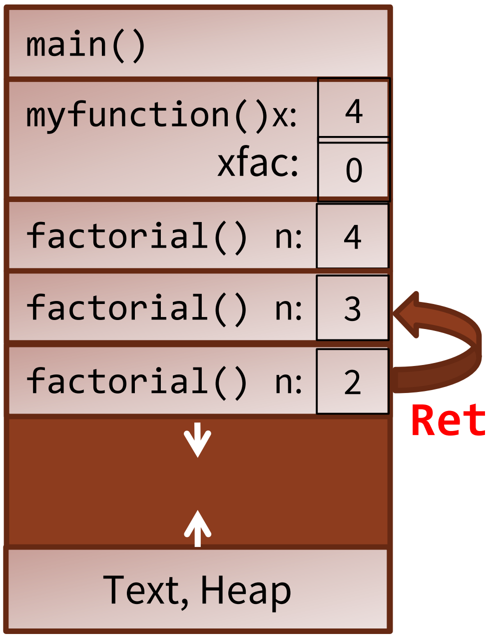


Return 1

Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}

void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

The "stack" part of memory is a stack



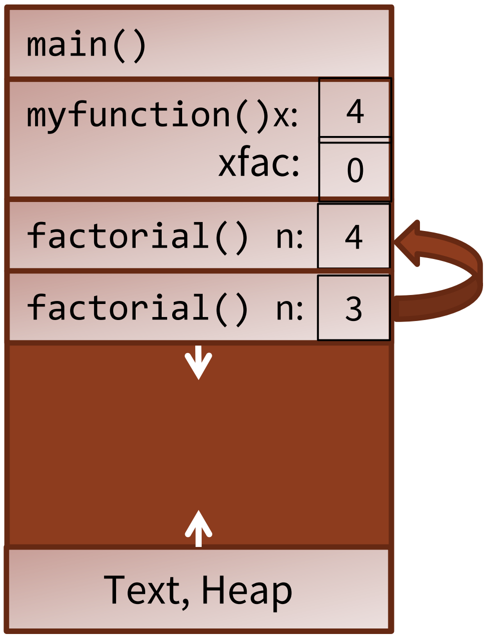
Return 2

Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}
```

```
void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

The “stack” part of memory is a stack



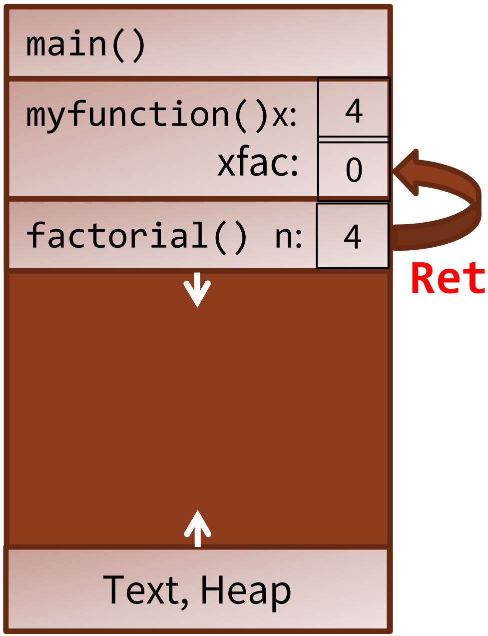
Recursive code

```
int factorial(int n) {
    cout << n << endl;
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}
```

Return 6

```
void myfunction(){
    int x = 4;
    int xfac = 0;
    xfac = factorial(x);
}
```

The "stack" part of memory is a stack



Return 24

Recursive code

```
int factorial(int n) {
     cout << n << endl;
     if (n == 1) return 1;
     else return n * factorial(n - 1);
}
```

Answer: 4th thing returned is 24

```
void myfunction(){
     int x = 4;
     int xfac = 0;
     xfac = factorial(x);
}
```

Factorial!

