Introduction to Recursion

What was the most challenging part of Assignment 1?
(in three words or less)

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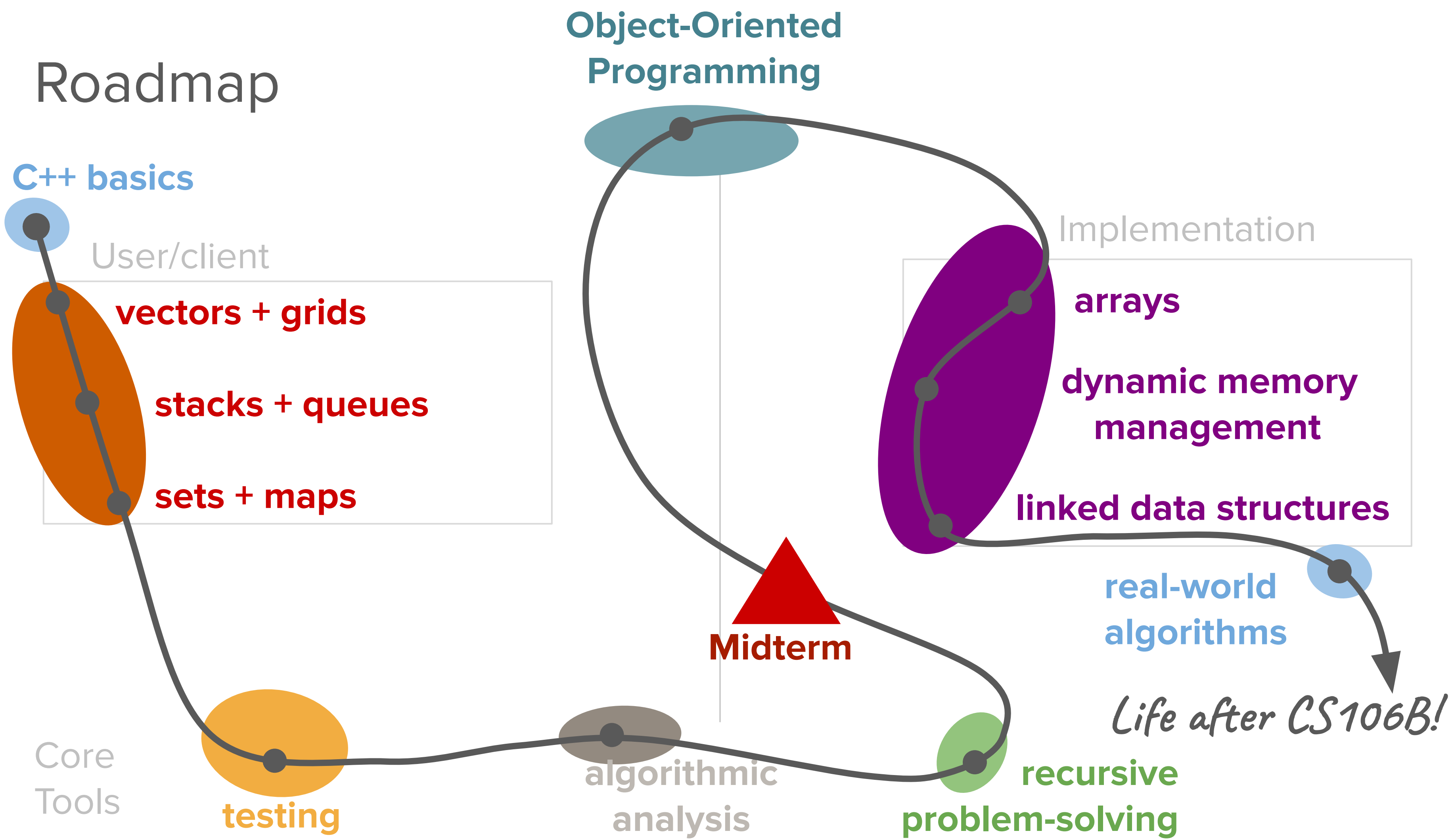
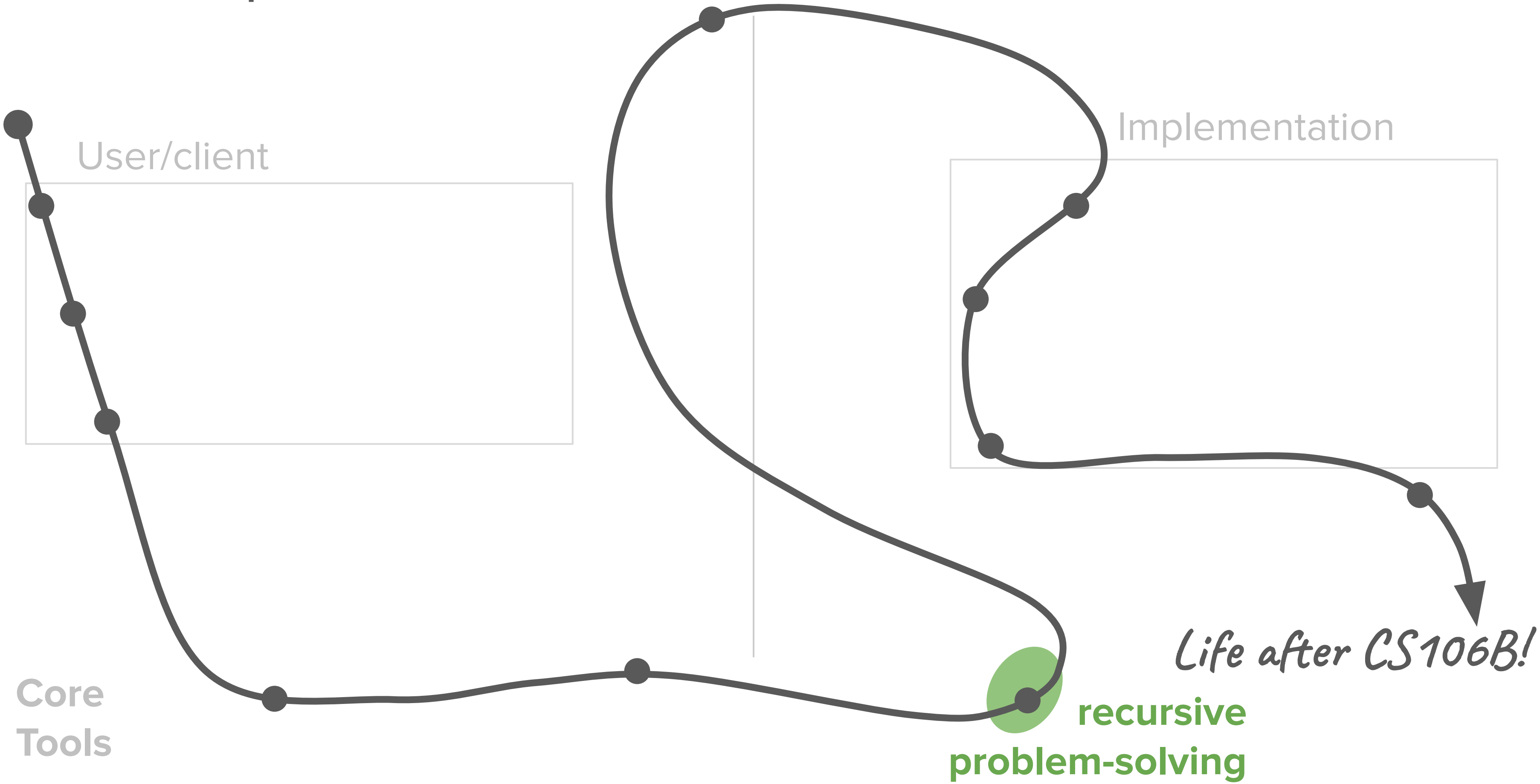
Roadmap



Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Midterm
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Roadmap


User/client
Implementation
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Tools
recursive
problem-solving
Life after CS106B!

Today's question

How can we take advantage of self-similarity within a problem to solve it more elegantly?

Today's topics

1. Review
2. Defining recursion
3. Recursion + Stack Frames (e.g. factorials)
4. Recursive Problem-Solving (e.g. string reversal)

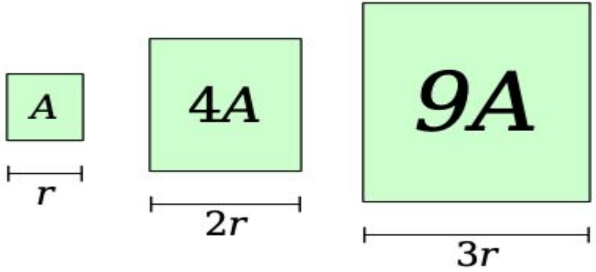
Review

(Big O)

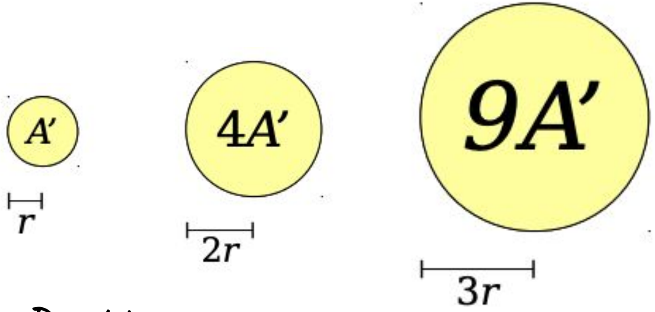
Big-O Notation

- **Big-O notation** is a way of quantifying the rate at which some quantity grows.
- Example:
  - A square of side length `r` has area `O(r²)`.
  - A circle of radius `r` has area `O(r²)`.

This just says that these quantities grow at the same relative rates. It does not say that they're equal!



Doubling r increases area 4x
Tripling r increases area 9x



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With respect to a given input variable!

Big-O Notation

- **Big-O notation** is a way of quantifying the **rate at which some quantity grows**.
- Example:
  - A square of side length r has area $O(r^2)$.
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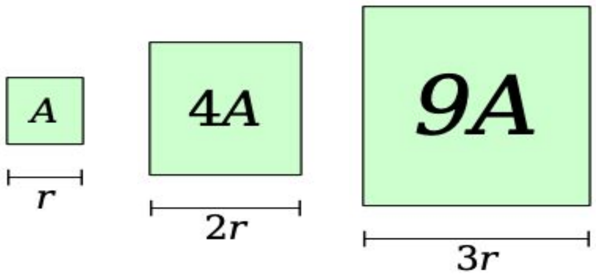

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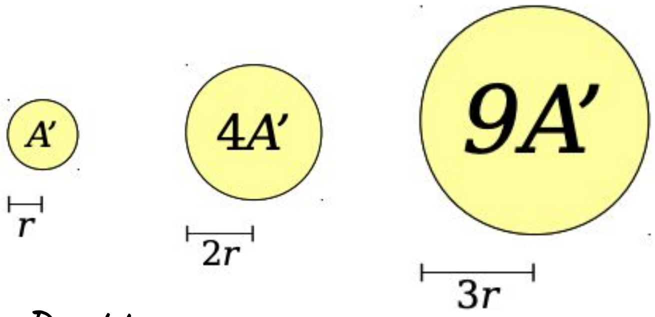

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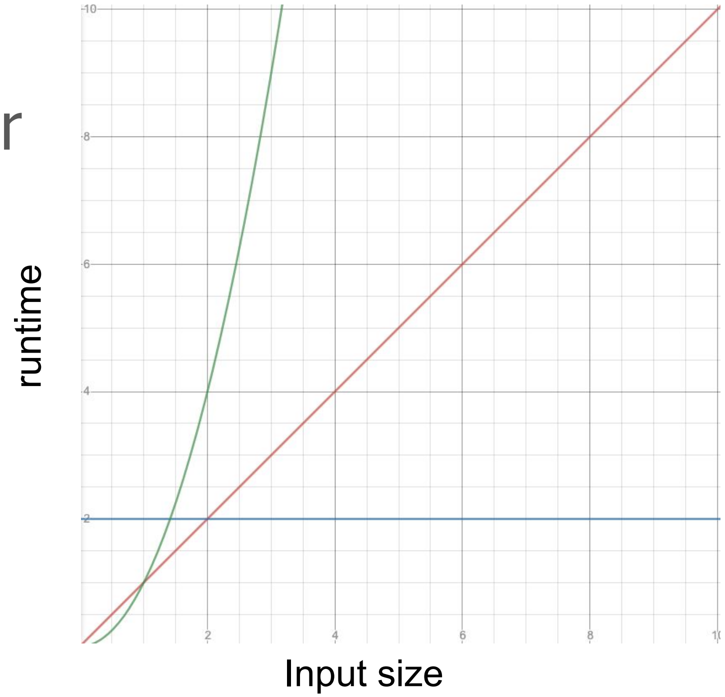
Efficiency Categorizations So Far



- Constant Time – O(1)
  - Super fast, this is the best we can hope for!
- Linear Time – O(n)
  - This is okay; we can live with this
- Quadratic Time – O(n^2)
  - This can start to slow down really quickly
  - Exhaustive Search for Perfect Numbers
- How do all the ADT operations we've seen so far fall into these categories?

