Nuclear Financial Economics[1]

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[1] Forthcoming in *Risk Management: Challenges and Solutions*, McGraw-Hill, 1994. This paper is based on notes prepared in 1993 by the author for the Stanford Graduate School of Business course in Portfolio Management. The material presented here is also intended to be incorporated in the author's work-in-progress, tentatively titled *Financial Economics and the Management of Investments.* The comments and suggestions of Stanford professors William Beaver, Ayman Hindy, and Paul Pfleiderer are gratefully acknowledged.

Introduction

An important subfield of physics -- *Nuclear Physics* -- deals with the smallest particles of which matter is composed. Constructs developed by Kenneth Arrow[2] and Gerard Debreu[3] provide a similar foundation for financial economics. With a bit of hyperbole, the approach may be termed *nuclear financial economics.*

In their pioneering work, Arrow and Debreu explored aspects of general equilibrium. After dealing with the economists' then-traditional world of certainty, they turned to issues related to *risk.*

Here we show how the Arrow-Debreu approach can be used to analyze risk in the domains of financial engineering, corporate finance, and investment analysis. The simplest possible cases, involving only two time periods (the present and a future date) are employed. While the results are quite general, no attempt is made to prove generality nor to extend the analyses to cover more complex cases. The relatively modest goal of the paper is to whet the appetite of the reader to consider the use of "nuclear financial economics" when analyzing issues involving risk in financial settings.

Contingent Contracts

To deal with risk, Arrow and Debreu introduced the concept of a *contingent contract* -- a contract "for delivery of goods or money contingent on the occurrence of [a] state of affairs"[4]; equivalently: a "contract for the transfer of a commodity [specifying], in addition to its physical properties, its location and date, an event on the occurrence of which the transfer is conditional."[5]

A special kind of contingent contract provides one unit of money or some notional commodity at one date if and only if one of the many possible states of the world at that date obtains. This may be termed a "pure" or "primitive" security, or simply a *time-state claim.* A *complete market* is one in which all such claims may be purchased or sold explicitly or synthetically at stated prices.

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[2]Kenneth J. Arrow, "The Role of Securities in the Optimal Allocation of Risk-bearing," *The Review of Economic Studies*, April 1964, pp. 91-96.

[3]Gerard Debreu, *Theory of Value*, The Cowles Foundation Monograph 17, 1959.

[4]Kenneth J. Arrow in William Breit and Roger W. Spencer, "*Lives of the Laureates: Ten Nobel Economists,* Second Edition, The MIT Press, Cambridge, Mass., 1990, p. 55.

[5]Debreu, *Theory of Value, op.cit.*.

The *price* of a contingent contract or time-state claim is ".. the amount paid.. initially by.. the agent who commits himself to accept.. delivery of one unit of that commodity. Payment is irrevocably made although delivery does not take place if specified events do not obtain."[6]

The concept of a contingent contract allows for the analysis of risk in simple, yet powerful ways, as we will show.

Financial Engineering

The International Association of Financial Engineers defines *financial engineering* as ".. the development and the creative application of financial technology to solve problems in finance and to exploit financial opportunities."[7]

The Arrow-Debreu approach may seem too far removed from the real world to be used for such purposes. There are hundreds of millions of contingencies for which no specific contingent contract exists, and it is difficult to imagine a way to synthesize most such contracts from securities that do exist. Nonetheless, the concept is in fact widely used in the practice of financial engineering under the assumption that markets are sufficiently complete for the states of the world that affect the securities being analyzed.

Those who employ so-called *binomial models of asset returns*[8] utilize contingent claims[9] explicitly. Those who employ continuous-time models, such as that of Black and Scholes,[10] do so implicitly. Curiously, however, many practitioners of financial engineering are unaware of the intellectual underpinnings of the methods that they employ.

Financial engineering is by no means the only area in which the Arrow-Debreu approach can be employed. With it one can address fundamental issues of corporate finance and investment analysis. In particular, it allows a very clear distinction between models that require only the assumption that markets are arbitrage-free and those that require additional assumptions about investor preferences and predictions.

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[6]Debreu, *Theory of Value, op. cit.*.

[7] *The Journal of Financial Engineering*, Volume 1, Number 1, June 1992.

[8] This concept was introduced in William F. Sharpe, *Investments*, First Edition, Prentice-Hall, Englewood Cliff, N.J., 1978, pp. 366-371, then expanded upon in John C. Cox, S.A. Ross and M. Rubenstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics*, September 1979, pp. 229-263.

