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SCALE CHANGES AND SHARED INFORMATION IN BARGAINING: AN EXPERIMENTAL STUDY*

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This paper reports one of a series of experiments designed to test aspects of various game-theoretic models of bargaining. The results of this experiment consolidate those of previous experiments, which are first reviewed. The principal new result of this experiment is the observation, under conditions of partial information, of systematic violations of the axiom of independence of equivalent utility representations. The adequacy of various theories of bargaining to describe the observed data is also tested and discussed.

Keywords: Bargaining game; bargaining solution; independence of equivalent utility representations.

1. Introduction

This paper concerns the family of bargaining theories which, in the tradition begun by Nash (1950), model a bargaining game entirely in terms of the von Neumann–Morgenstern utilities which the players can achieve. Because an individual's von Neumann–Morgenstern utility function is a numerical representation of his choice behavior containing certain arbitrary features, a theory which is intended to depend only on the choice behavior of the players as represented by their utility functions must be independent of those arbitrary features. However, a theory which is independent of the arbitrary features of the players' utility functions, but which is defined entirely in terms of those utility functions, must also be insensitive to certain substantive features of the underlying set of alternatives. One purpose of this paper is to investigate experimentally how these substantive features influence the outcome of bargaining, and to consider the consequences of these findings for theories of bargaining.

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The paper is organized as follows. Section 2 is devoted to a review of Nash's model of bargaining, and discusses the assumptions which are related to the issues we will consider. Section 3 reviews two earlier experimental studies, by Nydegger and Owen (1975); and by Roth and Malouf (1979). Section 4 reports a new experiment, designed to answer questions raised by the previous two studies. Section 5 discusses two bargaining solutions which have appeared in the literature and which seem appropriate for describing some aspects of the experimental results, and reports some additional experimental data which permits us to distinguish between these two solutions.

2. Nash' model of bargaining

Following Nash, we will consider two-player bargaining games defined by a pair (S, d), where d is a point in the plane, and S is a compact convex subset of the plane which contains d and at least one point x such that $x > d$. The interpretation is that S is the set of feasible expected utility payoffs to the players, any one of which can be achieved if it is agreed to by both players. If no such agreement is reached, then the disagreement point d is the result.

Nash proposed that bargaining between rational players be modelled by means of a function called a *solution*, which selects a feasible outcome for every bargaining game. That is, if we denote the class of all two-player bargaining games by B, a solution is a function $f: B \to R^2$ such that $f(S, d)$ is an element of S. Nash further proposed that a solution should possess the following property.

Property 2.1 (Independence of equivalent utility representations). *If* (S, d) *and* $(\hat{S}, \hat{d})$ *are bargaining games such that* $\hat{S} = \{(a_1 x_1 + b_1, a_2 x_2 + b_2) \mid (x_1, x_2) \in S\}$ *and* $\hat{d} = (a_1 d_1 + b_1, a_2 d_2 + b_2)$ *where* a_1, a_2, b_1 *and* b_2 *are numbers such that* a_1 *and* $a_2 > 0$, *then* $f(\hat{S}, \hat{d}) = (a_1 f_1(S, d) + b_1, a_2 f_2(S, d) + b_2)$.

In order to understand the significance of this property, we need to consider the set of underlying alternatives over which the bargaining is conducted. Suppose that two individuals are bargaining over some set of alternatives A, containing some prespecified disagreement outcome a^* (Note 1; see section Notes at the end of this paper). Then if these individuals have utility functions u_1 and u_2 over the set A, the resulting bargaining game (S, d) is given by

$$S = \{(u_1(a), u_2(a)) \mid a \in A\}, \qquad d = (u_1(a^*), u_2(a^*)). \tag{1}$$

Recall that an individual i's utility function u_i is a real-valued function defined on the set of alternatives A. It is a model of his choice behavior, in the sense that $u_i(a) > u_i(b)$ for two alternatives a and b if and only if he prefers a to b; i.e., if and only if he would chooose alternative a when faced with the choice between a and b. Von Neumann and Morgenstern (1944) demonstrated conditions on an individual's preferences which are sufficient so that his choice behavior over risky alternatives is the same as if he were maximizing the expected value of his utility function. Such a utility function is uniquely defined only up to an interval scale, which is to say that the origin (zero point) and unit of the utility function are arbitary. Thus if u_i is an expected utility function representing individual i's preferences, then another utility function v_i represents the same preference if and only if $v_i = a_i u_i + b_i$, where a_i is a positive number.

So Property 2.1 states that if a game $(\hat{S}, \hat{d})$ is derived from (S, d) by transforming the utility functions of the players to equivalent representations of their preferences, then the same transformations applied to the outcome of the game (S, d) should yield the outcome selected in $(\hat{S}, \hat{d})$. That is, if $(\hat{S}, \hat{d})$ is given by

$$\hat{S} = \{(v_1(a), v_2(a)) \mid a \in A\}, \qquad \hat{d} = (v_1(a^*), v_2(a^*)), \tag{1'}$$

where $v_i = a_i u_i + b_i$ for $i = 1, 2$, and if a solution f yields $f(S, d) = (u_1(b), u_2(b))$, then Property 2.1 requires that $f(\hat{S}, \hat{d}) = (v_1(b), v_2(b))$; i.e., that the payoff predicted by the solution f should correspond to the same alternative b in both games. Thus Property 2.1 states that a solution should depend only on the preferences of the players, and not on the arbitrary features of the utility functions representing those preferences.

Nash proposed that a solution should also possess the following three properties (Note 2).

Property 2.2 (Pareto optimality). *If $f(S, d) = x$ and $y \geq x$, then either $y = x$ or $y \notin S$.*

Property 2.3 (Symmetry). *If (S, d) is a symmetic game (i.e., if $(x_1, x_2) \in S$ implies $(x_2, x_1) \in S$ and if $d_1 = d_2$), then $f_1(S, d) = f_2(S, d)$.*

Property 2.4 (Independence of irrelevant alternatives). *If (S, d) and (T, d) are* bargaining games such that T contains S, and if $f(T, d) \in S$, then $f(S, d)) = f(T, d)$.

