The Evolution of General Relativity through the Lens of Curvature

Jeffrey Chang

June 12, 2017

Abstract

General relativity, Einstein's famous theory describing gravity in terms of the fabric of space and time, has grown to be an essential part of popular culture. The way we imagine it today, however, is immensely different than how Einstein first conceived it a century ago. A careful analysis of Einstein's papers, newspaper clippings, popular science expositions, and textbooks reveals that the theory has transformed over time from an abstract mathematical depiction to a visually intuitive one relying on metaphors of 'curvature'. We propose that this shift depended on general relativity's incredible popularity and the insatiable public hunger for a simple explanation. The power of cognitive metaphors implies that the modern language of curvature profoundly affects how scientists think, suggesting that popular science explanations can influence how a scientific theory evolves.

Introduction

Matter tells spacetime how to curve; spacetime tells matter how to move.

–John Wheeler, prominent 20th century physicist

In these twelve words lies the essence of humanity's most beautiful scientific theory: general relativity. The theory redefines our notions of space and time and sews gravity into the very fabric of our universe. Despite the abstruse mathematics behind the theory, it holds a central place in the public conscience: we're all familiar today with cosmological wonders such as mysterious black holes, the Big Bang, and 'fabric of spacetime'. But if a century ago, Einstein heard any of these modern 'popular' explanations of his theory, he would have found the language quite foreign! The theory of gravity remains more or less the same, but the way we explain it today is completely different from how Einstein first explained it back in the day—the metaphors are different, the premise is different, and even the general attitude towards the theory is different.

Perhaps the theory's 'taste' evolved because of how popular it became. Einstein was the most famous scientist of the 20th century, and his constant headlines created a public hunger demanding an explanation for his new theory of space and time. Writers faced a daunting task of distilling graduate-level physics concepts to a mathematically innocent audience, forcing them to invent creative analogies to make concepts as intuitive as possible. But did popular explanations affect how the scientists described general relativity among themselves?

The evolution of relativity's ideas can be revealed by peering through the lens of curvature. We'll take a journey through time and explore how various sources depict what we now call spacetime curvature. Newspapers and journals are the first to describe gravity in terms of curvature, and scientists slowly adopt it—hesitantly in the first few decades, but more acceptingly later on—and today the 'curvature metaphor' lies at the heart of general relativity, shaping the way scientists think about our universe. First, we'll see why the metaphor has become so popular: it is a lucid, insightful, and easy-to-understand way to qualitatively explain the theory.

The Curvature Metaphor

General relativity explains the nature of gravity, space, and time. Actually, space and time are two facets of the same thing: our sense of space and time changes when we move, causing length to contract and time to slow down on speeding spaceships. Space and time can intermix and 'rotate' into each other because they are intimately woven together into a four-dimensional **spacetime**. On this backdrop of spacetime is where stuff happens—where stars burn and where humans live.

Gravity is more complicated. Before Einstein's day, people saw gravity as an attractive force pulling together objects with mass. But today, gravity is no longer a force—it is a distortion in the fabric of spacetime arising from matter and energy. As Wheeler explained in his 1998 autobiography, "matter tells spacetime how to curve" (Wheeler and Ford, 1998). The presence of matter bends spacetime and induces some 'curvature'; the 'gravitational field' around an massive body is really the bent spacetime around it. Mathematically, spacetime curvature is described by the **Riemann curvature tensor**, written as $R^{\rho}_{\sigma\mu\nu}$, and the matter in space is described by the energy-momentum tensor $T^{\mu\nu}$. The Einstein field equations relate $R^{\rho}_{\sigma\mu\nu}$ to $T^{\mu\nu}$ and describe how matter bends spacetime.

