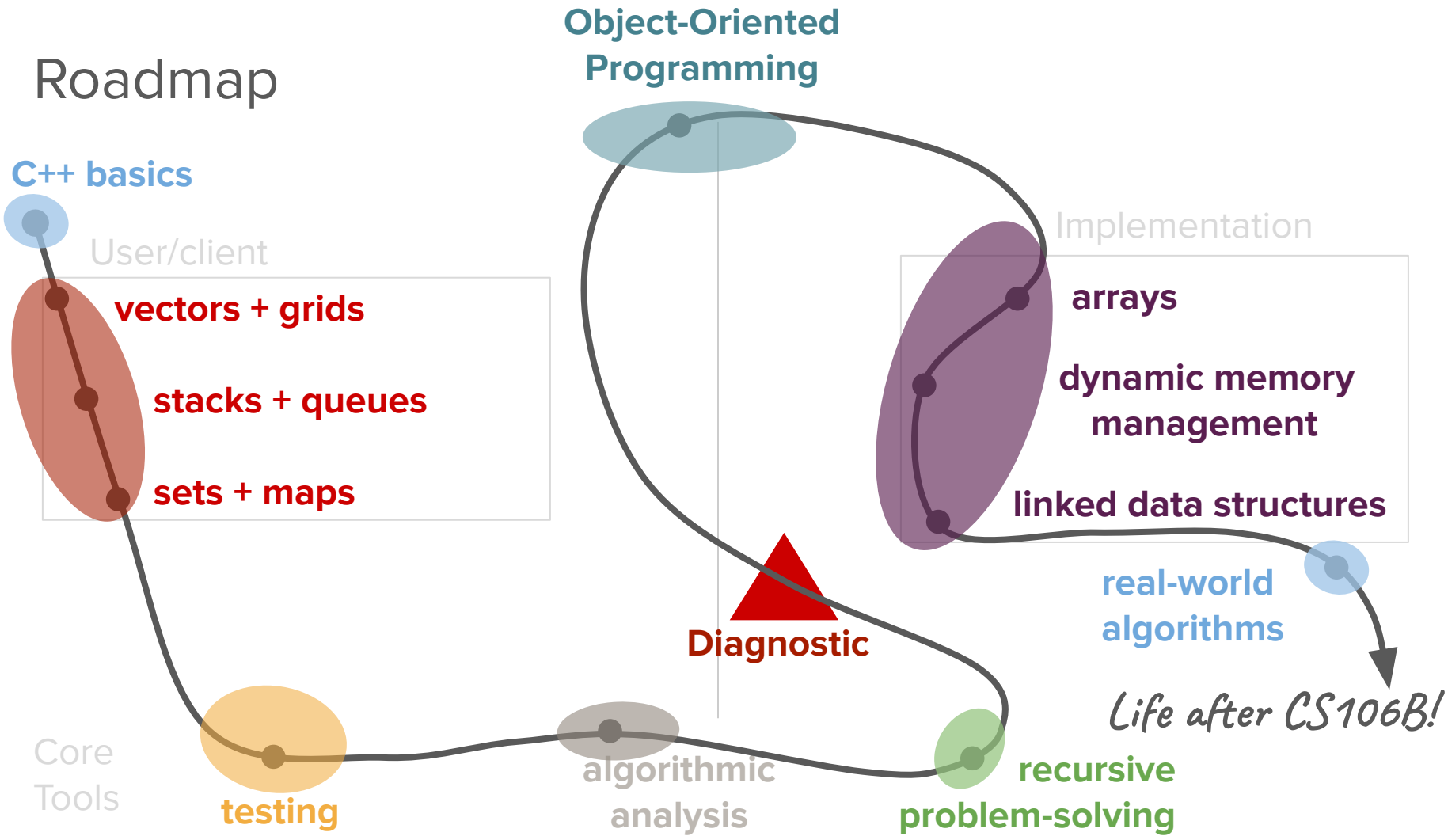
Introduction to Recursion

What was the most challenging part of Assignment 2?

(put your answers the chat)

Roadmap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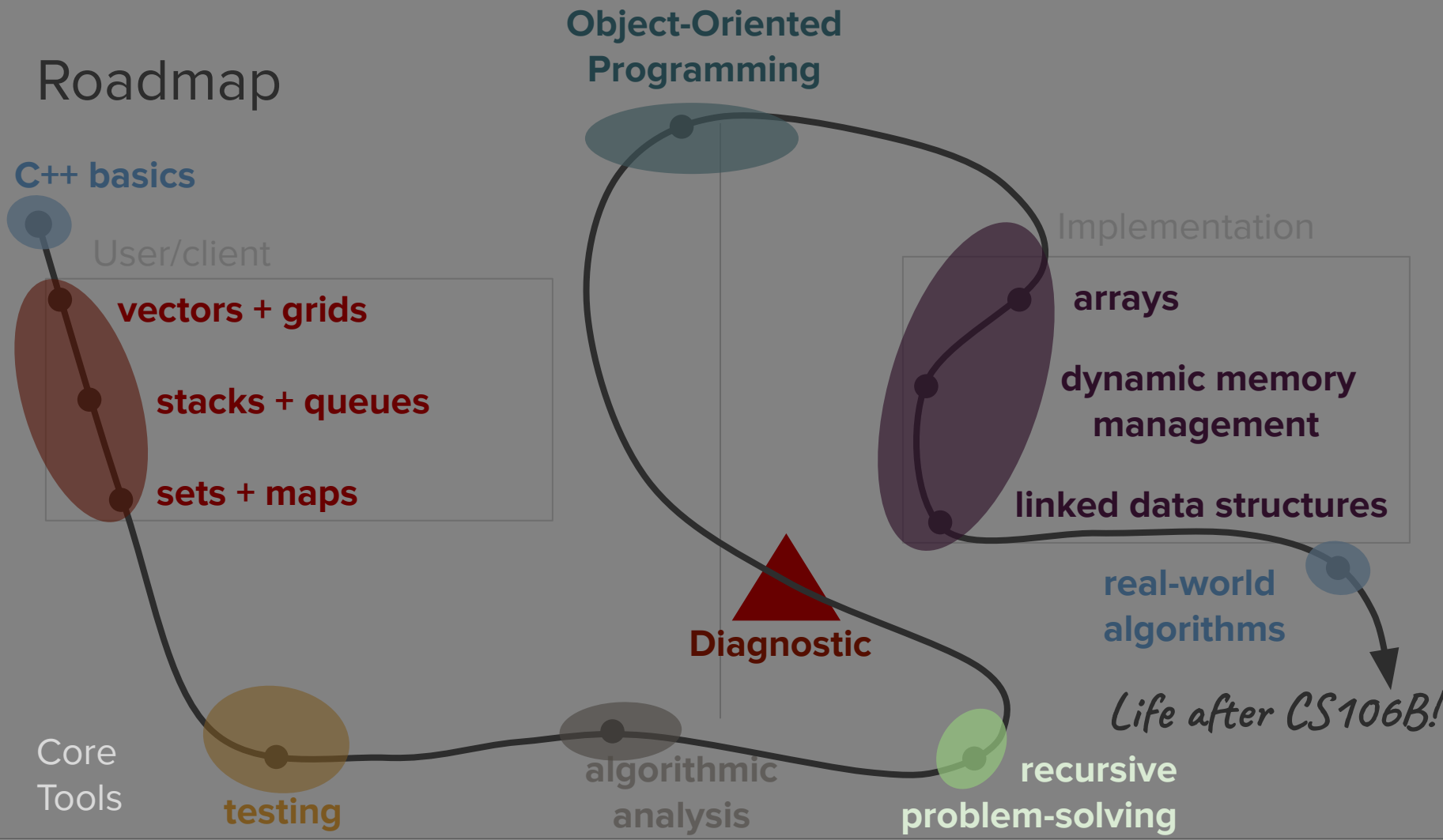
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Today's question

How can we take advantage of self-similarity within a problem to solve it more elegantly?

Today's topics

1. Review
2. Defining recursion
3. Recursion + Stack Frames (e.g. factorials)
4. Recursive Problem-Solving (e.g. string reversal)

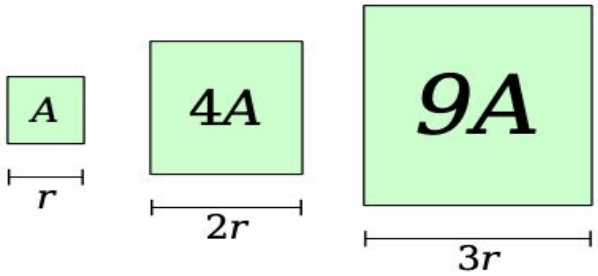
Review

(Big O)

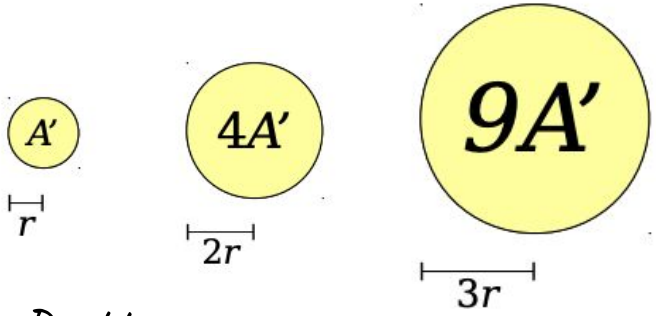
Big-O Notation

- **Big-O notation** is a way of quantifying the rate at which some quantity grows.
- Example:
  - A square of side length `r` has area $O(r^2)$.
  - A circle of radius `r` has area $O(r^2)$.

This just says that these quantities grow at the same relative rates. It does not say that they're equal!



Doubling r increases area 4x
Tripling r increases area 9x



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Big-O Notation

With respect to a given input variable!

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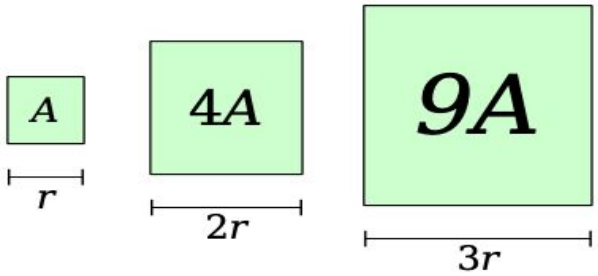

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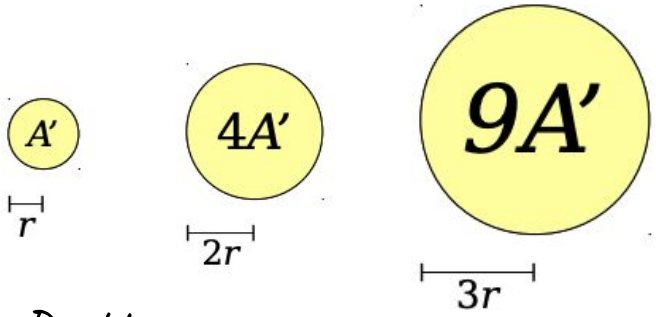

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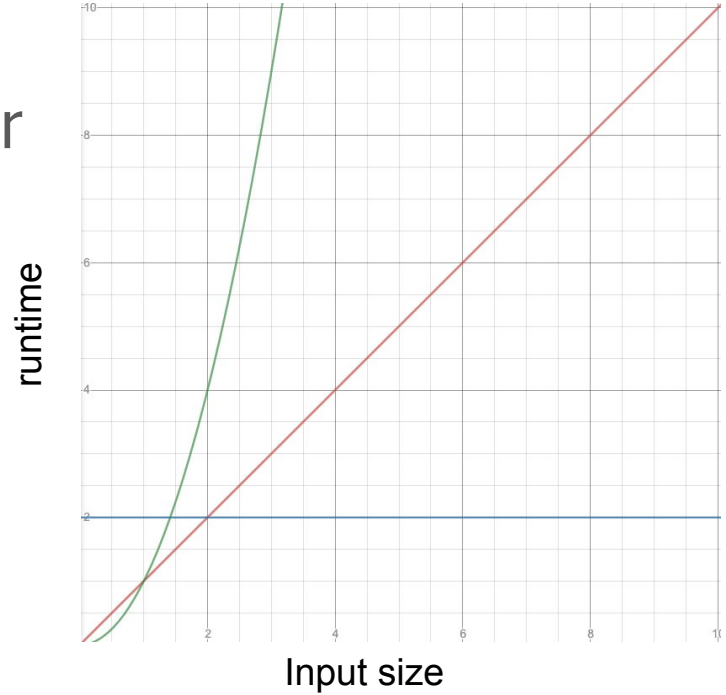
Efficiency Categorizations So Far

- Constant Time – O(1)
  - Super fast, this is the best we can hope for!
- Linear Time – O(n)
  - This is okay; we can live with this
- Quadratic Time – $O(n^2)$
  - This can start to slow down really quickly
  - Exhaustive Search for Perfect Numbers



- How do all the ADT operations we've seen so far fall into these categories?

ADT Big-O Matrix

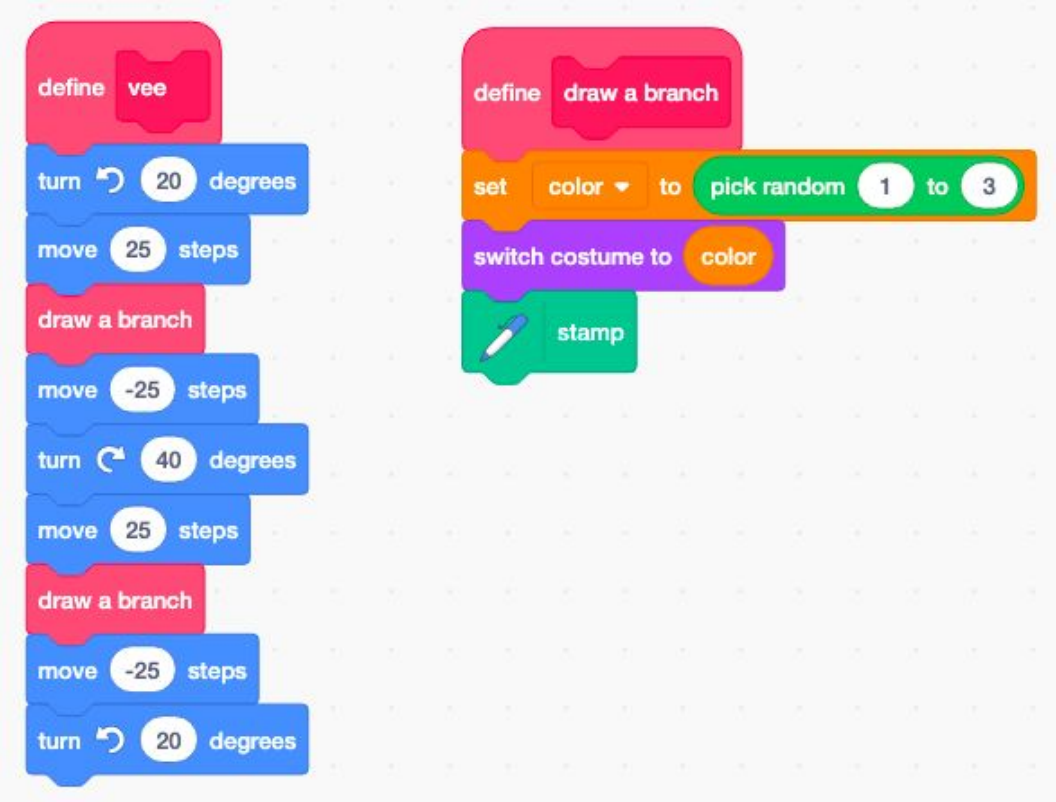
- Vectors
  - `.size() - O(1)`
  - `.add() - O(1)`
  - `v[i] - O(1)`
  - `.insert() - O(n)`
  - `.remove() - O(n)`
  - `.clear() - O(n)`
  - `traversal - O(n)}`
- Grids
  - `.numRows()/.numCols() - O(1)`
  - `g[i][j] - O(1)`
  - `.inBounds() - O(1)`
  - `traversal - O(n^2)`
- Queues
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.enqueue() - O(1)`
  - `.dequeue() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Stacks
  - `.size() - O(1)`
  - `.peek() - O(1)`
  - `.push() - O(1)`
  - `.pop() - O(1)`
  - `.isEmpty() - O(1)`
  - `traversal - O(n)`
- Sets
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `.add() - ???`
  - `.remove() - ???`
  - `.contains() - ???`
  - `traversal - O(n)`
- Maps
  - `.size() - O(1)`
  - `.isEmpty() - O(1)`
  - `m[key] - ???`
  - `.contains() - ???`
  - `traversal - O(n)`

What is recursion?

Activity: Vee

(https://scratch.mit.edu/projects/409796637/)

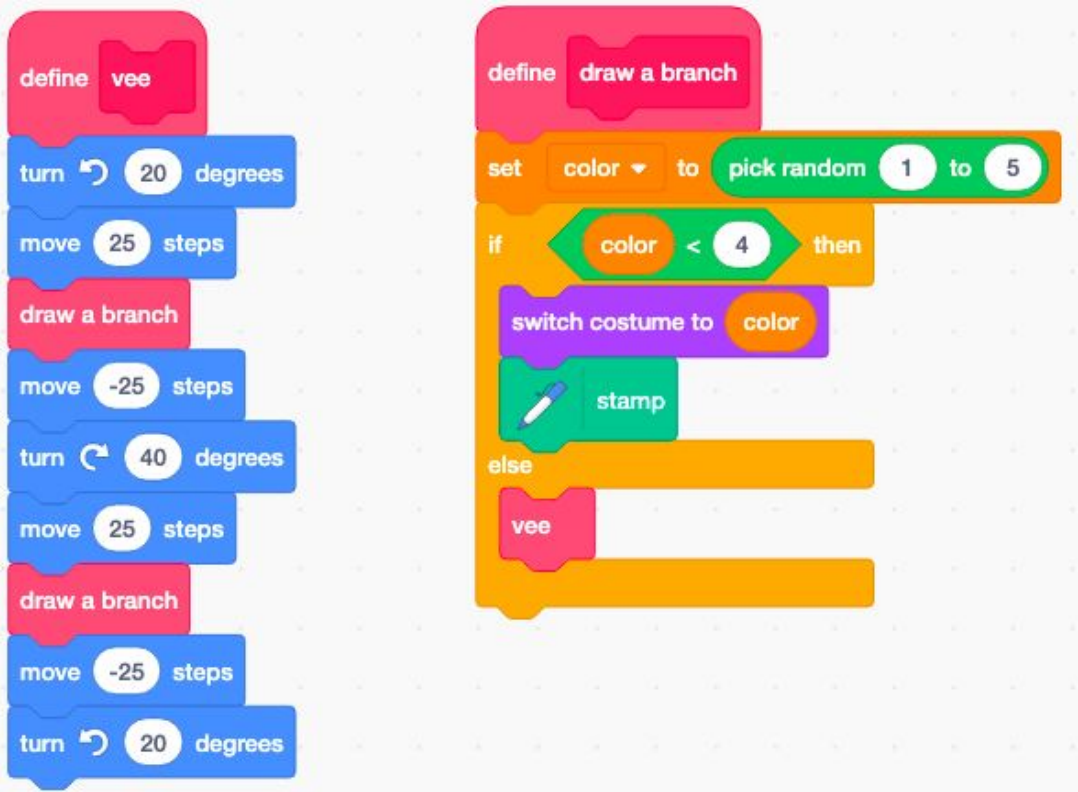
This code creates a “vee” shape with random colors.







Discuss in breakout rooms: What will this code do?