[9] Some use the term *time-state claim* instead of *contingent claim*.

[10] Fischer Black and M. Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, May-June 1973, pp. 141-183.

In the field of investment analysis, both academics and practitioners tend to favor the *mean-variance* approach introduced by Markowitz[11] over that of Arrow and Debreu. While there are good reasons for doing so, it seems unfortunate that the latter is often entirely omitted from investment courses. A preferred strategy utilizes the Arrow-Debreu model to develop fundamental ideas, then turns to mean-variance analysis and its use in practice.

The Economy

To illustrate the power of the Arrow-Debreu approach, we consider an extremely simple economy in which there is no currency. Apples are the only good and all trades involve apples. There are two time periods - *now* and *next year*. The only source of risk is the uncertainty about next year's weather. More precisely, there are two possible states of the world concerning next year: *good weather* and *bad weather*.

There are three distinct good available for trades at present: (1) Apples now, (2) Apples next year if the weather is good, and (3) Apples next year if the weather is bad

Note that the latter two are contingent contracts. For simplicity, we call these present apples, good-weather apples, and bad-weather apples.

In this economy, *dealers* stand ready to trade any one of the three goods for any other. Competition is so fierce that bid and ask prices are the same.

Initially, we assume that dealers will *swap* 0.4 present apples for 1.0 good apple. More precisely, if an individual will give up 0.4 apples today, he or she will receive a certificate promising delivery of 1.0 apple next year if (but only if) the weather during the coming year has been good; if the weather has been bad, no apples will be delivered next year.

Dealers will also swap 0.5 present apples for 1.0 bad apple. More precisely, if an individual will give up 0.5 apples today, he or she will receive a certificate promising delivery of 1.0 apple next year if (but only if) the weather during the coming year has been bad; if the weather has been good, no apples will be delivered next year.

Following convention, we call the terms of trade for swaps involving future and present goods *prices*. Thus the price of a good-weather apple is 0.40 (present apples) and the price of a bad-weather apple is 0.50 (present apples). Equivalently, the *present values* of such contingent claims are 0.40 and 0.50, respectively. Note that the term *present value* is simply a convention for stating terms at which market transactions can be made to convert claims for future payments to current payments. Absent competitive markets in which such swaps can be made, the concept has no particular usefulness.

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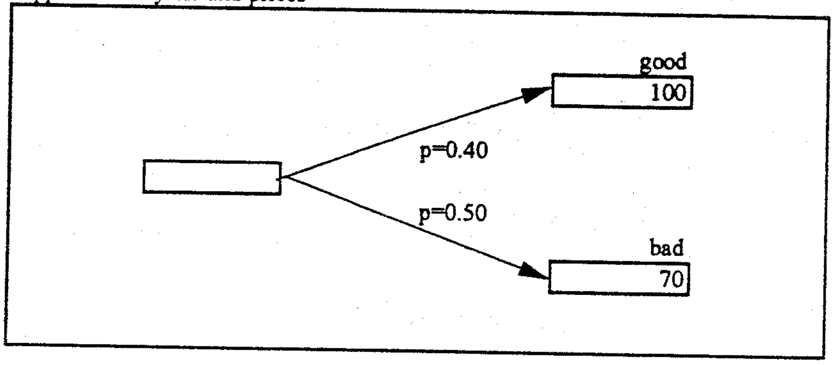
11 Harry M. Markowitz, "Portfolio Selection," *Journal of Finance*, March 1952, pp. 77-91.

Production in this economy comes from apple trees, each of which will produce 100 apples next year if the weather is good and 70 apples if the weather is bad. This is illustrated (naturally) via a *tree diagram* in Figure 1. For convenience, prices are also shown.

Figure 1
Apple Tree Payoffs and prices



Arbitrage-free Pricing

The two prices in Figure 1 show the terms of trade for swaps in which a present good is exchanged for a contingent future good. Not shown are the terms of trade for a swap in which one contingent future good is exchanged for another. However, *arbitrage* will insure that such trades are made using terms consistent with those already shown. Consider someone who wishes to trade 1.0 good-weather apples for some number of bad-weather apples. One way to do this is to trade 1.0 good-weather apple for 0.40 present apples while at the same time trading 0.40 present apples for 0.80 bad-weather apples. The net result is to *swap* 1.0 good-weather apples for 0.80 bad-weather apples. Such a swap has zero *net present value* and thus can be said to be "fair." If any dealer were willing to swap good-weather and bad-weather apples at any other terms of trade, an astute individual could exploit the dealers by engaging in combinations of trades that could provide (1) net receipt of apples in at least one time and state and (2) no net payout of apples at any time and state. Such an opportunity, termed an *arbitrage*, is rare in a well-functioning competitive capital market.