When we say that matter bends spacetime, we mean that the geometry of spacetime—the way we measure distances and time—has been altered by the 'stuff' in space. In empty space, spacetime is 'flat'. Flat geometry is something we're accustomed to, where you can measure distances with Pythagoras's familiar formula, $distance^2 = (\Delta x)^2 + (\Delta y)^2$. But once you put matter in spacetime, spacetime becomes curved, and geometry gets a lot tougher. Four right angles no longer make a square; a circle's area is no longer exactly πr^2. To measure distances, you need to know ten numbers at every point in space! These ten numbers, written as $g_{\mu\nu}$ and known as the **metric tensor**, tell you how to measure distances and times, and describe the geometry of spacetime.

Once you know the shape of spacetime, you can predict how objects move in it. This is given by the geodesic equation, the second part of Wheeler's quote, which tells us that 'spacetime tells matter how to move'. In our world, things try their best to move in a straight line, but since nothing is straight in a curved spacetime, they take the next-best alternative: the shortest possible path between points in spacetime, called a **geodesic**.

* * *

To summarize, the matter in the universe 'curves' spacetime by shaping its intrinsic geometry (the $g_{\mu\nu}$). In turn, objects try to move in a straight line, but end up moving in curved paths because spacetime itself is curved. Gravity is not really a force, but deeply 'built-in' to the curvature of spacetime itself.

This is our explanation of general relativity today, perfected over the years. It relies on the 'curvature metaphor', saying that gravity is akin to the bending of a sheet of fabric. In the early days of the theory, though, this metaphor didn't exist. As we'll see, Einstein initially described the theory completely differently.

Initial Expositions

General relativity was born in November 1915 after Einstein struggled through a heroic decade-long intellectual odyssey. Before then, as the theory was developing, Einstein wrote an incoherent and haphazard stream of papers that seemed to contradict themselves. So when he finalized the theory, he needed to tidy it up in a convincing, digestible, and elegant form to communicate it to his fellow scientists. The resulting masterpiece *The Foundation of The General Theory of Relativity*

(Einstein, 1916), is now one thought of as one of the greatest human works ever created (Gutfreund, J. Renn, and Stachel, 2015).

Einstein's 1916 paper was immensely influential. Many physicists thought his theory was "unconvincing" until this paper clarified his earlier ideas in a more logically compelling way (Jürgen Renn, Janssen, and Schemmel, 2007), suggesting that this work molded how they thought about the science. Because he was presenting to an audience of physicists, Einstein presented his equations in a direct mathematical way.

* * *

The Foundation of The General Theory of Relativity does not use metaphors to describe the flat or curved nature of spacetime. All explanations are mathematically literal. In the paper, Einstein simply defines different mathematical quantities like $g_{\mu\nu}$, $R^{\rho}_{\sigma\mu\nu}$, and $T^{\mu\nu}$, and explains how they relate to each other with equations. Nowhere do we find a concise, intuitive summary in the style of modern explanations—the reader must sit down and read through the technical language to learn what the theory describes.

His language draws no analogy to other concepts. For example, Einstein never calls the geometry of empty space 'flat spacetime'; instead he uses the direct math meaning that "$g_{\mu\nu}$ have constant certain values". He calls $R^{\rho}_{\sigma\mu\nu}$ the Riemann-Christoffel tensor rather than the Riemann curvature tensor since 'curvature' has not yet entered the theory. The romantic four-dimensional backdrop of space and time is a mathematical ploy rather than a geometric facet of the universe; Einstein only ever calls spacetime "the continuum". Clearly, the language is direct, the descriptions are mathematical, and he paints no grandiose images of the 'fabric of the cosmos.'

In this straightforward interpretation of the math, gravity is quite boring: it's just the ten numbers $g_{\mu\nu}$ at every point in space-time that he described as the "gravitation-potential." As far as Einstein was concerned, the $g_{\mu\nu}$ were just another field like the electric and magnetic fields. This gravitation-potential language suggests that Einstein's gravity played a rather limited role as a field.