Nash proved the following famous result.

Theorem 2.5. *There is a unique solution which possesses Properties 2.1–2.4. It is the solution F defined by $F(S, d) = x$ such that $x \geq d$ and $(x_1 - d_1)(x_2 - d_2) > (y_1 - d_1)(y_2 - d_2)$ for all y in S such that $y \neq x$ and $y \geq d$.*

Nash's solution F selects the outcome which maximizes the geometric average of the gains available to the bargainers over the set of feasible, individually rational outcomes.

3. Some empirical questions

Suppose that we have a theory of bargaining, embodied in a solution to the bargaining problem which is independent of equivalent utility representations (e.g., Nash's solution). Since a solution depends only on the utility payoffs available to the players, it yields the same prediction for a given game (S,d) no matter how that game arises; e.g., whether the game arises from bargaining over a set of alternatives A by individuals with utility functions u_1 and u_2, or from bargaining over an entirely different set of alternatives B by individuals with appropriate utility functions. Similarly, if $(\hat{S},\hat{d})$ is related to (S,d) as in the statement of Property 2.1, then the solution yields predictions for the two games which are related as in Property 2.1, regardless of whether $(\hat{S},\hat{d})$ differs from (S,d) only by a purely formal transformation (as in (1) and (1′)), or whether the two games have substantive differences, as when they arise from bargaining over a different set of alternatives.

Thus, to the extent that appropriate games can be constructed, experiments can be conducted to test the predictive value of solutions which are independent of equivalent utility representations. Reviewed below are two experiments designed with this in mind.

3.1. The experiment of Nydegger and Owen (1975)

A straightforward experiment was conducted by Nydegger and Owen (1975), who proposed to test each of Nash's properties by observing the results of a series of simple bargaining situations. In their experiment, 30 pairs of undergraduates each participated in a single bargaining encounter involving the distribution of monetary payoffs, about which they were fully informed. Nydegger and Owen interpreted the monetary payoffs in these games as being identical to the utility received by the bargainers (Note 3). In each of the three conditions of the experiment, the bargaining was conducted verbally and face-to-face, with the bargainers seated together at a table.

In their first condition, 10 pairs of bargainers were simply asked to split one dollar. They could divide the dollar any way they wished if they reached an agreement, but if they failed to reach an agreement neither of them would receive any share of the dollar. Nydegger and Owen report that each of the 10 pairs of bargainers reached agreement on dividing the dollar in half, so that each bargainer received 50 cents. Since the 50-50 split is the unique outcome which is both symmetric and Pareto optimal, this outcome is consistent with Nash's solution.

In their second condition, 10 pairs of bargainers were asked to split a dollar under the same rules, but with the additional restriction that one of them ('player B') would not be allowed to receive more than 60 cents, while the other ('player A') was under no such restriction. Thus the set of feasible agreements in this condition is a subset of those in the last condition. Once again, each of the 10 pairs of bargainers divided the dollar in half, 50 cents each. This outcome agrees with Nash's solution, and is consistent with the property of independence of irrelevant alternatives, which specifies that the potential agreements eliminated by the restriction should be irrelevant to the outcome.

In the third condition, 10 pairs of bargainers were asked to divide 60 poker chips, which would have a cash value of 1 cent per chip for player A, and 2 cents per chip for player B. Thus the maximum feasible payoffs to players A and B would be 60 cents and 120 cents, respectively, with the rate of exchange between them being 1 cent to 2 cents. That is, the set of feasible payoffs in this condition can be derived from the feasible set of payoffs in the first condition by multiplying player A's payoffs by 0.6 and player B's by 1.2. Nydegger and Owen report that in this condition all 10 bargaining pairs agreed to divide the chips in such a way as to give each player an equal monetary payoff: i.e., 40 chips for player A and 20 chips for player B, so that each player received 40 cents. This is contrary to the predictions of Nash's solution. Indeed, any symmetric and Pareto optimal solution which is also independent of positive linear transformations of the payoffs (i.e., which possesses property 1) must in this condition give players A and B respectively 0.6 and 1.2 times their payoffs in the first condition. That is, Nash's solution predicts that in this condition player A would receive 30 cents and player B 60 cents, which is the payoff that would result if each player received 30 chips.

Nydegger and Owen interpret these results as supporting the proposition that bargaining behavior is symmetric, Pareto optimal and independent of irrelevant alternatives, while contradicting the proposition that it is independent of equivalent utility representations. Specifically, in each of their games, the bargainers reached agreements which gave them equal monetary payoffs.

This supports the conclusion that, in bargaining for money with full information about payoffs, the scale of the monetary payoffs available has an effect on the agreements reached, and that comparison of the monetary payoffs received by each player plays a role in determining the outcome. The same conclusion is supported by the results of several other experiments in which the participants bargained for money. (A review of these and other experiments can be found in Roth and Malouf (1979, pp. 579–582).

3.2. *The experiment of Roth and Malouf* (1979)

In Roth and Malouf (1979) we reported an experiment designed to permit somewhat stronger conclusions to be drawn, by establishing more closely controlled experimental conditions. The primary aim of the experiment was to investigate the manner in which the information shared by the bargainers interacted with changes in the scale of the monetary payoffs available to each player. In order to be able to interpret the data unambiguously, the experiment was designed to permit the utility payoffs to be determined even when the players do not have utility functions which are linear in money. In order to explain how this was accomplished, it will be helpful to recall precisely what information is contained in an expected utility function.