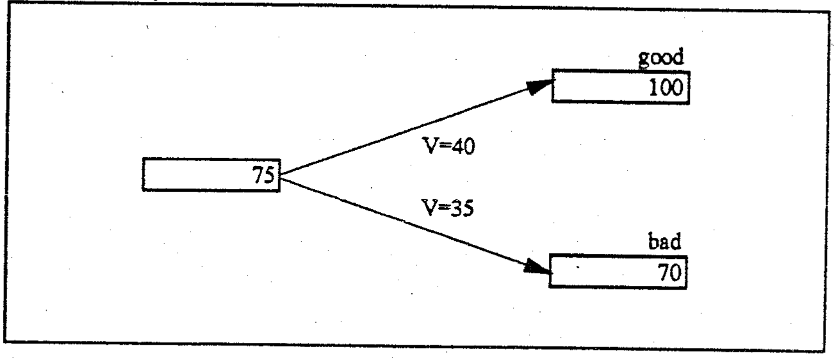
In an arbitrage-free economy with no transactions costs, any given time-state claim will sell for the same price, no matter how obtained. This will also be true for any "package" of time-state claims. This is known as the *law of one price*.

Most transactions involve complex combinations of present and future time-state claims. All are *swaps* in a broad sense. Those involving both present and future payments are often termed *investments*, with the term "swap" reserved for those that involve only future contingent payments. However, such terminology obscures the fact that all reflect trades of one set of valuable claims for another.

Valuation

How much is an apple tree worth? The answer is easily obtained by *valuing* each of the contingent claims that it provides. The 100 good-weather apples can be traded for 40 (0.40*100) present apples and the 70 bad-weather apples for 35 (0.50*70) present apples. Thus the (present) value of the tree is 75 (present) apples, as shown in Figure 2.

Figure 2
Apple Tree Payoffs and Values



Note that valuation is designed to answer the question: "For how much in present goods (or money) can the asset in question be traded -- either directly or indirectly." *In principle, all standard valuation methods rest on the use of market prices to answer such questions.*

Financing

Consider the apple-tree owner who issues two claims. The first promises the holder 60 apples next year. Since the amount promised is fixed, the associated *security* can be termed a *bond*. The other claim promise the holder all the apples left over after the bondholder has been paid. The associated security represents a *residual claim* and can be termed a (common) *stock*.

Table 1 provides the computations required to determine the value of the bond. Note that it is *riskless*, since the promised amount can be paid no matter what the state of the world (weather) may be. An equivalent method for valuing a riskless security *discounts* the promised payment using the *riskless discount factor*. For a riskless security, the factor is simply the sum of the prices of all the time-state claims for the date in question, for this is the cost of obtaining one unit at that date, no matter what the state of the world. In this economy, the discount factor is 0.90 (0.40+0.50). This is sometimes stated in terms of a *rate of interest*. In this case the rate of interest is equal to 0.10/0.90, or 11.11%, since 0.90 today can be converted to 1.00 next year (and vice-versa) with certainty.

Table 1

Time	State	Payment	Price	Present Value
1	good	60	0.40	24
1	bad	60	0.50	30
				54

Whichever method is employed, the result is the same. The value of the bond is 54 present apples.

Table 2 provides the computations required to determine the value of the stock. It is *worth* 21 present apples since the same set of contingent payments could be obtained with 21 apples by judiciously making swaps with dealers.

Table 2

Time	State	Payment	Price	Present Value
1	good	40	0.40	16
1	bad	10	0.50	5
				21

Note that the value of the bond plus the value of the stock is 75 present apples. Not surprisingly, this is the value computed earlier for the tree as a whole.

What if the tree had been financed by issuing "risky debt?" Consider a case in which the bondholder has been promised 80 apples next year. In fact, he or she will receive 80 apples if the weather is good, but only 70 if the weather is bad. Clearly the bond cannot be valued by discounting the promised payment. Table 3 shows how this should be done. Note that the actual value (67 present apples) is considerably less than the value that the bond would command if the promised payments were guaranteed (0.90*80, or 72 present apples).

Table 3

Time	State	Payment	Price	Present Value
1	good	80	0.40	32
1	bad	70	0.50	35
				67

With this sort of financing (high leverage), the stock is risky indeed -- it will pay 20 apples if the weather is good but nothing if the weather is bad (i.e. the firm will be bankrupt). Such a stock will be worth 8 apples, as shown in Table 4.