* * *

Evidently, gravitation in Einstein's continuum doesn't have quite the poetic charm that today's 'fabric of the cosmos' does. The theory is literal; space and formed a "continuum" mathematically described with "absolute differential calculus"—no more, no less—and gravity was just a field, the ten numbers $g_{\mu\nu}$ at any point in space and time. Yes, the $g_{\mu\nu}$ were "special" numbers—they happened to determine the lengths of "measuring-rods and clocks"—but this interpretation doesn't emphasize the intimate relation between gravity and the fundamental curvature of our universe's geometry. After all, at this stage in the theory's development, only physicists needed to learn it, so a direct mathematical presentation of the equations sufficed.

Realizing the mathematical barrier to understanding his theory, Einstein attempted a semi-popular explanation in his pamphlet *Relativity: The Special and the General Theory* (Einstein, 1920), dedicated to readers with a university education but "not conversant with the mathematical apparatus." Even though he sidestepped more difficult technical material, he still labors his audience with jargon such as the "space-time continuum" and "Euclidean or non-Euclidean." But without the full mathematical context, these words are are meaningless terms inside the reader's head! Explainers still had more work to do—more terms to invent, more metaphors to construct—before mathematical innocents could easily digest the theory.

For many theories, a successful popular explanation is not a pressing concern, but for general relativity, it definitely was. For in 1919, Einstein was suddenly launched into international fame.

Newspapers and Popularizations

"Revolution in Science," proclaims a headline on the London *Times* of November 7, 1919, "New Theory of the Universe—Newtonian ideas overthrown." A few months prior, a British expedition had verified a crucial prediction of general relativity—how much the sun's gravity would deflect light from distant stars—and the moment they reported their success, the Einstein legend was born. Einstein was no longer just "somewhat of a public celebrity" in German-speaking countries, but became an international superstar, from Japan to the Netherlands to the United States. Not a single year between 1919 and his death passed without *The New York Times* mentioning his name (Pais, 1982). Einstein became the most famous scientist of the twentieth century, and his theory became the most publicized scientific theory of the time.

In its earliest public days, the mystery shrouding the theory led to the myth that no more than twelve people in the world could understand it. Einstein himself insisted to the *Times* the "difficulty of making himself understood by laymen" (Pais, 1982). In spite of its difficulty—or perhaps because of it—the theory remained fully entrenched in the public conscience. The media continued to make references, caricatures, and cartoons about the absurdity of Einstein's theory for the next few decades (Price, 2012). Their authors needed to describe ideas that supposedly nobody could understand—and unsurprisingly, they borrowed the curvature metaphor.

* * *

It is hard to track down when exactly newspapers and journals began to explain gravity in terms of curvature. But the metaphor was so effective that editors to employed it again and again. By 1921, it had become so commonplace that the British comedy magazine *Punch* even parodied curvature to give advice to the poorly-performing English cricket team. "The earth does not emit a gravitational force which pulls the cricket ball down," the writer quite helpfully proclaims. "It imposes a curvature on the surrounding space so that the path of the cricket ball appears curved, although it pursues the shortest course available to it." (Price, 2012) The absurd context highlights just how widely Einstein's theory had spread into popular culture, and how engrained the 'curvature-represents-gravity' idea had become. Moreover, just two years after Einstein's theory went public, the way the media employed curvature to explain gravity was already pretty much how we do today!

Accompanying the newspaper articles and magazine comics about relativity was a "boom" of popular-science books. For following decade, entertaining books about the wonder of science sprung up everywhere, culminating in James Jeans' influential 1932 book *The Mysterious Universe* (Jeans, 1932). It became immensely popular: 70,000 copies were sold in the UK in the first month alone (Whitworth, 1996). Again, Jeans borrowed the curvature metaphor to explain how gravity worked: he explains that "space itself is curved, much in the same way in which the surface of the earth is curved," and that the "presence of matter" is what produces a "curving of space."

