Risk Aversion and the Negotiation of Insurance Contracts

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ABSTRACT

A game-theoretic model is used to study the effect of risk aversion on the outcome of bargaining over the terms of an insurance contract. When the insurer is risk netural, it prefers to bargain with the more risk averse of any two potential clients, since that client will agree to spend more, for less insurance, than will a less risk averse client. Bargaining over insurance contracts leads to results that differ from those obtained in a competitive insurance market. In a competitive market, clients seeking to insure against the same loss choose the same insurance contract, regardless of their risk posture.

Introduction

This paper uses a game-theoretic model to study the insurance contracts reached through direct negotiation, in a non-competitive context. In this situation, a single insurer insures many risks similar to and approximately independent of that being analyzed. This permits the insurer to diversify these risks, to behave as though it were risk neutral, and to insure at essentially fair rates. The client, on the other hand, faces a relatively small number of these risks and therefore cannot self-insure at fair rates. The client therfore bargains as though he or she were risk averse. The fact that clients bargain rather than behaving competitively is justified if the client is a relatively large client who does repeat business and who faces large risks.[1] Industrial insurance of large companies or marine insurance of oil tankers are examples of situations to which this analysis might, at least as an approximation, be applied.

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[1] Competitive behavior is price taking. In other words, a client who behaves competitively takes prices as given.

In other interesting situations, the assumption of insurer risk neutraility cannot be justified by the above or other arguments. In these situations, the approach developed in this paper can be viewed as a first step toward a theory of negotiated insurance contracts. Subsequent steps would, of course, require an extension of the results presented below to the case of risk averse insurers.

This paper focuses on the effect of the clients' risk aversion on the outcome of bargaining about the terms of an insurance contract. In particular, consider a situation in which a risk averse individual, faced with a possible financial loss, bargains with a risk neutral insurance company. The two parties bargain about the amount of insurance to be provided as well as its price. In such a situation it seems reasonable to expect that the insurance company will be more successful in bargaining against a more risk averse client than against one who is less risk averse. The formal analysis of this situation that follows considers a game theoretic model of bargaining whose predictions are consistent with this expectation. This result is contrasted with results obtained when insurance contracts are determined through exchanges in competitive insurance markets. When insurance markets are competitive, risk aversion need be of no disadvantage to the insured. In fact, as is well known, in a competitive market equilibrium the price of insurance is actuarily fair regardless of the risk aversion of the insured, as long as the insurer is risk netural. Futhermore, at this price the risk averse insured always chooses to be fully covered.

The formal model used to obtain these results has the following features. Risk aversion of the insured is introduced by assuming that he or she maximizes the expected value of a concave von Neumann-Morganstern utility function. As usual, risk neutrality of the insurer is interpreted to mean that it maximizes expected income.[2] Associated with each possible insurance contract is an expected utility for the insured and an expected income for the insurer. The set of all expected utility-expected income pairs associated with all possible contracts describes a Nash bargaining game in which the "disagreement point" is the expected utility-expected income pair that results when no insurance is provided. This means that if bargaining breaks down and no agreement is achieved, no insurance is provided. The outcome of bargaining is assumed to be the insurance contract predicted by the Nash solution to this bargaining game.

In this situation, increases in the risk aversion of the insured are introduced using the Arrow-Pratt risk aversion measure. Using arguments similar to those in Kihlstrom, Roth and Schmeidler [1979], it is possible to describe the effect of an increase in the Arrow-Pratt risk averseness of the insured on the set of expected utility-expected income pairs that defines the Nash bargaining game and on the Nash solution to this game. In particular, it will be shown that the insurer obtains a higher expected income when it bargains against a more

[2] That is, the insurer maximizes the expected value of a linear von Neumann-Morganstern utility function, so that his or her utility for a given lottery is equal to the expected income received from that lottery.

risk averse client than when it bargains against a less risk averse client seeking to insure against the same potential loss. This is shown to imply that more risk averse clients who bargain against risk neutral insurers pay higher insurance premiums for less insurance than less risk averse clients in the same situation.

Before the insurance situation is described in Section 3, Section 2 discusses the bargaining game and Nash's solution. Section 4 begins with a derivation of the bargaining game implicit in the insurance situation. The manner in which the game depends on the risk aversion of the client is then characterized, and this characterization is used to determine the influence of an increase in the client's risk aversion on the insurance contract that yields the Nash solution to the bargaining game.

A Model of Bargaining

Nash [1950] modeled any two-player bargaining game by a pair (S,d), where S is a compact and convex subset of the plane, representing the set of feasible expected utility payoffs to the players in the event of disagreement. He also assumed that S contains at least one point s such that $s > d$. This confines our attention to games in which it is possible for both players to gain from an agreement. The rules of the game are that any payoff vector in S will be the result of the game if it is agreed to by both players, and if no agreement is reached, each player receives his disagreement payoff. That is, if the players agree on a point $y = (y_1, y_2)$ in S, the resulting utility payoffs to the two players are y_1 and y_2, and in the event that no agreement is reached, the players receive d_1 and d_2, respectively. Thus the rules of the game give each player a veto over any outcome different from d, and it will be natural here to think of the disagreement outcome d as corresponding to the "status quo," which can only be altered by an agreement between the bargainers.