Table 4

Time	State	Payment	Price	Present Value
1	good	20	0.40	8
1	bad	0	0.50	0
				8

Note that the sum of the values remains the same as before (67+8=75). This reflects the *principle of value additivity*. In a complete market with no transactions costs, the value of the sum of the claims will be the same, no matter how those claims are structured. In this setting, "financing doesn't matter." This is the essence of the famous Modigliani-Miller Proposition Number I[12].

In the real world, of course, transactions costs (broadly construed) do matter. Nonetheless, the principle of value additivity remains useful if a broad enough view is taken of the set of claimants on a firm's prospects. In particular, governments (which impose taxes on firms and on those who receive income from firms) must be included, as must lawyers, accountants, investment bankers and others who may be more likely to absorb some of a firm's cash flows under some financial arrangements than under others.

In a sense, the questions to be asked concerning alternative financing procedures have more to do with the division of the pie than with its overall size. The latter tends to be determined more by non-financial decisions than by financial ones. However, there are cases in which financial decisions can significantly alter incentives for those charged with "corporate governance" to maximize the size of the pie.

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12 Franco Modigliani and M.H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review, June 1958, pp. 261-297.*

Synthetic Securities

In the real world, people rarely make explicit agreements for payments to be made if one and only one state of the world obtains. Most traded securities represent patterns of payments over many states. Equivalently, they may be thought of as packages of pure time-state claims.

To illustrate this, we return to the case of the firm that has issued a riskless bond promising a payment of 60 apples (as described in Table 1) and a stock representing a claim for the remaining apples (as described in Table 2).

Consider an economy in which the only traded goods are this firm's bond and stock. We assume each can be divided into smaller holdings and that dealers are willing to trade the bond for 54 present apples and the stock for 21 present apples. No other markets exist.

Given the current price of a security and the payments it will provide in each state of the world, it is possible to determine the *value-relative*, or future payment per unit invested, that will be received if that state obtains. Tables 5 and 6 show these computations for the bond and stock, respectively. Note that the percentage *realized return* associated with a given state of the world is simply 100 times [the value-relative minus 1.0].

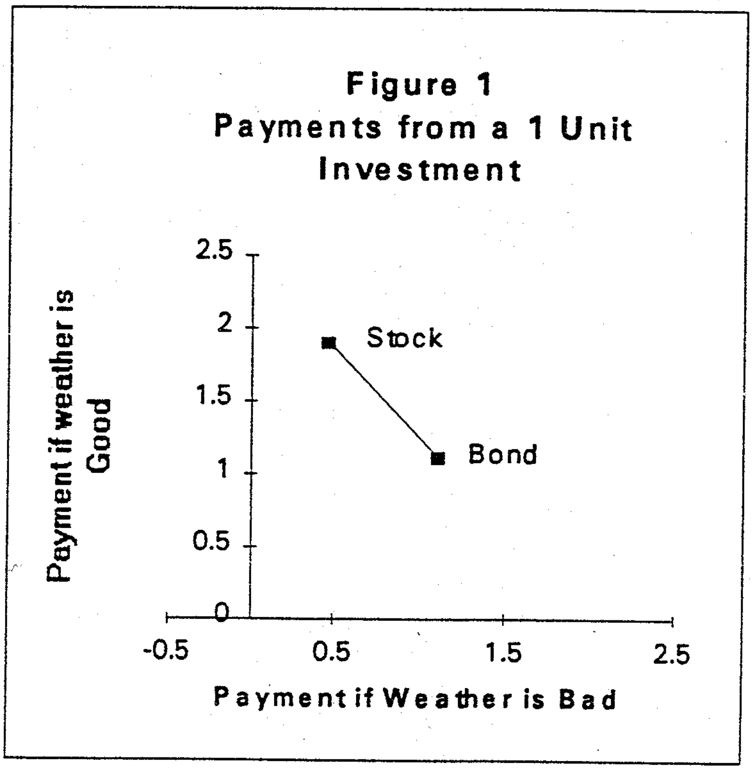
Table 5

Time	State	Bond Payment	Price	Value Relative	% Return
1	good	60	54	1.1111	11.11
1	bad	60	54	1.1111	11.11

Table 6

Time	State	Stock Payment	Price	Value Relative	% Return
1	good	40	21	1.9048	90.48
1	bad	10	21	0.4762	-52.38

Figure 1 plots the value-relatives for the two securities. Since the bond is riskless, it lies on a 45-degree line from the origin -- indicating that the payoff per unit invested is the same, no matter what happens in the future. Any security that plots at a point not falling on such a line is *risky* -- the payments differ in different states of the world and hence the amount to be received is not fully predictable in advance. In this case, the stock is risky and the bond is not.