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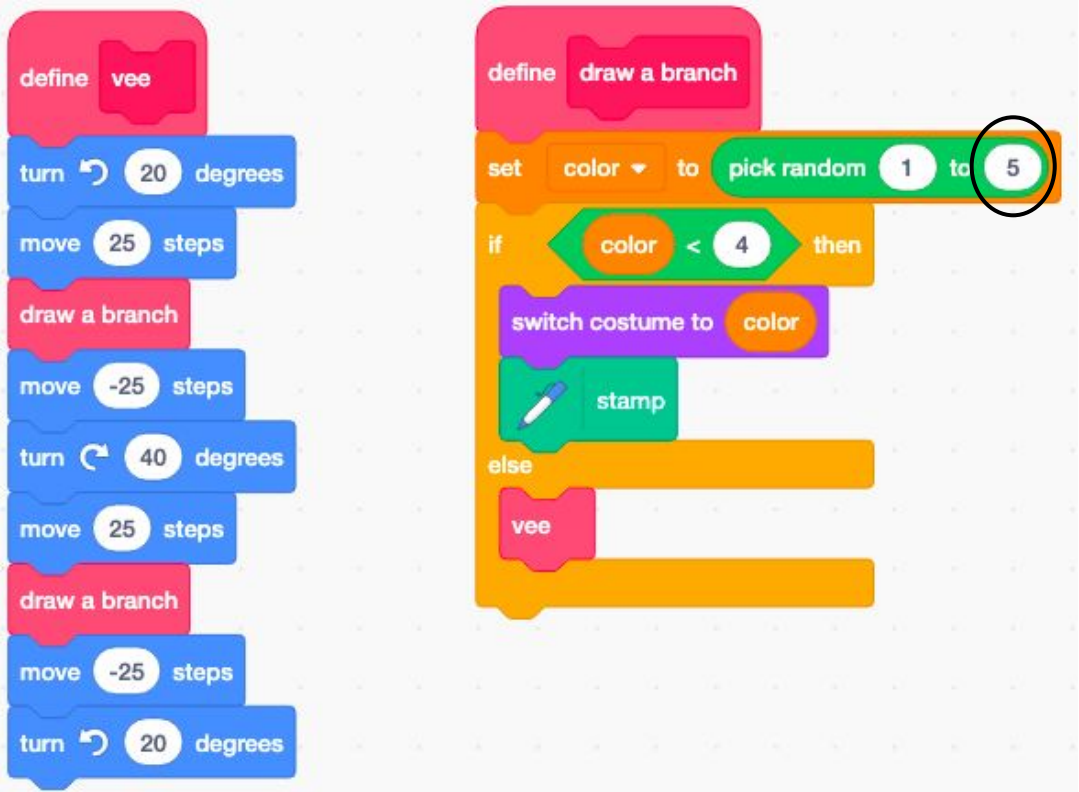
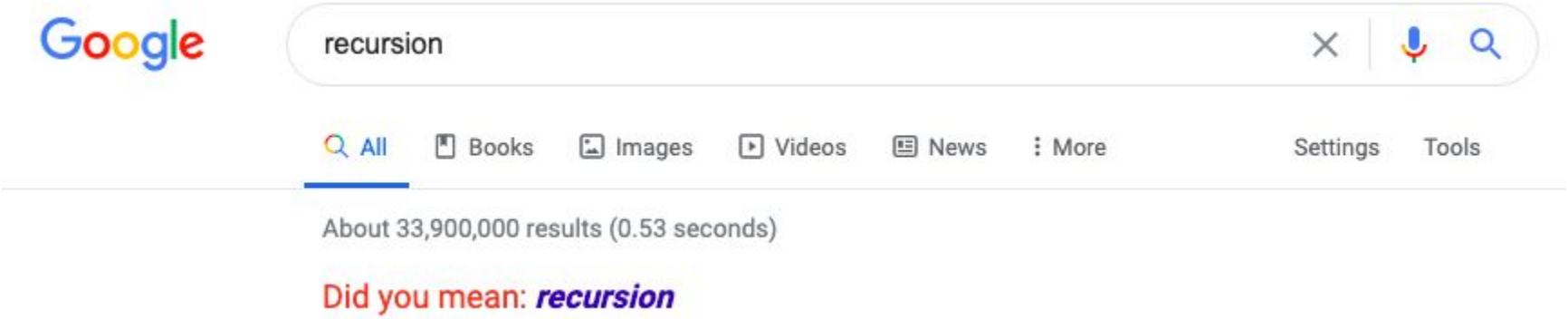


Demo: Recursive Vee

(https://scratch.mit.edu/projects/409785610/)

What is recursion?

Wikipedia: “Recursion occurs when a thing is defined in terms of itself.”



Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

What is recursion?

- A powerful substitute for iteration (loops)
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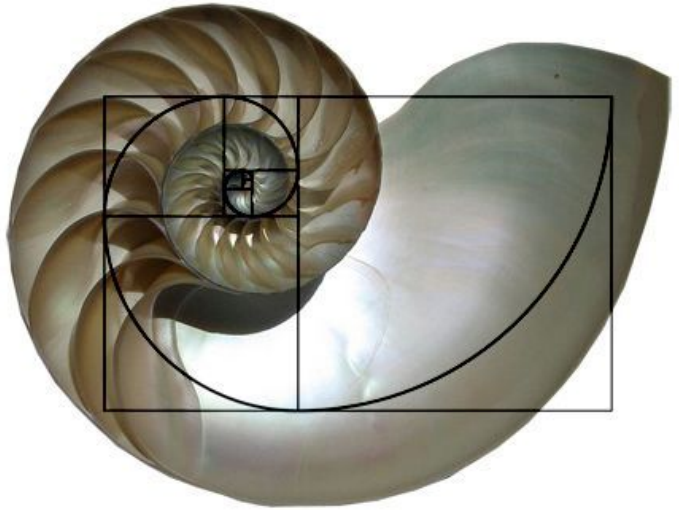

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- Will be part of many of our future assignments!

How many students are in a lecture hall?

A [non-COVID] analogy

How many students are in the lecture hall?

- Let’s suppose I want to find out how many people are at lecture today, but I don’t want to walk around and count each person.

- I want to recruit your help, but I also want to minimize each individual’s amount of work.

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We can solve this problem recursively!

How many students are in the lecture hall?

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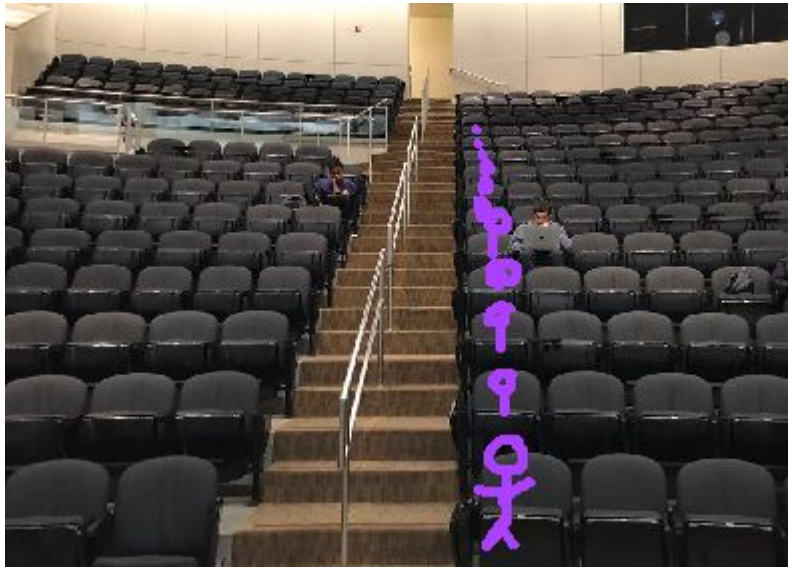

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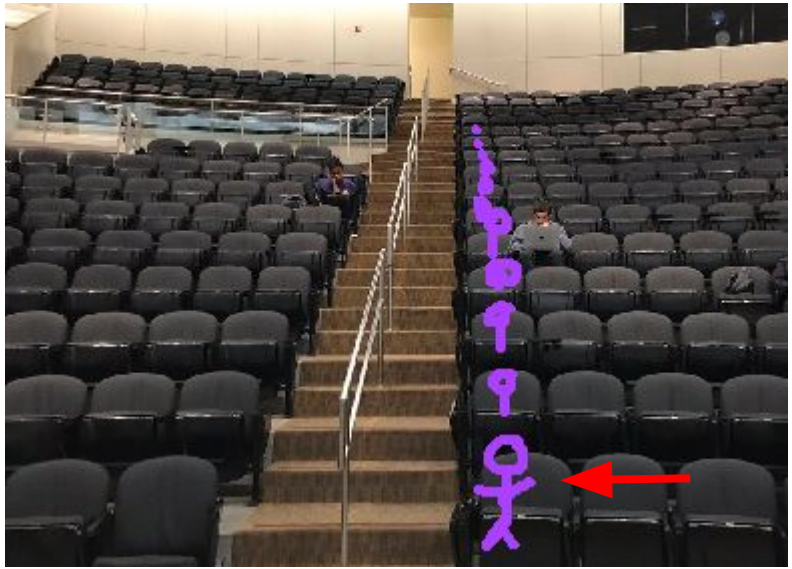

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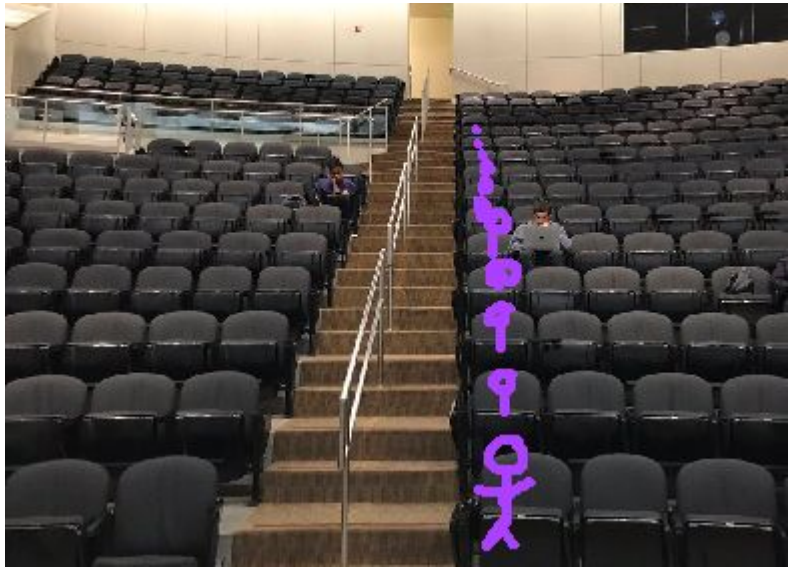

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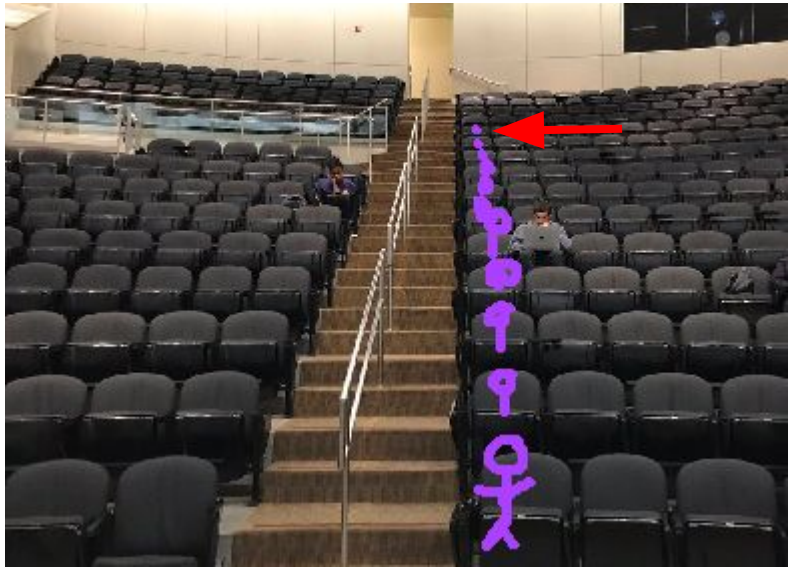

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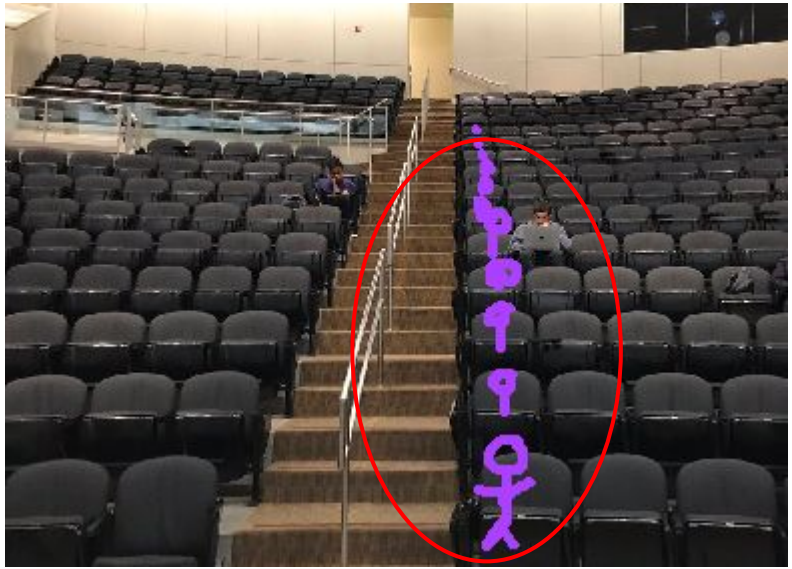

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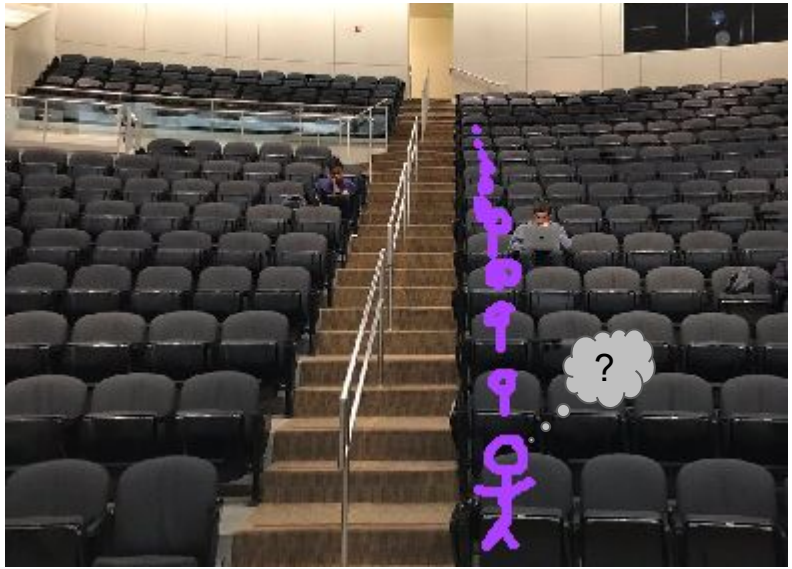

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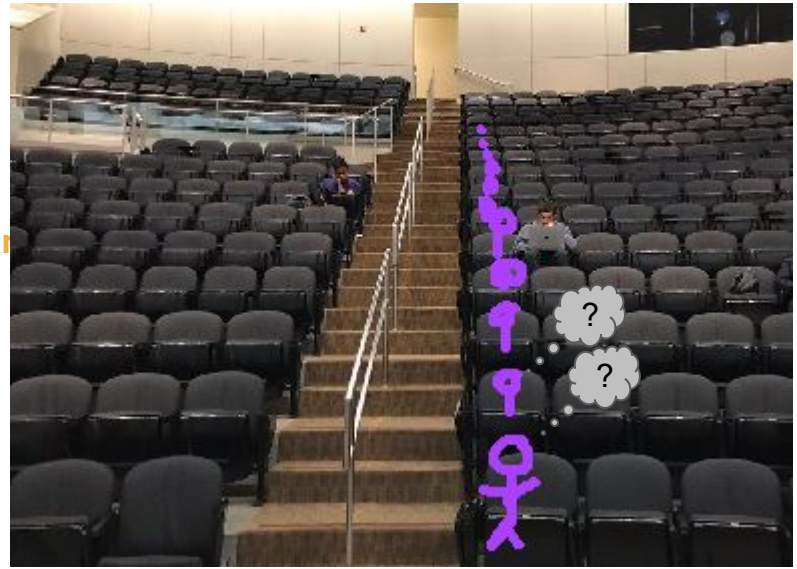

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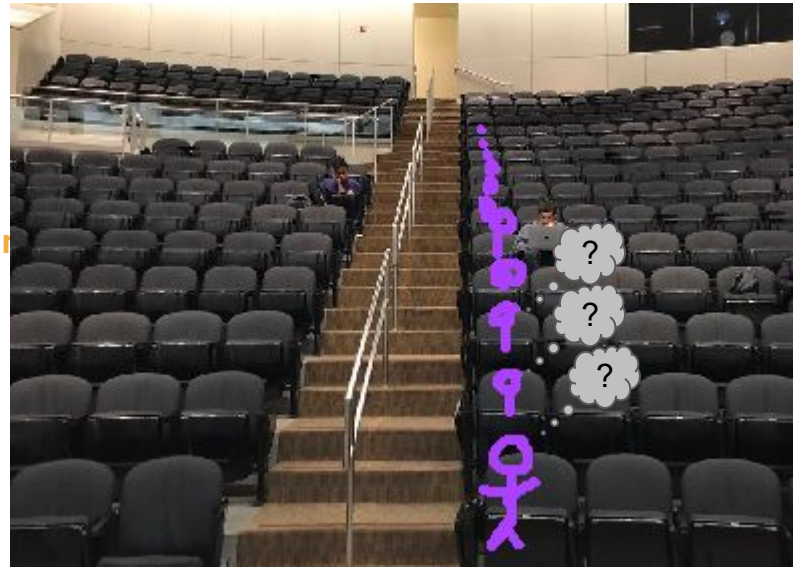

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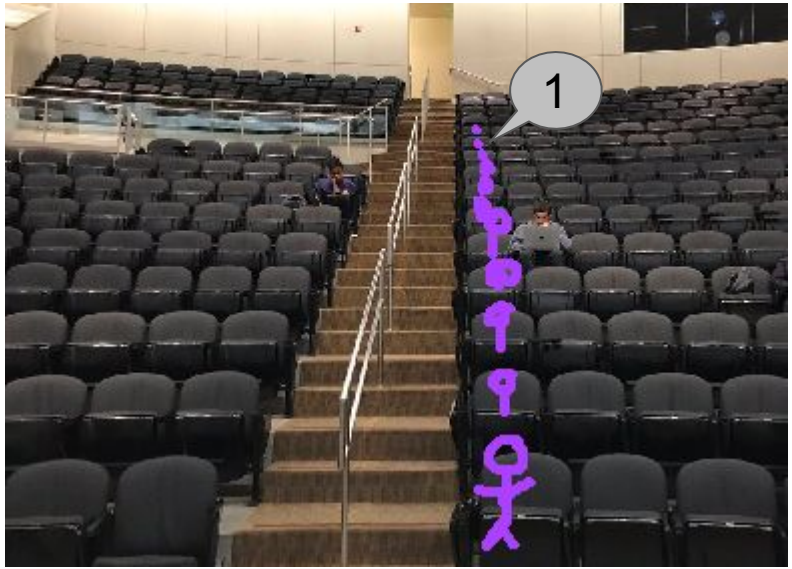

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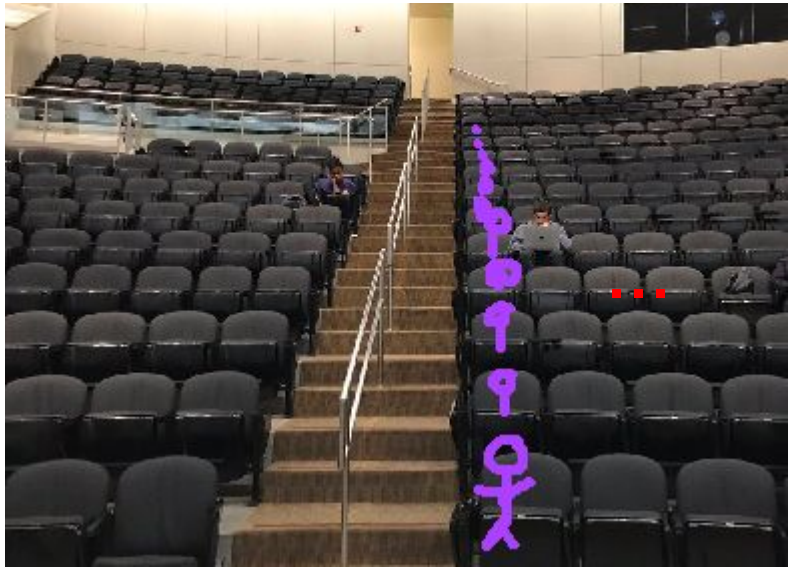

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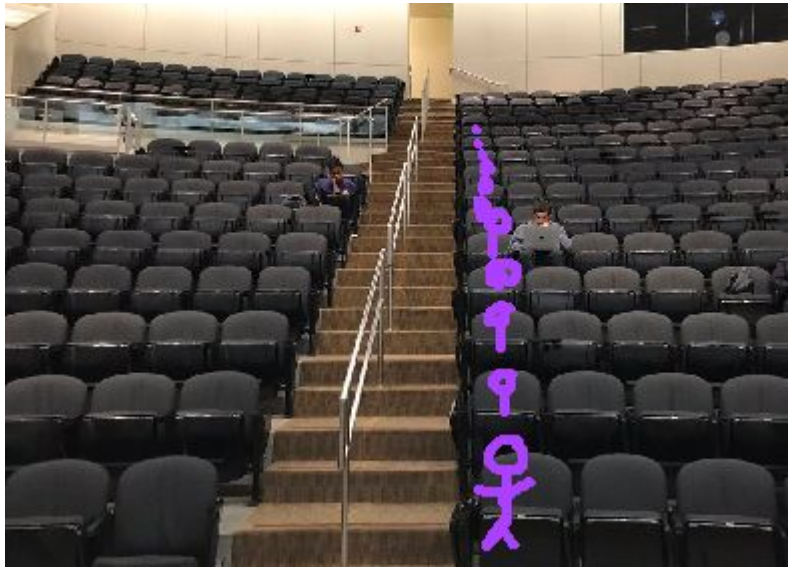

- Can generalize to the entire lecture hall!

Definition

recursion

A problem-solving technique in which tasks are completed by reducing them into repeated, smaller tasks of the same form.

Two main cases (components) of recursion

- Base case
  - The simplest version(s) of your problem that all other cases reduce to
  - An occurrence that can be answered directly

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“If someone is sitting behind me...”