Iterative version

```
int factorial(int n) {
    int f = 1;
    while (n > 1) {
        f = f * n;
        n = n – 1;
    }
    return f;
}
```

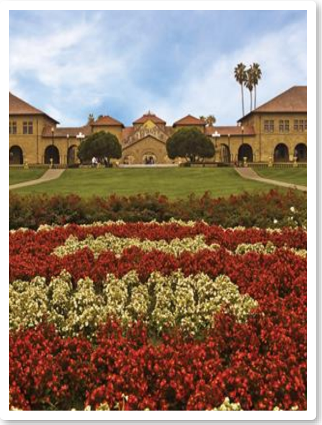
Recursive version

```
int factorial(int n) {
    if (n == 1) return 1;
    else return n * factorial(n – 1);
}
```

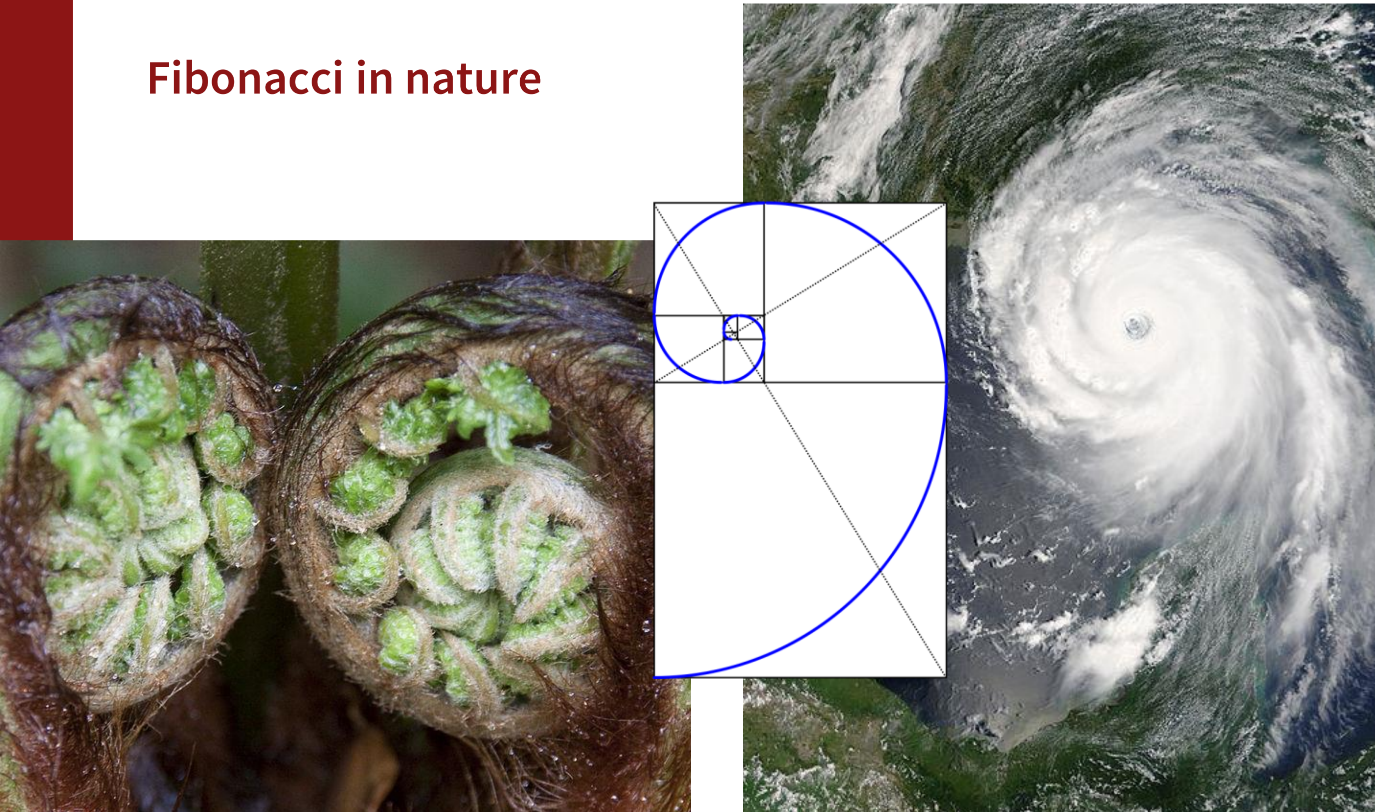
NOTE: sometimes **iterative can be much faster** because it doesn't have to push and pop stack frames. Method calls have overhead in terms of space *and* time (to set up and tear down).