An *investor* may prefer the bond over the stock or vice-versa. However, these are by no means the only alternatives. By choosing a *portfolio* that includes proportionate shares of the bond and the stock with a total present value of 1 (apple), an investor can obtain any point on the line connecting the two points in Figure 1.

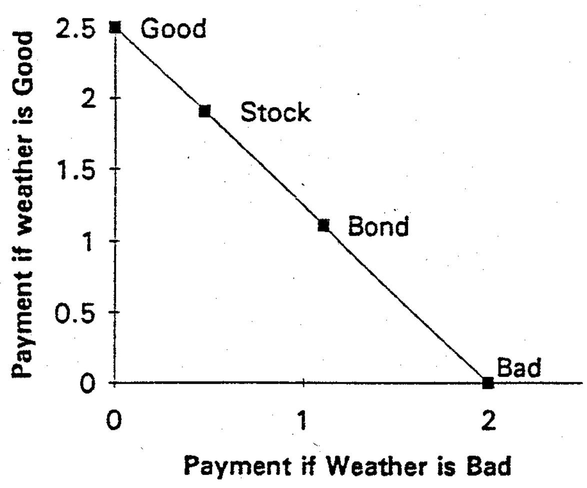
It is helpful to write the relationships among portfolio *holdings*, payments in the two states of the world and portfolio *value*. Letting N_s and N_b represent the number of stocks and bonds held, P_G and P_B the payments received in good and bad weather, and V the present value of the portfolio:

$$40N_s + 60N_b = P_G$$
$$10N_s + 60N_b = P_B$$
$$21N_s + 54N_b = V$$

The equation of the line in Figure 1 is obtained by setting V equal to 1, then varying N_s and N_b over the range in which neither is negative.

But these are not the only alternatives in a well-functioning capital market. Directly or indirectly, it should be possible to take a *negative* position in a security (e.g. "sell it short") as long as an investor's overall portfolio does not involve commitments that would lead to obligations involving negative *net* payments in any state of the world. By judicious use of such positions, an investor should be able to obtain any point on the line through the two points extended to the axes, as shown in Figure 2.

Figure 2
Payments from a 1 Unit Investment



In practice, such arrangements require procedures to guarantee "counterparty creditworthiness." Frequent marking of positions to market, the use of impound accounts, the guaranteeing of credit, and reliance on credit ratings made by outside agencies represent institutional approaches to this issue.

Note that by combining a "long" position in the bond with a short position in the stock in precisely the right proportions, one can *construct* a (pure) "good-weather claim." This is represented by the point labeled *good* in Figure 2. Not surprisingly, it provides 2.5 good-weather apples per 1.0 present apples. Equivalently, the price of 1.0 good-weather apple, obtained in this manner, is 0.40 present apples (as in our earlier example). By combining *existing securities* (the bond and the stock), one can *synthesize* a security that does not exist (a good-weather security). The result is often termed a *derivative security*, since it is derived from existing securities.

The point labeled "bad" shows the payments received from a synthetic bad-weather security with a present value of 1. Not surprisingly, the associated price of 1.0 bad-weather apple is 0.50 (again, as in our earlier example).

In principle, any desired set of payments in the two states of the world can be created using only the two "standard" securities. Given desired payments P_G and P_B, one simply solves the set of simultaneous equations:

$$40N_s + 60N_b = P_G$$
$$10N_s + 60N_b = P_B$$

The value (cost) of obtaining the payments can then be determined by "pricing out the portfolio" with the final equation:

$$21N_s + 54N_b = V$$

In financial engineering terms, the portfolio (N_s and N_b) is sometimes termed a *replicating strategy*.[13]

It is a simple matter to determine the prices of pure time-state claims implicit in a set of existing securities. First, the cost of a strategy for which $P_G = 1$ and $P_B = 0$ is determined. This is the price of a claim to receive 1 unit if and only if the "G" state obtains. In this economy it is 0.40. Then the cost of a strategy for which $P_G = 0$ and $P_B = 1$ is determined. This is the price of a claim to receive 1 unit if and only if the "B" state obtains. In this economy it is 0.50. Henceforth, any desired set of payments can be "priced" by simply multiplying the amounts to be paid by these prices and then summing. In this case:

$$V = 0.40P_G + 0.50P_B$$

Only when a replicating strategy needs to be employed will it be necessary to solve the original set of simultaneous equations[14].