ADT Big-O Matrix

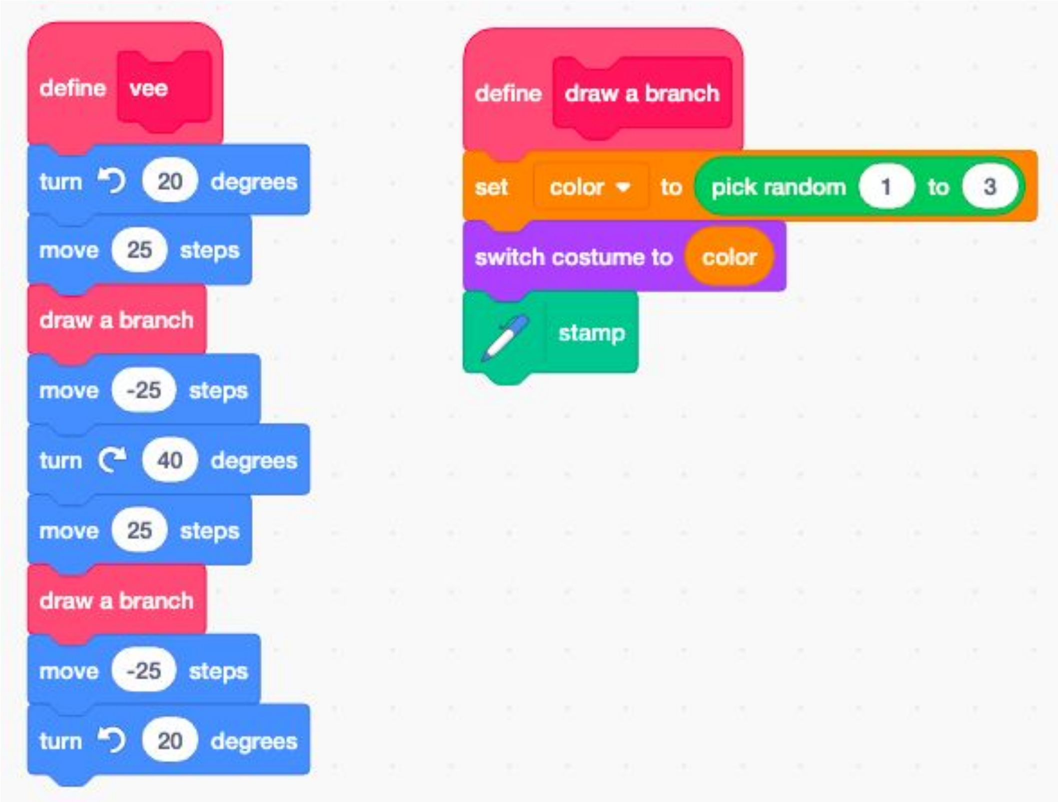
- Vectors
  - `.size() - O(1)`
  - `.add() - O(1)`
  - `v[i] - O(1)`
  - `.insert() - O(n)`
  - `.remove() - O(n)`
  - `.clear() - O(n)`
  - `traversal - O(n)}`
- Grids
  - `.numRows()/.numCols() - O(1)`
  - `g[i][j] - O(1)`
  - `.inBounds() - O(1)`
  - `traversal` - $O(n^2)$
- Queues
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.enqueue() - O(1)`
  - `.dequeue() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Stacks
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.push() - O(1)`
  - `.pop() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Sets
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `.add() - ???`
  - `.remove() - ???`
  - `.contains() - ???`
  - `traversal - O(n)`
- Maps
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `m[key] - ???`
  - `.contains() - ???`
  - `traversal - O(n)`

What is recursion?

Activity: Vee

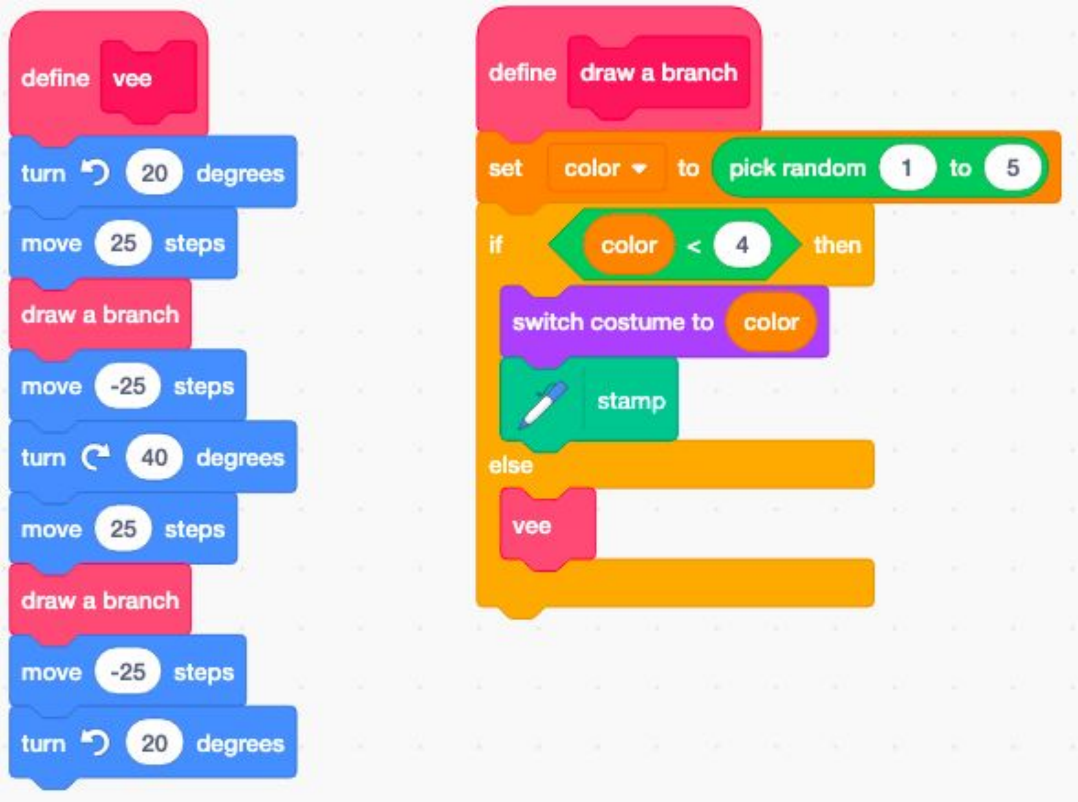
(https://scratch.mit.edu/projects/409796637/)

This code creates a “vee” shape with random colors.

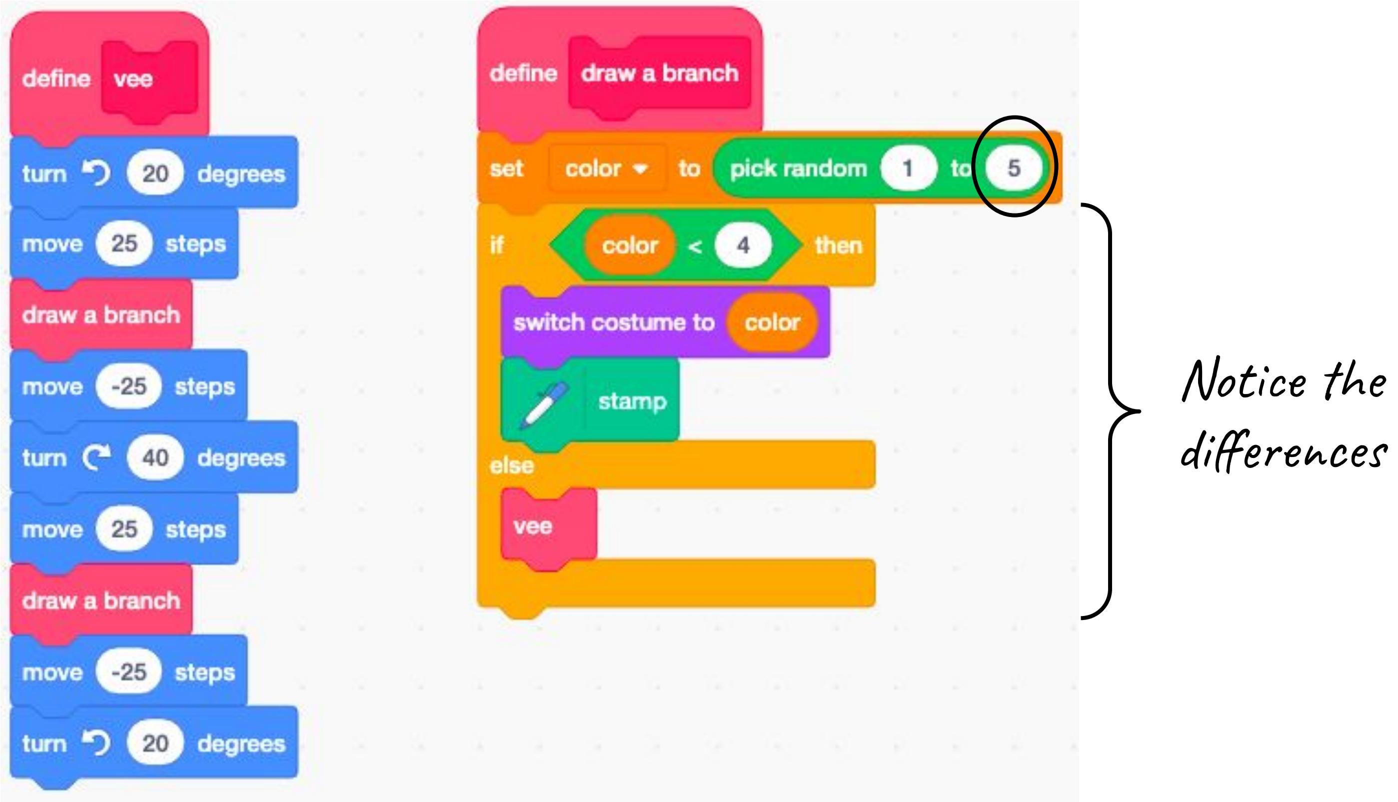


Based on Vee by Dan Garcia

What about this version of “vee”?



Discuss with a partner: What will this code do?

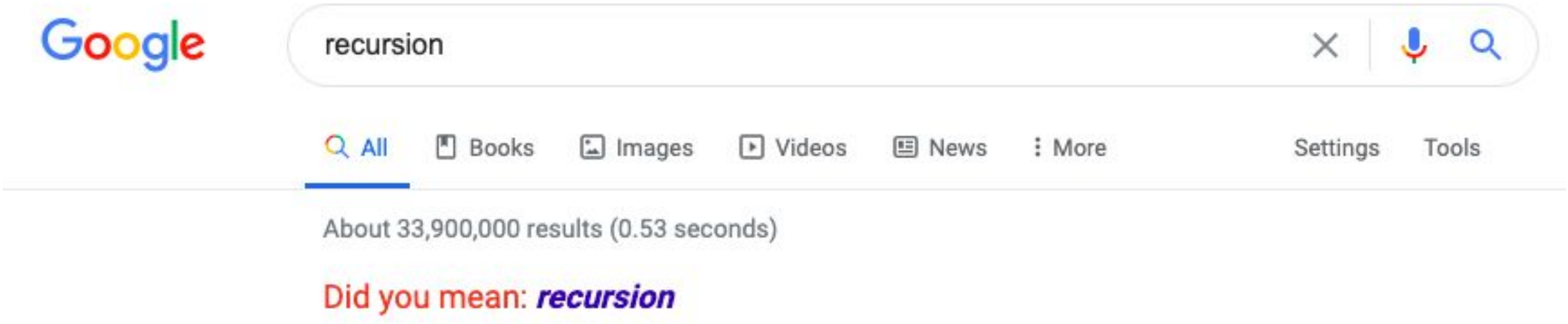


Demo: Recursive Vee

(https://scratch.mit.edu/projects/409785610/)

What is recursion?

Wikipedia: “Recursion occurs when a thing is defined in terms of itself.”



Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

What is recursion?

- A powerful substitute for iteration (loops)
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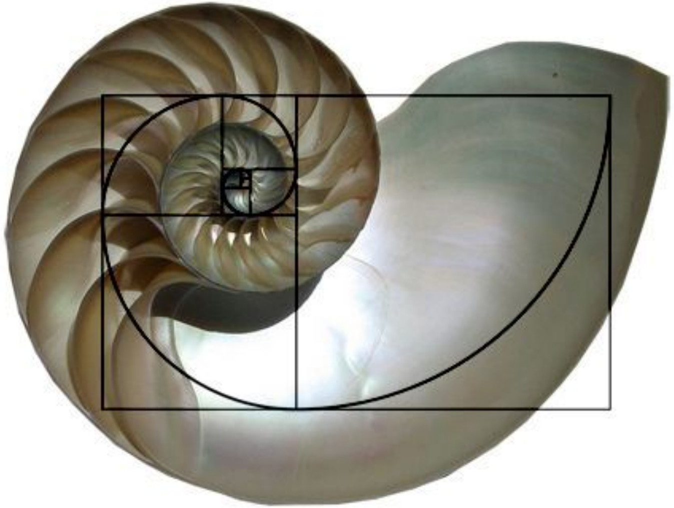

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- Often applied to sorting and searching problems and can be used to express patterns seen in nature

- Will be part of many of our future assignments!

How many students are in a lecture hall?

An analogy

How many students are in the lecture hall?

- Let’s suppose I want to find out how many people are at lecture today, but I don’t want to walk around and count each person.

- I want to recruit your help, but I also want to minimize each individual’s amount of work.

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We can solve this problem recursively!

How many students are in the lecture hall?

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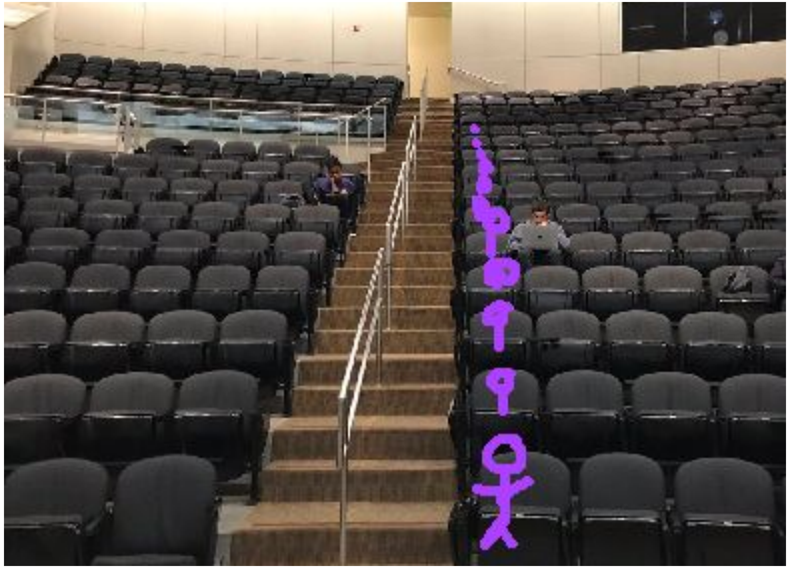

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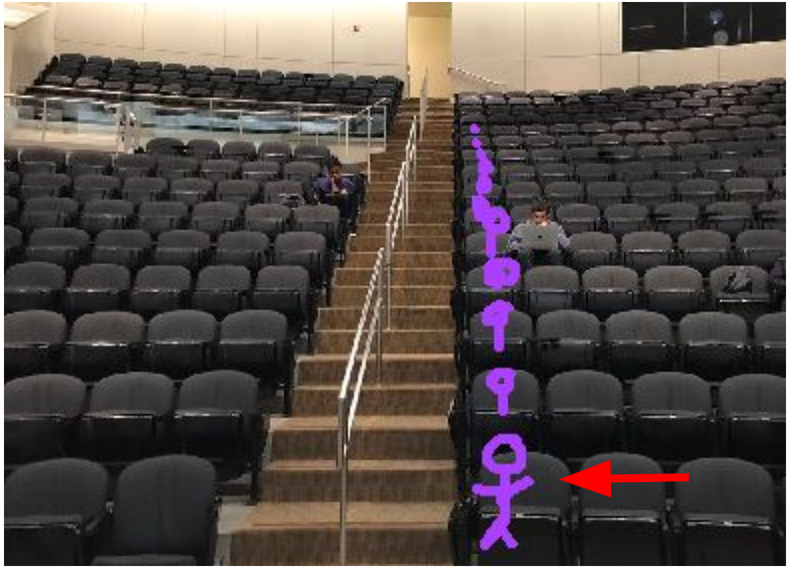