Consider the case in which the set A of alternatives contains elements a and c such that a is strictly preferred to c, and for any alternative $b \in A$, the player likes a at least as well as b, and b at least as well as c. Then if u is a utility function representing this individual's preferences over the set of alternatives A, it must have the property that $u(a) \geq u(b) \geq u(c)$. Since u is defined only up to an interval scale, we may arbitrarily choose its unit and zero point, and in particular we may take $u(a) = 1$ and $u(c) = 0$. The problem of determining $u(b)$ then becomes the problem of finding the appropriate value between 0 and 1 so that all those lotteries over alternatives that the individual prefers to b have a higher expected utility, and all those lotteries to which b is preferred have a lower exptected utility. If we denote by $L(p) = [pa; (1-p)c]$ the lottery that with probability p yields alternative a and with probability $(1-p)$ yields alternative c, then the utility of participating in the lottery $L(p)$ is its expected utility, $pu(a) + (1-p)u(c) = p$. If p is the probability such that the individual is indifferent between b and $L(p)$, then their utilities must be equal, and so, $u(b) = p$. Thus when we say that the utility of alternative b to a given individual is known, we mean that the probability p is known such that the individual is indifferent between having alternative b for certain or having the risky alternative $L(p)$.

Binary lottery games

Since knowing an individual's expected utility for a given agreement is equivalent to knowing what lottery he or she thinks is as desirable as that agreement, in a bargaining game in which the feasible agreements are the appropriate kind of lotteries, knowing the utilities of the players at a given agreement is equivalent to simply knowing the lottery they have agreed on. In our experiment, therefore, each player i was told about 2 monetary prizes: a large prize $l(i)$ and a small prize $s(i)$. In each game the players bargained over the *probability* $p(i)$ that they would receive their large prize $l(i)$. Specifically, they bargained over how to distribute 'lottery tickets' that would determine the probability that each player would win his or her personal lottery (i.e., a player i who received 40% of the lottery tickets would have a 40% change of winning his or her large monetary prize $l(i)$ and a 60% change of winning his or her small prize $s(i)$). In the event that no agreement was reached in the allotted time, each player i received his or her small prize $s(i)$. In other words, a player would receive his large prize only if an agreement is reached on splitting the lottery tickets and if he wins the ensuing lottery. Otherwise he is always assured of getting his small prize. We will refer to games of this type as *binary lottery games* (Note 4).

To interpret the set of feasible outcomes of a binary lottery game in terms of each player's utility function for money, recall that if we consider each player's utility function to be normalized so that the utility for receiving his large prize is 1, and the utility for receiving his small prize is 0, then the player's utility for any lottery between these two alternatives is the probability of winning the lottery. That is, an agreement which gives player i $p(i)$ percent of the lottery tickets gives him a utility of $p(i)$ (Note 5).

Note that a change in the prizes is therefore equivalent to a change in the origin and scale of the player's utility functions. This makes it possible to use binary lottery games to experimentally investigate the circumstances under which the bargaining process can indeed be described by a solution which is independent of equivalent utility representations.

Design of the experiment

Each player played four games, in random order, against different opponents. Each player played all four games under one of two information conditions: *full information*, or *partial information*. In the full information condition, each player was informed of the value of his own potential prizes and of his opponent's potential prizes. In the partial information condition, each player was informed only of the value of his own prizes (Note 6). Players were seated at isolated computer terminals, and were allowed to communicate freely by teletype, but they were unaware of the identity of their opponents (Note 7).

In all four games, the small prize of both players was equal to zero. In game 1 no restriction was placed on the percentage of lottery tickets which each player could receive, and both players had the same large prize of $1.00. Game 2 was played with the same prizes as game 1, but one of the players (player 2) was restricted to receive no more than 60% of the lottery tickets. Game 3 was played with the same rules as game 1, but with different large prizes for the two players: $1.25 for player 1, and $3.75 for player 2. Game 4 was played under the same rules as game 2, with the same prizes as game 3 (see Table 1).

Table 1
Large prizes and feasible distributions for games 1-4

Game	Large prize for player 1	Large prize for player 2	Maximum % allowed player 1	Maximum % allowed player 2
1	$1	$1	100	100
2	$1	$1	100	60
3	$1.25	$3.75	100	100
4	$1.25	$3.75	100	60

Note that game 1 is related to game 3, and game 2 is related to game 4 by a change in the prizes, and hence by a scale change as in Property 2.1. So if the bargaining process obeys Property 2.1, we should observe the same outcomes in these pairs of games. In fact, Nash's solution predicts that in each game the players will divide the lottery tickets equally, so that $p_1=p_2=50\%$.

If we denote the difference between the probabilities received by the two players by $D=p_1-p_2$, then Nash's solution predicts that we will observe $D=0$ in all four games. On the other hand, if we supposed that the players would reach agreements which equalized their expected monetary payoffs, then we would expect to observe $D=0$ for games 1 and 2, and $D=50$ for games 3 and 4 (corresponding to $p_1=75$, $p_2=25$), which would contradict the predictions of Property 2.1.

The principal hypothesis of the experiment was that the observed outcomes would be consistent with Property 2.1 in the partial information condition, but not in the full information condition. The data, presented in Table 2, supports that hypothesis.

Table 2
Means and standard deviations for $D=p_1-p_2$

Statistic \ Game	1	2	3	4
Full information (11 pairs)				
M	0.00	1.91	34.60	21.64
SD	0.00	12.17	19.28	22.48
Partial information (8 pairs)				
M	0.00	-1.32	-2.50	2.50
SD	0.00	8.33	4.63	4.11

Statistical analysis confirms that, in the partial information condition D is not significantly different between game 1 and 3, or between games 2 and 4, while in the full information condition these differences are significant. This offers tentative support to the conclusion that the bargaining process obeys Property 2.1 in the partial information condition, but it clearly demonstrates that this is not the case in the full information condition. In the full information condition the observed outcomes show a distinct shift in the direction of equal expected income - i.e., the fact that D is significantly greater in game 3 than in game 1, and in game 4 than in game 2 in this condition reflects the fact that, of the two players, the player with the smaller prize tended to receive a higher probability of receiving his prize.

4. A new experiment

The results reported in Roth and Malouf (1979) clearly show that the effect of scale changes in the underlying alternatives in a bargaining game interacts with the information shared by the bargainers. However, these results also raise a number of questions, particularly when considered together with the results reported by Nydegger and Owen.