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13 This terminology is more appropriate when multiple time-periods are involved. In such a situation, some of the decision variables represent investments that are programmed to take place at various dates in specific circumstances in the future. The solution then determines not only the current set of investments but also investments to be undertaken over time. While such problems can be solved by determining the solution to a set of simultaneous equations, it is generally preferable to take advantage of special aspects of the structure of an assumed payment tree to reduce the computational burden.

14 Alternatively, the solutions for the pure time-state claim replicating strategies can be mixed in the appropriate proportions.

Forward Prices

A *forward price* can be defined as a price to be paid *with certainty* at some specified future date for delivery of one or more goods, where the amount delivered may be contingent on events occurring on or before that date. An important special case is the *forward price of a (pure) time-state claim.*

In our economy, the expenditure of 0.40 apples today will purchase a security promising delivery of 1.0 apple tomorrow if the weather during the forthcoming year is good. A forward contract for the same security would promise the payment of some amount f_G next year, *no matter what.* For example, one party might promise to pay 0.4444 apples next year (no matter what) in exchange for a promise from a counterparty that the latter will deliver 1.0 apples if the weather has been good and nothing if it has been bad. Equivalently, after netting, if the weather has been good, the counterparty will deliver 0.5556 apples; on the other hand, if the weather has been bad, the original party will deliver 0.4444 apples.

In the absence of any payments or receipts during the year (as in our example), a very simple relationship should hold between the present price and the associated forward price. The forward price will simply equal the present price plus associated interest. Equivalently, the present price will equal the forward price divided by the discount factor for the payment date. In this case:

$$0.4444 = 0.40 / 0.90$$

If this relationship does not hold, clever arbitrageurs can exploit others in the market, thereby literally making "something for nothing." Such opportunities are fleeting, at best.

Forward prices for pure time-state claims can also be used to value desired payments. First the *forward value* (F) for the set of payments is determined. Here:

$$F = f_G P_G + f_B P_B$$

This is the amount that one should be willing to commit to pay next year for the set of payments in question.

To find the present value, the forward value is simply discounted. Here:

$$V = \frac{F}{0.90}$$

This two-step procedure is widely used by financial engineers. Unfortunately, the underlying economics is obscured by the use of terms that are at best confusing and at worst misleading. Forward prices of time-state claims are termed *risk-neutral probabilities* (or just *probabilities*). The forward value of the set of payments is called the *expected future value,* and the present value the *discounted expected future value.*

While the procedures employed by those who use such terms provide correct answers, it is dangerous to confuse forward prices with probabilities and forward values with expected values. In a world of risk-averse investors, probabilities and forward prices will generally differ, as will forward and expected values.

There is every reason to believe that investors are risk-averse. This, in turn, gives rise to differences in expected returns. To construct an appropriate investment strategy, one must attempt to understand such differences.

Investment Choice

The astute reader will have noted that none of the conclusions reached thus far used the concept of *probability* in any way! Issues of valuation, replication, the construction of swaps, and so on can in principle be addressed without assuming or measuring probabilities associated with various states of the world. All that is required is a sufficiently complete market and price information for enough securities to allow purchase of pure time-state claims, either directly or synthetically[15].

In effect, once a tree representing the payments provided by a sufficient set of existing securities has been drawn, further analysis proceeds from the sole assumption that markets are arbitrage-free. This is the proper domain of financial engineering[16].

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[15] In general, as many "independent" securities as there are states of the world will suffice. In our economy, with two states of the world, only two are required. Formally, it must be possible to invert the matrix of payments from the securities in the various states of the world. This will not be possible, if, for example, the payments from one of the securities can be obtained from some combination of one or more of the others.

[16] In practice, the distinction is not this clean-cut. Financial engineers usually construct payment trees based on assumptions about security attributes (for example standard deviations of returns) that incorporate probabilities. In many cases, additional assumptions are employed (e.g., that each branch of a binomial tree is equally probable). If the collective set of such assumptions is in error, the resulting valuation and replication strategies may be incorrect. The potential magnitudes of such errors are basically empirical issues. In any event, the subject is beyond the scope of this paper.