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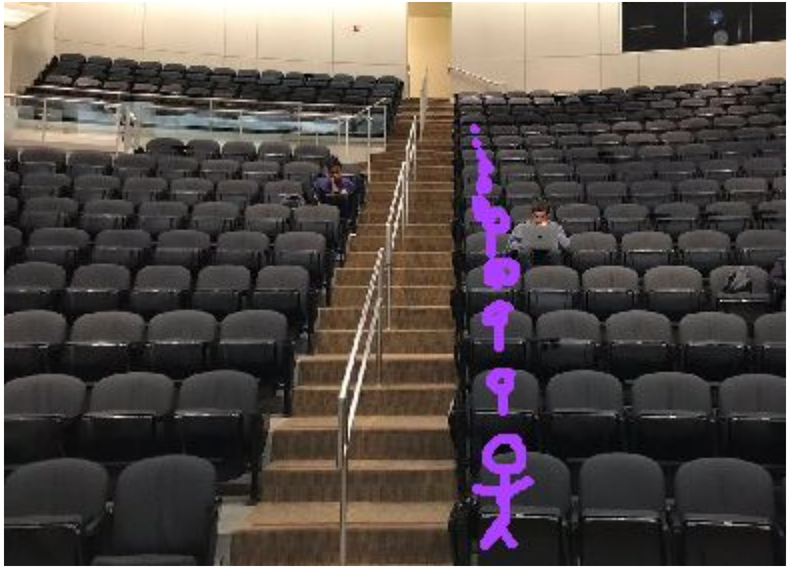

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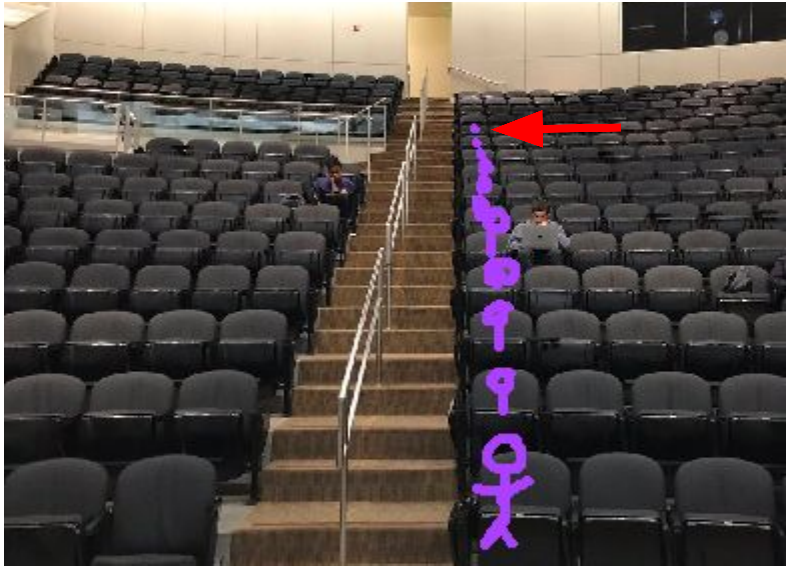

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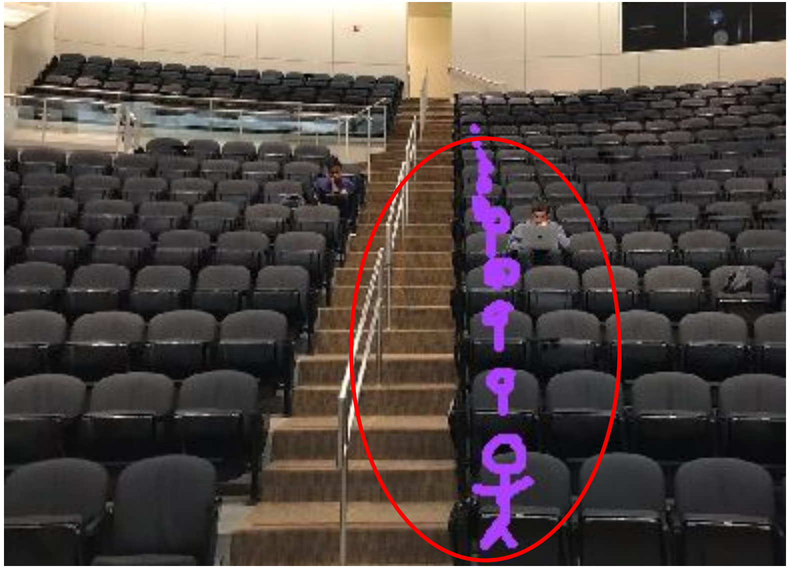

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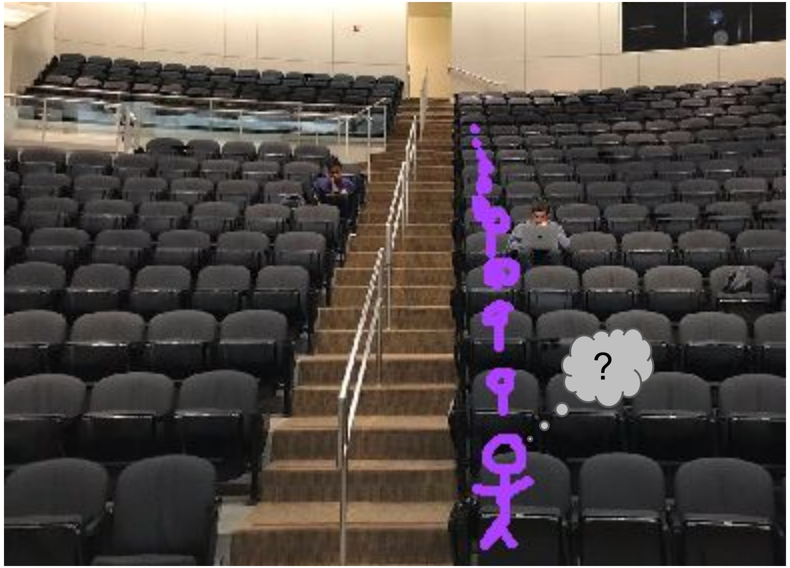

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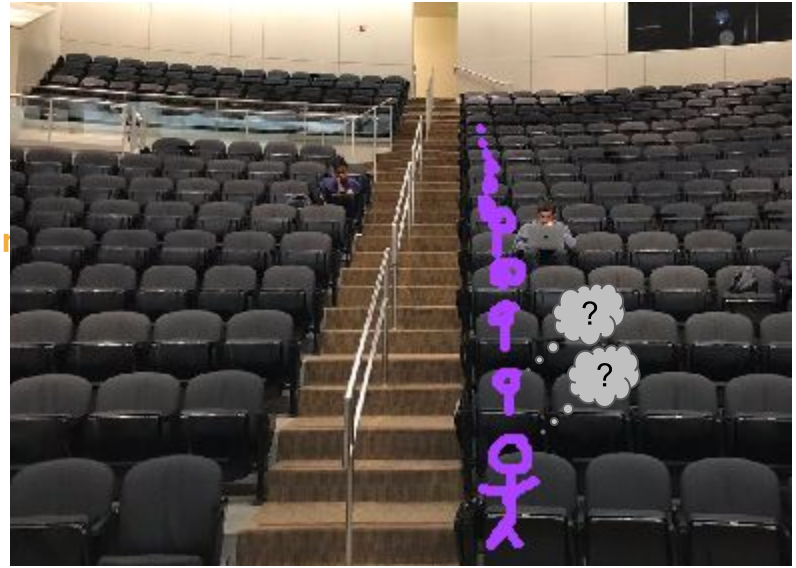

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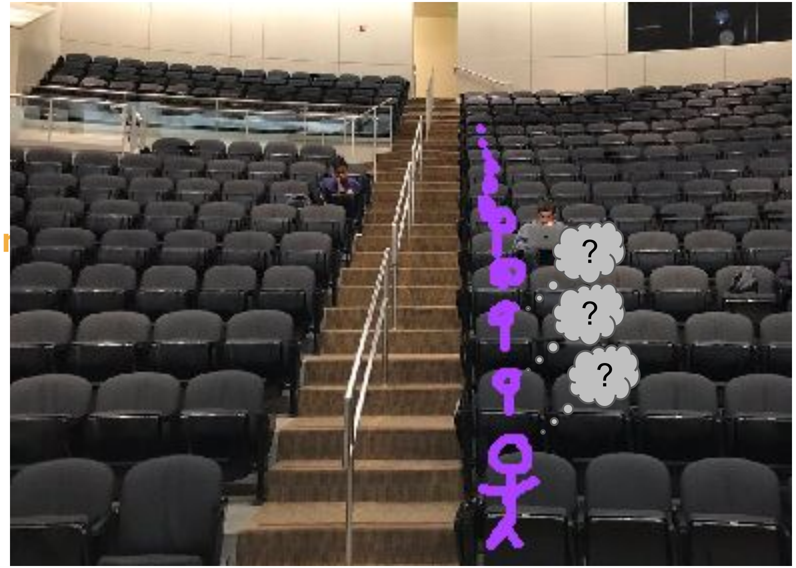

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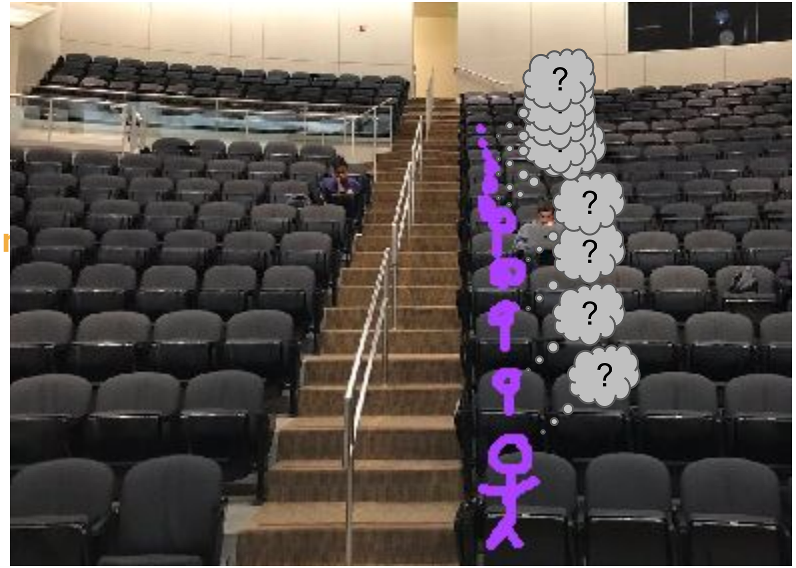

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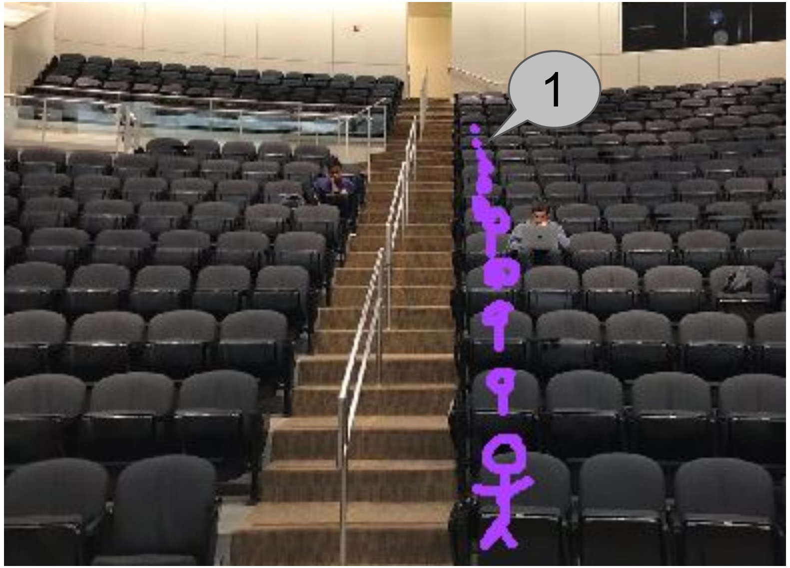

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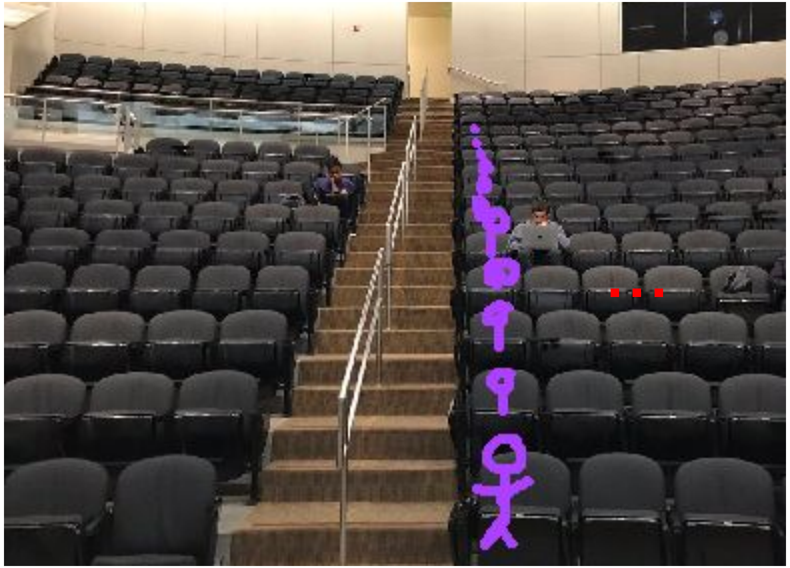