Nash modeled the bargaining process as a function called a *solution,* which selects, for any bargaining game, a unique feasible outcome. Letting B denote the class of all bargaining games, a solution f is a function $f: B \rightarrow R^2$ such that, for every game (S,d) in B, f(S,d) is a point contained in S. (So a solution f models the bargaining process by predicting the outcome that will result.)

Nash proposed that a solution intended to model bargaining among rational players should possess the following properties.

Property 1: Pareto optimality. If $f(S,d) = z$ and $y \geqslant z$, either $y = z$ or else y is not contained in S.

Property 2: Symmetry. If (S,d) is a symmetric game (i.e., if $d_1 = d_2$ and if $(x_1,x_2) \in S$ implies $(x_2,x_1) \in S$), $f_1(S,d) = f_2(S,d)$.

Property 3: Independence of irrelevant alternatives.[3] If (S,d) and (T,d) are games such that T contains S and f(T,d) is an element of S, $f(S,d) = f(T,d)$.

Property 4: Independence of equivalent utility representatives. If (S',d') is related to (S,d) by the transformations $d' = (a_1d_1 + b_1, a_2d_2 + b_2)$ and $S' = \{(a_1y_1 + b_1, a_2y_2 + b_2) \mid (y_1,y_2) \in S\}$ where a_1, a_2, b_1 and b_2 are numbers such that $a_1, a_2 > 0$, f(S',d') is related to f(S,d) by the same transformations. That is, if $f(S',d') = z'$ and $f(S,d) =$

[3] Perhaps a more descriptive name for this property is "independence of alternatives other than the disagreement point" (cf. Roth [1977b]).

$z, z' = (a_1 z_1 + b_1, a_2 z_2 + b_2)$.

Because these properties have been discussed elsewhere at great length (cf. Nash [1950], Luce and Raiffa [1957], Roth [1979]), this paper will not discuss them further, except to note that Property 4 is the only property that is directly motivated by the fact that the payoffs to the players are assumed to be expressed in terms of their expected utility functions. Since each player's utility function is uniquely defined only up to the arbitrary choice of its origin and scale, Property 4 requires that the utility payoff selected by the solution for a player should be defined with respect to the same origin and scale as are the other feasible payoffs for that player. Nash proved the following important theorem:

Theorem 2.1: There is a unique solution which possesses Properties 1-4. It is the solution $f = F$ defined by $F(S,d) = z$ such that $z \geqslant d$ and $(z_1 - d_1)(z_2 - d_2) > (y_1 - d_1)(y_2 - d_2)$ for all y in S such that $y \geqslant d$ and $y \neq z$.

That is, Nash's solution F selects the (unique) outcome z in S that is individually rational (i.e., $z \geqslant d$) and that maximizes the geometric average (i.e., the product) of the gains that the players achieve by agreeing instead of disagreeing.

A well-known alternative characterization of Nash's solution F is that it selects the unique point $z = F(S,d)$ such that the line joining d to z has the negative slope of some tangent to S at z. That is, let ϕ be the function such that all Pareto optimal points $y = (y_1, y_2)$ of S can be represented as $y = (y_1, \phi(y_1))$. Then the tangent to S at z has slope $\phi'(z_1)$, and the following lemma results. (For simplicity, the result is stated for the case of ϕ differentiable at z_1.)

Lemma 2.1: For any game (S,d), $F(S,d) = z$ is the point such that $(\phi(z_1) - d_2)/(z_1 - d_1) = -\phi'(z_1)$.

Note that the individual rationality of Nash's solution is consistent with the assumption that each player's payoffs are expressed in terms of his or her expected utility function, which models the choice behavior. The disagreement outcome can be chosen by either player acting alone, and so if Player i is faced with a choice of agreeing on an outcome z or taking the disagreement payoff, he or she will choose z only if $z_i \geqslant d_i$. That is, because each player chooses between a potential agreement z and the disagreement payoff so as to maximize his or her utility, only if $z \geqslant d$ can it be agreed to by both players. Roth [1977a, 1979] has recently shown that when the individual rationality of the players is made explicit, it is not necessary to assume Pareto optimality in order to characterize Nash's solution. That is, the following theorem results.

Theorem 2.2: There are precisely two solutions f which are individually rational (i.e., $f(S,d) \geqslant d$) and which possess properties 2-4. One is Nash's solution $f = F$, and the other is the *disagreement solution* $f = D$, defined by $D(S,d) = d$ for every game (S,d).

Consequently, there are only two individually rational modes of behavior consistent with Properties 2-4; one of which yields disagreement in every game, while the other yields Nash's solution. In other words, Nash's solution

F is the unique individually rational solution that is consistent with Properties 2-4 and that yields an outcome other than disagreement for at least one game.