First, we need to be able to account for the difference observed between the results in the full information condition of Roth and Malouf and those of Nydegger and Owen, since in both cases the players bargained with full information about the monetary value of their opponent's alternative. Although both studies reported a strong tendency for the observed agreements to be those having equal monetary value to both players, Nydegger and Owen reported zero variance in their results, while the results of Roth and Malouf show considerable variance. One possible cause for this difference is that the bargaining in the Roth and Malouf study was conducted anonymously through computer terminals, while in the Nydegger and Owen study, bargaining was conducted face-to-face (Note 8). One goal of the experiment reported here was to clarify this issue.

More importantly, the data from the Roth and Malouf study, while demonstrating that there is a tendency towards equal-payoff agreements in the full information condition, is still open to conflicting interpretations in the partial information condition. One hypothesis consistent with the data is that, in bargaining situations resembling the partial information condition, the bargaining process does indeed possess Property 2.1, so that Nash's solution, for example, might approximately describe the kind of agreements which will be observed. An alternative hypothesis, also consistent with the data, is that, in bargaining situations resembling the partial information condition, the agreements reached will tend to be those which give the bargainers equal shares of the commodity being divided (e.g., equal percentages of lottery tickets in the Roth and Malouf study (Note 9). If this latter hypothesis is correct, then we would expect that violations of Property 2.1 could be consistently observed, if changes of scale were implemented in an appropriate manner. The experiment described below is designed to help distinguish between these two hypotheses.

Design of the experiment

Each player played four games, in random order, against different opponents. Each game involved the distribution of a commodity ('chips') having a monetary value to the players. Each player played all four games under one of the following four experimental conditions.

(1) Full information; equal payoff.
(2) Partial information; equal payoff.
(3) Full information; unequal payoff.
(4) Partial information; unequal payoff.

The experimental variables thus consisted of 'information' and 'payoff'. The information variable was defined by:

(i) Full information; both players knew the monetary value of each other's chips.
(ii) Partial information; each player knew the monetary value of his own chips only.

The payoff variable was defined by:

(i) Equal payoff; both players received 1 cent per chip.
(ii) Unequal payoff; player one received 1 cent per chip, player two received 3 cents per chip.

Thus, the first two conditions gave the players equal payoff, i.e. 1 cent per chip for both players, while the last two conditions had unequal payoff, giving the first player 1 cent per chip and the second player 3 cents per chip. The first and third conditions gave the players full information, i.e. the players knew the monetary value of each other's chips, while conditions 2 and 4 had only partial information, with each player knowing how many chips were available to his opponent, but only knowing the value of his own chips.

The set of feasible distributions of chips for each of the four games is shown in Fig. 1. To help in the analysis, the games can be classified as allowing equal or unequal 'tradeoffs' (depending on the slope of the Pareto surface) and as being 'restricted' or 'unrestricted' (depending on the size of the feasible set), as summarized in Table 3. (The restricted games are included to provide a test of the property of independence of irrelevant alternatives.)

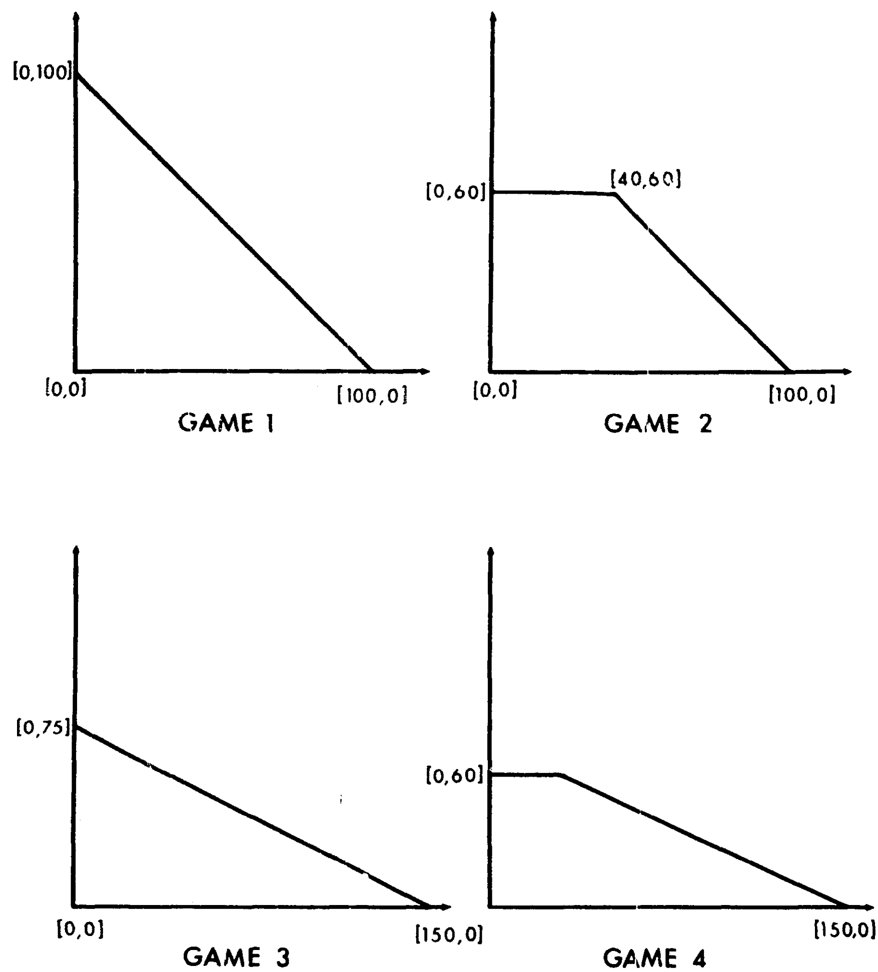

Fig. 1

Table 3

	Trade-off	Restricted	Feasible region[a]	Max chips allowed	
				player 1	player 2
Game 1	equal	no	$x_1+x_2\leq 100$	100	100
Game 2	equal	yes	$x_1+x_2\leq 100$	100	60
Game 3	unequal	no	$x_1+2x_2\leq 150$	150	75
Game 4	unequal	yes	$x_1+2x_2\leq 150$	150	60

[a] x_1: chips of player 1; x_2: chips of player 2.