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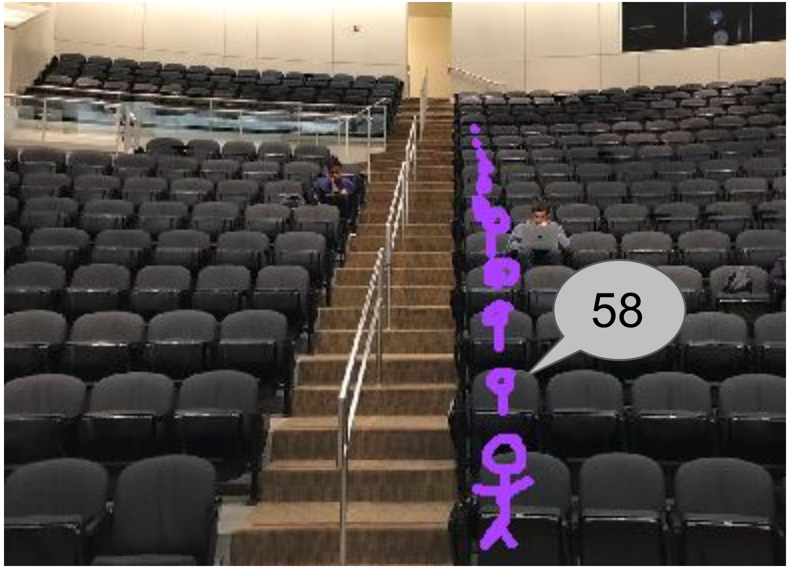

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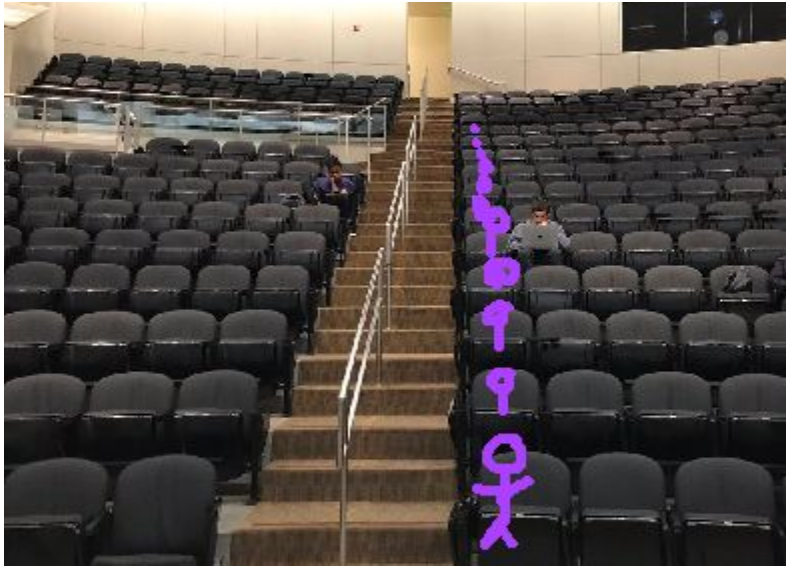

- Can generalize to the entire lecture hall!

Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

Two main cases (components) of recursion

- Base case
  - The simplest version(s) of your problem that all other cases reduce to
  - An occurrence that can be answered directly

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“If someone is sitting behind me…”

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Announcements

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- Assignment 2 is due at **11:59pm PDT this Thursday, July 7**.
- Our midterm will be **during lecture next Monday, July 11**. More info coming on the next slides!
- Assignment 3, which will cover recursion, will be released **after the midterm** next Monday. Final project guidelines will also be released next week.
- **Quick note about Ed**: We don't get notifications when you comment/reply on a classmate's comment on an old post. Please make a new thread if you want to make sure a staff member will see your question!

Midterm logistics

- The goal is to simulate a timed coding challenge where **compilability of your code doesn't matter**.

- We care most about **evaluating your problem-solving and conceptual understanding**! In other words, we won't take off points for typos or things like incorrectly named methods (e.g. using `set.add()` vs. `set.append()`), and partial credit will be given for pseudocode.

- To encourage growth and celebrate struggle, there will be an ***optional* post-exam reflection and check-in** with your section leader. Completing these thoughtfully will earn you back ⅓ of the points you missed.

Midterm logistics

We'll be releasing logistical information and practice materials later today on the course website! But here's a tl;dr:

- The midterm will be open-notes, open-book, and open-course website, but **you cannot communicate with any other human being during the exam.**
- We'll be providing you with .cpp files so you can use a code editor to type, but **we discourage you from running your code.** File submission will happen via Gradescope.
- We won't ask you to turn off or not use the internet, but like with assignments, you should not be taking any code from external online sources, and we will run similarity detection on exams.
- You'll need to bring your own laptop or rent one from Lathrop Tech Desk so please plan ahead.

Factorial example

Factorials

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  - `4! = 4 × 3 × 2 × 1 = 24.`
  - `5! = 5 × 4 × 3 × 2 × 1 = 120.`
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- Factorials show up in unexpected places. We'll see one later this quarter when we talk about sorting algorithms.
- Let's implement a function to compute factorials!

Computing factorials

```
5! = 5 x 4 x 3 x 2 x 1
```

Computing factorials

`5! = 5 x 4 x 3 x 2 x 1`

Computing factorials

$$5! = 5 \times \underbrace{4 \times 3 \times 2 \times 1}_{4!}$$

Computing factorials

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4! = 4 x 3 x 2 x 1
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3 x 2 x 1
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3 x 2 x 1
```

$$4! = 4 \times \underbrace{3 \times 2 \times 1}_{3!}$$

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2 x 1
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2 x 1
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2 x 1
```

2!

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
0! = 1
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
0! = 1
```

By definition!

Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

This is a "**stack frame**." One gets created each time a function is called.

- The "stack" is where in your computer's memory the information is stored.
- A "frame" stores all of the data (variables) for that particular function call.

Recursion in action

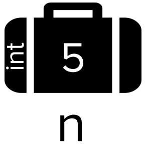
```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



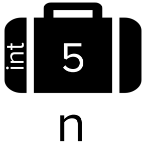
When a function gets called, a new stack frame gets created.

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

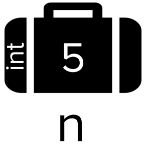


Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

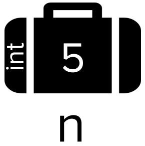


Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

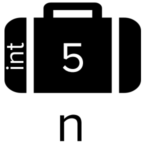


Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



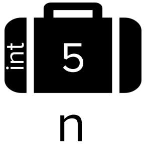
5

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          5
}
```



Recursion in action

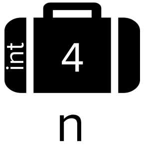
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Every time we call **`factorial()`**, we get a new copy of the local variable **n** that's independent of all the previous copies because it exists inside the new frame.

Recursion in action

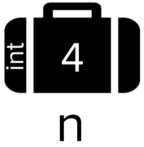
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

Recursion in action

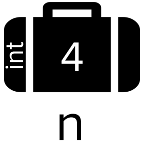
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

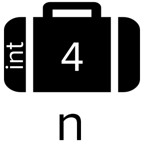
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

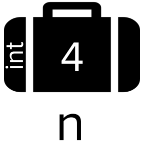
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }              4
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          4
}
```

Recursion in action

```
int main() {

}
```

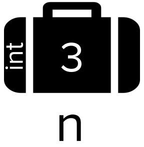
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
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```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
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    }
}
```

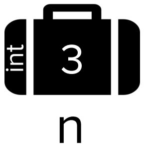

Recursion in action

```
int main() {
}
```

```
int factorial (int n) {
}
```

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int factorial (int n) {
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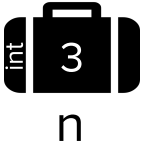
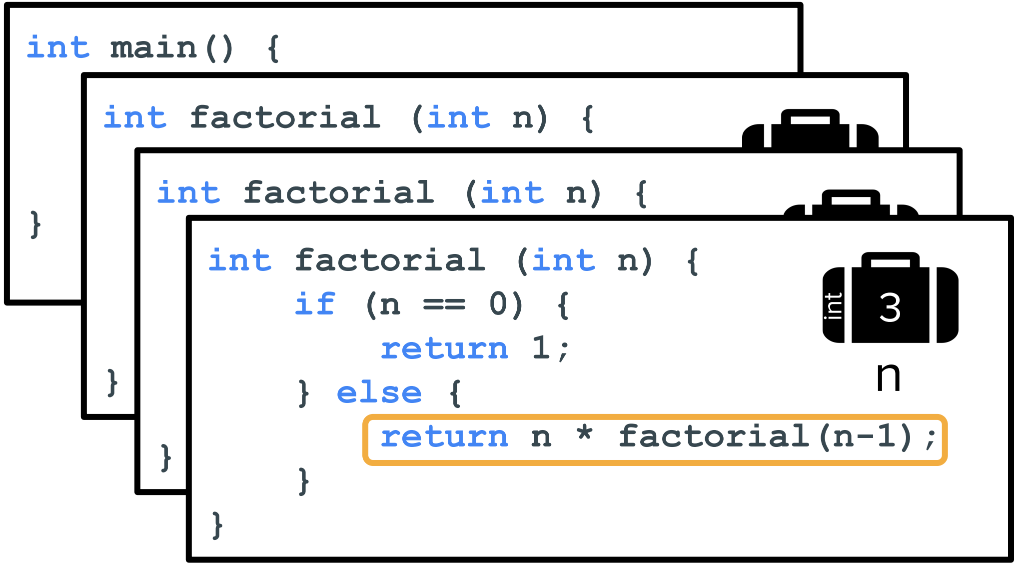
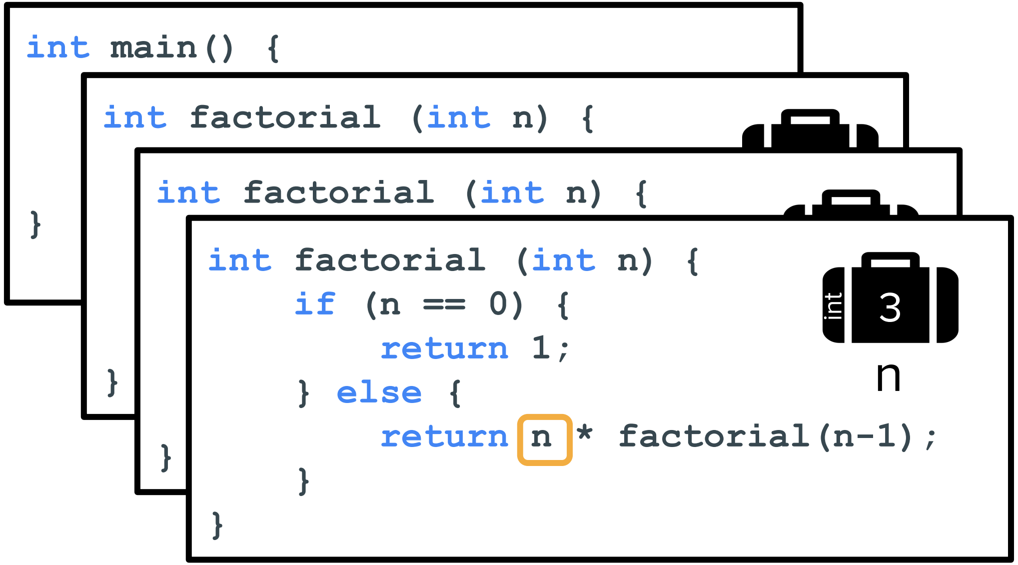


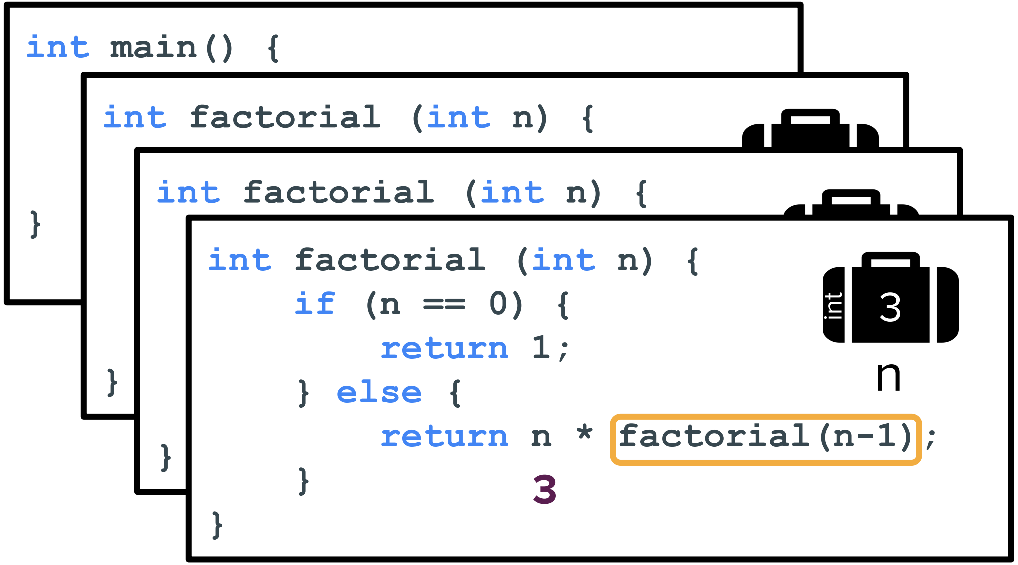
Recursion in action



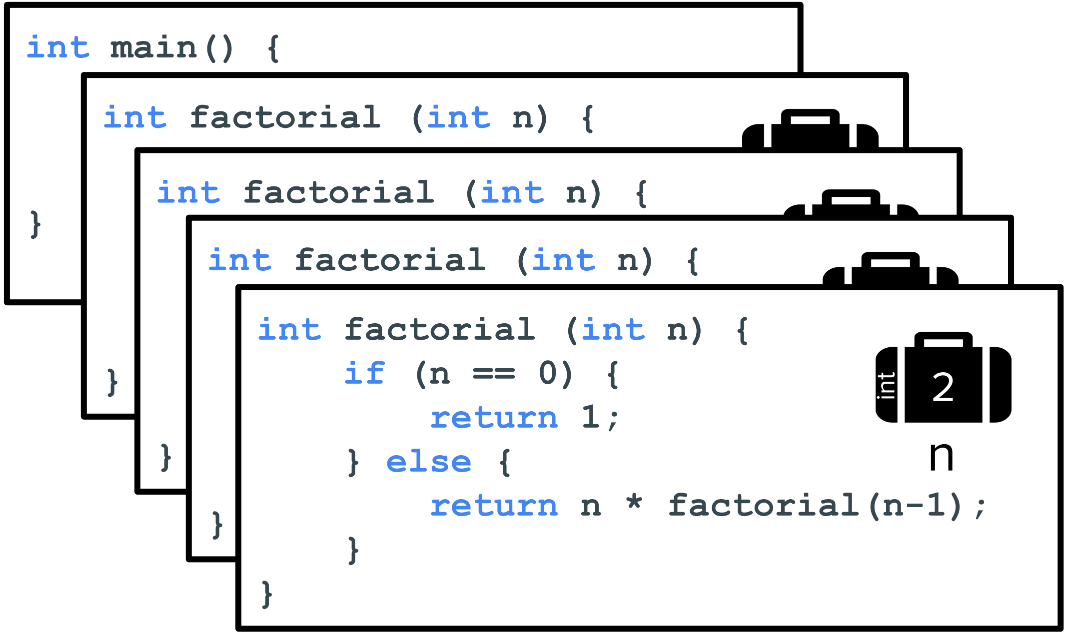
Recursion in action



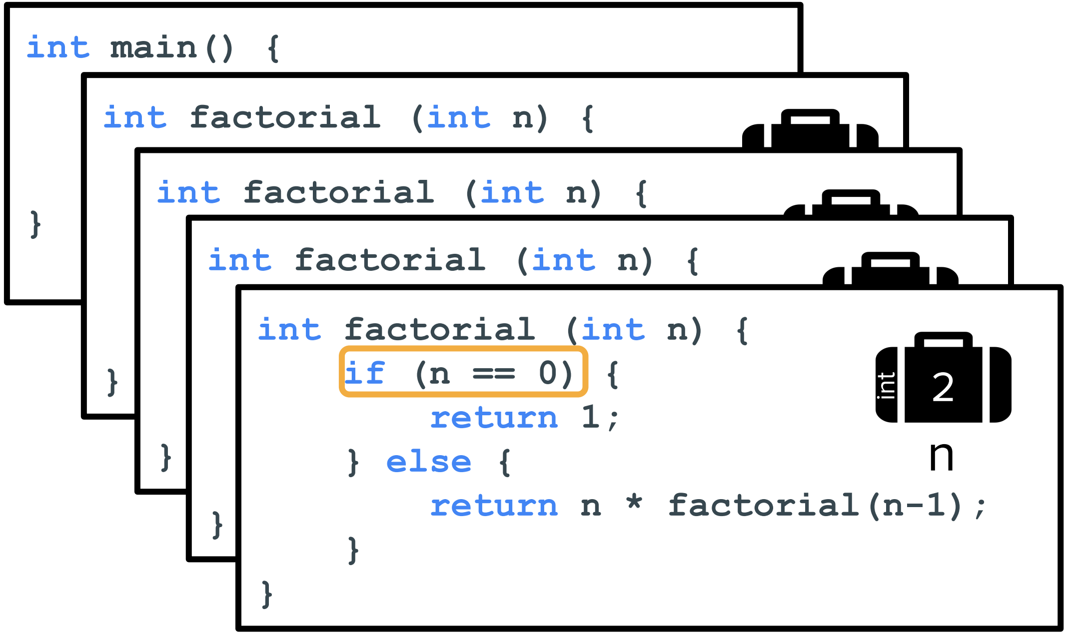
Recursion in action



Recursion in action



Recursion in action



Recursion in action