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Announcements

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- Assignment 2 is due by 11:59pm PDT tonight.

- Assignment 3 will be released **after lecture on Monday**.
  - Note the change (instead of today): We realized that because of the holiday, you won't have learned enough about recursion to start the assignment until then anyway!
  - The assignment is due on Monday, July 19 so you'll still have a week to complete it.
  - Instead, take a look at **this week's section recursion problems** to get practice with recursion!

- Waitlist update: Some folks are being admitted off the waitlist this week (yay!). If you are admitted this week and haven't submitted Assignment 1, please reach out to me and Nick.
  - **Today at 5pm PDT** is the final study list deadline (add/drop deadline).

Factorial example

Factorials

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- Factorials show up in unexpected places. We'll see one later this quarter when we talk about sorting algorithms.

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  - $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$
  - $0! = 1.$ **(by definition)**
- Factorials show up in unexpected places. We'll see one later this quarter when we talk about sorting algorithms.
- Let's implement a function to compute factorials!

Computing factorials

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

Computing factorials

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

Computing factorials

$$5! = 5 \times \underbrace{4 \times 3 \times 2 \times 1}_{4!}$$

Computing factorials

$$5! = 5 \times 4!$$

Computing factorials

$$5! = 5 \times 4!$$

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3 x 2 x 1
```

Computing factorials

$$5! = 5 \times 4!$$

$$4! = 4 \times 3 \times 2 \times 1$$

Computing factorials

$$5! = 5 \times 4!$$

$$4! = 4 \times \underbrace{3 \times 2 \times 1}_{3!}$$

Computing factorials

$$5! = 5 \times 4!$$

$$4! = 4 \times 3!$$

Computing factorials

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5! = 5 x 4!
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5! = 5 x 4!
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5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2 x 1
```

Computing factorials

$$5! = 5 \times 4!$$

$$4! = 4 \times 3!$$

$$3! = 3 \times \underbrace{2 \times 1}_{2!}$$

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
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Computing factorials

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5! = 5 x 4!
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Computing factorials

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5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
```

Computing factorials

```
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
```

Computing factorials

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5! = 5 x 4!
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0! = 1
```