The Fibonacci Sequence

MATH NERD REJOICING INTENSIFIES

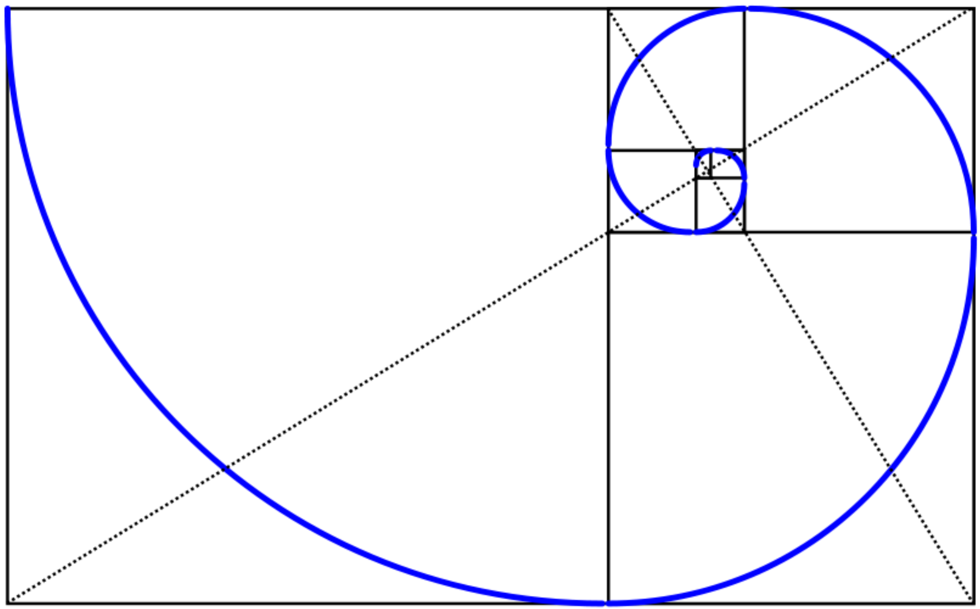


Fibonacci in nature



Fibonacci

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,



Fibonacci

0	1	2	3	4	5	6	7	8	9	10	11
0	1	1	2	3	5	8	13	21	34	55	89

$$Fib_N = \begin{cases} 0 & if\ N = 0 \\ 1 & if\ N = 1 \\ Fib_{N-1} + Fib_{N-2} & otherwise \end{cases}$$

Basic Recursive Function Design Pattern

Always two parts:

Base case:

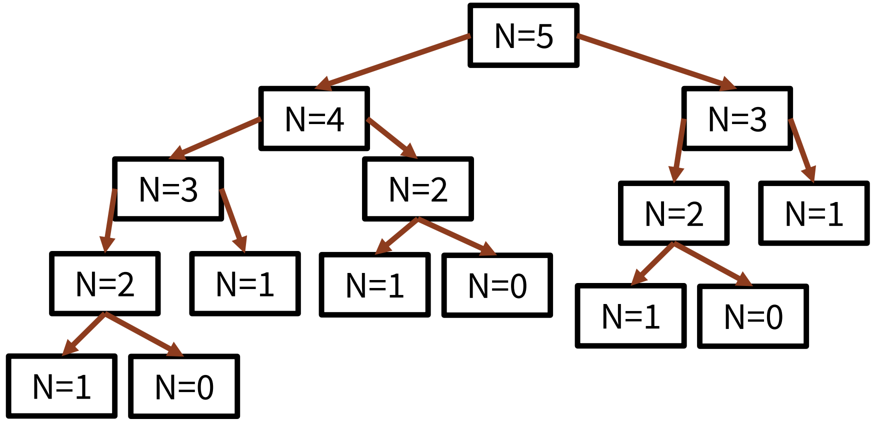
- This problem is so tiny, it's hardly a problem anymore! Just give answer.