```
int main() {
}
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```
int factorial (int n) {
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        return 1;
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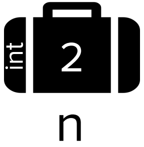
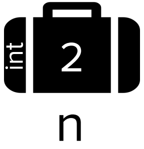
    }
}
```

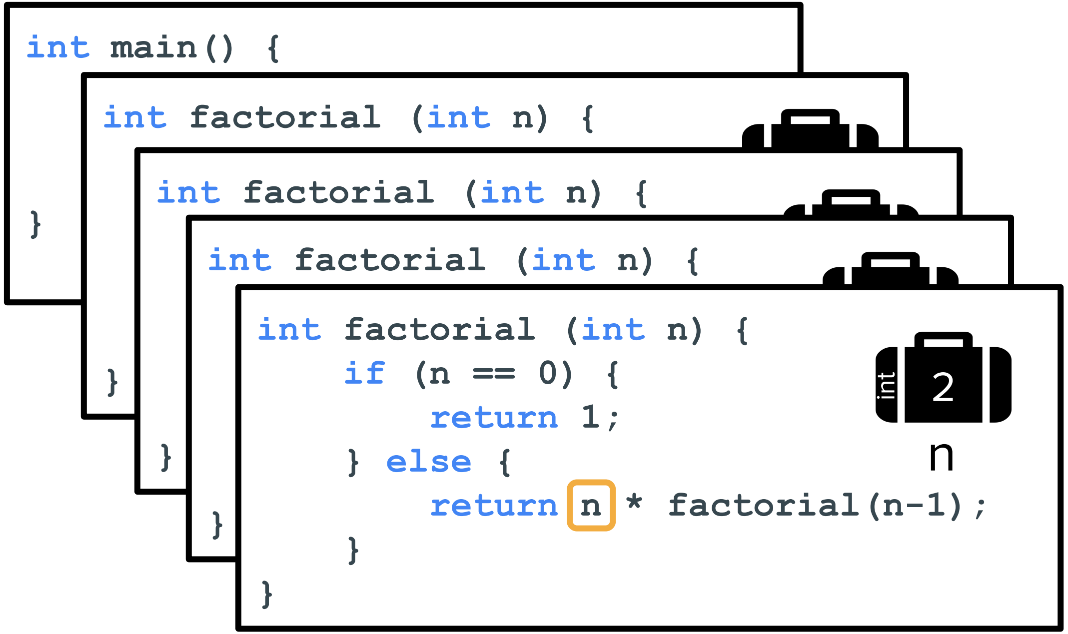


Recursion in action

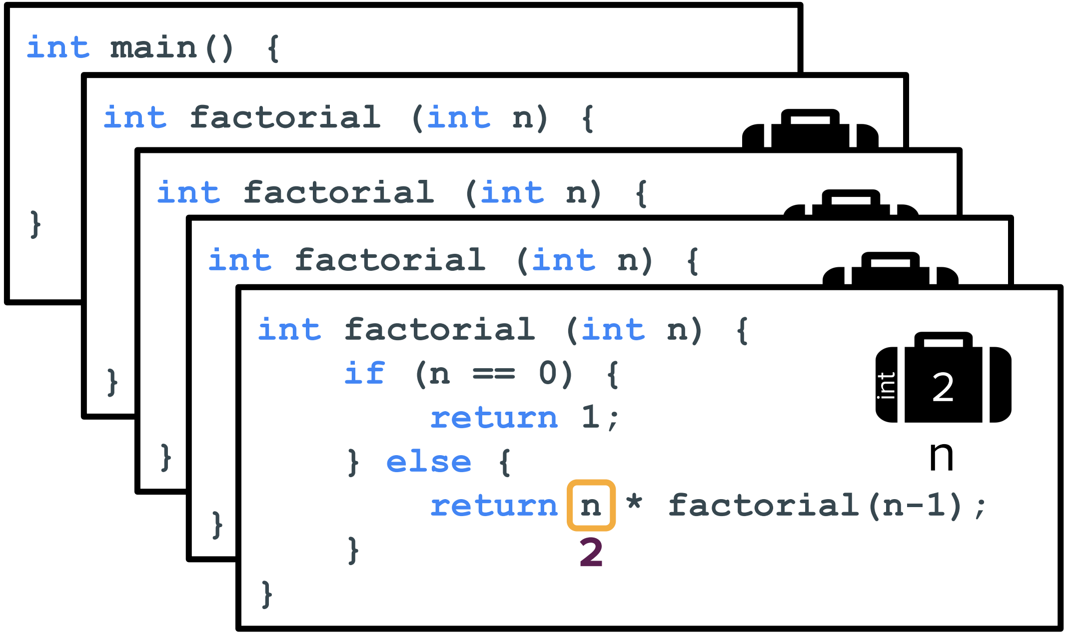
```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    if (n == 0) {
                        return 1;
                    } else {
                        return n * factorial(n-1);
                    }
                }
            }
        }
    }
}
```



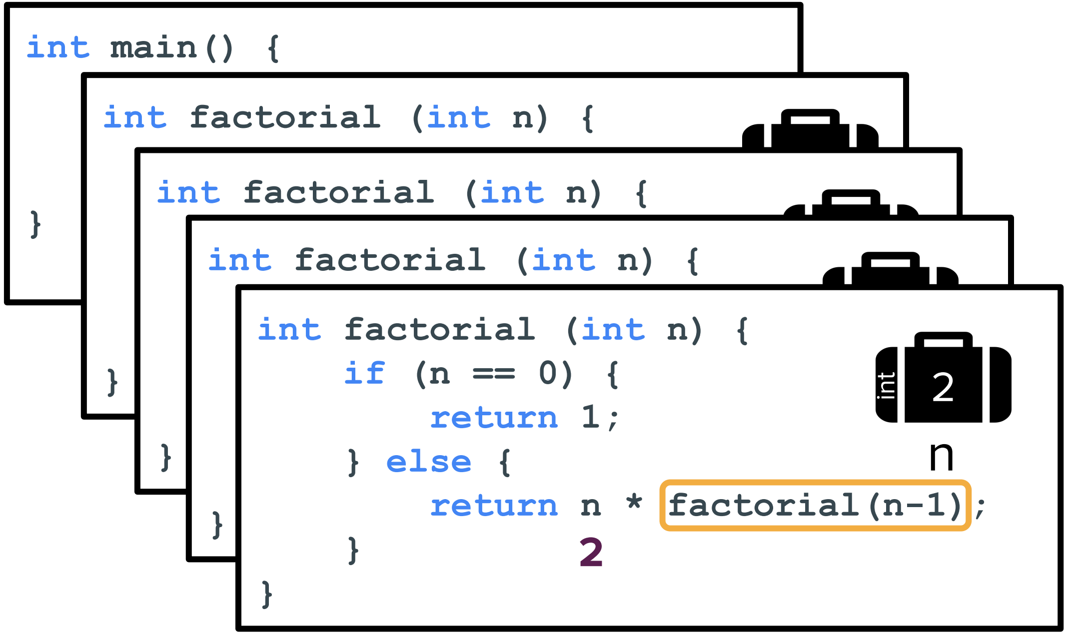
Recursion in action



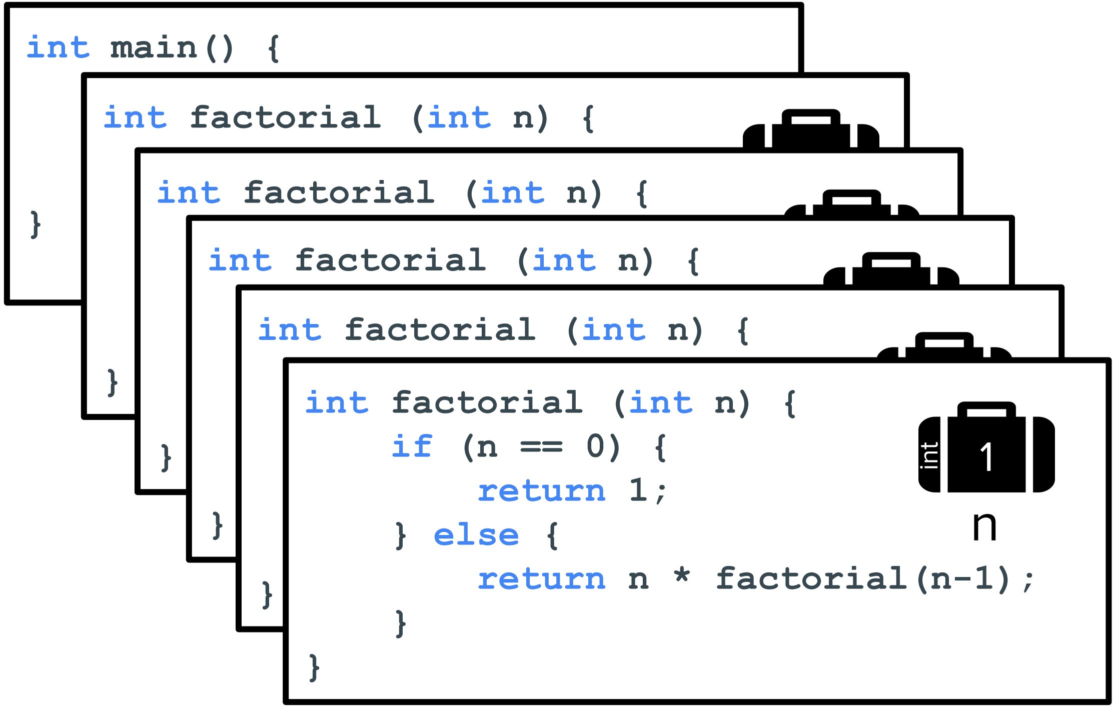
Recursion in action



Recursion in action



Recursion in action



Recursion in action

```
int main() {
}

int factorial (int n) {
}

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}

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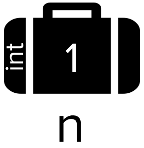
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

```
int main() {
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        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    int factorial (int n) {
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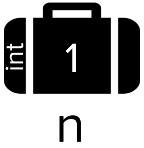
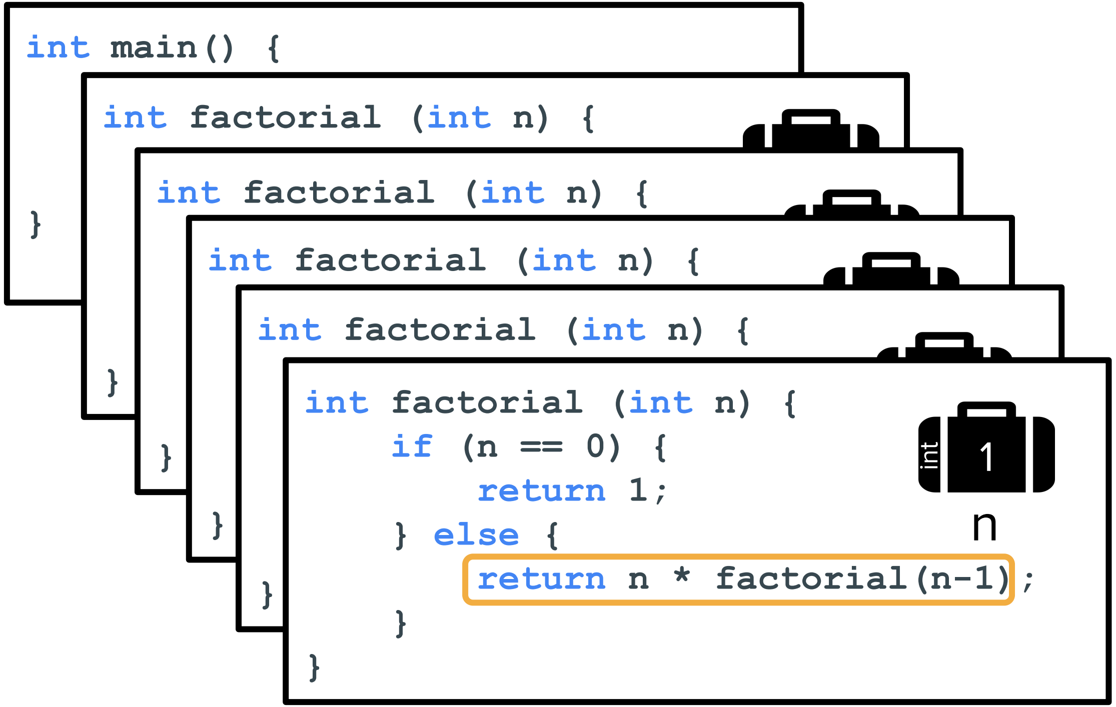
                            return 1;
                        } else {
                            return n * factorial(n-1);
                        }
                    }
                }
            }
        }
    }
}
```