Investment choice deals with the selection of a *portfolio* of investments. Fundamentally, it is concerned with choosing an appropriate combination of time-state claims given the investor's overall wealth. In the Arrow-Debreu framework, wealth (including potential future earnings from human capital) can be stated in present-value terms. The *consumption-investment* decision is to select the best combination of present and state-contingent future goods (for example, apples) from among all combinations that can be obtained given overall wealth. Letting P_N represent the payment now (that is, consumption of present apples), EU() the investor's "expected utility" function, and W his or her wealth, the goal is to:

$$\text{Maximize: } EU(P_N, P_G, P_B)$$

$$\text{Subject to: } 1.00P_N + 0.50P_G + 0.40P_B \leq W$$

Clearly, the utility that an investor expects to receive from holdings of contingent claims on future consumption will depend on his or her assessment of the probabilities that the associated state of the world will actually come about. Hence investment choice must deal with probabilities.

Moreover, while the financial engineer may be able to take security prices as given, investors will wish to consider their determinants -- including productive opportunitues and the opinions and preferences of others. In a sense, the current price of a time-state claim reflects a consensus of all investors' *predictions* of the *probability* of the associated state and their *feelings* about the *desirability* of obtaining goods in that state. A specific investor will, in the first instance, compare his or her own assessment of the probability of a state and of the desirability of obtaining goods in it with the price that must be paid for the associated contingent contract. At a deeper level, however, the investor is comparing his or her own probability assessment and utility with that of the consensus of other investors.

Space precludes a detailed analysis of these relationships here. Instead we focus on two central aspects of investment analysis: the existence of *risk premiums* and the *type* of risk that is likely to be rewarded with such premiums.

Consensus Probabilities

There is no reason to believe that individuals agree on the probabilities associated with various future states of the world. Thus the very notion of, say, "the probability" of good weather is ill-defined. As a practical matter, it is frequently assumed that markets are "efficient" in a particular sense. A market can be said to be *efficient relative to a given set of information* if security prices are the same as they would be if all investors had carefully analyzed that information. The set of probabilities assessed by a "careful analyst" after having processed a body of information can be termed "fully informed" relative to that set of information. Theorists sometimes assume, for example, that prices "incorporate all publicly-available information." Roughly, this is equivalent to assuming that prices "reflect" probabilities that would be assigned by a sophisticated analyst after having studied all such information.

A more modest set of assumptions holds that security prices reflect a set of "consensus probabilities," representing weighted averages of investors' predictions. In this process, the predictions of those with large amounts of wealth are generally weighted more heavily than the predictions of those with small amounts of wealth.

Formal models of investment behavior generally assume that all investors hold the same set of opinions concerning probabilities. Results are then derived concerning security prices, expected returns, and the like. Such results can be compared with the opinions of a particular investor, who may assign different probabilities, either due to the use of a different information set or a different analysis of the same information. Recommended consumption and investment decisions can arise from any of three types of differences between the investor and the "average investor": differences in *preferences*, differences in *circumstances*, or differences in *predictions*.

For the purposes of this paper, we assume that there are no differences of opinion concerning the probabilities of possible states of the world. More precisely, everyone agrees that the probability of good weather is 0.50, as is the probability of bad weather.

Forward Prices, Probabilities, and Expected Returns

Properly constructed probabilities have two well-known properties: they are non-negative and sum to 1.0 when all possible states of the world are included.

Forward prices for pure time-state claims have precisely the same properties. As long as the goods in question are indeed "good", each forward price will be non-negative. Moreover, as argued earlier, the sum of all such forward prices will be 1.0 when all possible states of the world are included.

These two facts imply that either (1) every pure time-state claim forward price will equal its probability, or (2) some forward prices will be greater and others less than the associated probabilities.

The *expected payment* from a pure time-state claim is simply the probability that the associated state will occur, since an expected value is obtained by multiplying a payment (here, 1.0) by a probability. The ratio of the expected payment to the current price will equal the *expected value relative*, or [1 + the *expected return*] for the time-state claim in question.

Table 7 shows the calculations for our economy. The expected return for a good-weather apple contract is 25% ; that for a bad-weather apple contract is 0%. Note that the former exceeds the riskless rate of interest (11.11%) while the latter falls below it.