Note that, in ranges for which utility can be taken to be linear in money, there is more than one way in which a scale change of the kind considered in Property 2.1 can be implemented. Specifically, in this experiment, the relevant kind of scale changes are implemented in two distinct ways. On the one hand, the change from an equal trade-off game to an unequal trade-off game involves transforming the bargaining region by multiplying player 1's payoffs by 1.5 and player 2's by 0.75. Game 1 is related to game 3 in this way. On the other hand, the change from equal to unequal payoff conditions (i.e., from conditions 1 and 2 to 3 and 4) involves multiplying player 1's payoffs by a factor of 3. Whereas the first kind of scale change is visible to both players under both information conditions, the second is only fully apparent to both players in the full information conditions.

The methods by which this experiment was implemented, described below, are essentially the same as those used in the Roth and Malouf study described in the previous section. (The major differences are those associated with the fact that, in this experiment, the bargaining concerned the division of chips rather than lottery tickets.)

Methods

Each participant was seated at a visually isolated terminal of a computer-assisted instruction system developed at the University of Illinois, called PLATO, whose features include advanced graphic displays and interactive capability. The experiment was conducted in a room containing over 70 terminals, most of which were occupied at any given time by students uninvolved in this experiment. Participants were seated by the experimenter in order of their arrival at scattered terminals throughout the room, and for the remainder of the experiment they received all of their instructions, and conducted all communication, through the terminal.

The subject pool was from an introductory business administration course mostly take by college sophomores. Pretests were run with the same subject pool to make sure that the instructions to participants were clear and easily understandable.

Background information was first presented. The main tools of the bargaining were then introduced: these consisted of messages or proposals. A proposal was a pair of numbers, the first of which was the number of chips demanded by the sender and the second was the number of chips being offered to the receiver. The use of the computer enabled any asymmetry in the presentation to be avoided. PLATO also computed the monetary value of each proposal and displayed the proposal on a graph of the feasible region. After being made aware of these computations, the bargainer was given the option of cancelling the proposal before its transmittal. Proposals were binding on the sender, and an agreement was reached whenever one of the bargainers returned a proposal identical to the one he had just received.

Messages were not binding. Instead, they were used to transmit any thoughts which the bargainers wanted to convey to each other. To insure anonymity, the monitor intercepted any messages that revealed the identity of the players. In the partial information condition the monitor also intercepted messages containing information about the available prizes. The intercepted message was returned to the sender with a heading indicating the reason for such action.

To verify their understanding of the basic notions, the subjects were given some drills followed by a simulated bargaining session with the computer. As soon as all the participants finished this portion of the experiments, they were paired at random and the bargaining started.

At the end of 12 minutes or when agreement was reached (whichever came first), the subjects were informed of the results of that game and were asked to wait until all the other bargainers were finished. For the subsequent game there were new random pairings, and the bargaining resumed. The cycle continued until all four games were completed. At no point in the experiment were the players aware of what the other participants were doing, or of the identity of their opponents.

The bargaining process consisted of the exchange of messages and proposals, and participants were instructed that "your objective should be to maximize your own earnings by taking advantage of the special features of each session". Only if the bargainers reached agreement on what number of chips each would receive were they able to get any payoff for the particular game being played. All transactions were automatically recorded.

After each game was completed, each player was informed of the outcome and the amount of his winnings. A brief explanation of the purpose of the experiment was given at the end, and subjects were offered the opportunity to type any comments, questions etc., and were directed to the monitor who paid them.

Results

The data were analyzed in terms of the quantity D, defined as the number of chips received by player 1 minus the number of chips received by player 2. The only potential disadvantage of using D is that it conceals non-Pareto optimal agreement, i.e., a (60,40) agreement and a (50,30) agreement would both yield $D=20$. Since no appreciable difference was found between analyses that included and excluded the non-Pareto optimal agreements, this potential difficulty had little effect (Note 10).

There were 7 disagreements out of a total of 184 games played (4%), and of the 177 agreements, 164 were Pareto optimal (92.6%) (Note 11). A 2 (information) $\times$ 2 (payoff) $\times$ 2 (restriction) $\times$ 2 (trade-off) analysis of variance on the number of disagreements and the number of non-Pareto optimal outcomes showed no significant effects and no significant interactions. We may therefore neglect the effect of disagreements and non-Pareto optimal agreements in subsequent analysis. The unaggregated data is shown in Table 6. The means and standard deviations of D are shown in Table 4 (Note 12).

Note that, in all conditions except the full information; unequal payoff condition, D is near zero, indicating that in these conditions the agreements reached tended to divide the chips equally between the two bargainers.

A 2 (information) $\times$ 2 (payoff) $\times$ 2 (restriction) $\times$ 2 (trade-off) analysis of variance on D showed a significant main effect for information, $F(1,167)=113.74$,

Table 4
Means and standard deviations for D

	Games			
	1	2	3	4
	Equal trade-off		Unequal trade-off	
	Unrestricted	Restricted	Unrestricted	Restricted
Full info equal pay (11 pairs)	0.00 (4.47)	1.82 (6.03)	2.09 (5.39)	1.36 (4.52)
Partial info equal pay (11 pairs)	2.00 (6.00)	0.91 (3.02)	7.27 (10.81)	2.09 (4.91)
Full info unequal pay (12 pairs)	35.36 (19.52)	37.17 (20.14)	44.75 (31.29)	51.50 (17.62)
Partial info unequal pay (12 pairs)	0.00 (0.00)	1.17 (3.01)	3.75 (9.32)	2.50 (8.66)

$p<0.001$, for payoff, $F(1,167)=109.37$, $p<0.001$, and for trade-off, $F(1,167)=6.429$, $p<0.012$. The effect of trade-off was probably cumulative since it yields no significant effect when analyzed across experimental conditions. The only significant interaction was that of information X payoff, $F(1,167)=123.94$, $p<0.001$.