Recursion in action



Recursion in action

```
int main() {
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    } else {
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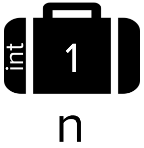
    }
}
```



Recursion in action

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}
```
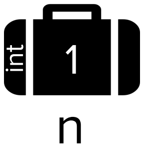
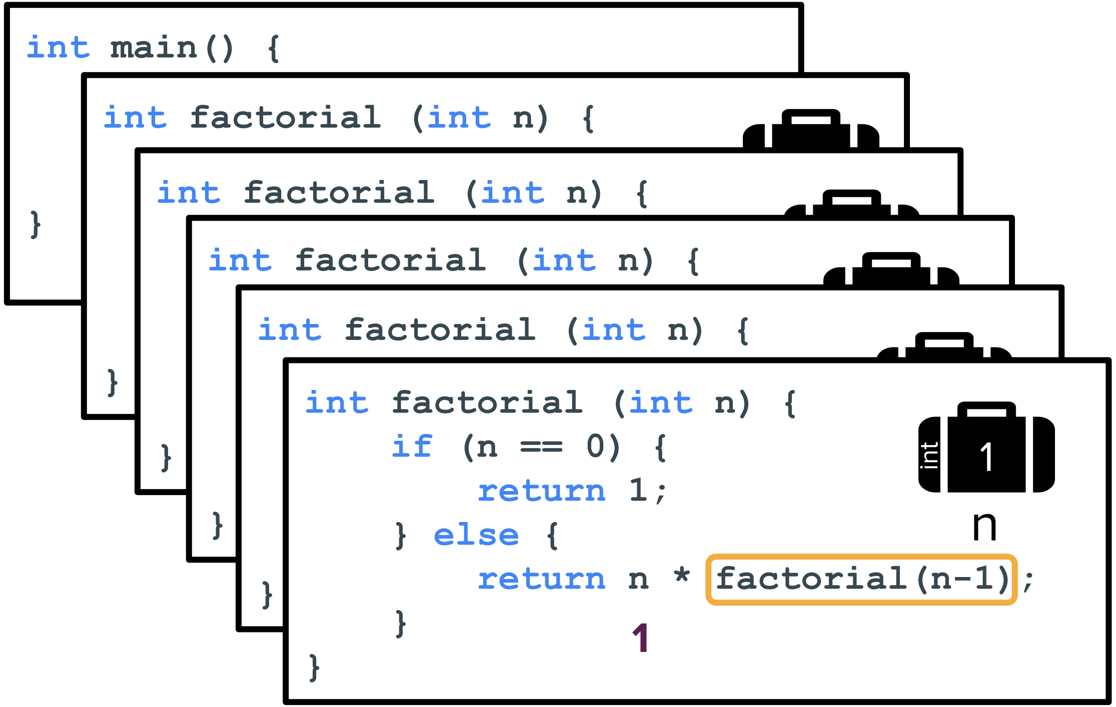
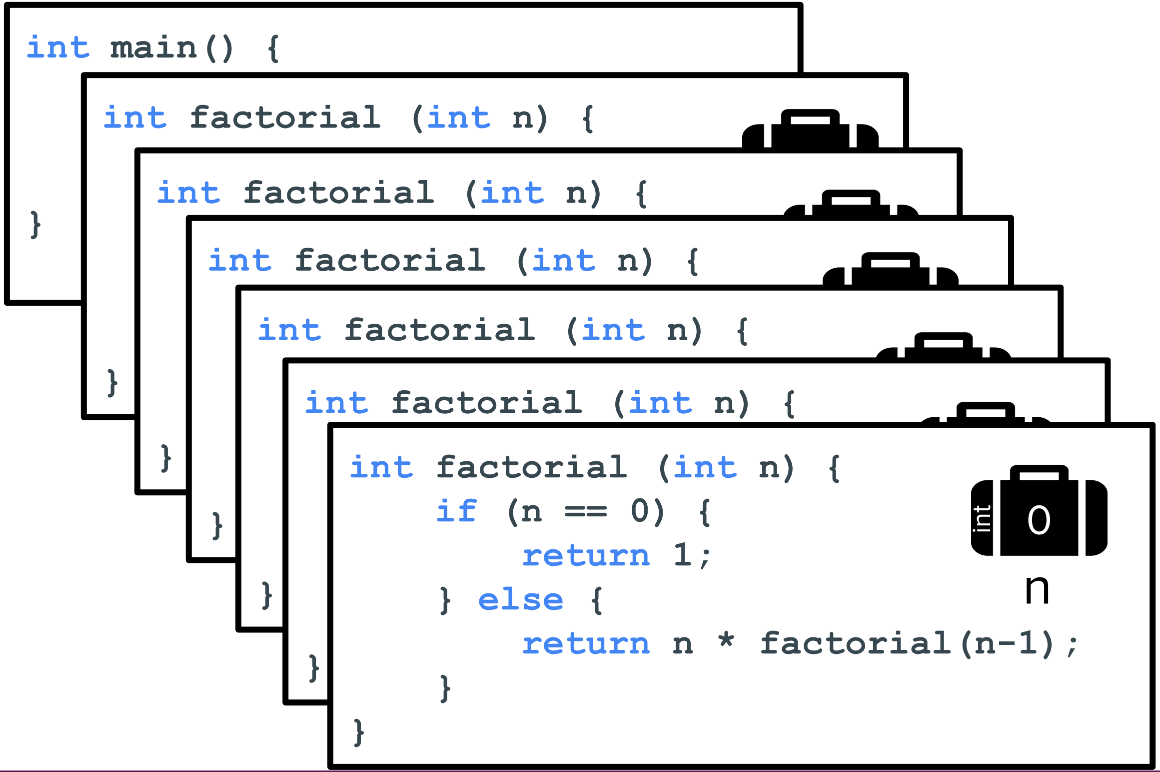

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
                1
    }
}
```



Recursion in action



Recursion in action



Recursion in action

```
int main() {

}
```

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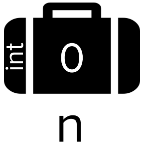
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



Recursion in action

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int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    int factorial (int n) {
                        int factorial (int n) {
                            if (n == 0) {
                                return 1;
                            } else {
                                return n * factorial(n-1);
                            }
                        }
                    }
                }
            }
        }
    }
}
```
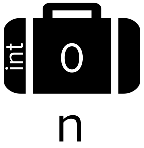
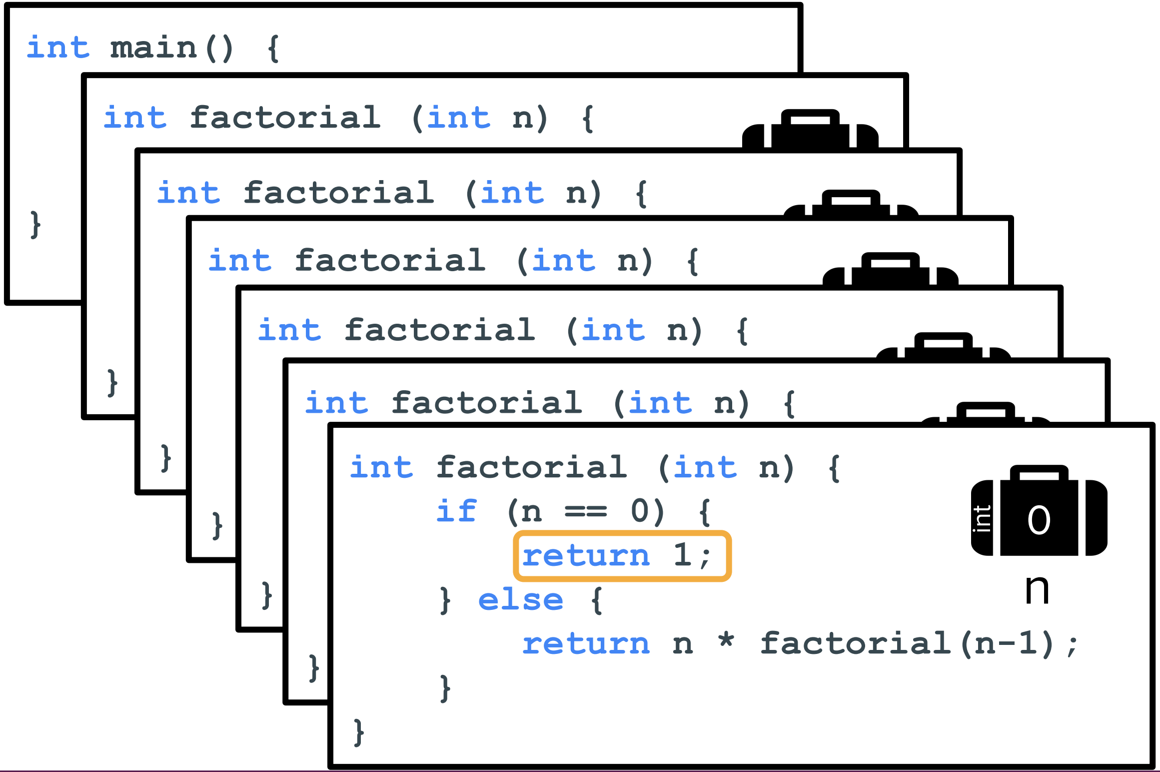
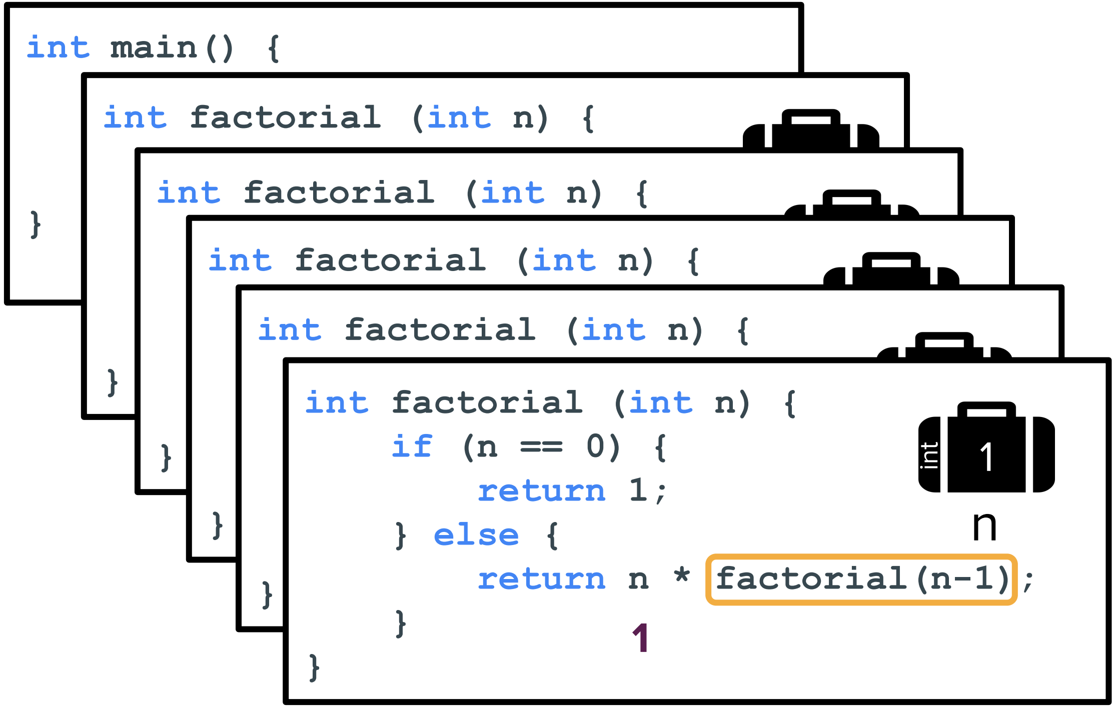



Recursion in action

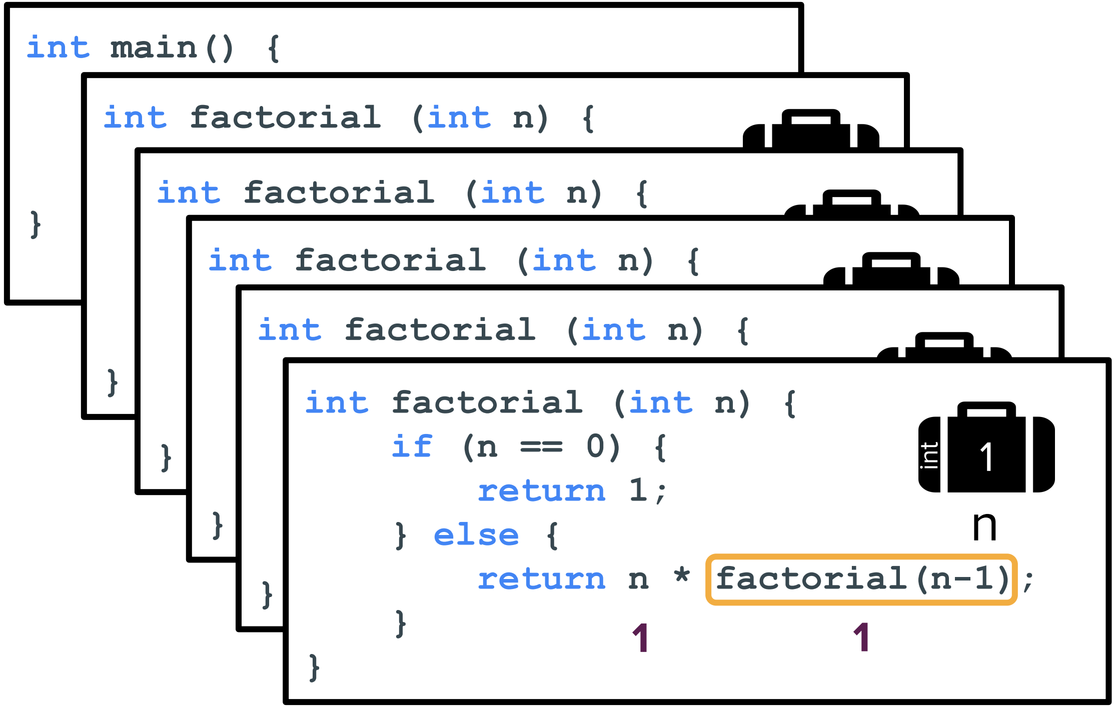


Stack frames go away (get cleared from memory) once they return.

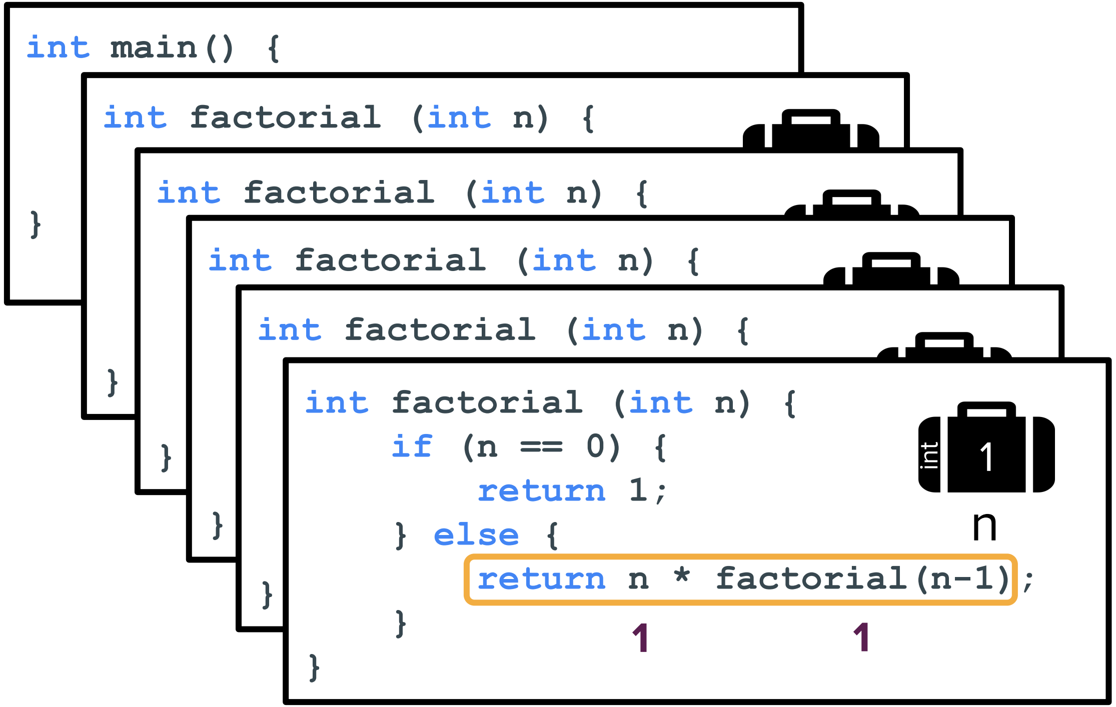
Recursion in action



Recursion in action

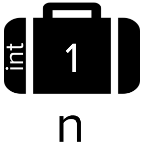


Recursion in action



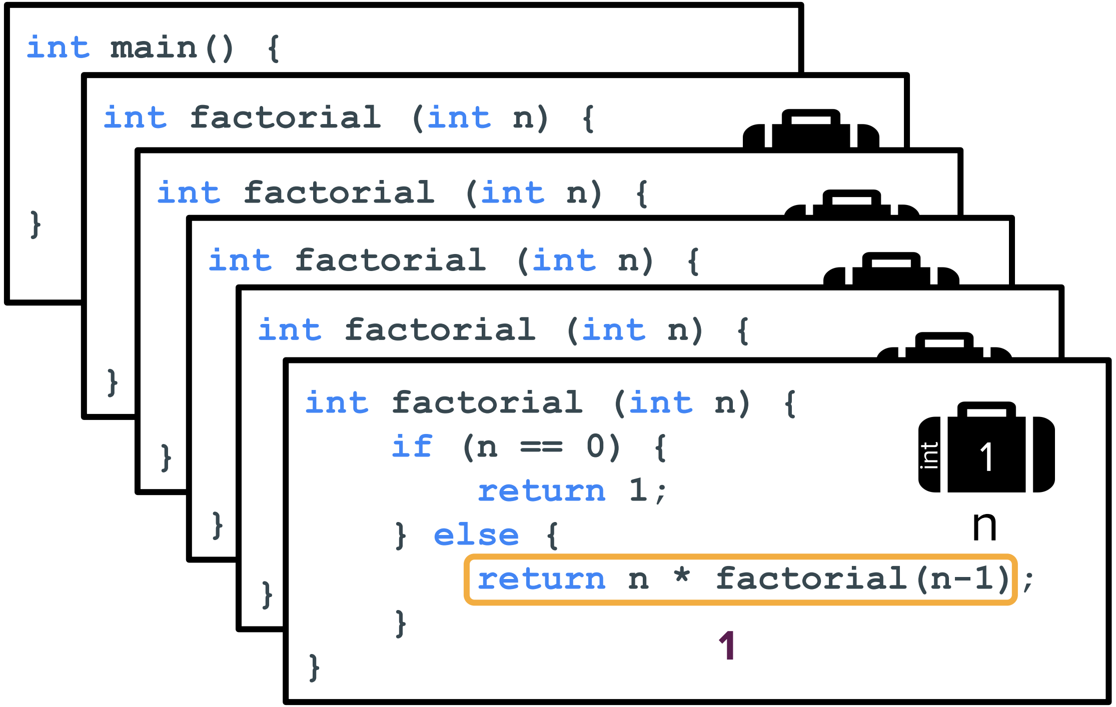
Recursion in action