Recursive case:

- This problem is still a bit large, let's (1) bite off just one piece, and (2) delegate the remaining work to recursion.

$$Fib_N = \begin{cases} 0 & if\ N = 0 \\ 1 & if\ N = 1 \\ Fib_{N-1} + Fib_{N-2} & otherwise \end{cases}$$

Fibonacci



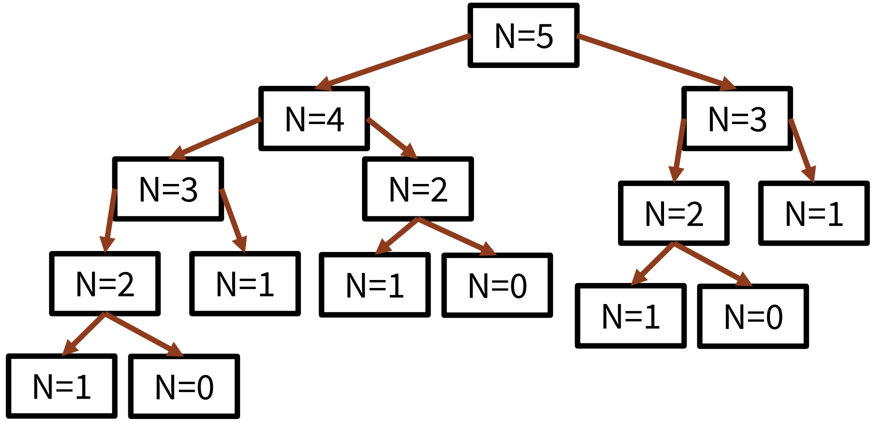

```
int fib(int n)
{
    if (n == 0) {
        return 0;
    } else if (n == 1) {
        return 1;
    } else {
        return fib(n – 1) + fib(n – 2);
    }
}
```

Observation: work is duplicated throughout the call tree

- fib(2) is calculated 3 separate times when calculating fib(5)!
- 15 function calls in total for fib(5)!

Fibonacci

fib(2) is calculated 3 separate times when calculating fib(5)!



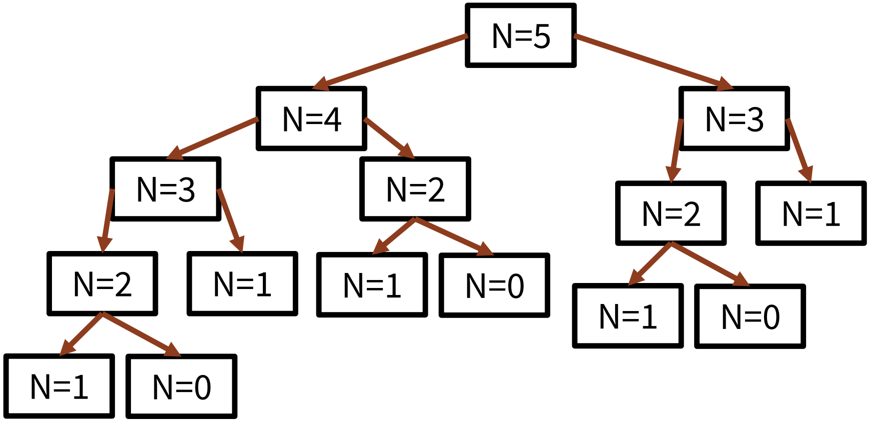
How many times would we calculate fib(2) while calculating fib(6)? ***See if you can just "read" it off the chart above.***

A. 4 times
B. 5 times
C. 6 times
D. Other/none/more

pollev.com/cs106b

Fibonacci

N	fib(N)	# of calls to fib(2)
2	1	1
3	2	1
4	3	2
5	5	3
6	8	5
7	13	8
8	21	13
9	34	21
10	55	34



Efficiency of naïve Fibonacci implementation

When we **added 1** to the input N, the number of times we had to calculate fib(2) **nearly doubled** (~1.6* times)

- Ouch!

Goal: predict how much time it will take to compute for arbitrary input N.

Calculation: "approximately" $\mathbf{(1.6)^N}$

* This ~1.6 number is called the "Golden Ratio" in math—cool!