A Newman–Keuls post hoc test was performed on the information X payoff interaction, showing that the full information-unequal payoff condition is significantly different from the other 3 conditions, which are not significantly different from each other (see Table 5, in which cells with common superscripts are not significantly different from one-another using the Newman–Keuls test). Under this condition the values of D seem to lie somewhere between the equal division of chips and the equal division of money. Of the 41 Pareto optimal agreements under the full information and unequal payoff condition, 37 (90%) were situated between the point of equal chips and that of equal monetary payoffs. (Equal monetary payoffs in the equal tradeoff games would result from agreements giving 75 chips to player 1 and 25 chips to player 2, with $D=50$. In the unequal tradeoff games, equal monetary payoffs come from a (90, 30) division of chips, with $D=60$.)

Table 5

Information \ Payoff	Equal	Unequal	Mean
Full	1.32[a]	42.34[b]	22.51
Partial	3.07[a]	1.85[a]	2.43
Mean	2.19	21.88	

The property of independence of irrelevant alternatives is supported since there was no significant effect for restriction. Under the partial information condition, the findings confirm that no change in the division of chips occurs when the payoff is changed; this is consistent with the results of Roth and Malouf (1979), since the information about any payoff differences is not available to both players.

Discussion

Note that the experiment of Nydegger and Owen (1975) is essentially paralleled here by games 1 and 2 under the condition of full information-equal payoff together with game 1 under the condition of full information-unequal payoff, the chief difference being that, in this study, the games were played anonymously. The four games considered by Roth and Malouf (1979) under conditions of both full and partial information, are essentially paralleled here by games 1 and 2 in the two equal payoff conditions and in the two unequal payoff conditions (Note 13), the chief difference being that, in this study, the games involve a division of a valuable commodity (chips) instead of lottery tickets.

Thus the results concerning games 1 and 2 (Table 6) in this experiment consolidate the results of these two previous studies (Note 14). These results provide strong support for the hypothesis that, in the full information condition of this experiment, there is a marked tendency for agreements to move in the direction of equal monetary gains for the players (Note 15). Thus, under conditions of full information, the bargaining process can be consistently observed to violate Property 2.1.

The results concerning games 3 and 4 (Table 6) in this experiment permit us (for the first time) to draw similar conclusions about bargaining under the partial information conditions. Specifically, in the partial information conditions no significant differences were observed between the equal tradeoff games (1 and 2) and the unequal tradeoff games (3 and 4). (In fact, if we take $D=0$ for games 1 and 2 in the partial information conditions, then the prediction of Property 2.1 is that we should find $D=37.5$ for games 3 and 4, which lies outside of the 99.9% confidence interval based on the observations in these conditions. That is, the hypothesis that Property 2.1 is descriptive of the mean outcomes in this case can be rejected with $p<0.001$.)

Thus the evidence suggests that in the partial information conditions of this experiment, the agreements tend to be those which give each player an equal number of chips. This contrasts with the full information conditions, in which the agreements show a pronounced shift in the direction of those which give each player an equal monetary payoff.

5. Proportional versus maximin behavior

If we wish to use some solution to the bargaining problem to describe behavior of the sort reported in the previous two sections, it will have to be a solution which is not independent of the scale in which the payoffs are expressed. Any solution which selects outcomes giving the players equal payoffs according to some measure must be dependent on that measure.

Thus, for instance, in the full information conditions of the experiment reported

Table 6
Summary of final agreements

		Game 1 Player			Game 2 Player			Game 3 Player			Game 4 Player		
	Group	1	2	*D*	1	2	*D*	1	2	*D*	1	2	*D*
	3	55	45	10	50	50	0	60	45	15	50	50	0
		50	50	0	50	50	0	50	50	0	50	50	0
	8	50	50	0	50	50	0	45	50	-5	50	50	0
Full		50	50	0	50	50	0	50	50	0	50	50	0
info	10	50	50	0	50	50	0	50	50	0	50	50	0
-		0	0	0	60	40	20	50	50	0	60	45	15
Equal		50	50	0	50	50	0	50	50	0	50	50	0
pay	16	50	50	0	50	50	0	55	47	8	50	50	0
		45	55	-10	50	50	0	50	50	0	50	50	0
		50	50	0	0	0	0	50	50	0	0	0	0
		50	50	0	50	50	0	50	45	5	45	45	0
	4	50	50	0	50	50	0	50	50	0	50	50	0
		51	49	2	55	45	10	70	40	30	50	50	0
	5	50	50	0	50	50	0	50	50	0	50	50	0
Partial	11	50	50	0	50	50	0	50	50	0	50	50	0
info		50	50	0	50	50	0	60	45	15	50	50	0
-	15	50	50	0	50	50	0	50	50	0	50	50	0
Equal		50	50	0	0	0	0	50	50	0	50	50	0
pay	18	50	50	0	50	50	0	60	45	15	50	50	0
		50	50	0	50	50	0	60	40	20	60	45	15
		60	40	20	50	50	0	50	50	0	55	47	8
		50	50	0	50	50	0	50	50	0	50	50	0
	2	75	25	50	78	22	56	90	30	60	92	29	63
		60	40	20	75	25	50	90	30	60	90	30	60
	9	50	50	0	75	25	50	20	65	-45	60	45	15
Full		70	25	45	0	0	0	90	30	60	90	30	60
info		0	100	-100	75	25	50	75	25	50	90	30	60
-	13	0	0	0	75	25	50	90	30	60	90	30	60
Unequal		74	26	48	60	40	20	90	30	60	90	30	60
pay		75	25	50	50	50	0	88	31	57	80	35	45
	17	72	28	44	70	30	40	90	30	60	90	30	60
		70	30	40	75	25	50	90	30	60	90	30	60
		70	30	40	65	35	30	75	35	40	60	45	15
		76	24	52	75	25	50	60	45	15	80	20	60
	1	50	50	0	50	45	10	50	50	0	50	50	0
	7	50	50	0	50	50	0	50	50	0	40	40	0
		50	50	0	50	50	0	40	40	0	50	50	0
Partial	12	50	50	0	50	50	0	45	45	0	70	40	30
info		50	50	0	50	50	0	70	40	30	50	50	0
-	14	50	50	0	52	48	4	50	50	0	50	50	0
Unequal		50	50	0	0	0	0	50	50	0	50	50	0
pay		50	50	0	50	50	0	60	45	15	50	50	0
	19	50	50	0	50	50	0	50	50	0	50	50	0
		50	50	0	50	50	0	50	50	0	50	50	0
		50	50	0	50	50	0	50	50	0	50	50	0
		50	50	0	50	50	0	50	50	0	50	50	0

in Section 4, if (S,d) represents one of the bargaining games in terms of the feasible *monetary* payoffs to the players (Note 16), then the observed outcomes tended in the direction of the Pareto optimal outcome $x=(x_1,x_2)$ such that $x_1=x_2$. The same behavior was observed in the study of Roth and Malouf (1979) where (S,d) represents the feasible *expected* monetary payoffs to the players.