```
int main() {
    int factorial (int n) {
        int factorial (int n) {
            int factorial (int n) {
                int factorial (int n) {
                    int factorial (int n) {
                        if (n == 0) {
                            return 1;
                        } else {
                            return n * factorial(n-1);
                        }
                    }
                }
            }
        }
    }
}
```



1 x 1

Recursion in action



Recursion in action

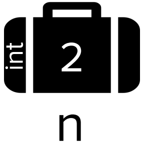
```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }           2
}
```



Recursion in action

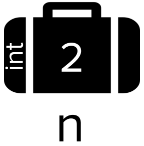
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



2 1

Recursion in action

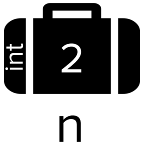
```
int main() {
}
```

```
int factorial (int n) {
}
```

```
int factorial (int n) {
}
```

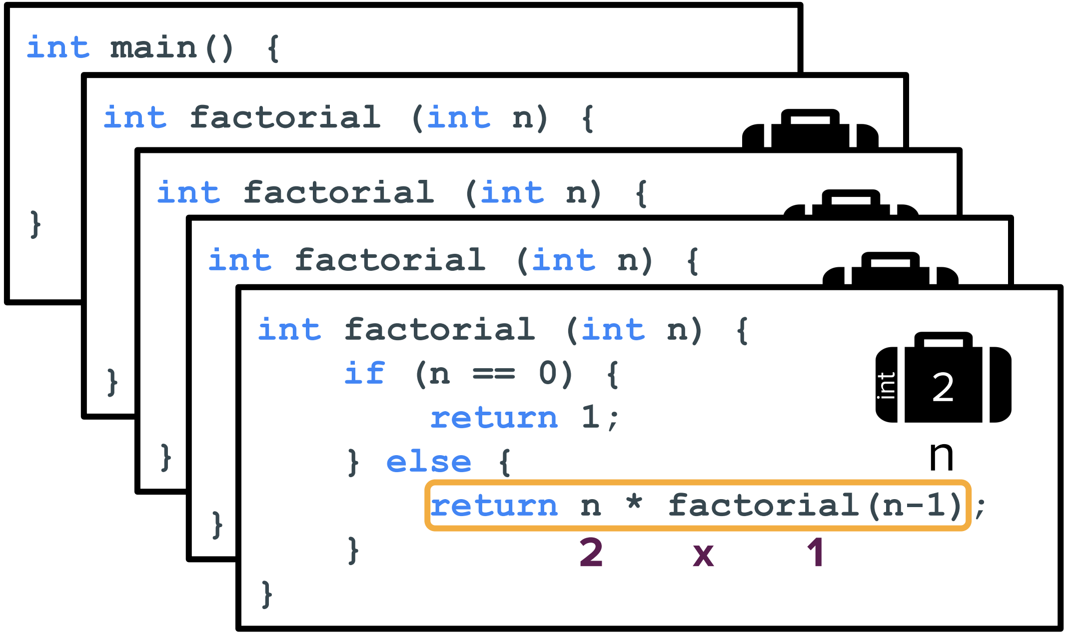
```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



2 **1**

Recursion in action



Recursion in action

```
int main() {

}
```

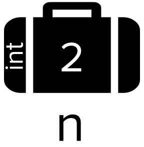
```
int factorial (int n) {

}
```

```
int factorial (int n) {

}
```

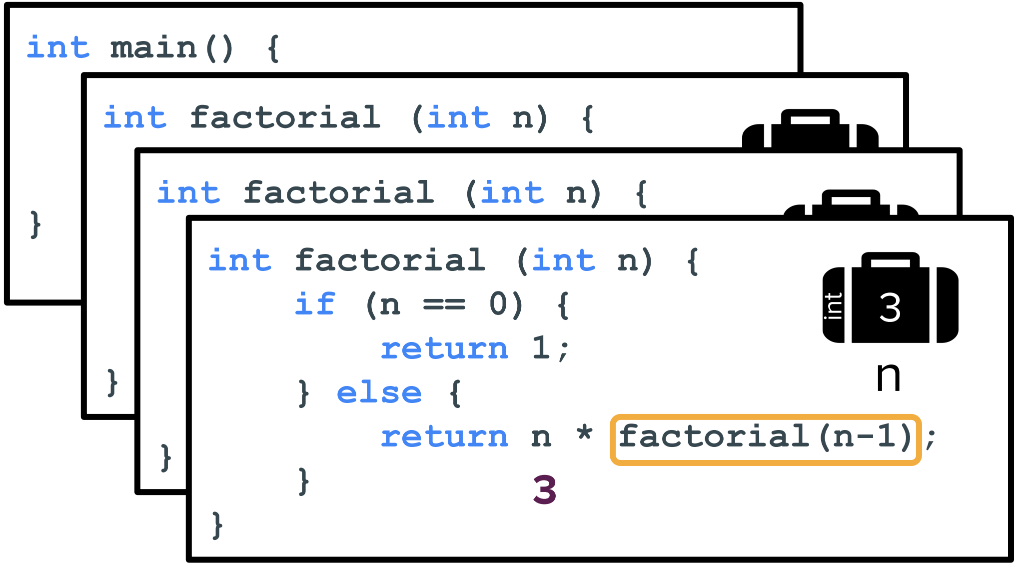
```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

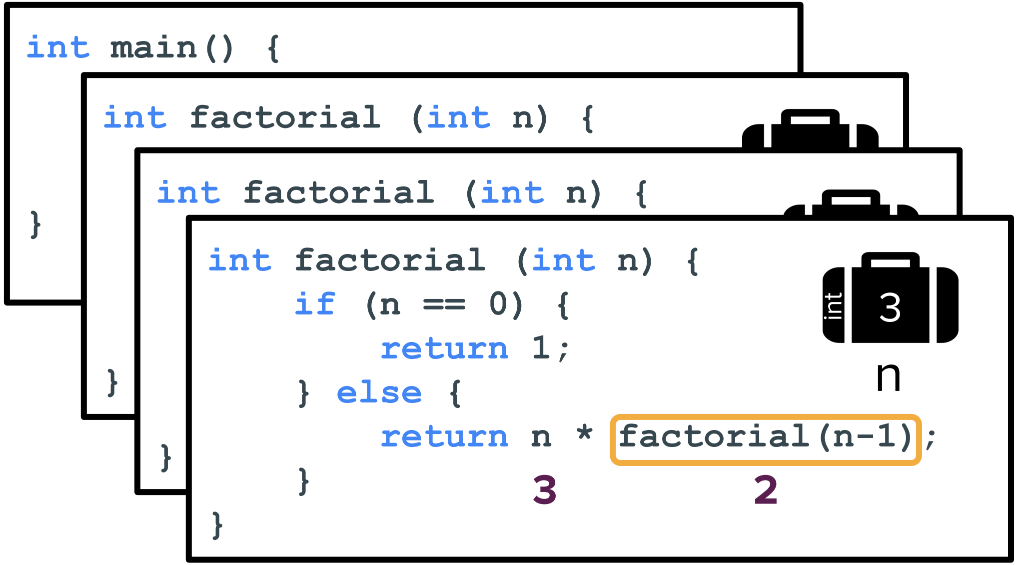
2



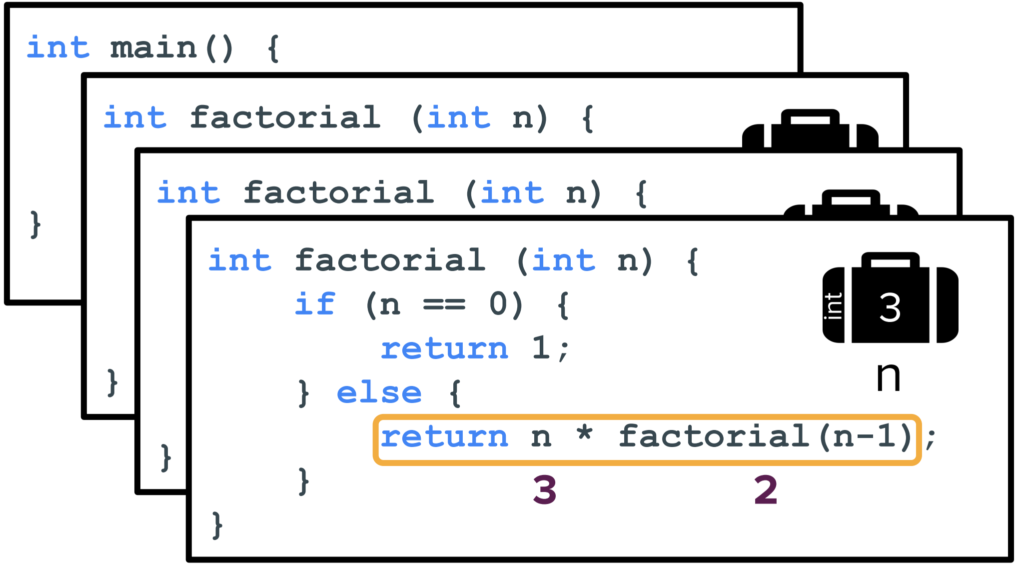
Recursion in action



Recursion in action



Recursion in action



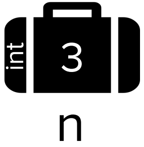
Recursion in action