Computing factorials

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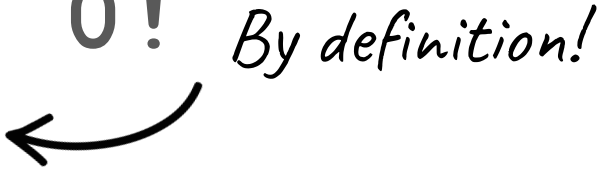
5! = 5 x 4!
4! = 4 x 3!
3! = 3 x 2!
2! = 2 x 1!
1! = 1 x 0!
0! = 1
```



Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

Another view of factorials

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \times (n-1)! & \text{otherwise} \end{cases}$$

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

This is a “**stack frame**.” One gets created each time a function is called.

- The “stack” is where in your computer’s memory the information is stored.
- A “frame” stores all of the data (variables) for that particular function call.

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {

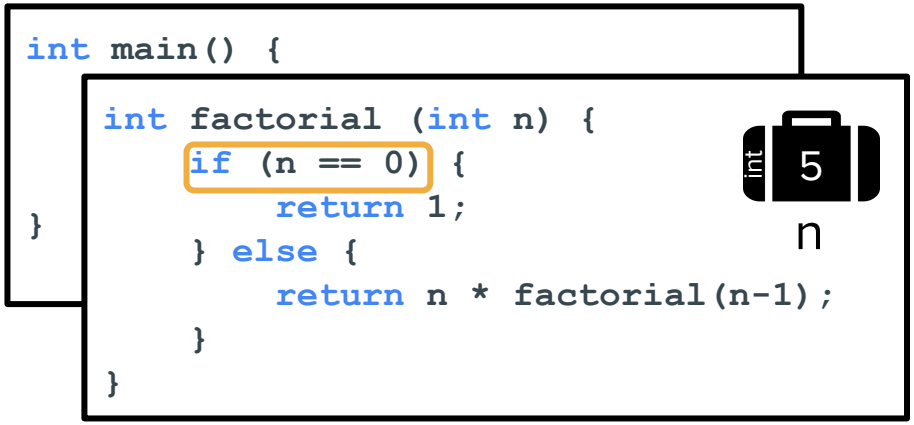
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

When a function gets called, a new stack frame gets created.

Recursion in action



Recursion in action

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int factorial (int n) {
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Recursion in action

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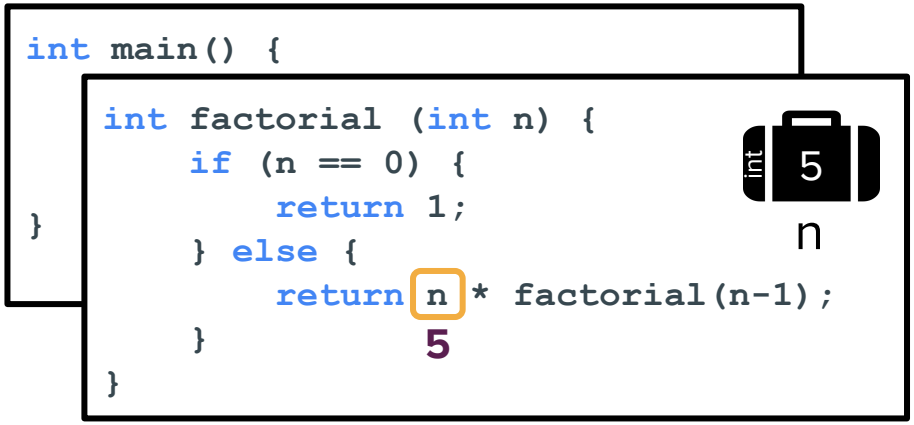
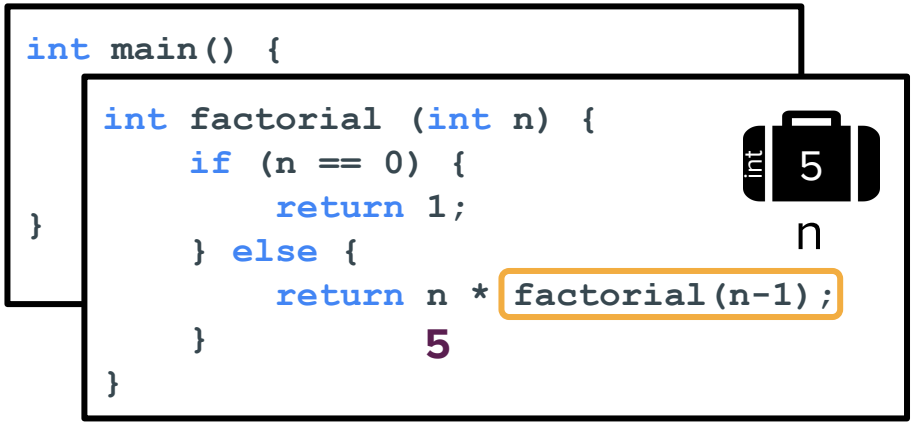
n

Recursion in action



Recursion in action



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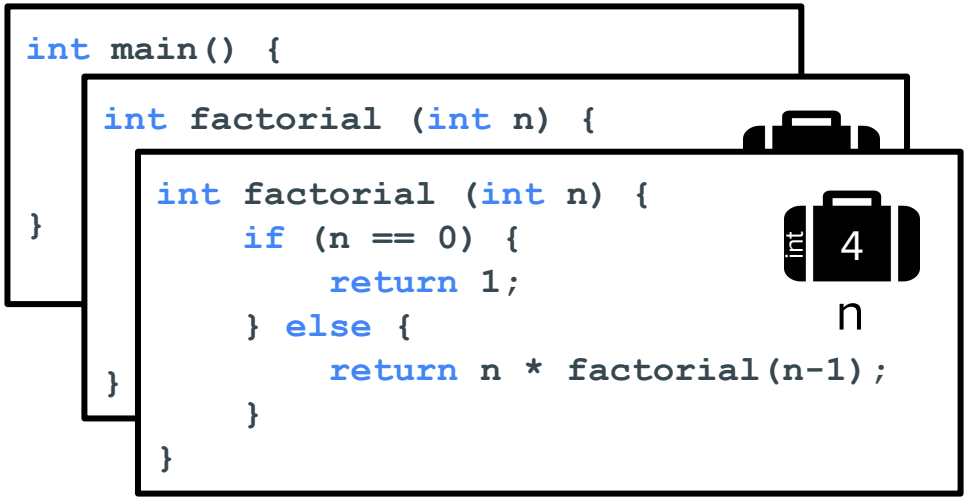
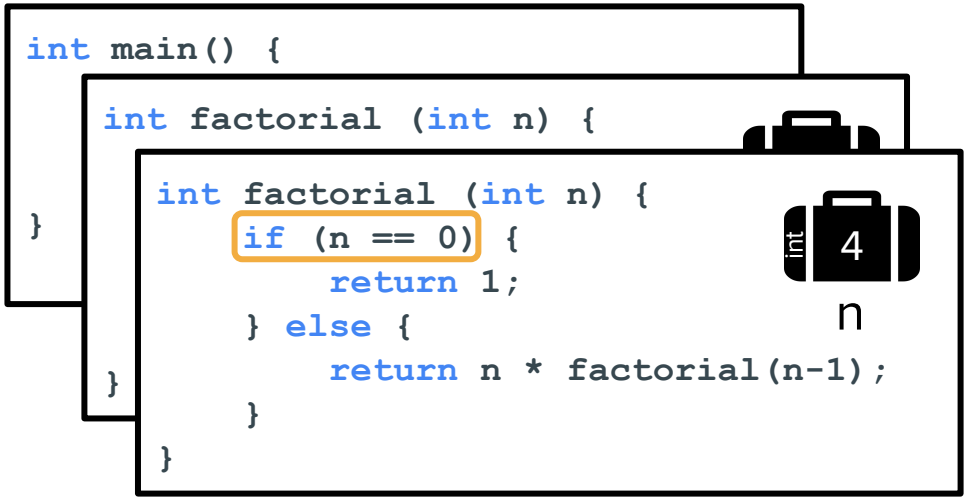
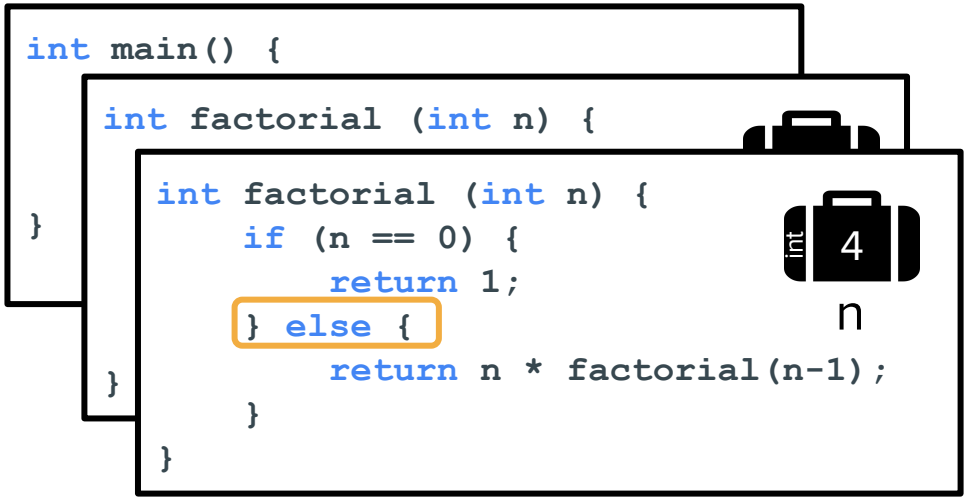


Every time we call **`factorial()`**, we get a new copy of the local variable **`n`** that's independent of all the previous copies because it exists inside the new frame.