Other popularizations of the 1920's also borrowed similar rhetoric to explain the new nature of space and time to the masses. The astronomer Eddington said that gravity "can be pictured as a curvature of space and time" in his best-seller *The Nature of the Physical World* (Eddington, 1928), as did Whitehead in the popular *Science and Modern World* (Whitehead, 1926). With all the consistent exposure from newspapers, journals, and books, the attitude that gravity was the curvature of spacetime must have become quite popular. But even though the curvature metaphor established itself rather quickly in the media, scientists were initially hesitant to adopt it in their technical writings.

Early textbooks

Not everybody approved of the popular science books of the 1920's. According to popular science scholar Michael Whitworth, the scientific community had "serious criticisms" about popular works such as Jeans' *The Mysterious Universe*; by the 1930's they felt that the genre's entertaining mysticism and philosophical questioning were "vulgar debasements" of the science. Accordingly, the reputations of these popular science writers "sank" (Whitworth, 1996).

In this cultural context, authors of general relativity textbooks were in a rather peculiar situation. They knew their textbooks would be crucial, since textbooks 'formalize' theories and mold the minds of future scientists (Kuhn, 1970). These authors had to carefully choose their language: they knew the students were familiar with the curvature metaphor from all the popular expositions, so they chose to borrow 'curvature' to help to explain the math—but at the same time, they couldn't completely embrace it, since contemporary scientists were so critical of such popular rhetoric.

* * *

Tolman's 1934 *Relativity, Thermodynamics and Cosmology* (Tolman, 1934), used by Caltech undergraduates through the 1950's, hesitantly borrowed parts of the curvature metaphor to describe the geometry of spacetime. Tolman readily talks about "flat" or "curved" spacetime, but nearly every time he uses the words, he puts them in quotations to remind the student that they are just a metaphor for the math. For example, he explains that relativity involves

> ...the general notion that 'flat' space-time corresponds to the absence of intrinsic gravitational action and that 'curved' space-time corresponds to the presence of permanent gravitational fields.

Tolman's depiction of spacetime curvature sounds quite modern, but the quotations around the words "flat" and "curved" make the metaphor feel uncertain or even uncomfortable. We can hear a hesitance in Tolman's voice because he faced a dilemma. The metaphor made equations easier to understand, but he couldn't overtly rely on it because he had to maintain the textbook's rigor and formality. Quotation marks were thus a perfect strategy for Tolman to use the words 'flat' and 'curved' without risking sounding too much like a popular science book. Evidently, curvature had a peculiar status in these early days—useful as an analogy for the math, but only ever so much.

In other respects, though, Tolman's 1934 textbook is still not very modern. Even though he had progressed from Einstein's math-heavy language by using the non-technical terms 'flat' and 'curved', he still shared Einstein's direct interpretation of the $g_{\mu\nu}$. Today, the $g_{\mu\nu}$ are just the metric tensor, and gravity is built into the fabric of spacetime. But Tolman claimed the $g_{\mu\nu}$ was both: it had a "dual" nature of the metrical tensor and the gravitational potentials. In his view, gravity was still a field, and not quite a fundamental aspect of the universe's geometry. So this critical part of the metaphor today is still missing from Tolman's early textbook.

* * *

Like Tolman, other early textbook authors also borrowed curvature as a metaphor to explain gravity—but with reservations, as if they were afraid that the students would read the metaphor too literally like a popular science book. Intriguingly, Eddington, who only timidly used the curvature metaphor in his 1923 textbook *The Mathematical Theory of Relativity*, readily employed it in his popular work *The Nature of the Physical World.* So he was clearly aware of the metaphor, but decided it was inappropriate for a scientific audience at the time.

By the 1960's, though, attitudes likely started changing. Scientists started to drop the quotes around 'curved' and applied the metaphor more readily. For instance, the Nobel laureate Richard Feynman, well-known for his charismatic and intuitive explanations, titled his 1961 lecture about general relativity "Curved Space" (Feynman, Leighton, and Sands, 1965), and quite directly tells a technical Caltech undergraduate audience that "space and time...are *curved* near heavy masses." As the rhetoric of curvature became more and more prevalent in the scientific discourse, people started to lose sight that 'flat' and 'curved' were just convenient words to describe the underlying math. Curvature was less and less a pedagogical tool to *represent* gravity—it started *becoming* gravity.