However, in the partial information conditions of the experiment reported in Section 4, the bargaining game would have to be represented in terms of the number of chips each player could receive, in order to be able to describe the outcomes as giving the players equal payoffs. That is, if $(\hat{S},\hat{d})$ represents one of the bargaining games in terms of the payoffs to the players in chips, then the observed outcomes tended to be close to the Pareto optimal outcome $y=(y_1,y_2)$ such that $y_1=y_2$. The same behavior was observed in the study of Roth and Malouf (1979) where $(\hat{S},\hat{d})$ is expressed in terms of the probability which each player i has of winning his prize $l(i)$; i.e., in terms of the payoff to the players in lottery tickets (Note 17).

Of course, equalizing behavior of this sort can arise in a number of different ways. Two solutions which can be interpreted as describing the observed data have been studied in the literature: the *proportional solution* with equal weights (Kalai, 1977; Myerson, 1977; Roth, 1979a;b) and the *maximin* (or *equal gains*) solution (Roth and Malouf, 1979; Roth, 1979b). The equally weighted proportional solution P is defined for any game (S,d) as follows.

$$\begin{aligned}P(S,d)=x \text{ such that } & x_1-d_1=x_2-d_2 \text{ and}\\ & x\geq y \text{ for all } y \text{ in } S \text{ such that } y_1-d_1=y_2-d_2.\end{aligned}$$

That is, P selects the outcome which gives the players the greatest payoff consistent with the constraint that the gains of the two players relative to their disagreement payoffs are exactly equal.

The maximin solution E is defined for any game (S,d) as follows.

$$\begin{aligned}E(S,d)=x \text{ such that } & x \text{ is Pareto optimal in } S \text{ and}\\ & \min\{x_1-d_1,x_2-d_2\}\geq\min\{y_1-d_1,y_2-d_2\} \text{ for all } y \text{ in } S.\end{aligned}$$

That is, $E(S,d)=P(S,d)$ when $P(S,d)$ is Pareto optimal in S; otherwise $E(S,d)$ is the Pareto optimal point (Note 18) in S which comes closest to giving the players equal gains (Note 19).

Thus, in games which have no Pareto optimal outcome yielding equal gains to the players, the solution P sacrifices Pareto optimality in order to achieve exact equality, while the solution E sacrifices equality for Pareto optimality. However, all the data reported so far have concerned games on which P and E coincide, and so the results of these experiments can be summarized in terms of either solution. In order to distinguish between these two kinds of equalizing behavior, we need to consider games for which P and E yield different predictions.

Data from two such games are reported below. (These games were part of a larger study, concerned with the frequency of disagreement in bargaining, which is reported in Malouf and Roth (1981).) Both games were binary lottery games conducted under the condition of partial information described in Section 3, using the methods of Roth and Malouf (1979), which are essentially those described here in Section 4. Each player played four games (each of which ended after at most eight minutes), in random order, against different, anonymous opponents. The two games for which the solutions E and P give distinct predictions are described in Table 7.

Table 7

	Small prizes	Large prizes	Feasible region	E	P
Game A	$s(1)=s(2)=\$5$	$l(1)=l(2)=\$10$	$x_1 \le 60,\ x_2 \le 30$ $x_1, x_2 \ge 0$	(60, 30)	(30, 30)
Game B	$s(1)=s(2)=\$5$	$l(1)=l(2)=\$10$	$x_1+x_2 \le 90,\ x_2 \le 40$ $x_1, x_2 \ge 0$	(50, 40)	(40, 40)

Both games give each player a small prize of \$ 5, and a large prize of \$ 10. In game A (Note 20) the unique (strongly) Pareto optimal point is the outcome (60, 30), which gives player 1 a 60% chance and player 2 a 30% chance of winning the large prize. The solution E thus selects (60, 30) as the outcome of game A, while the solution P selects the outcome (30, 30). In game B, the set of (strongly) Pareto optimal outcomes is the line segment joining the points (90, 0) and (50, 40). The solution E thus selects the outcome (50, 40) in game B, while the solution P selects the outcome (40, 40). The observed outcomes are given in Table 8.

Table 8
Final outcomes

	Game A		Game B	
	Player		Player	
Group	1	2	1	2
1	30	30	40	40
	60	30	0	0
	60	30	50	40
2	60	30	50	40
	60	30	0	0
	60	30	50	40
	60	30	55	35
3	60	30	50	40
	60	30	50	40
	60	30	54	36
	60	30	0	0
4	60	30	50	40
	40	30	39	39
	60	30	59	31
	60	30	50	40
5	60	30	50	40
	60	30	50	40
	60	30	0	0
	60	30	50	40

In 19 trials of game A, the agreement predicted by E was observed 17 times, the agreement predicted by P was observed 1 time, 1 other agreement was observed, and there was 0 disagreements. In 19 trials of game B, the agreement predicted by E was observed 10 times, the agreement predicted by P was observed 1 time, 4 other agreements were observed, and there were 4 disagreements.

Thus, while the agreements selected by both the solutions E and P were observed in both games, E was observed at least ten times more frequently than P. This data thus lends support to the hypothesis that the equalizing behavior reported earlier in this paper bears more resemblance to the behavior modelled by the solution E than to that modelled by the solution P.