Recursion in action



Recursion in action



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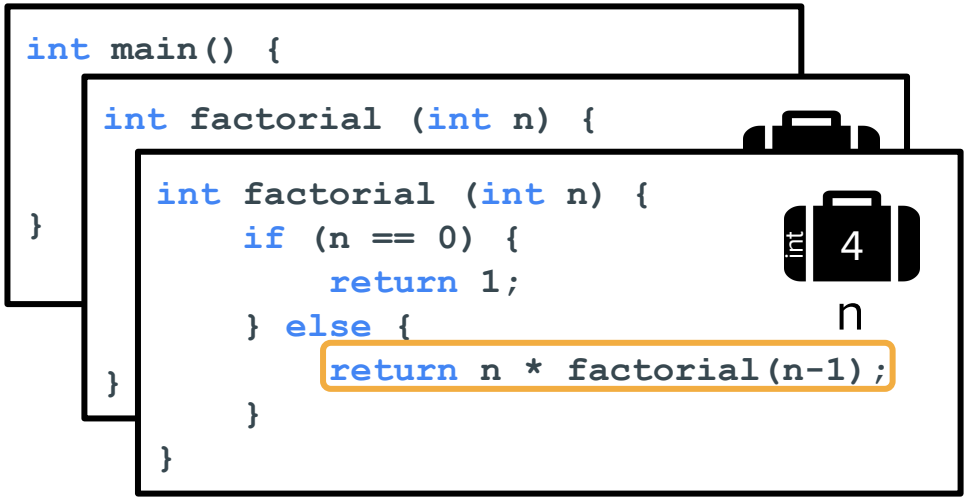

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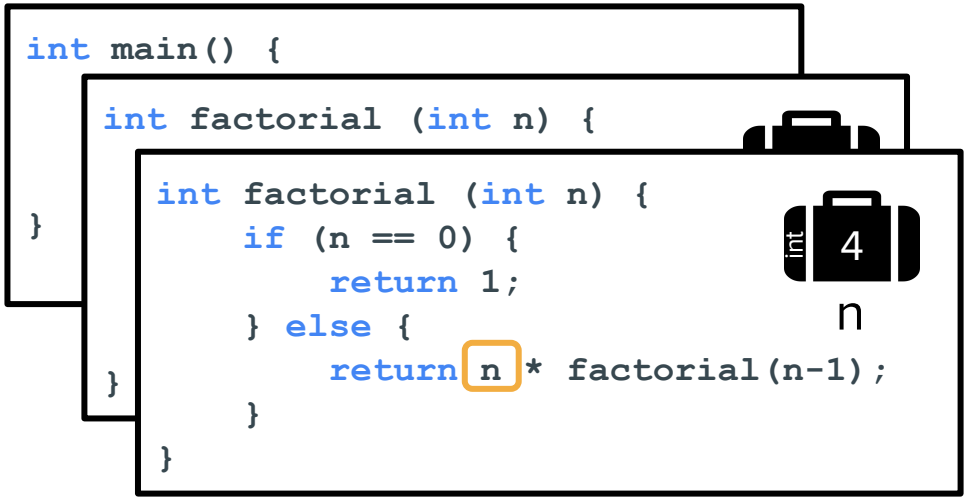

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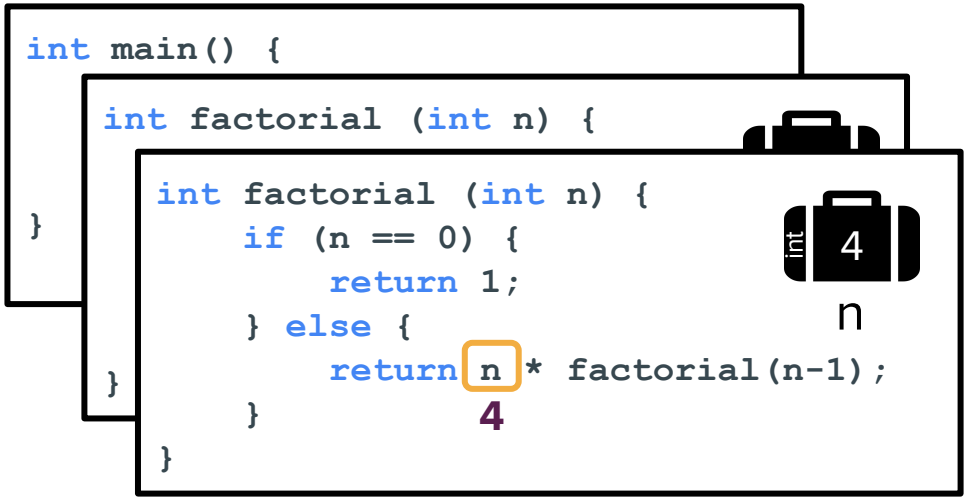

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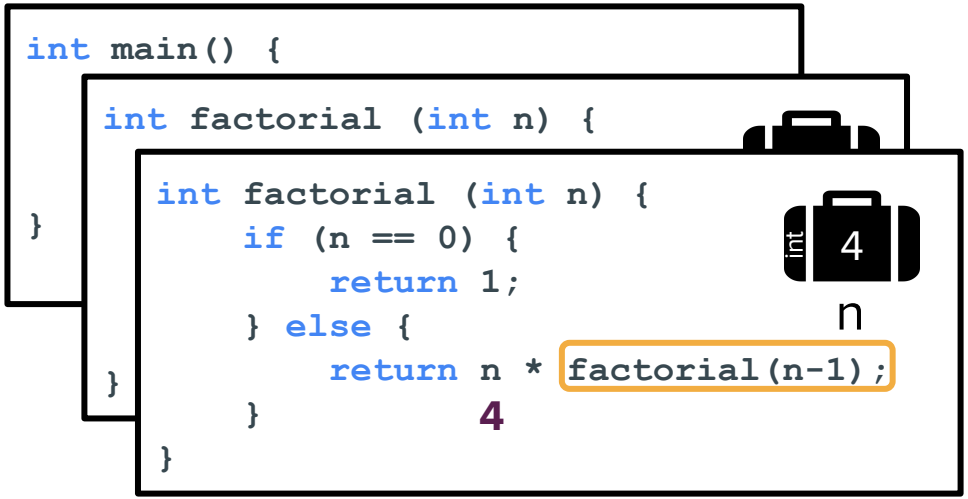

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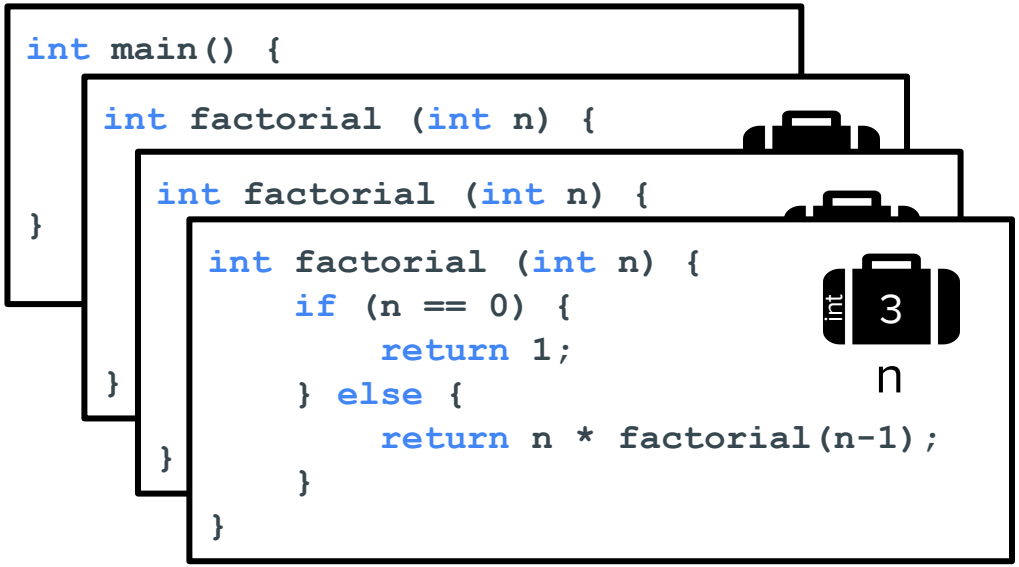

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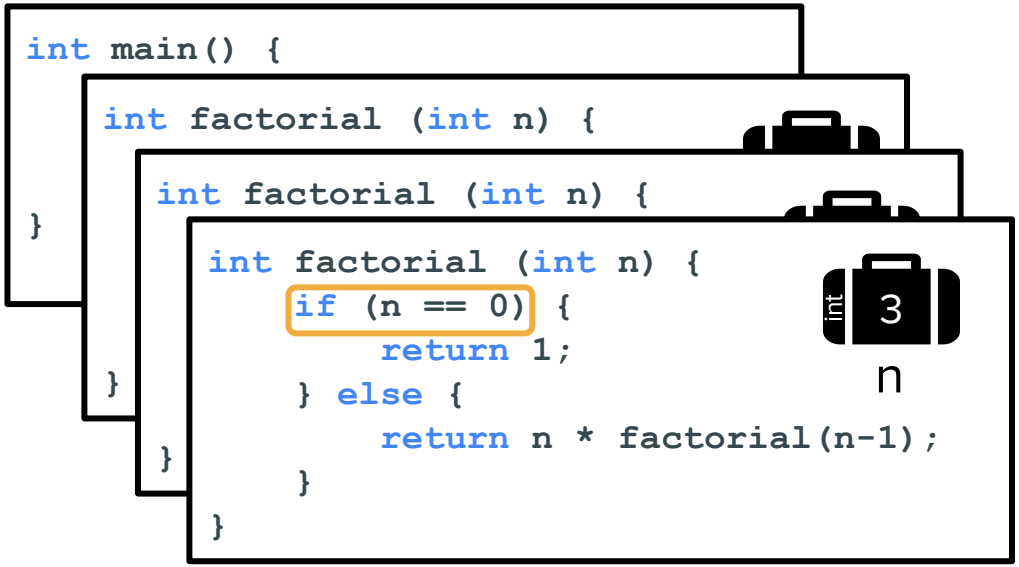

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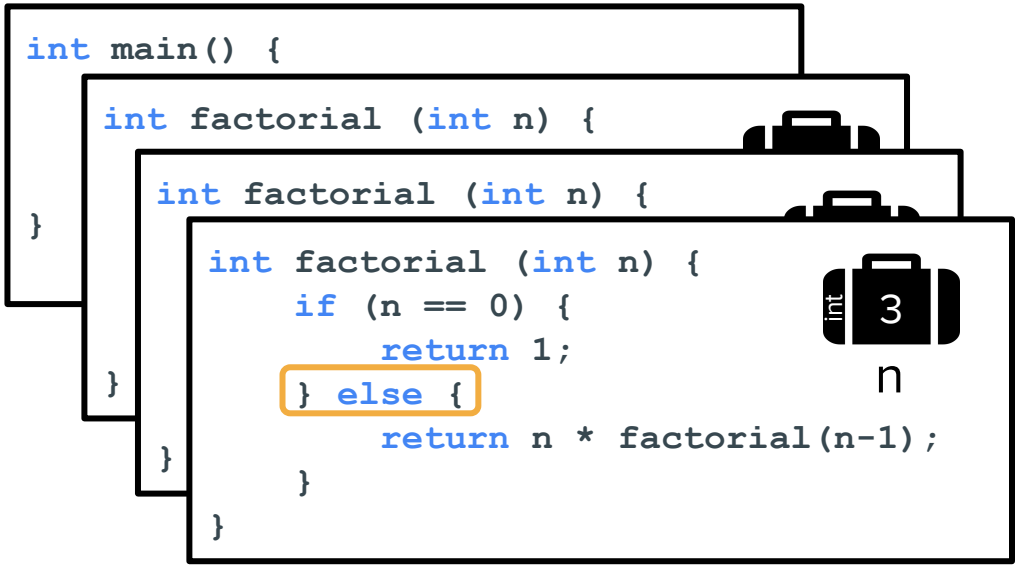

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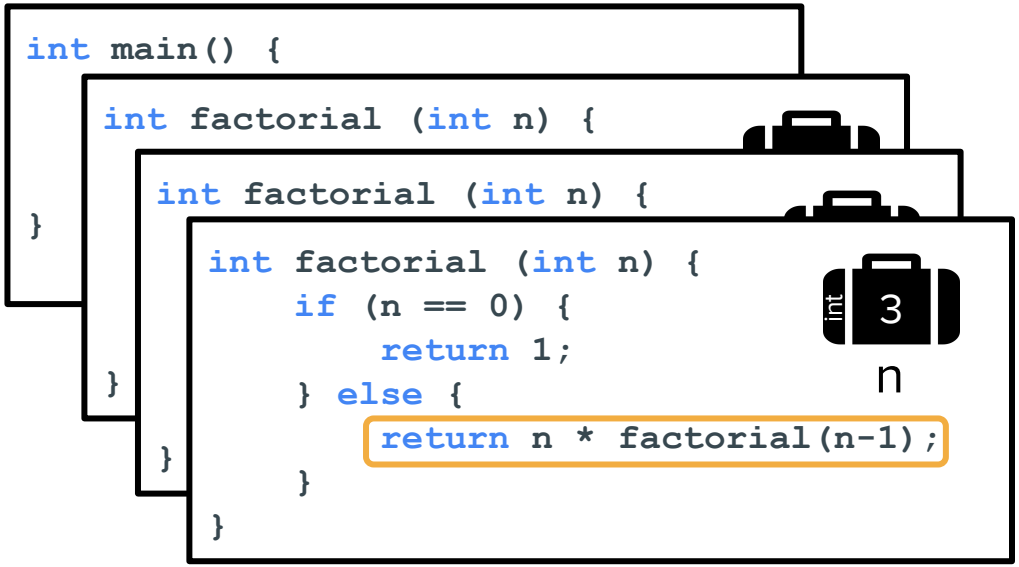

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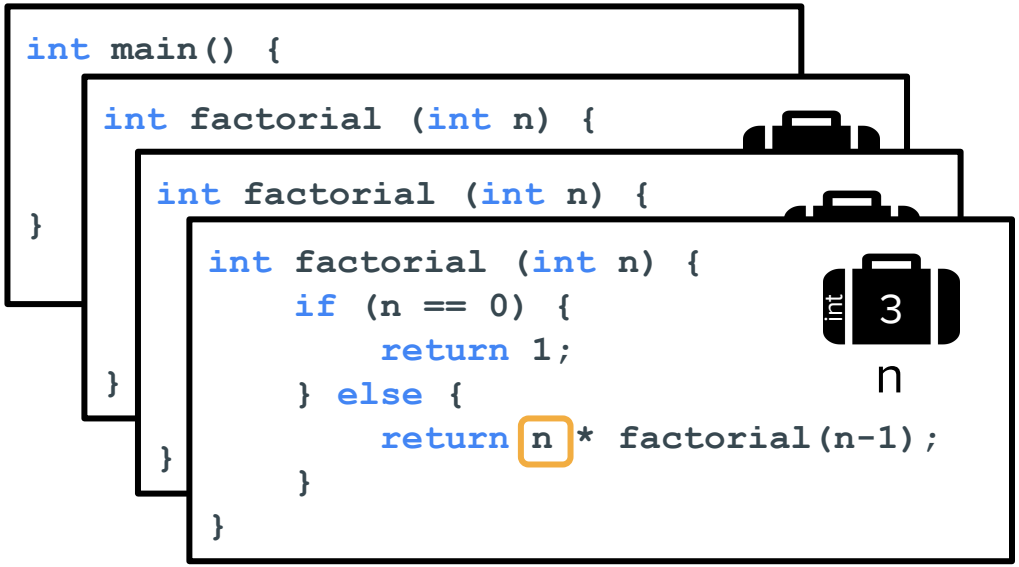

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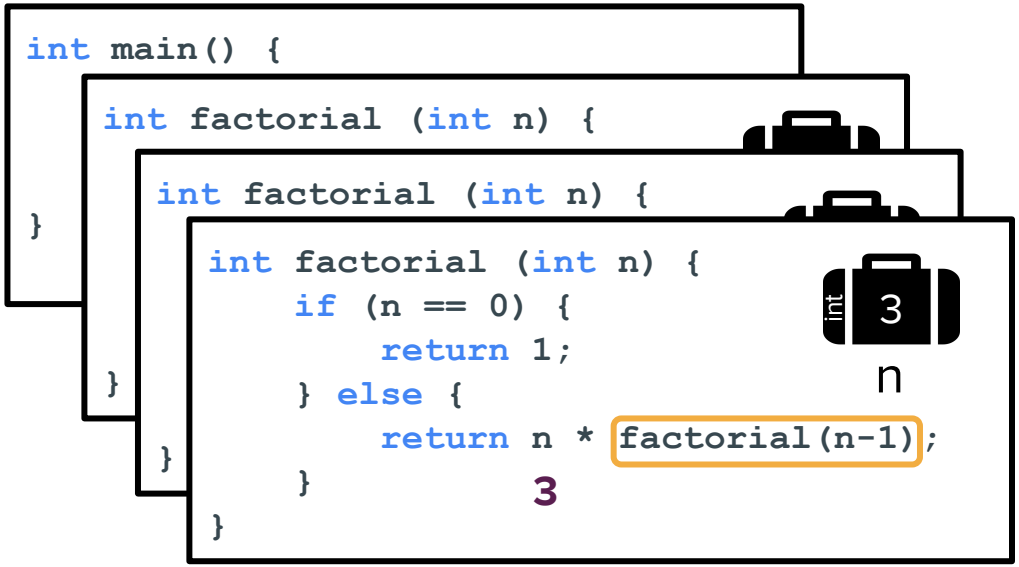

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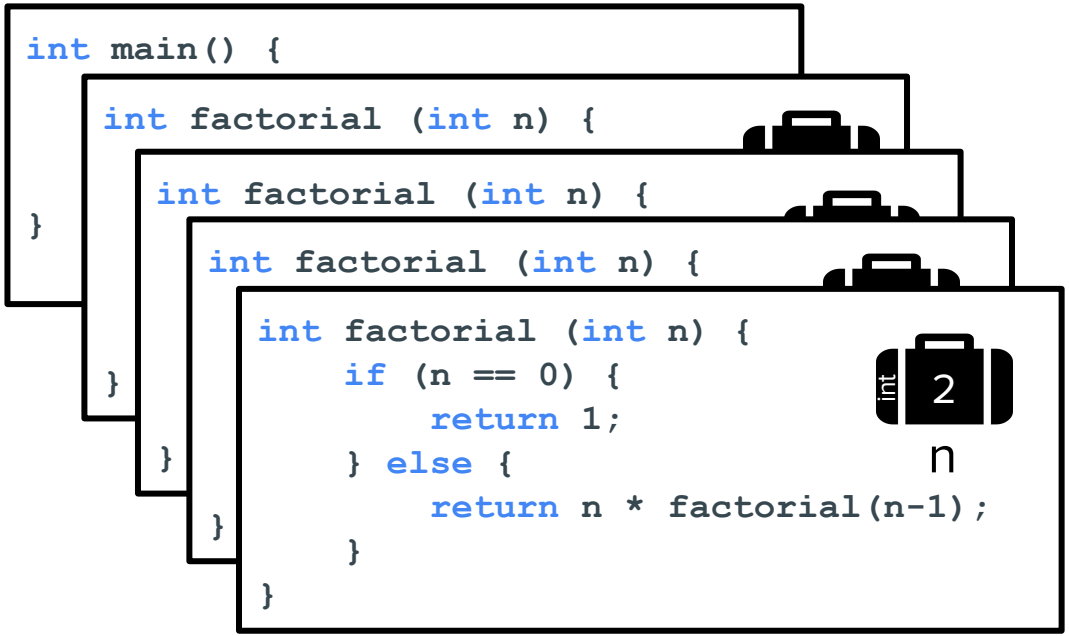

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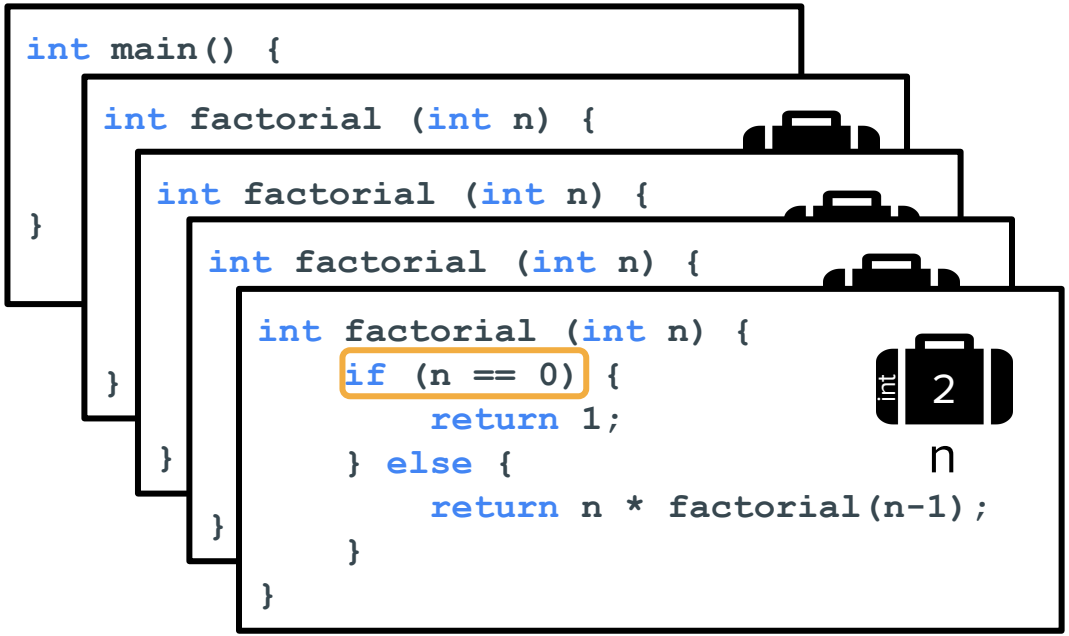

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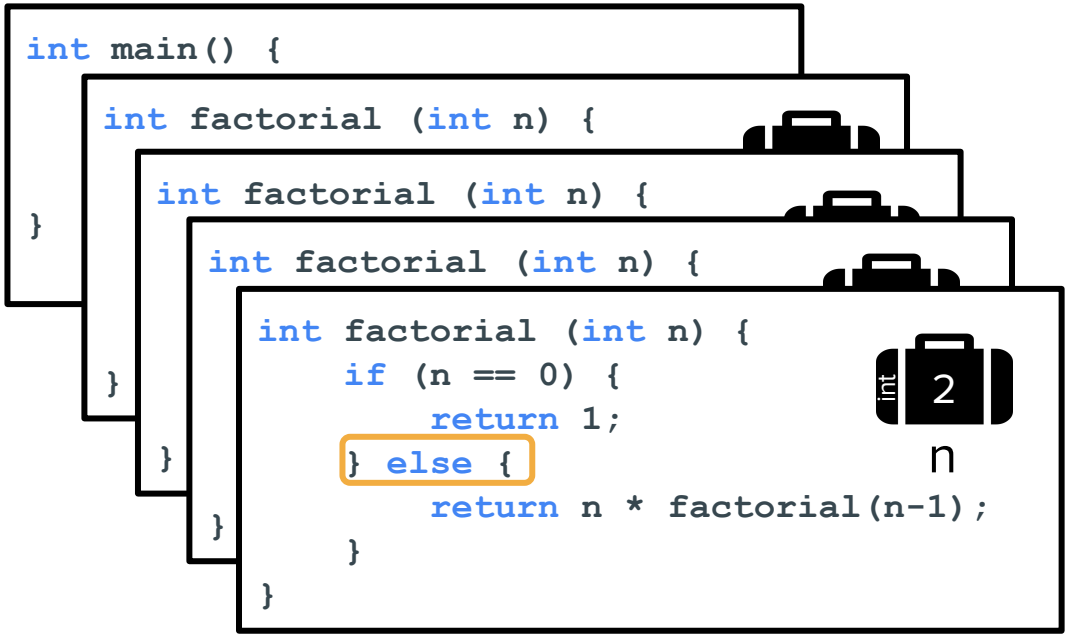

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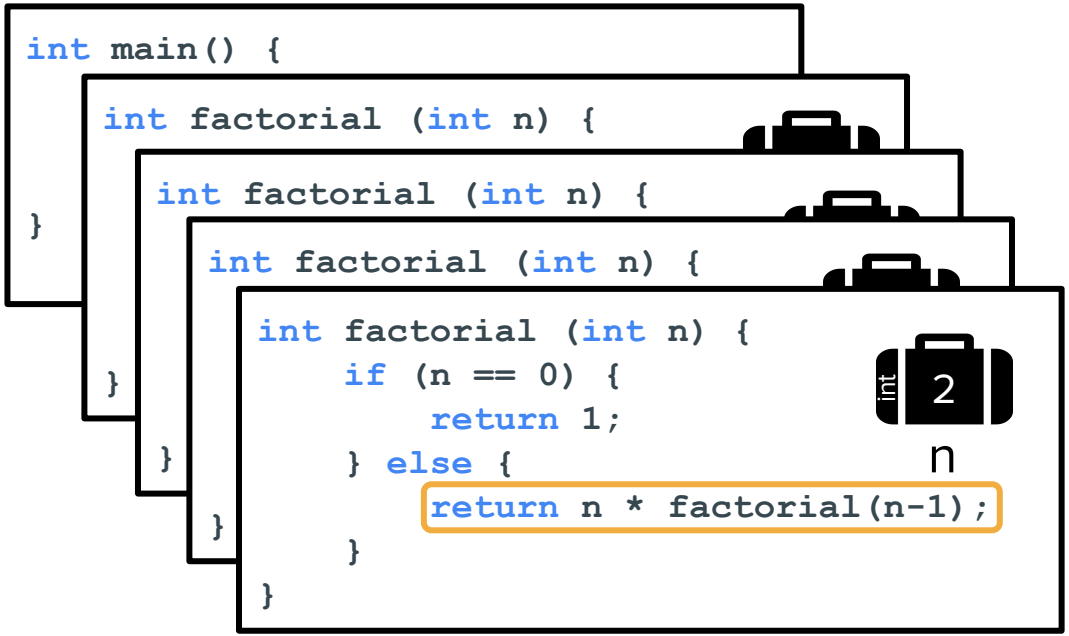

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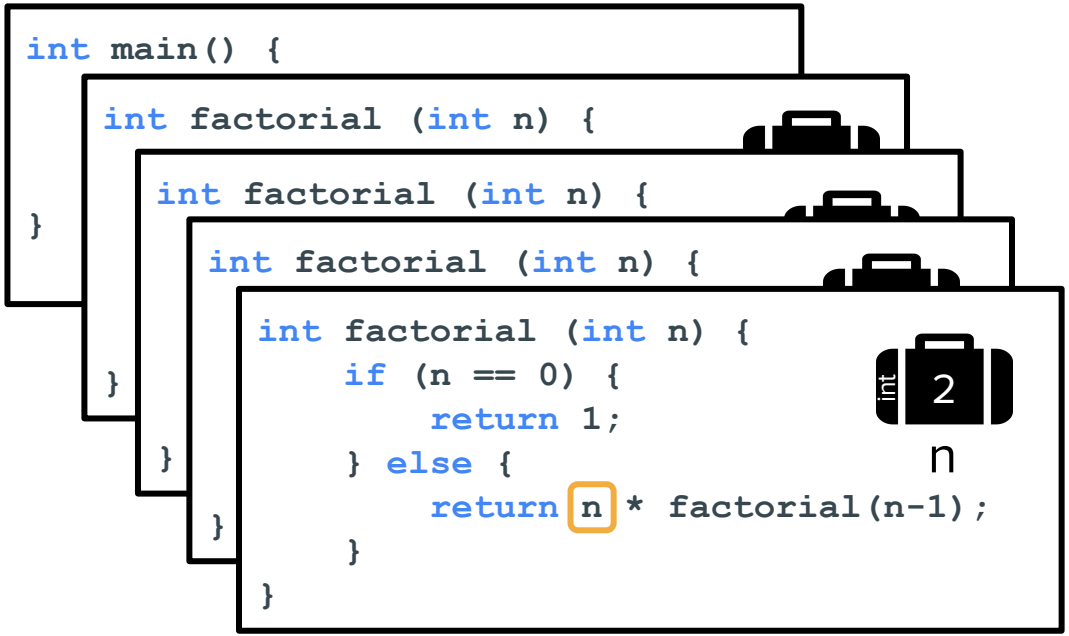