Table 7

Time	State	Price	Payment	1+Exp.Ret.
1	good	0.40	0.50	1.25
1	bad	0.50	0.50	1.00

Recall that arbitrage insures that a forward price will equal the present price times (1+i), where *i* is the riskless rate of interest. It follows that if the probability of a state equals the forward price of a claim to receive one unit in that state, then the expected return on the claim will equal the riskless rate of interest. Using this as a watershed, we can easily conclude that for any pure time-state claim:

Probability > Forward Price <=======> Expected Return > Riskless Rate
Probability = Forward Price <=======> Expected Return = Riskless Rate
Probability < Forward Price <=======> Expected Return < Riskless Rate

In our economy, the good-weather state represents a case of the first type, and the bad-weather state represents a case of the third type.

If all forward prices equaled probabilities, all time-state claims would have expected returns equal to the riskless rate of interest. But then, so would all securities! In such a world there would be no reason to expect to do better with, say stocks, than with treasury securities.

We can define a *risk premium* as the difference between the expected return on a security and the riskless rate of interest. In a society in which all time-state claim forward prices equaled probabilities there would be no risk premiums.

Simple experience, introspection and many empirical studies suggest that in actual economies, prices will be set so that some securities should be *expected* to outperform others. In the vernacular, such securities should provide better "long-term performance".

To offer a superior expected return, a security must provide disproportionate amounts of payments in the states of the world for which the probability exceeds forward price of the associated time-state claim. Such a security will have a *risk* for which there is a *reward* (risk premium). Our good-weather apple security provides an example.

But, as we have argued, if there are states for which the probability exceeds the forward price, there must be states for which the reverse holds. Securities providing disproportionate amounts of payments in such states will produce what may be termed a *risk discount* -- their expected returns will be less than the riskless rate. Such securities will have a risk for which there is a *penalty*, rather than a reward. Our bad-weather apple security provides an example.

The Societal Risk Premium

In our economy, good-weather apples are cheap -- their forward price is less than the expected payment. Conversely, bad-weather apples are expensive -- their forward price is greater than the expected payment. This is perfectly plausible. If the weather is bad there will be fewer apples. In general, the value of an additional apple will be greater, the fewer there are to consume. Investors will be willing to sacrifice more present consumption to provide for additional consumption "for a rainy day" than they will to provide for consumption for a sunny day. More directly to the point, an additional unit of a commodity is likely to be worth more in a *state of scarcity* than in a *state of plenty*.

In well-functioning economies, bad news generally accompanies good news. If there is good news about a security (high expected returns) there is likely to be bad news as well (really bad returns in states of scarcity). Securities with high expected returns are likely to be "fair weather friends," while those with low expected returns are likely to be "foul weather friends."

What of the *market portfolio* -- made up of all securities representing claims for future goods and services. What will be the *societal risk premium* associated with investment in a portfolio that includes proportional shares of all securities?

Given the fact that some time-state claims must provide risk discounts if others are to provide risk premiums one might erroneously conclude that the societal risk premium would equal zero. However, this is unlikely to be the case. The expected return on a portfolio will be a value-weighted average of the expected returns on the underlying securities. More fundamentally, it will equal a value-weighted average of the expected returns on the underlying time-state claims. In general, a greater amount of value will be associated with "states of plenty" than with "states of scarcity." Since the former provide risk premiums and the latter risk discounts, the value-weighted average expected return is likely to exceed the riskless rate of interest. There will be a societal risk premium to compensate those who hold proportionate shares in the market as a whole for their willingness to take on the risk of doing badly in bad times.

Summary

The main goal of this paper was to show the power of using contingent claims analysis to analyze the impact of risk on financial decisions. In the process, we reached some conclusions concerning a simple economy -- conclusions that can be shown to hold for far more general economies.

Risk can usefully be characterized in terms of the pattern of payments obtained over different future states of the world. If markets are sufficiently complete, it is possible to transform any such set of payments into any other set that has the same present value. Moreover, the cost of obtaining any desired set of payments can be computed directly, and a strategy that will replicate the payments using existing securities can be constructed. The technology for doing this provides the backbone of the profession of Financial Engineering. The creation and valuation of derivative securities represent direct applications of the methods discussed here.

It is not enough, however, to know how to construct and value sets of payments. Ultimately, someone must select the best feasible payment set for a given investor. In general, one can expect to earn higher returns in the long run from certain strategies than from others. However, it is not risk *per se* that is likely to be rewarded in a well-functioning capital market -- only *the risk of doing badly in bad times*.

Financial Engineers can now reshape future contingencies into many different patterns. Investors are thus faced with larger and larger menus from which to choose. More than ever, it is important that they have as clear an understanding as possible about the determinants of the security prices that form the inputs for the activities of the Financial Engineer. Nuclear Financial Economics, concerned as it is with "first principles," can provide a solid base for such an understanding.