Modern Textbooks

Today, curvature is pervasive. The 1973 standard graduate-level book *Gravitation* (Misner, Thorne, and Wheeler, 1973), which has molded the minds of the last two generations of physicists, unapologetically explains the theory's mathematics through the curvature metaphor. The authors label sections of the book with "flat spacetime" and "curved spacetime," without hesitant quotation marks, and the explanations in the chapters rely heavily on this rhetoric. (By now, too, 'spacetime' has become such a common concept that it became one word, rather than a hyphenated 'space-time.') Furthermore, gravitation has become intimately tied to the fabric of the universe: the authors stress that "there is no real gravitational field," in direct contrast to Einstein's 1916 view that the $g_{\mu\nu}$ were "ten functions representing the gravitational field." In the modern portrayal, 'gravity' is not merely the ten numbers $g_{\mu\nu}$; it is a profound consequence of the innate geometry of spacetime.

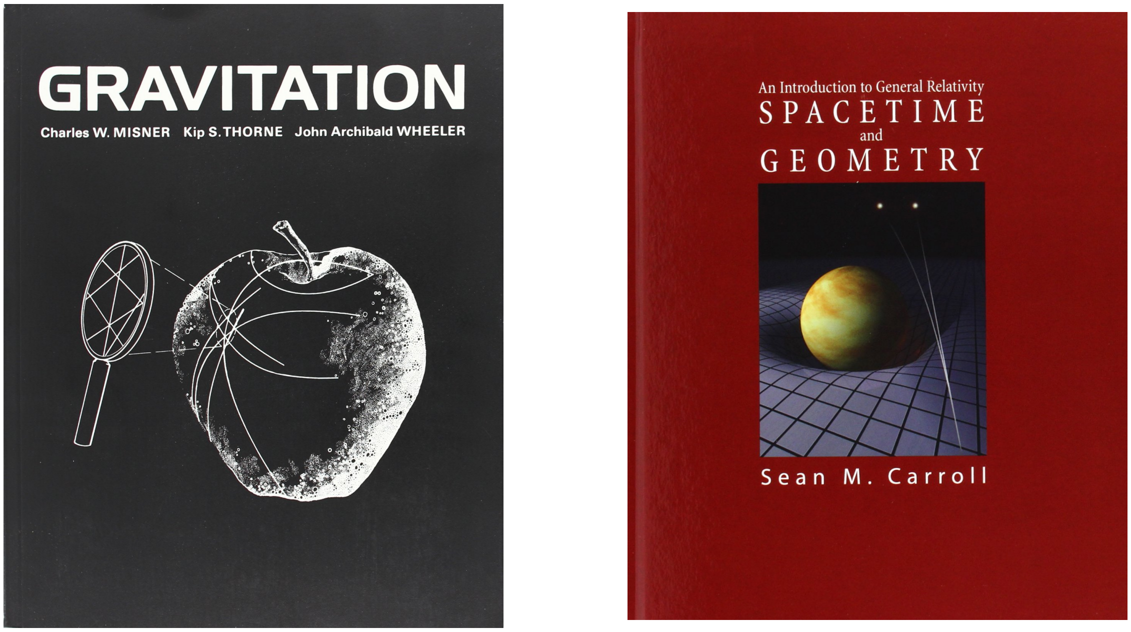
As generations pass, it is easy to forget that curvature is only a *representation* of the underlying concepts. Explainers first borrowed the metaphor as a tool to explain gravity, but amazingly, through the constant public exposure to its visualizations, curvature is no longer just a metaphor but gaining the status of a real phenomenon. The well-known theoretical physicist Sean Carroll's 2004 textbook *Spacetime and Geometry* titles an entire chapter with one word: "Curvature". Curvature can exist, or not exist; he says that says that "there is curvature" when $R^{\rho}_{\sigma\mu\nu}$ is not zero. Gravity is now the "manifestation of the curvature of spacetime." We have imbued spacetime curvature with a new, rather privileged status.