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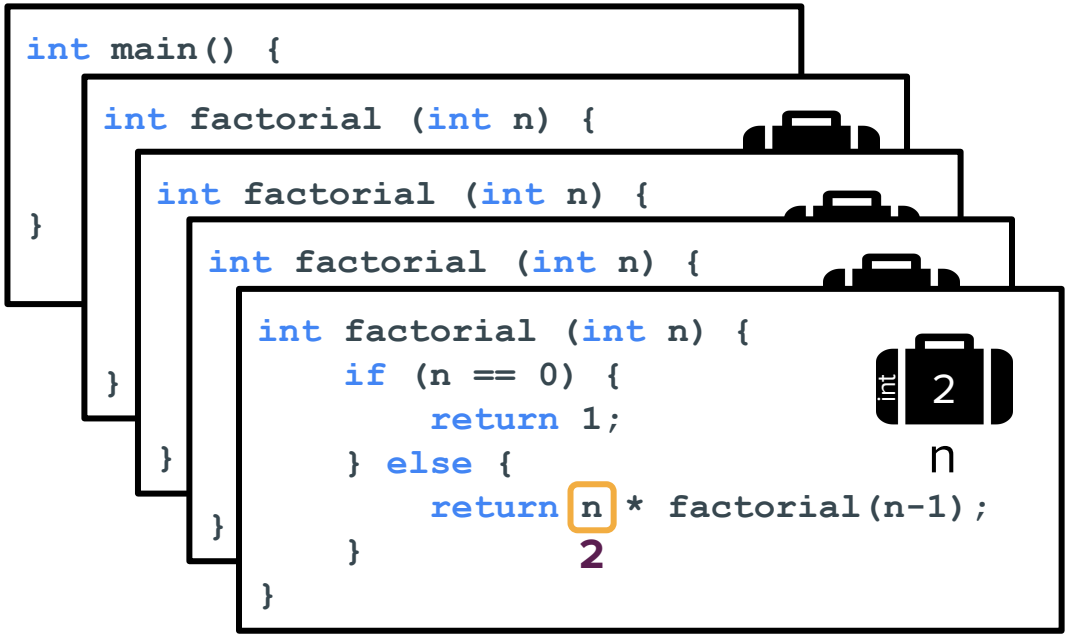

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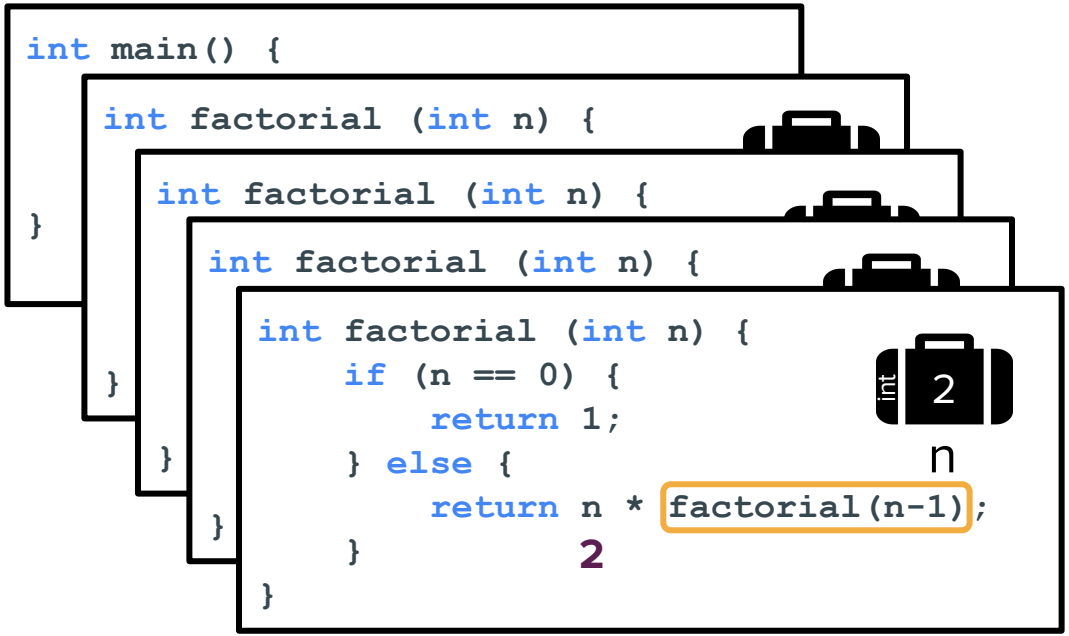

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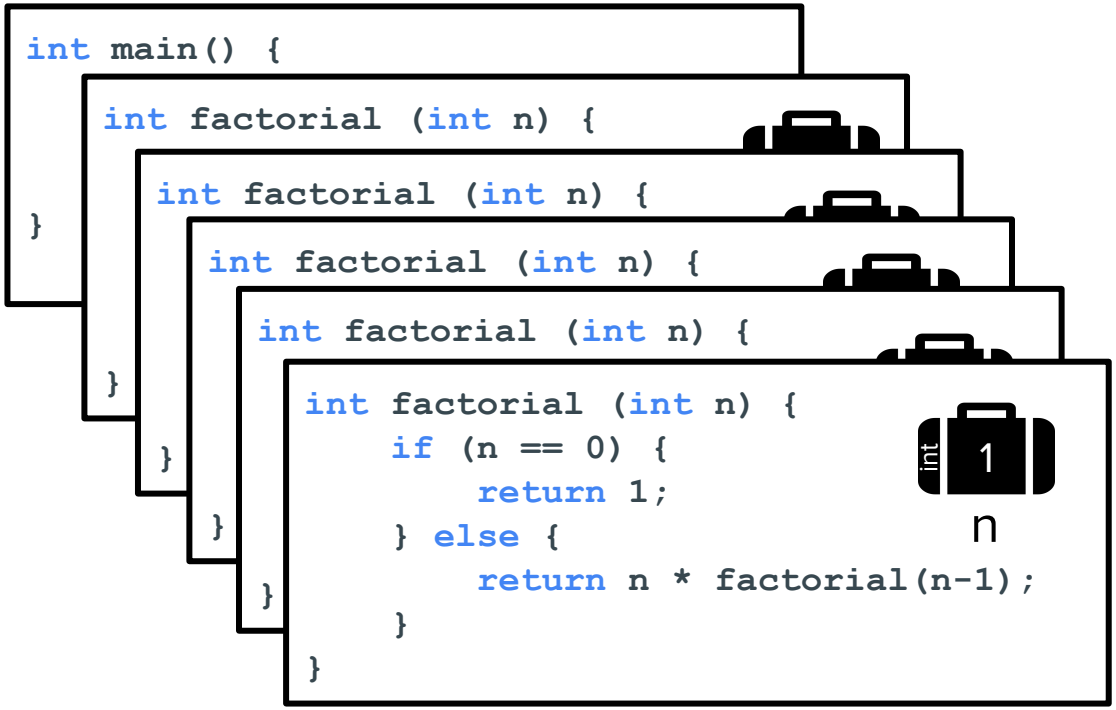

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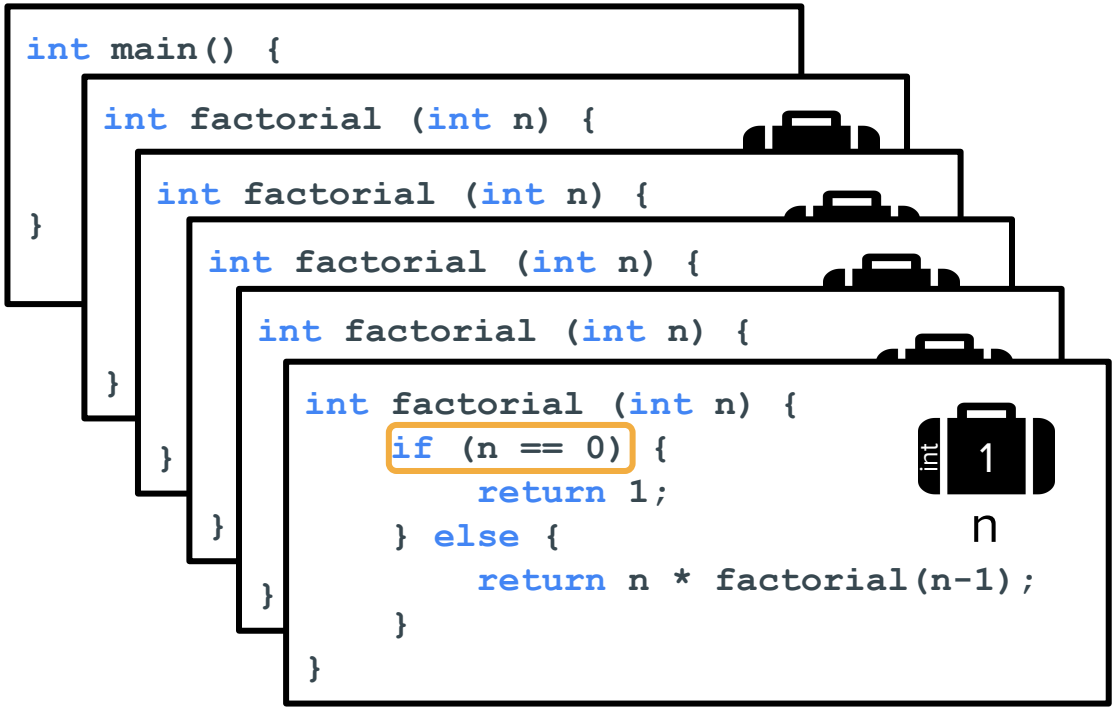

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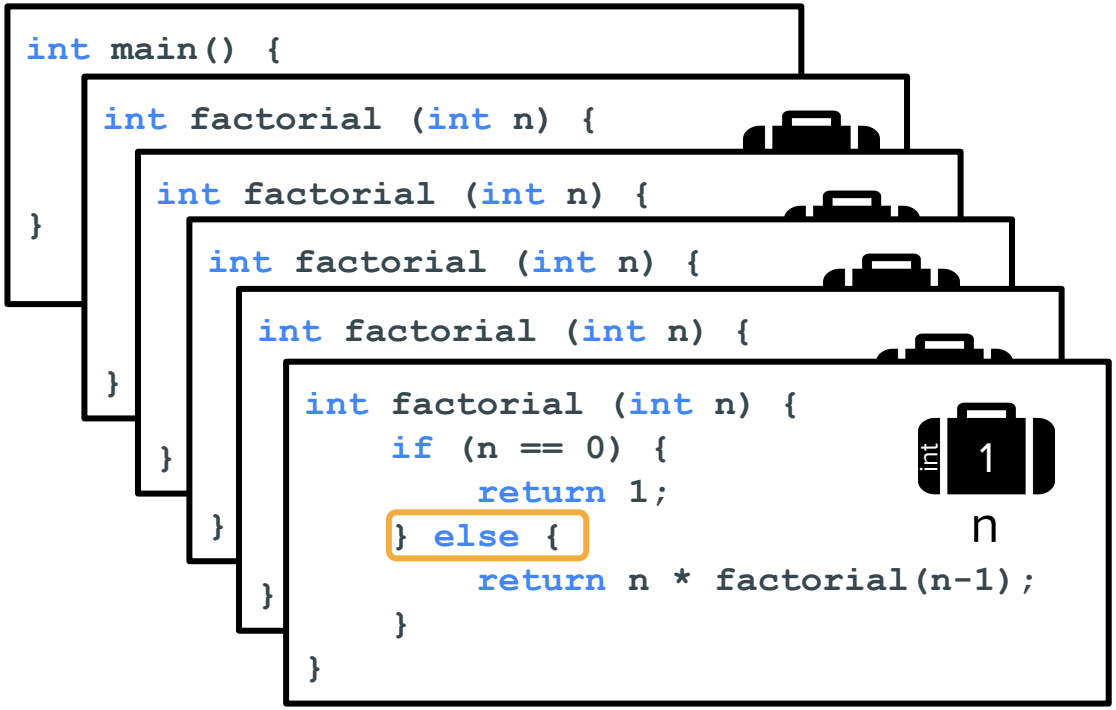

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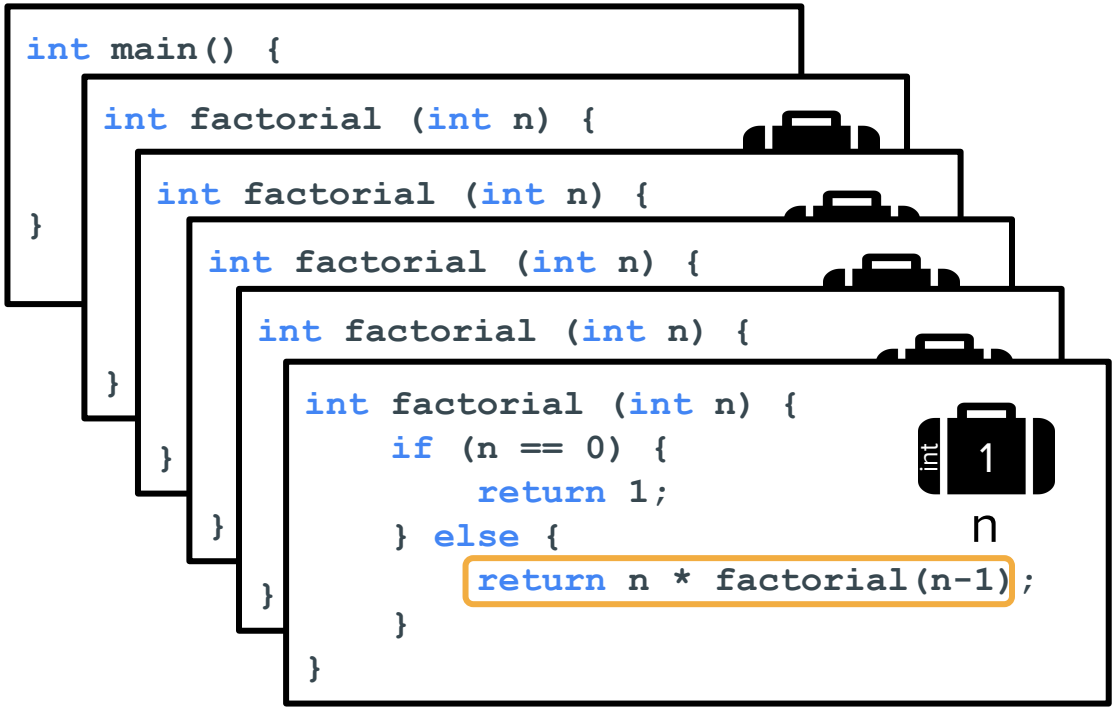

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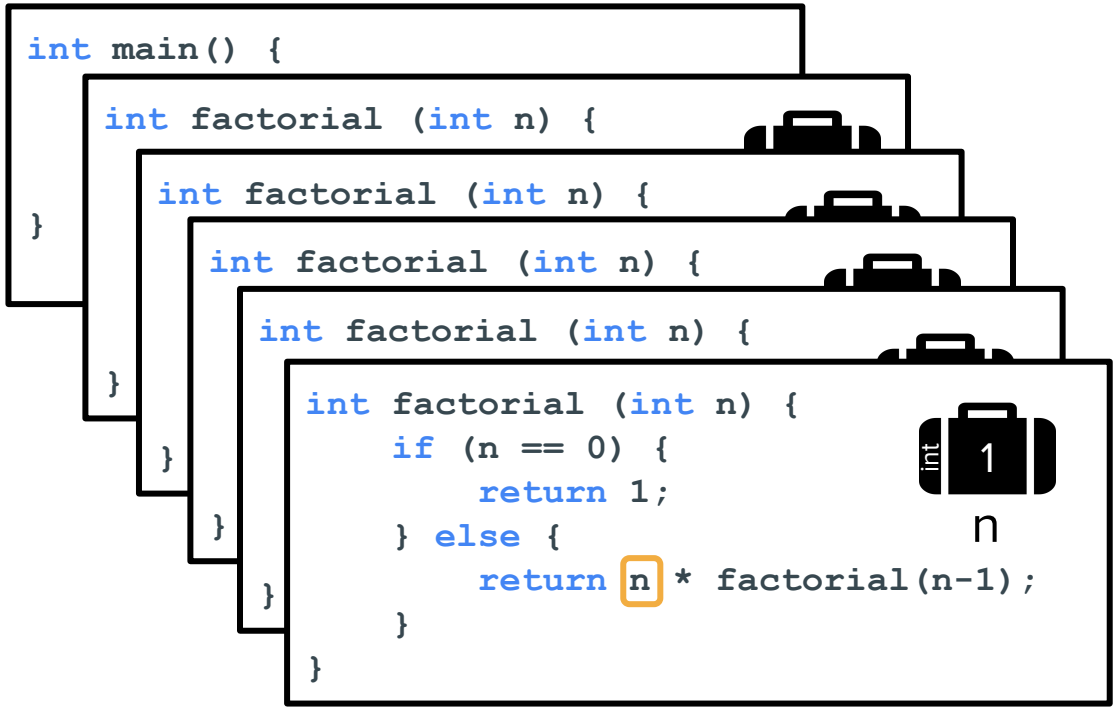

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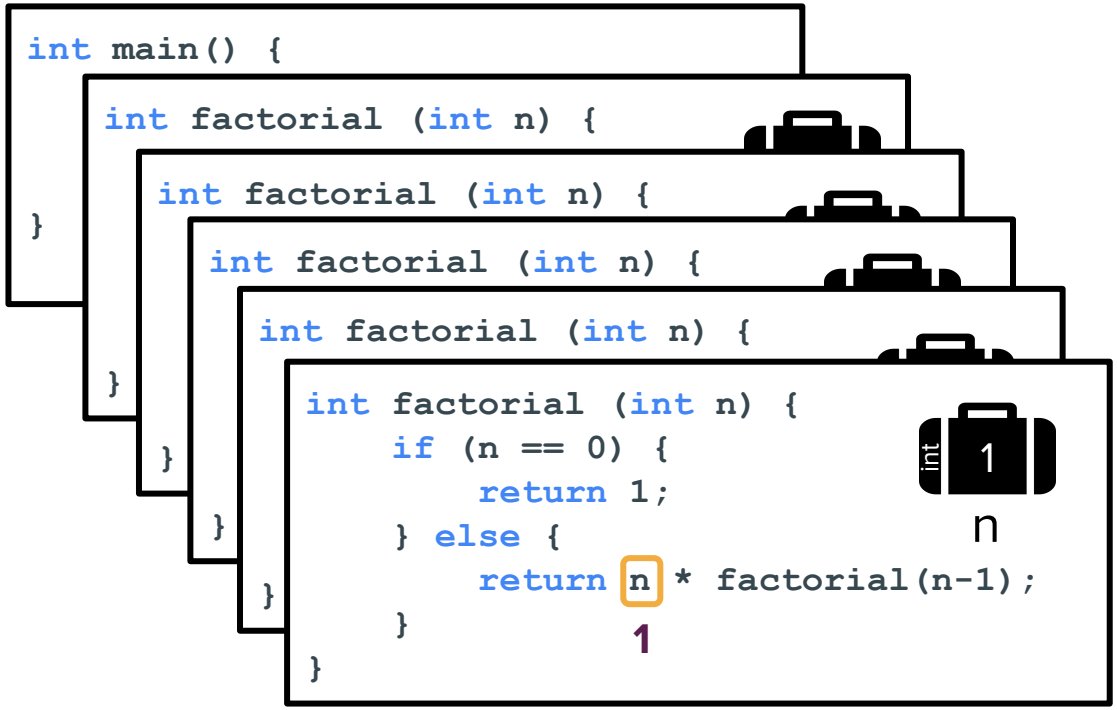

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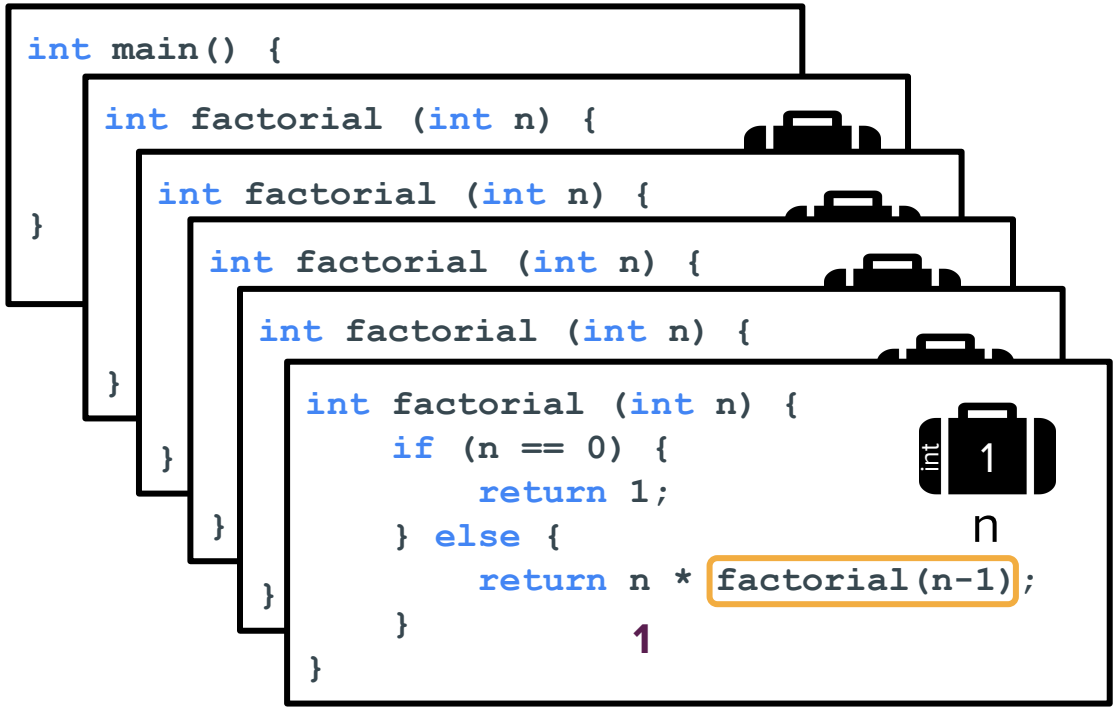

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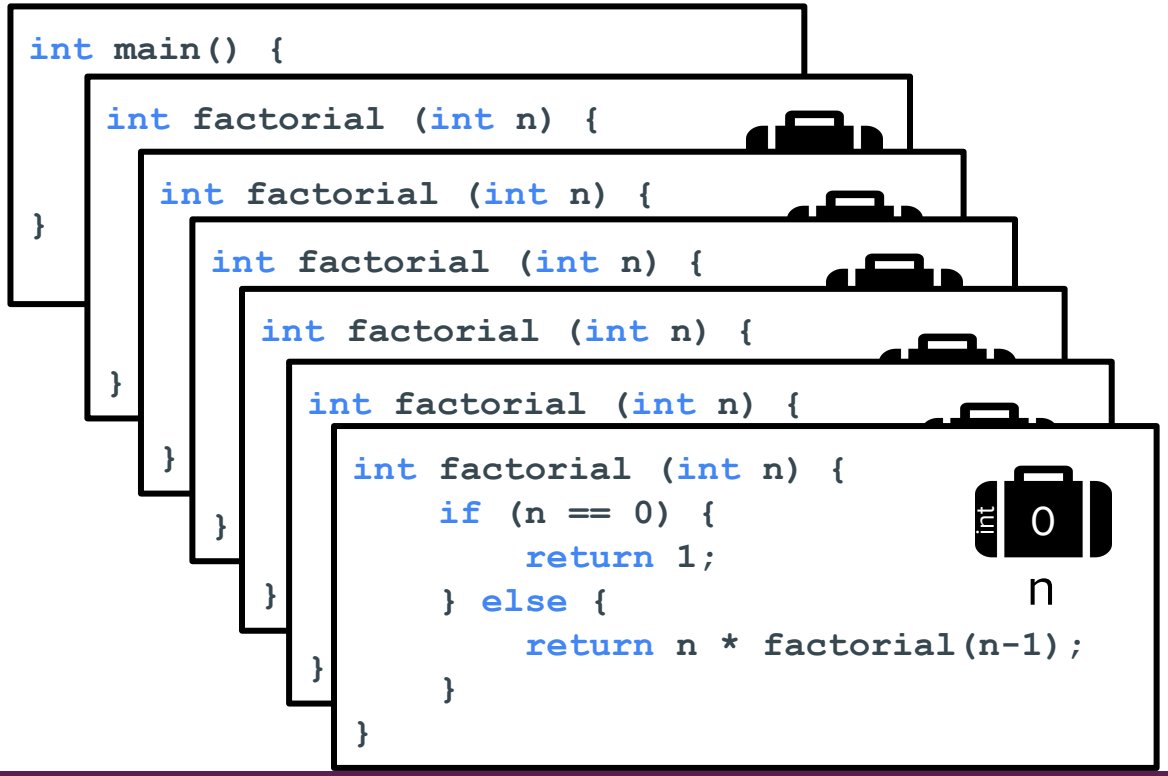

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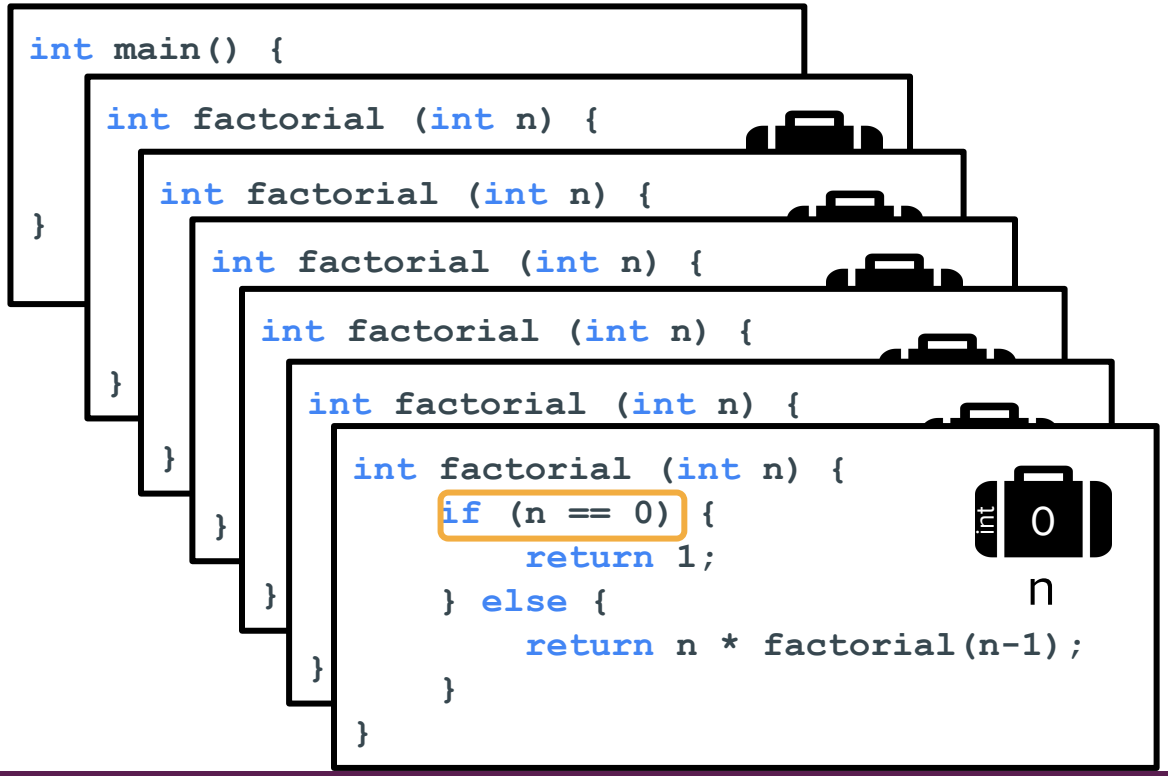

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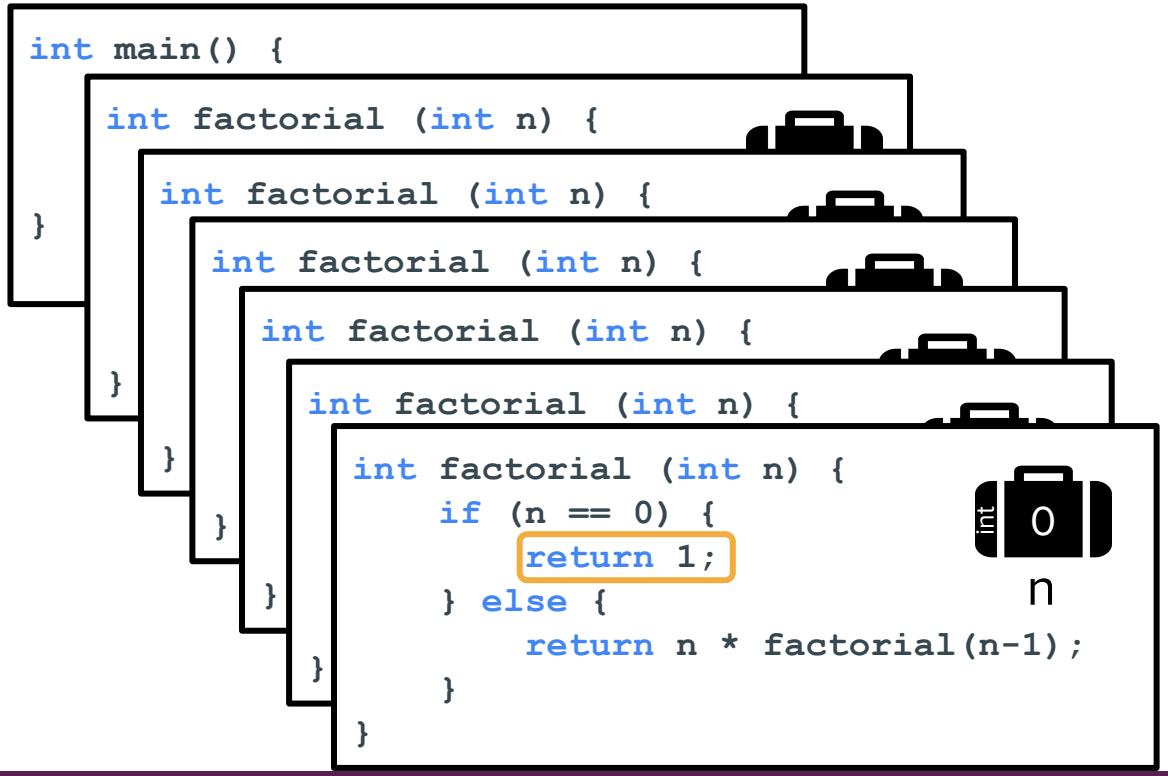

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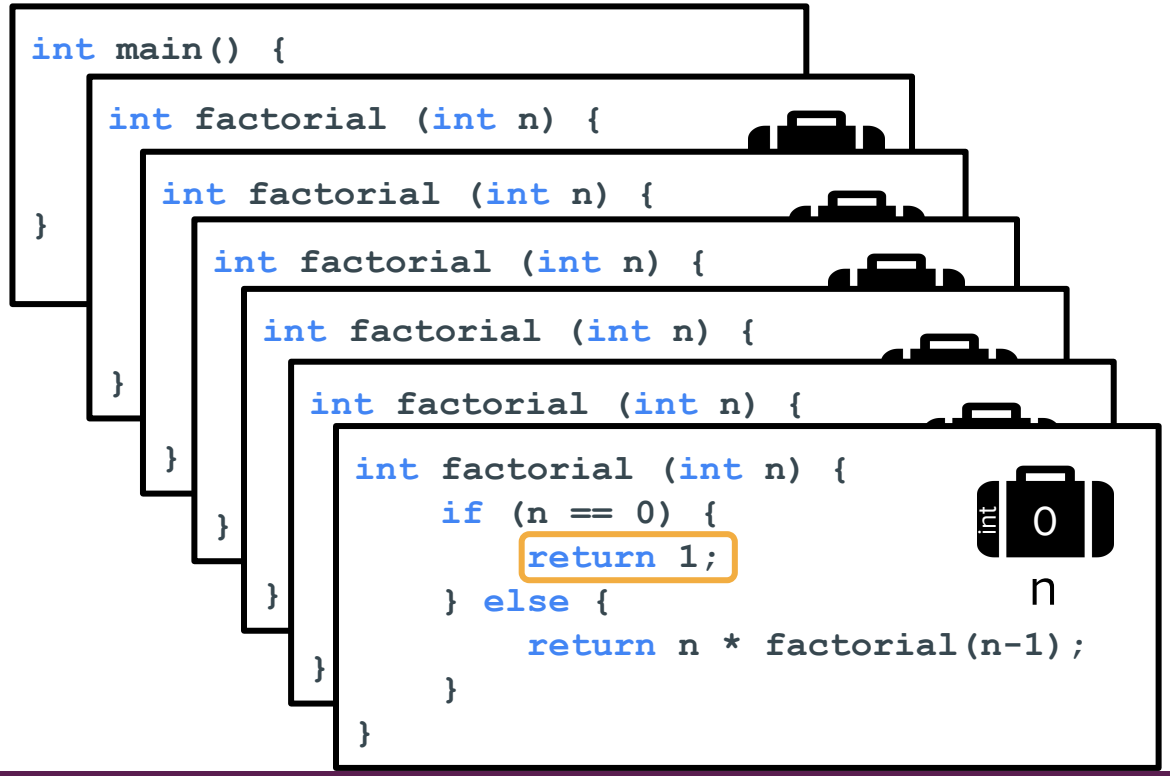
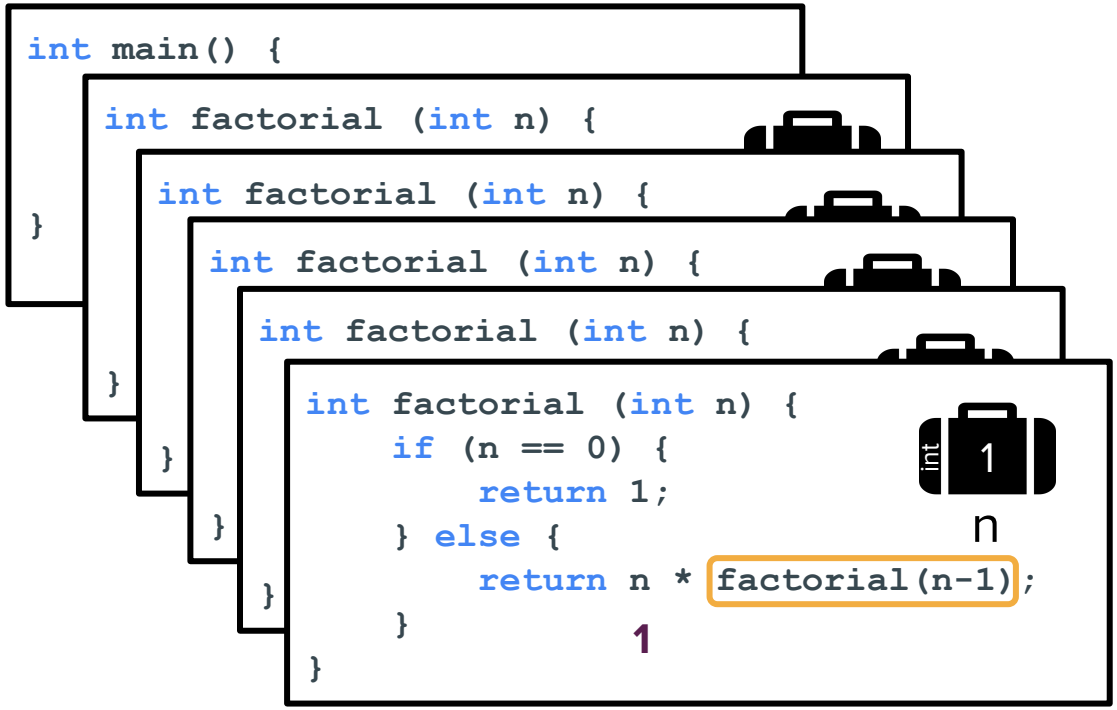


Stack frames go away (get cleared from memory) once they return.

Recursion in action



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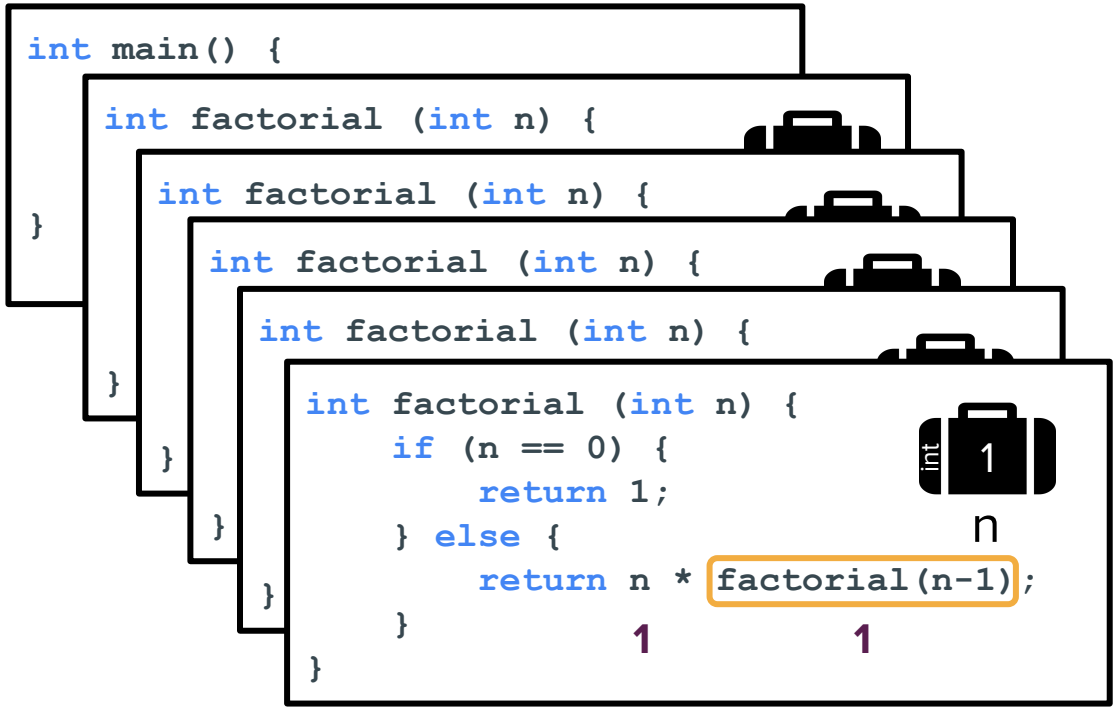

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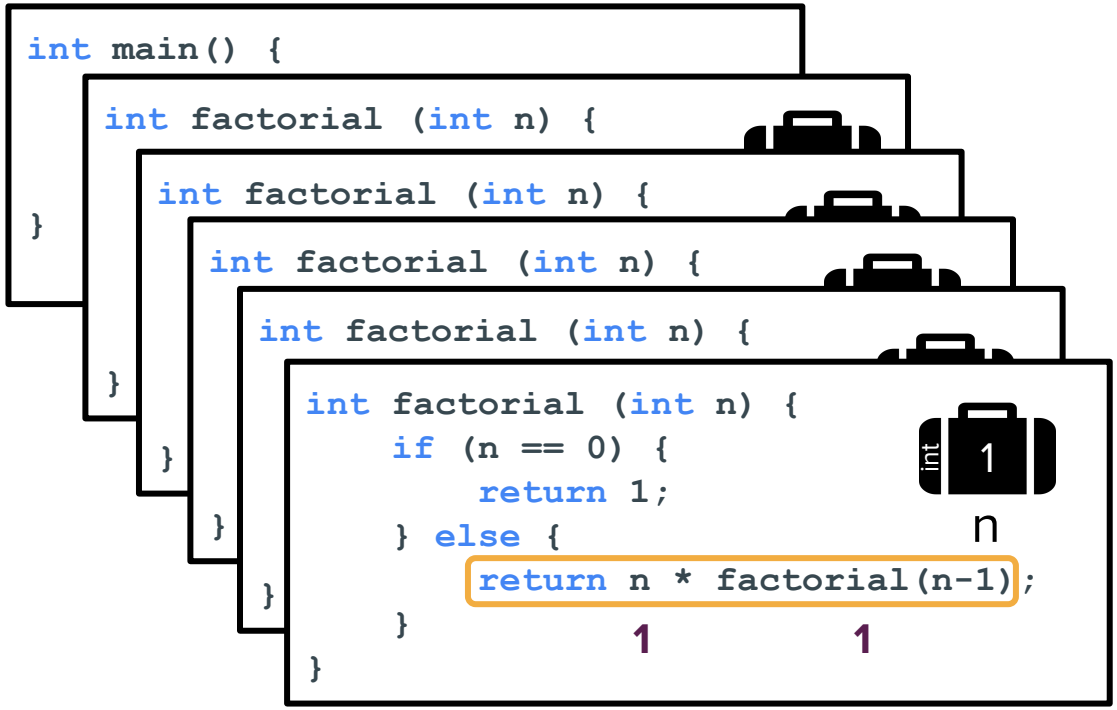