```
int main() {
}
```

```
int factorial (int n) {
}
```

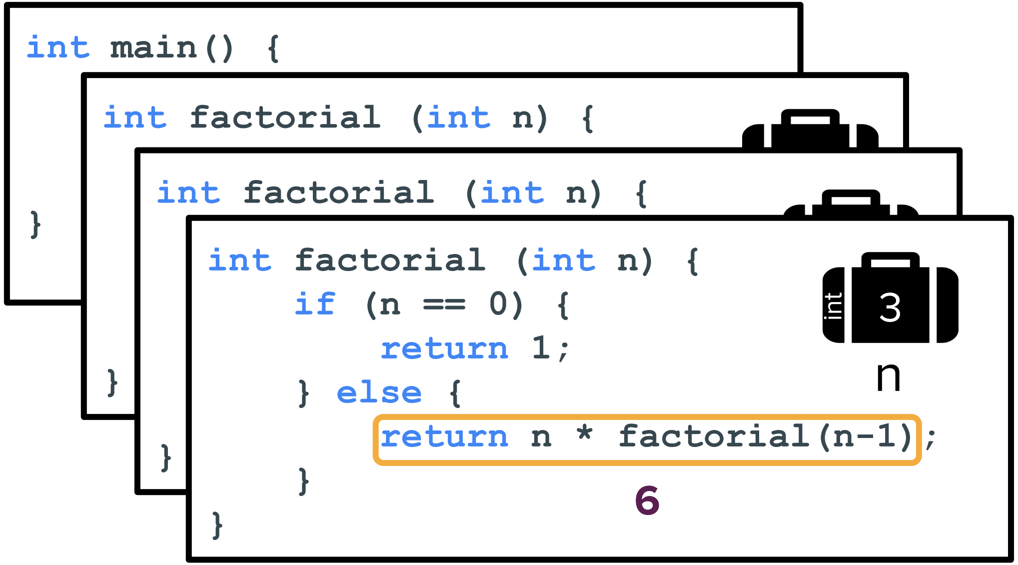
```
int factorial (int n) {
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

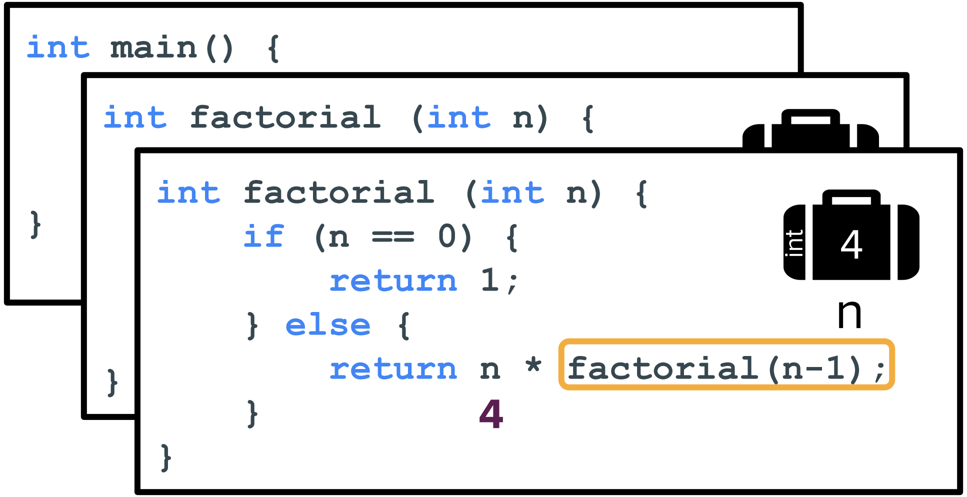


3 x 2

Recursion in action



Recursion in action

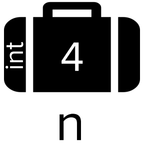


Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          4          6
}
```



Recursion in action

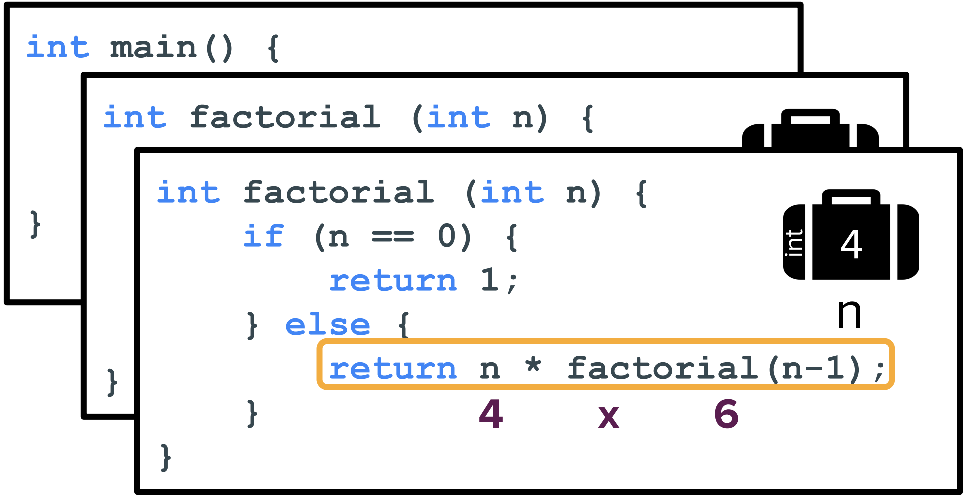
```
int main() {

}
```

```
int factorial (int n) {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

4 6

Recursion in action



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {

}
```

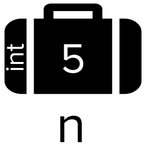
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

24

Recursion in action

```
int main() {

}
```

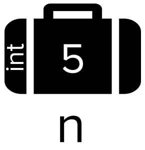
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }       5
}
```



Recursion in action

```
int main() {

}
```

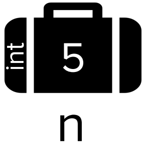
```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }          5          24
}
```



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

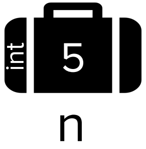


5 24

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

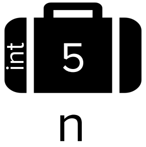


5 x 24

Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```



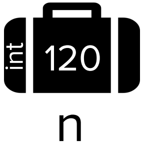
120

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```



Recursive vs. Iterative

[Qt Creator]

Reverse string example

How can we reverse a string?

Suppose we want to reverse strings like in the following examples:

“dog” ➔ “god”

“stressed” ➔ “desserts”

“recursion” ➔ “noisrucer”

“level” ➔ “level”

“a” ➔ “a”

Approaching recursive problems

- Look for self-similarity.

- Try out an example.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **`n`** in each **`factorial`** stack frame).

- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

Discuss:

What are the base and recursive cases?

Attendance ticket:

https://tinyurl.com/stringrecursion

Please don't send this link to students who are not here. It's on your honor!

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - What’s the first step you would take to reverse “stressed”?

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.

How can we reverse a string?

- Look for self-similarity: **stressed ➜ desserts**
    - Take the s and put it at the end of the string.
    - Then reverse “tressed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
    - Take the s and put it at the end of the string.
    - Then reverse “tressed”:
        - Take the t and put it at the end of the string.
        - Then reverse “ressed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
    - Take the s and put it at the end of the string.
    - Then reverse “tressed”:
        - Take the t and put it at the end of the string.
        - Then reverse “ressed”:
            - Take the r and put it at the end of the string.
            - Then reverse “essed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed ➔ desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - ...
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” ➔ get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed ➔ desserts**
  - **Take the s and put it at the end of the string.**
  - **Then reverse "tressed":**
    - Take the t and put it at the end of the string.
    - Then reverse "ressed":
      - Take the r and put it at the end of the string.
      - Then reverse "essed":
        - ...
          - Take the d and put it at the end of the string.
          - **Base case**: reverse "" ➔ get ""

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - **reverse(“stressed”) = reverse(“tressed”) + ‘s’**
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - ...
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - **Take the t and put it at the end of the string.**
    - **Then reverse “ressed”:**
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed ➔ desserts**
  - reverse("stressed") = reverse("tressed") + 's'
    - **reverse("tressed") = reverse("ressed") + 't'**
      - Take the r and put it at the end of the string.
      - Then reverse "essed":
        - ...
          - Take the d and put it at the end of the string.
          - **Base case**: reverse "" ➔ get ""

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed ➜ desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - reverse(“tressed”) = reverse(“ressed”) + ‘t’
      - **Take the r and put it at the end of the string.**
      - **Then reverse “essed”:**
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” ➜ get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - reverse("stressed") = reverse("tressed") + 's'
    - reverse("tressed") = reverse("ressed") + 't'
      - **reverse("ressed") = reverse("essed") + 'r'**
        - ...
          - Take the d and put it at the end of the string.
          - **Base case**: reverse "" → get ""

How can we reverse a string?

- Look for self-similarity: **stressed ➔ desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - reverse(“tressed”) = reverse(“ressed”) + ‘t’
      - reverse(“ressed”) = reverse(“essed”) + ‘r’
        - ...
          - **Base case**: reverse(“”) = “”

How can we reverse a string?

- **Recursive case:** reverse(str) = reverse(str without first letter) + first letter of str
- **Base case**: reverse(“”) = “”

How can we reverse a string?

- **Recursive case:** reverse(str) = reverse(str without first letter) + first letter of str
- **Base case**: reverse(“”) = “”

Depending on how you thought of the problem, you may have also come up with:

- **Recursive case:** reverse(str) = last letter of str + reverse(str without last letter)
- **Base case**: reverse(“”) = “”

Let’s code it!

(live coding)

Summary

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
    - A recursive operation (function) is defined in terms of itself (i.e. it calls itself).

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.
  - Base case: Simplest form of the problem that has a direct answer.
  - Recursive case: The step where you break the problem into a smaller, self-similar task.

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.
  - The base case will define the “base” of the solution you’re building up.
  - Each previous recursive call contributes a little bit to the final solution.
  - The initial call to your recursive function is what will return the completely constructed answer.

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

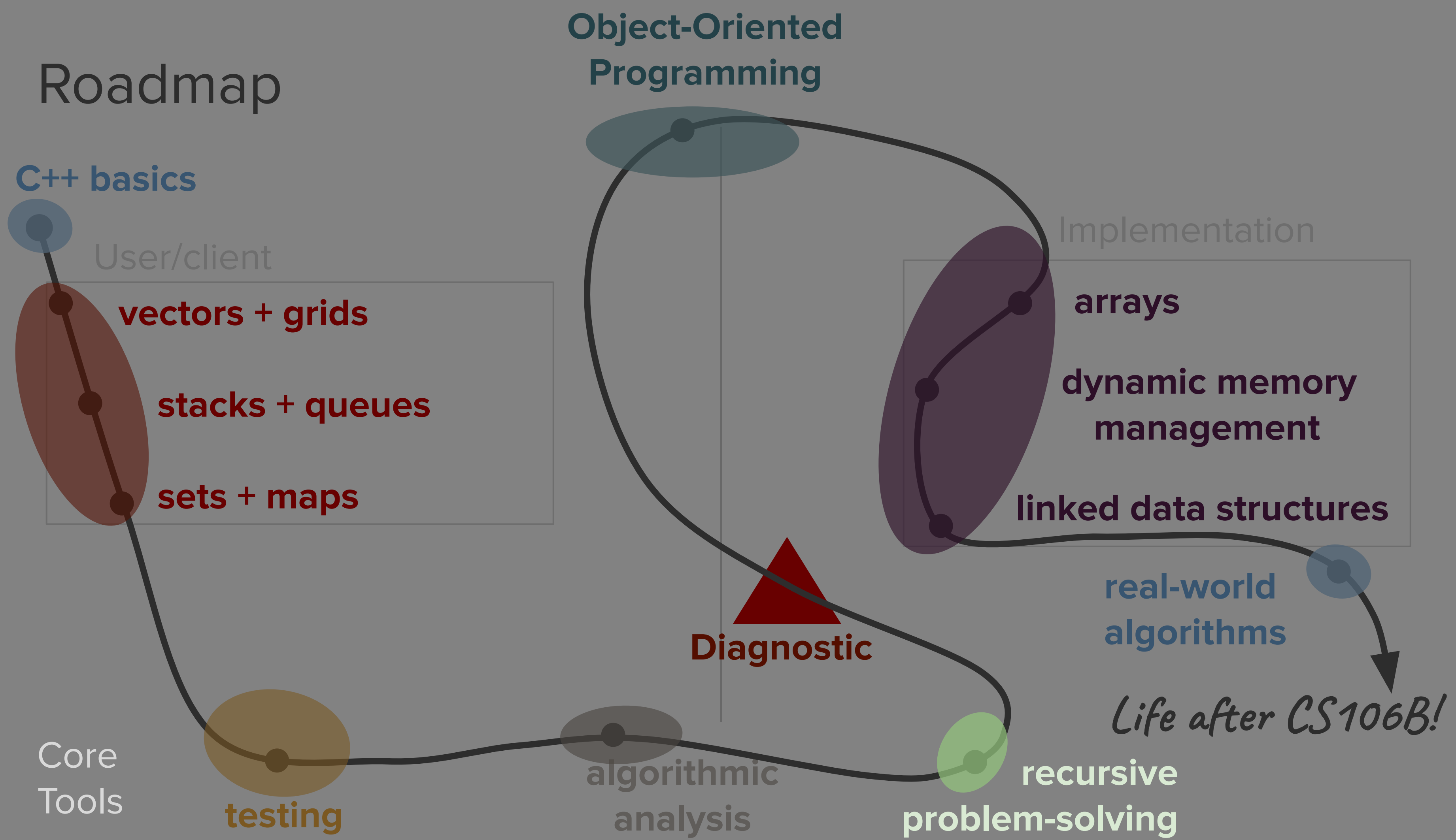
- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.

- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

What’s next?

Roadmap



C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Object-Oriented Programming
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Fractals