* * *

The front cover artwork of some modern general relativity textbooks (figure 1) reveal just how literal this interpretation of "curvature" has become. The 1973 *Gravitation* cover shows a bug on a fruit. The fruit-world looks curved to us, but the bug thinks its fruit-world is flat, because it lives within the fruit-world. Analogously, even though spacetime has global curvature, it appears locally flat to us since we live in its fabric. The old bug-on-a-fruit analogy has a indirect and subtle explanation which reminds us that concept of curvature is at best a *metaphor* for the underlying math.

However, the more recent 2004 text *Spacetime and Geometry* depicts the curvature of space in an completely literal sense. Its cover art resembles a scene in popular science documentary, with a mesh representing the 'fabric of spacetime', a cartoon planet creating a dimple warping this fabric, and two particle trajectories showing how 'warped spacetime' bends the particle's path. This sort of visualization plays a crucial role in popularizations of general relativity, since it explains the premise of the theory in such an intuitive way. But, like any analogy, it oversimplifies the concept of curvature and ignores crucial nuances. The visualization seems to suggest that there is a *physical* deformation in the geometry of spacetime. There is a subtle distinction between saying that a visualization *represents* gravity and saying that it *is* gravity—and this shift profoundly affects how scientists themselves understand the theory.

* * *



(a) (Misner, Thorne, and Wheeler, 1973)

(b) (Carroll, 2004)

Figure 1: Modern relativity textbooks visualize curvature as a warping of spacetime

Metaphor lies at the heart of scientific understanding. As the influential cognitive scientist Lakoff states in his work *Metaphors We Live By*, metaphors are far more than “a mere matter of words”—they are at their core a *conceptual* lens through which we understand the world. More and more evidence from psychological studies today “reveals the central role of metaphor in abstract thought” (Lakoff and Johnson, 1980).

If metaphors indeed are the way the human brain understand concepts, the curvature metaphor plays a very profound role in how we understand the universe. As Lakoff beautifully states,

> Beyond middle-level physical experience [...] our basic-level concepts utterly fail us. To conceptualize such experience requires the magnificent tool of conceptual metaphor.

General relativity lies incredibly far outside the realm of our everyday lives: nobody has felt the direct physical experience of its peculiar predictions, such as non-Euclidean geometry around a black hole or the redshift (altered colors of light) in a strong gravitational field. The curvature metaphor is *essential* if we want to make any sense at all out of a theory so out of touch with common sense.

In this light, tracing language use in textbooks is much more than mere word-counting. The authors’ choice of words are a lens for us to see how they think, and how they choose to convey these thoughts to the next generation. The thoughts corresponding to the words represent the scientific community’s shared conception of what general relativity means at the time. The words and images in textbooks reveal not only how scientists at the time choose to describe the theory, but how they actually think.

For a concept as counter-intuitive, revolutionary, and bizarre as general relativity, metaphors have an especially profound impact on the way scientists think. The curvature metaphor and its popular cartoon visualizations are driving the next generation of scientists to see the curvature of spacetime as a real, physical artifact. With modern visualizations, we are becoming more and more acquainted with this unusual 4-dimensional feature of our world. Armed with this intuitive grasp, scientists today can complement mathematical descriptions to visualize phenomena and propose new ideas in a way that previous generations of scientists could not.

Conclusion

What a different world we live in today. Gravity is no longer just a field permeating space that determine notions of 'distance' and 'time'; it is tied intimately to the very curvature of our universe. Matter no longer just determines the ten numbers of the gravitational potential; it actively warps and distorts space and time itself. With all the visualizations of weights warping sheets in popular media and textbooks, curvature itself is becoming a real, *physical* phenomenon, rather than just a mathematical analogy.