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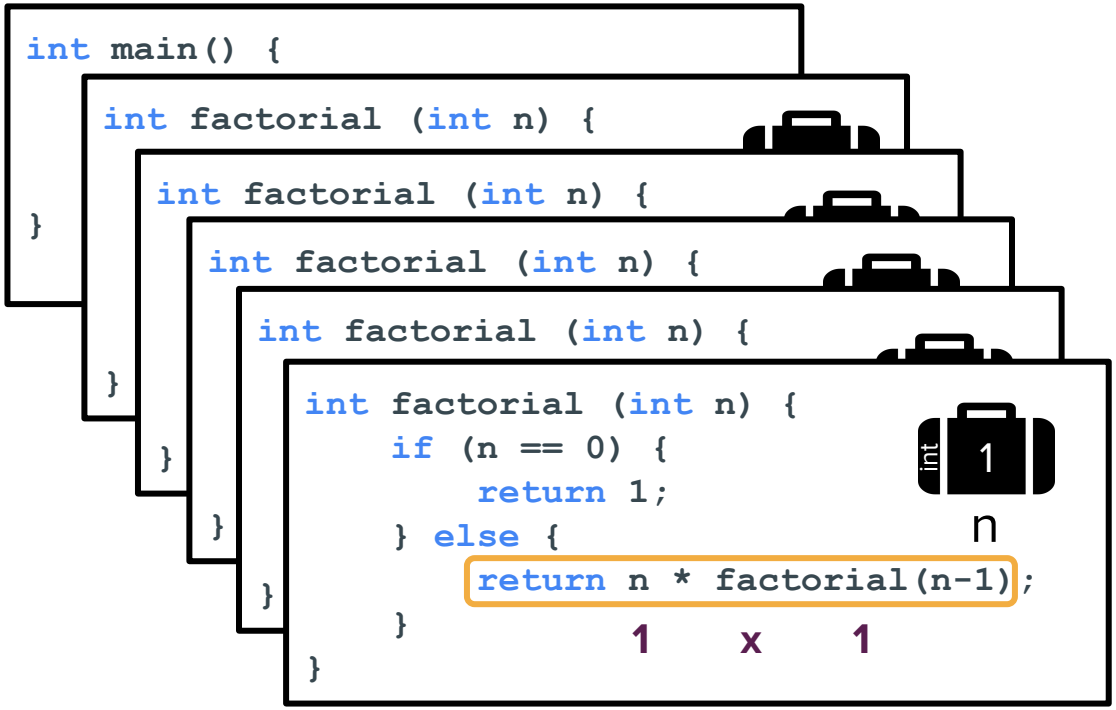

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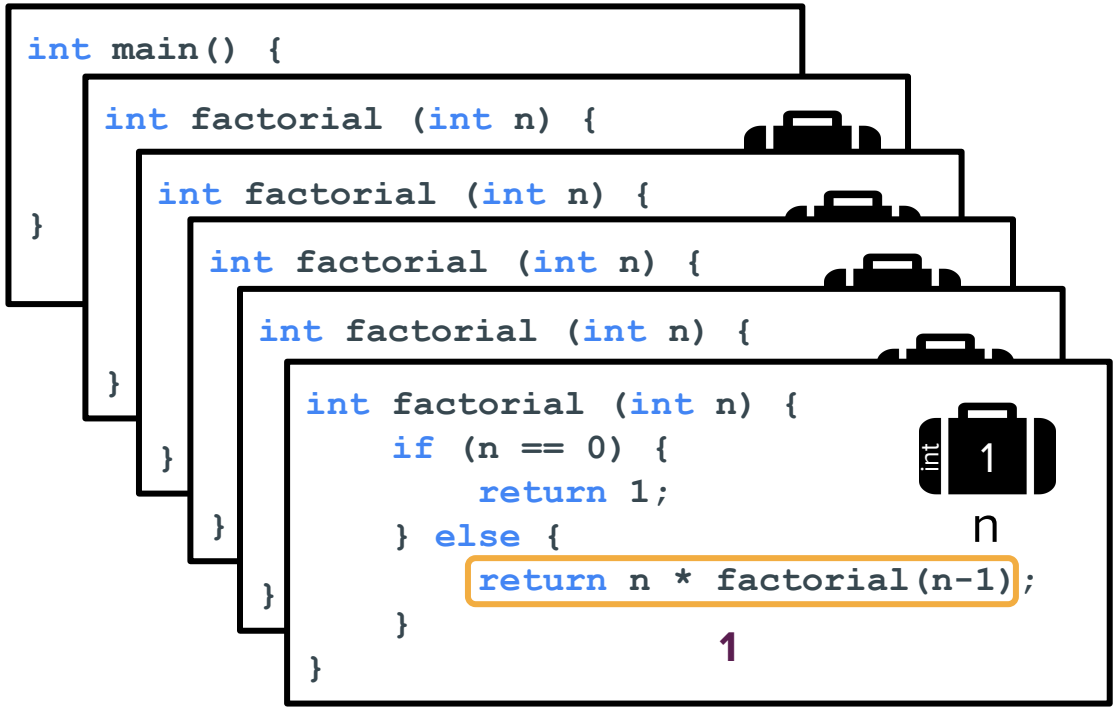

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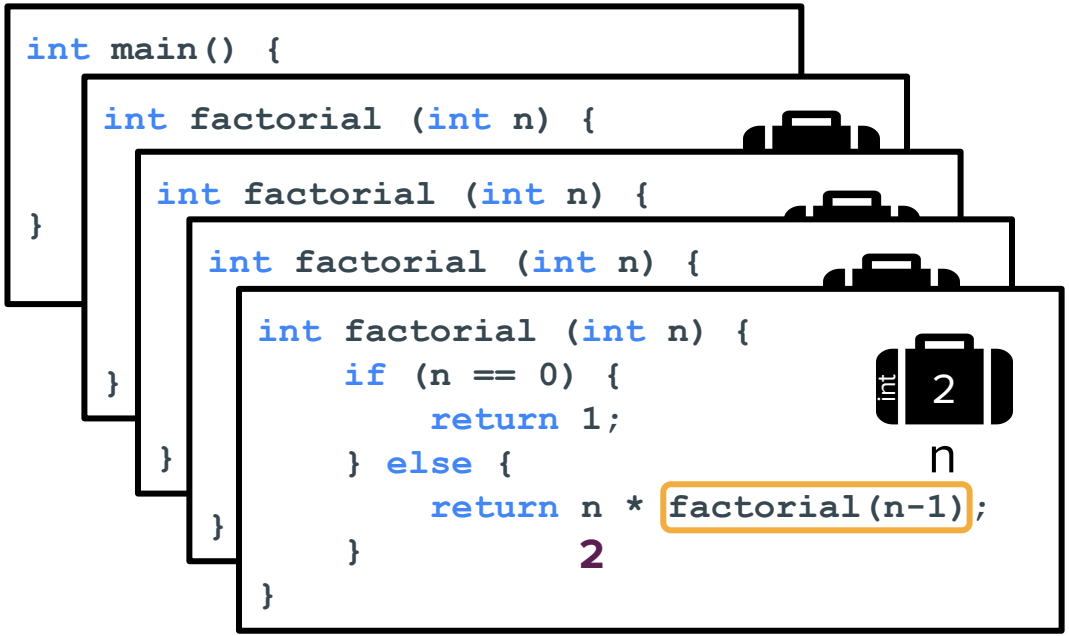

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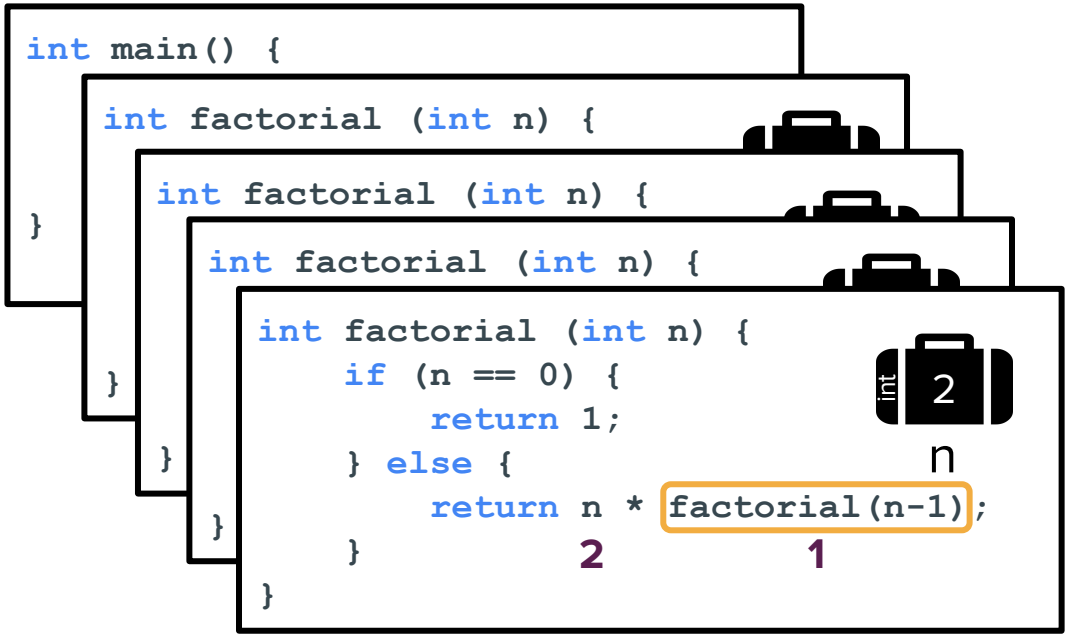

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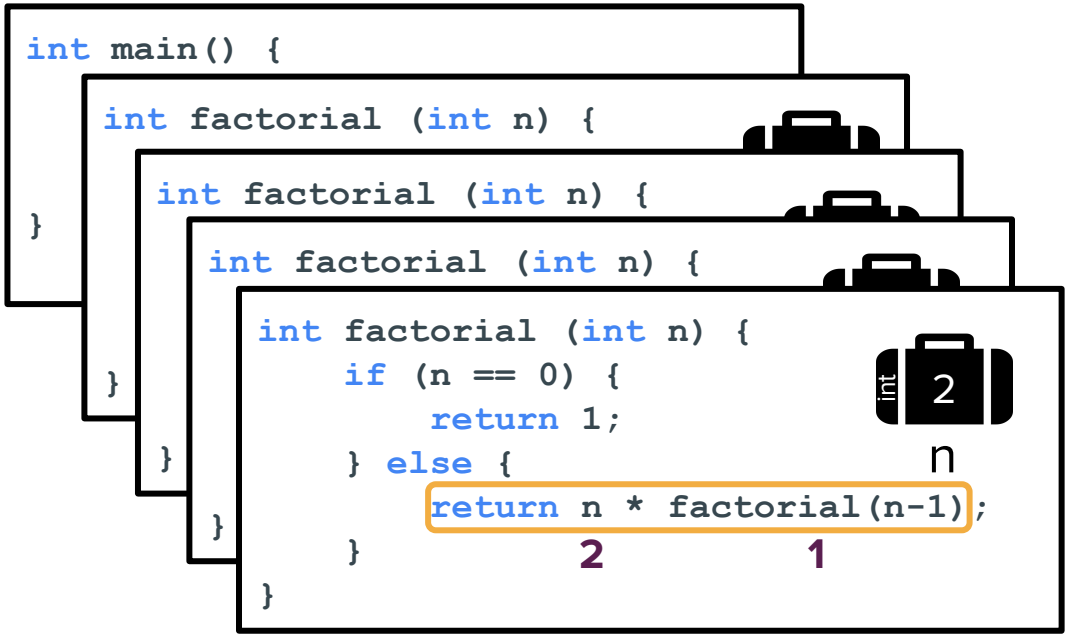

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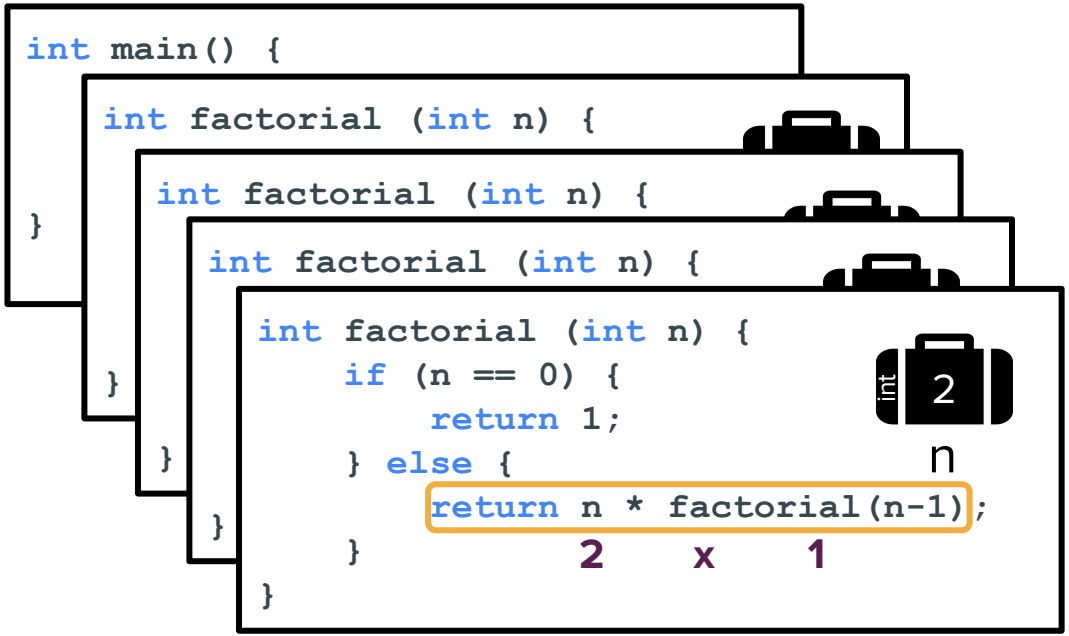

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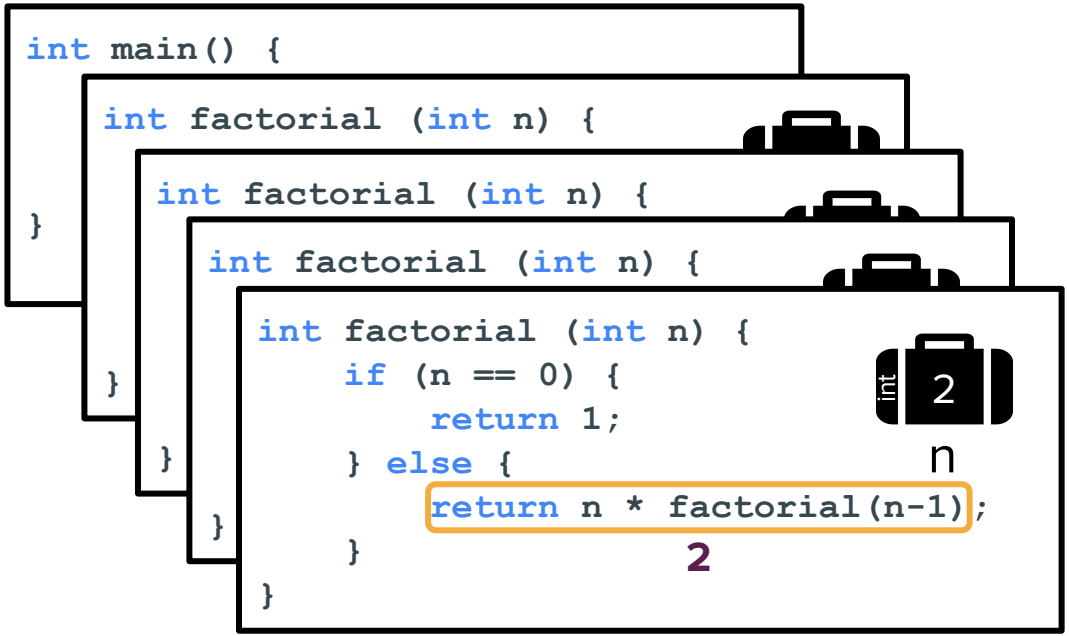

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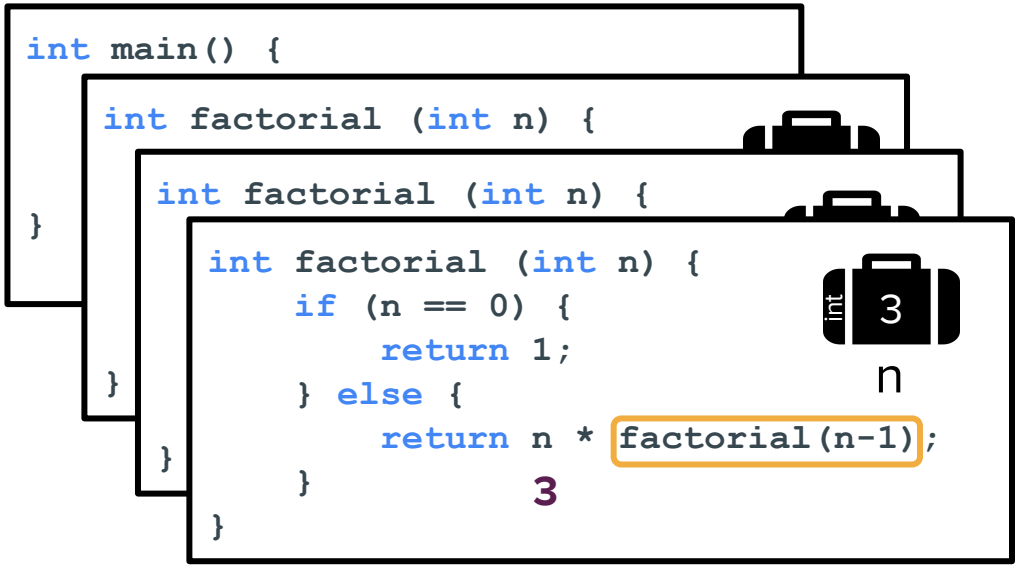

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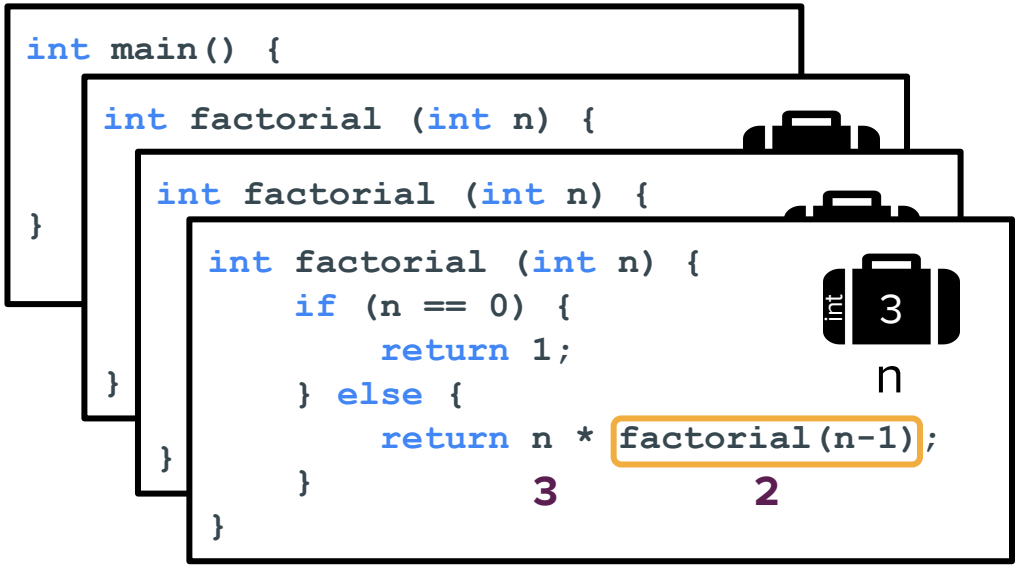

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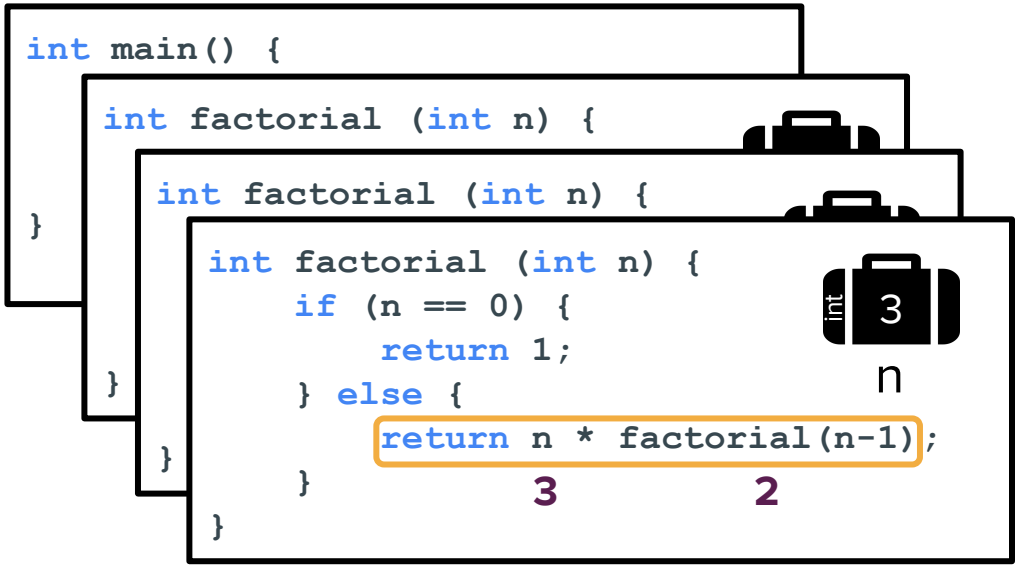
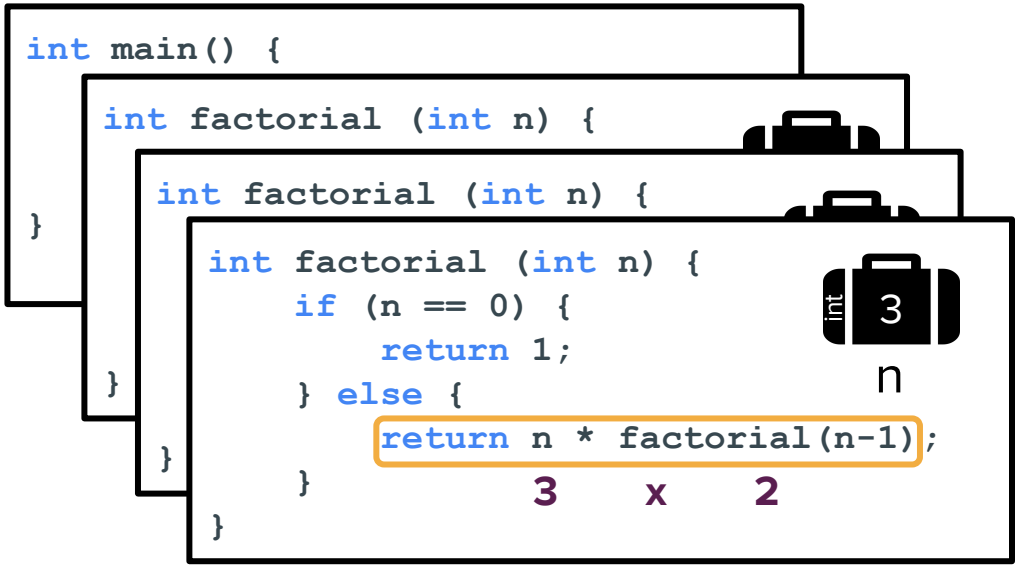
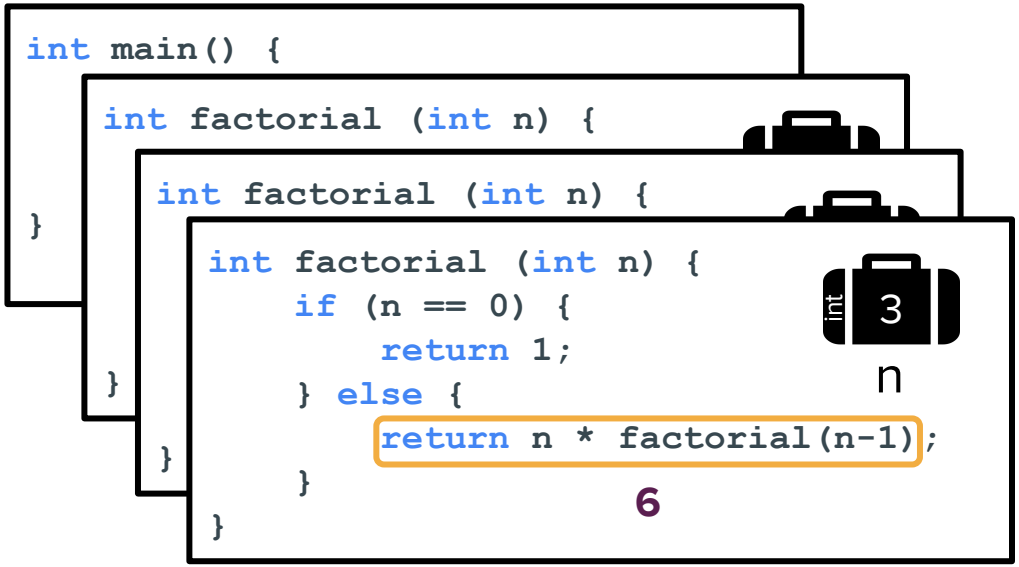


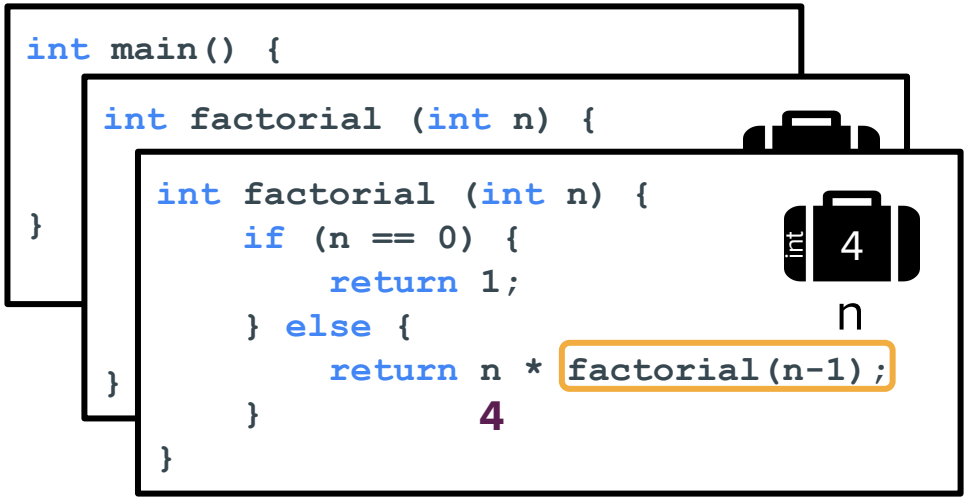
Recursion in action



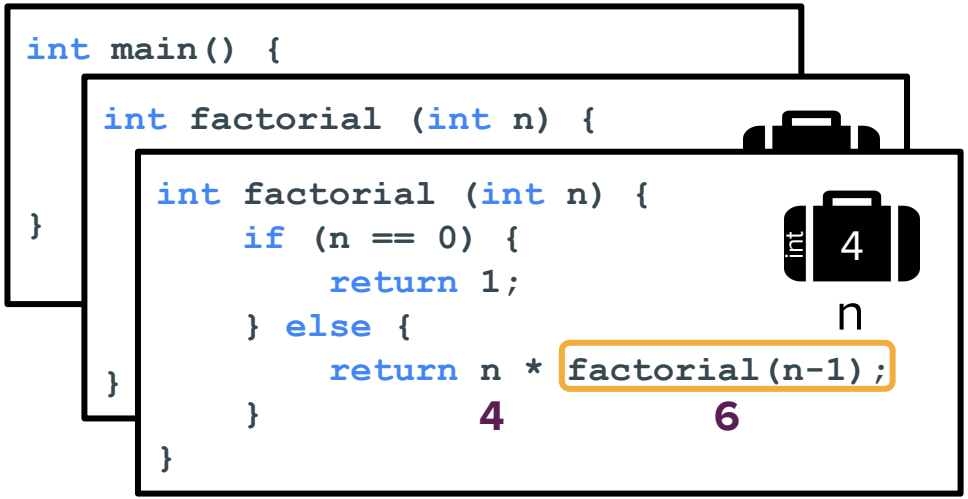
Recursion in action



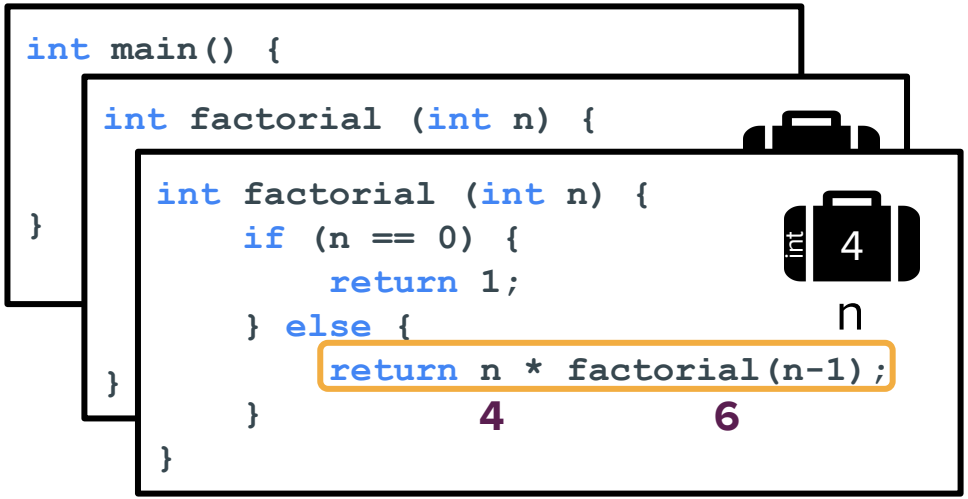
Recursion in action



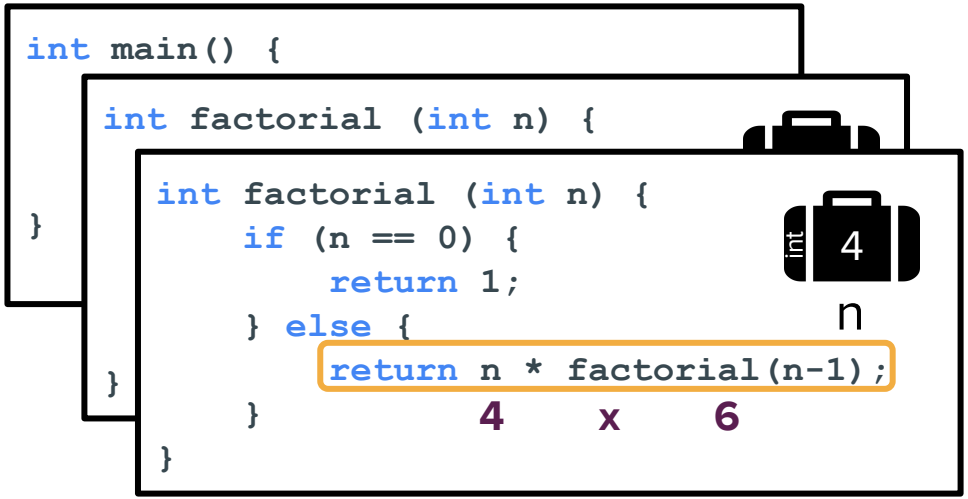
Recursion in action



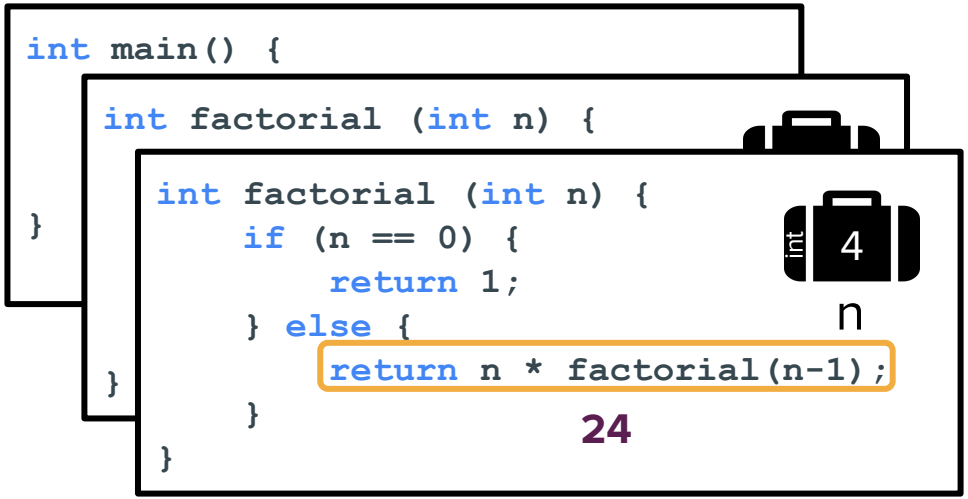
Recursion in action



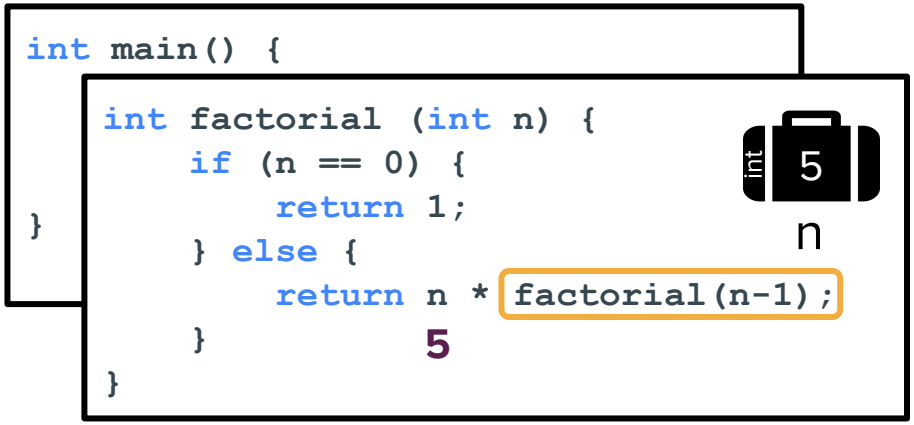
Recursion in action



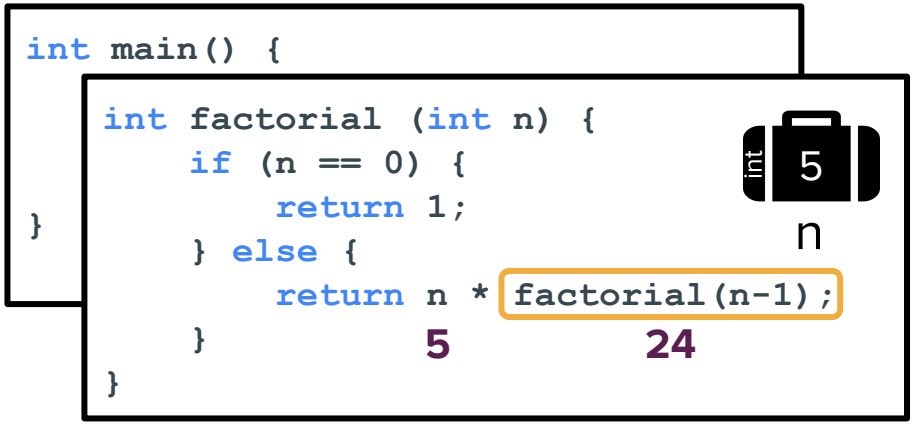
Recursion in action



Recursion in action



Recursion in action



Recursion in action