6. Summary

The evidence discussed here clearly demonstrates that pronounced and systematic violations of Property 2.1 can be observed in bargaining under conditions both of partial and full information. For bargaining over the division of a single commodity (e.g. lottery tickets or chips) under conditions of partial information, the observed outcomes have been closely approximated by the maximin solution E, acting on games (S, d) defined in terms of the commodity being divided.

For bargaining under conditions of full information the situation seems somewhat more complicated. Except for the data collected by Nydegger and Owen from face-to-face encounters, the outcomes observed under this condition have exhibited relatively high variance. For bargaining games under full information, concerned with the division of a single commodity of the sort discussed here, the mean outcome seems to fall about half-way between an equal division of the commodity (lottery tickets or chips) and an equal division of the (expected) monetary value to the players. That is, the mean outcome which we observed under this condition for the games reported here can be roughly approximated by a convex combination (Note 21) $\frac{1}{2}E(S,d)+\frac{1}{2}E(\hat{S},\hat{d})$, where (S,d) represents the game in terms of the (expected) monetary payoffs to the players, and $(\hat{S},\hat{d})$ represents the game in terms of the commodity payoffs.

Notes

Note 1. That is, the rules of the game are that any alternative a in the set A will be the outcome of the game if both players agree on it, otherwise a^* will be the outcome. Thus the rules on the game give both players a veto over any outcome other than a^*.

Note 2. These properties have been discussed at length elsewhere (cf. Roth (1979b)), so we will forego further discussion.

Note 3. That is, they implicitly assumed that the utility functions of all the bargainers were linear in money.

Note 4. Other experimental investigations of bargaining which employ binary lottery games are reported in Roth, Malouf and Murnighan (1981), Roth and Murnighan (1982), and Roth and Schoumaker (1982).

Note 5. Note that we are considering the feasible set of utility payoffs to be defined in terms of the utility function of each player for the lottery which he receives, independently of the bargaining which has taken place to achieve this lottery, and even independently of the lottery which his opponent receives. In doing so we are taking the point of view that, while the progress of the negotiations may influence the utilities of the bargainers for the agreement eventually reached, the description of any effect which this has on the agreement reached belongs in the model of the bargaining *process*, rather than in the model of the bargaining *situation*. Considerable confusion in the literature has resulted from attempts to interpret bargaining models in terms of the players' utilities for outcomes *after* the bargaining has ended, since no bargaining model can be falsified by experimental evidence if, after an outcome has been chosen, the utilities of the players can be interpreted as having changed in whatever way is necessary to be consistent with the model. In order to have predictive value, bargaining theories must be stated in terms of parameters which can be measured independently of the phenomena which the theories are designed to predict.

Note 6. Note that in both the full information and in the partial information conditions, the resulting games meet the usual assumption that the game is one of *complete* information: i.e., both players have sufficient information to determine one another's expected utility for every outcome. Of course in the full information condition, the players have additional information, since they know one another's monetary payoffs as well.

Note 7. The methods by which these conditions were implemented in practice will be discussed in Section 4, since they are essentially the same as those used in the experiment discussed there.

Note 8. It has been well established by social psychologists (cf. Murnighan (1981)) that face-to-face encounters exert considerable social pressure. Thus, when the potential monetary awards are quite small (as they need to be if the approximation that the bargainers have linear utility functions is to be accurate), it is likely that the primary incentives to the bargainers arise from the social situation rather than from the monetary awards, when bargaining is face-to-face.

Note 9. At least in those cases in which the bargainers have utility functions which are approximately linear in the commodity being divided.

Note 10. If disagreements are not excluded, they would add to $D=0$, but in this experiment their number is small enough to be disregarded, and their effect is not significant as indicated by the analysis of variance that follows.

Note 11. Of all outcomes, 89.3% were Pareto optimal.

Note 12. The means and standard deviations are reported after the removal of an outlier ($D=-100$ resulting from a (0,100) agreement) that is 6.9 standard deviation from the mean.

Note 13. That is, games 1 and 2 in the full and partial information conditions of Roth and Malouf (1979) are paralleled here by games 1 and 2 in experimental conditions 1 and 2. Games 3 and 4 in Roth and Malouf (1979) are similarly paralleled here by games 1 and 2 in experimental conditions 3 and 4.

Note 14. The fact that these results are qualitatively so similar to those of Roth and Malouf (1979) suggests that any non-linearity in the players' utility for money was sufficiently small in this experiment so as not to introduce any noticeable distortions. Also, it tends to confirm the hypothesis that the absence of variance in the results of Nydegger and Owen (1975) is attributable to the face-to-face bargaining procedure which they employed.

Note 15. That is, under full information, the agreements reached were significantly closer to giving the players equal monetary payoffs than under partial information.

Note 16. That is, if $(x_1, x_2) \in S$ corresponds to an outcome at which players 1 and 2 receive x_1 dollars and x_2 dollars, respectively.

Note 17. Note that if $(\hat{S}, \hat{d})$ and (S, d) represent the same game, then $(\hat{S}, \hat{d})$ is related to (S, d) as in the statement of Property 2.1, but the outcomes x and y described above are not in general related in the same way. Thus, not only is Property 2.1 violated within each information condition, as we observed earlier, but it is also violated across information conditions.

Note 18. The definition of Pareto optimality which we are using (as defined in the statement of Property 2.2) is sometimes referred to as *strong* Pareto optimality.

Note 19. We have defined the solutions P and E here in terms of the *gains* rather than the absolute payoffs of the players. The alternative definition would also be consistent with the data, since in the experiments considered here, the disagreement payoffs of the players have been equal. The formulation used here is chosen merely for simplicity; the relative merit of the two formulations as descriptive theories of behavior remains an open question.

Note 20. The games named A and B here are referred to in Malouf and Roth (1981) as games 1 and 2.

Note 21. Of course, a convex combination of this sort is Pareto optimal in the games considered here only because the Pareto surface is linear, which occurs only so long as the utility functions of the players are linear in the commodity being divided.

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