Of course, the curvature metaphor is not the only change in general relativity in the last century; the science itself has progressed enormously—we've detected gravitational waves propagating from far corners of our universe, theorized black holes and wormholes and ridiculous spacetime geometries, measured residual radiation from the birth of our universe, and far more. And there is definitely much more physics to come; as Sean Carroll puts it, "Nobody believes that general relativity is the final word" (Carroll, 2004).

Without the public demanding an easy-to-understand explanation of general relativity, would today's powerful metaphors of curvature have entered the scientific conscience? I think not. We ought not to discredit popularizations of science as bastardizations of rigorous scientific theory, but instead appreciate how their intuitive explanations help scientists understand their work. Physicists are becoming more and more intimately familiar with the utterly bizarre nature of our world, and hopefully our intuition today will lead to an even deeper understanding of the universe tomorrow.

References

Carroll, S.M. (2004). *Spacetime and Geometry: An Introduction to General Relativity.* LCCB: 2004296148. Addison Wesley. ISBN: 978-0-8053-8732-2.

Eddington, A.S. (1928). *The Nature of the Physical World.* Gifford lectures. LCCB: 29015261. The Macmillan Company.

Einstein, A. (1916). "Die Grundlage Der Allgemeinen Relativitätstheorie." In: *Annalen der Physik* 354.7, pp. 769–822. ISSN: 1521-3889. DOI: `10.1002/andp.19163540702`.

— (1920). *Relativity [Über Die Spezielle Und Die Allgemeine Relativitätstheorie. Engl.]:* Henry Holt.

Feynman, R.P., R.B. Leighton, and M.L. Sands (1965). *The Feynman Lectures on Physics.* Addison-Wesley world student series. LCCB: 63020717. Addison-Wesley Publishing Company. ISBN: 978-0-201-02117-2.

Gutfreund, H., J. Renn, and J. Stachel (2015). *The Road to Relativity: The History and Meaning of Einstein's "The Foundation of General Relativity", Featuring the Original Manuscript of Einstein's Masterpiece.* Princeton University Press. ISBN: 978-1-4008-6576-5.

Jeans, J.H. (1932). *The Mysterious Universe.* Rede lecture. LCCB: 32005017. The Macmillan Company.

Kuhn, T.S. (1970). *The Structure of Scientific Revolutions.* Foundations of the unity of science. LCCB: 79107472. University of Chicago Press. ISBN: 978-0-226-45803-8.

Lakoff, G. and M. Johnson (1980). *Metaphors We Live By.* Phoenix books. LCCB: lc80010783. University of Chicago Press. ISBN: 978-0-226-46800-6.

Misner, C.W., K.S. Thorne, and J.A. Wheeler (1973). *Gravitation*. Gravitation. LCCB: 78156043. W. H. Freeman. ISBN: 978-0-7167-0344-0.

Pais, Albert (1982). *'Subtle Is the Lord...' The Science and the Life of Albert Einstein*. Oxford University Press.

Price, Katy (2012). *Loving Faster than Light : Romance and Readers in Einstein's Universe /*. Chicago ; The University of Chicago Press, ISBN: 978-0-226-68073-6.

Renn, Jürgen, M. Janssen, and Matthias Schemmel, eds. (2007). *The Genesis of General Relativity*. Boston studies in the philosophy of science v. 250. Dordrecht: Springer. 4 pp. ISBN: 978-1-4020-3999-7 978-1-4020-4000-9.

Tolman, R.C. (1934). *Relativity, Thermodynamics and Cosmology*. International series of monographs on physics. LCCB: lc34032023. The Clarendon Press.

Wheeler, J.A. and K.W. Ford (1998). *Geons, Black Holes, and Quantum Foam: A Life in Physics*. LCCB: 97044566. Norton. ISBN: 978-0-393-04642-7.

Whitehead, A.N. (1926). *Science and the Modern World*.

Whitworth, Michael (1996). “The Clothbound Universe: Popularphysics Books, 1919-39.” In: *Publishing History* 40, p. 53.