```
int main() {

}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

5 24

Recursion in action

```
int main() {

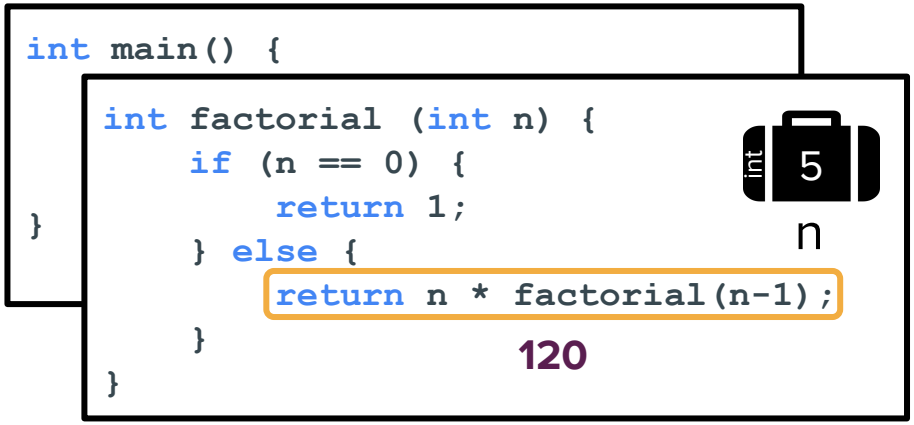
}
```

```
int factorial (int n) {
    if (n == 0) {
        return 1;
    } else {
        return n * factorial(n-1);
    }
}
```

n

5 x 24

Recursion in action



Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

Recursion in action

```
int main() {
    int n = factorial(5);
    cout << "5! = " << n << endl;
    return 0;
}
```

n

Recursive vs. Iterative

[Qt Creator]

Reverse string example

How can we reverse a string?

Suppose we want to reverse strings like in the following examples:

“dog” ➔ “god”

“stressed” ➔ “desserts”

“recursion” ➔ “noisrucer”

“level” ➔ “level”

“a” ➔ “a”

Approaching recursive problems

- Look for self-similarity.

- Try out an example.
  - Work through a simple example and then increase the complexity.
  - Think about what information needs to be “stored” at each step in the recursive case (like the current value of **`n`** in each **`factorial`** stack frame).

- Ask yourself:
  - What is the base case? (What is the simplest case?)
  - What is the recursive case? (What pattern of self-similarity do you see?)

Discuss:

What are the base and recursive cases?

(breakout rooms)

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - What's the first step you would take to reverse "stressed"?

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”

How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - Take the s and put it at the end of the string.
  - Then reverse “tressed”:
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - **Take the s and put it at the end of the string.**
  - **Then reverse “tressed”:**
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - **reverse(“stressed”) = reverse(“tressed”) + ‘s’**
    - Take the t and put it at the end of the string.
    - Then reverse “ressed”:
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - …
          - Take the d and put it at the end of the string.
          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

- Look for self-similarity: **stressed → desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - **Take the t and put it at the end of the string.**
    - **Then reverse “ressed”:**
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      - Then reverse “essed”:
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How can we reverse a string?

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- Look for self-similarity: **stressed → desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - **reverse(“tressed”) = reverse(“ressed”) + ‘t’**
      - Take the r and put it at the end of the string.
      - Then reverse “essed”:
        - ...
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          - **Base case**: reverse “” → get “”

How can we reverse a string?

How can we express the recursive case?

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  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
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How can we reverse a string?

How can we express the recursive case?

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  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - reverse(“tressed”) = reverse(“ressed”) + ‘t’
      - **reverse(“ressed”) = reverse(“essed”) + ‘r’**
        - …
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How can we reverse a string?

- Look for self-similarity: **stressed → desserts**
  - reverse(“stressed”) = reverse(“tressed”) + ‘s’
    - reverse(“tressed”) = reverse(“ressed”) + ‘t’
      - reverse(“ressed”) = reverse(“essed”) + ‘r’
        - …
          - **Base case**: reverse(“”) = “”

How can we reverse a string?

- **Recursive case:** reverse(str) = reverse(str without first letter) + first letter of str
- **Base case**: reverse(“”) = “”

How can we reverse a string?

- **Recursive case:** reverse(str) = reverse(str without first letter) + first letter of str
- **Base case**: reverse(“”) = “”

Depending on how you thought of the problem, you may have also come up with:

- **Recursive case:** reverse(str) = last letter of str + reverse(str without last letter)
- **Base case**: reverse(“”) = “”

Let’s code it!

(live coding)

Summary

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
  - A recursive operation (function) is defined in terms of itself (i.e. it calls itself).

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.
  - Base case: Simplest form of the problem that has a direct answer.
  - Recursive case: The step where you break the problem into a smaller, self-similar task.

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.

- Recursion has two main parts: the **base case** and the **recursive case**.

- The solution will get built up **as you come back up the call stack**.
  - The base case will define the “base” of the solution you’re building up.
  - Each previous recursive call contributes a little bit to the final solution.
  - The initial call to your recursive function is what will return the completely constructed answer.

Summary

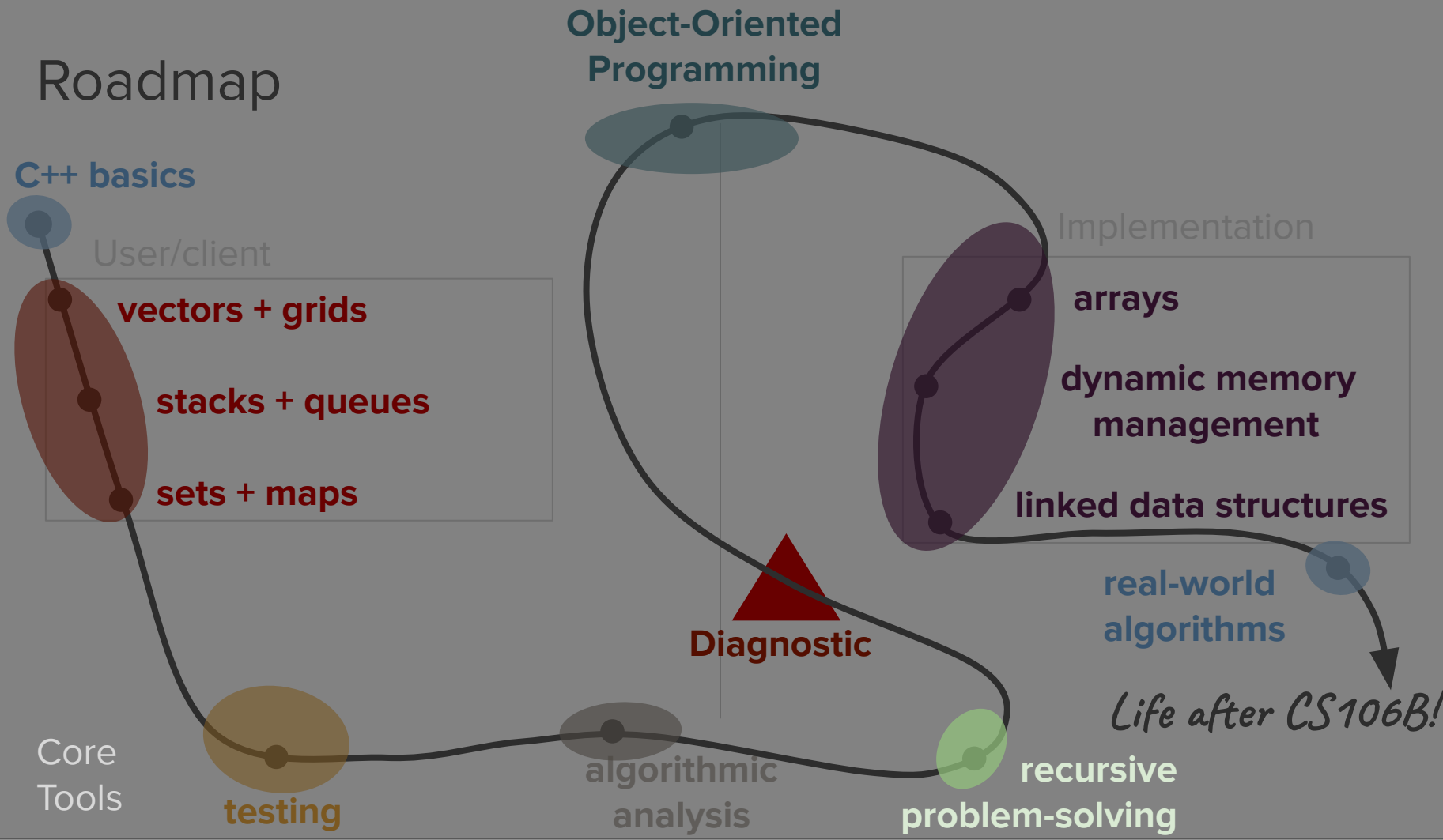
- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
- Recursion has two main parts: the **base case** and the **recursive case**.
- The solution will get built up **as you come back up the call stack**.
- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

Summary

- Recursion is a problem-solving technique in which tasks are completed by reducing them into **repeated, smaller tasks of the same form**.
- Recursion has two main parts: the **base case** and the **recursive case**.
- The solution will get built up **as you come back up the call stack**.
- When solving problems recursively, look for **self-similarity** and think about **what information is getting stored in each stack frame**.

What’s next?

Roadmap


Object-Oriented Programming
C++ basics
User/client
vectors + grids
stacks + queues
sets + maps
Implementation
arrays
dynamic memory management
linked data structures
real-world algorithms
Diagnostic
Life after CS106B!
Core Tools
testing
algorithmic analysis
recursive problem-solving

Fractals