Thus Nash's solution is intimately associated with the individual rationality of the players, which is an ordinal property of their utility functions. Although Nash made no explicit use of individual rationality, it can essentially replace the assumption of Pareto optimality, which can be viewed as an assumption of collective rationality.

Of course, the expected utility functions of the players convey more than just ordinal information about the players' preferences. In particular, each player's expected utility function also summarizes his or her preferences over risky alternatives. A recent paper by Kihlstrom, Roth and Schmeidler [1979] showed that, for bargaining games that arise from bargaining over riskless alternatives, Nash's solution is responsive in an intuitively plausible way to changes in the risk posture of the bargainers. It was further shown that this property of "risk sensitivity" could be used to replace Property 4 in the assumptions used in Theorem 2.1 to characterize the solution F on games that arise from bargaining over riskless alternatives.

This paper studies the responsiveness of Nash's solution to changes in the risk posture of the players in bargaining games that arise from bargaining over risky alternatives. In particular, the paper will examine bargaining games that arise from bargaining over possible insurance contracts. The paper will use the notion of risk posture first introduced by Arrow [1965,1971] and Pratt [1964]. For the case of utility functions a single variable (such as money), a utility function $\hat{w}$ will be said to be *at least as risk averse* as another utility function w if $\hat{w} = k(w)$, where k is an increasing concave function. The utility function $\hat{w}$ is said to be (strictly) *more risk averse* than w if the function k is strictly concave.

Note that for any model of bargaining that depends in a non-trivial way on the expected utility function of the bargainers, the underlying assumption is that the risk aversion of the bargainers influences the outcome of bargaining. That is, the risk aversion of the bargainers influences the decisions they make in the course of negotiations, which in turn influence the outcome of bargaining. (See Roth [1979], for an explicit treatment.) Consequently, when the paper considers the effect that risk aversion has on the outcome of bargaining, it does not assume that the bargainers need to know one another's risk posture.

An Insurance Problem

Envision a situtión with two individuals, one of whom faces a possible financial loss. This individual will be referred to as the client, and his or her wealth (in dollars) will be $\omega_C > 0$ if the loss fails to occur. If, however, the loss occurs, it will amount to L dollars and his or her resulting after-loss wealth will be $\omega_C - L > 0$. The other individual will be referred to as the insurer. The insurer's wealth is ω_I dollars and he or she is not faced with the possibility of any exogenously determined losses. The insurer may however agree to bear some of the burden of the client's loss in the event it arises; i.e., he or she may agree to insure the client. Of course, the insurer must be induced to assume this risk. The inducement comes in the form of a premium payment made from the client to the insurer in the event that no loss is incurred. If the insurer assumes A dollars of the client's loss and receives a premium payment of P dollars in the event of no loss, his or her resulting wealth will be

$$x_{I\ell} = \omega_I - A \qquad (1)$$

if the loss occurs and

$$x_{In} = \omega_I + P \qquad (2)$$

if no loss occurs.[4] With this insurance contract in force, the client's wealth is

$$x_{C\ell} = \omega_C - L + A \qquad (3)$$

if the loss is incurred and

$$x_{Cn} = \omega_C - P \qquad (4)$$

when no loss is incurred.

Following Arrow [1963-4] and Debreu [1959], a claim to wealth that is contingent on the event that there is no loss can be considered to be a different good than a claim to wealth contingent on the event of a loss. If the event "no loss" is called "state n" and the event "loss" is called "state ℓ," then x_{is} is individual i's claims to wealth contingent on the occurence of state s. The variable i can equal I or C and s can equal ℓ or n.

The economy composed of these two individuals has $\omega_C + \omega_I$ contingent claims to wealth in state n and $x_C + \omega_I - L$ contingent claims to wealth in state ℓ. Before an insurance contract is agreed to, $x_{I\ell} = \omega_I = x_{In}$ while $x_{Cn} = \omega_C$ and $x_{C\ell} = \omega_C - L$. If (A,P) is the agreed upon insurance contract, x_{is} is given by (1)–(4) for i = I and C and s = ℓ and n.

The contingent claims allocations $(x_{In}, x_{I\ell}, x_{Cn}, x_{C\ell})$ which are feasible for this economy in the sense that

$x_{In} + x_{Cn} = \omega_C + \omega_I$

and

$x_{I\ell} + x_{C\ell} = \omega_C + \omega_I - L$

are described as points in the Edgeworth Box represented in Figure 1.

In Figure 1, the initial allocation of contingent claims is denoted by α and the allocation of these claims implied by some insurance contract (A, P) is shown as β. In fact, any allocation can be achieved from α by an insurance contract (A, P). However, only a small subset of these insurance contracts would be entered into voluntarily by both the client and the insurer. This subset is the shaded region in Figure 1, if $J_I(\alpha)$ is the indifference curve of the insurer through the initial allocation α and if $J_C(\alpha)$ is the indifference curve of the client through the initial allocation.

[4] A is, in fact, the net payment made by the insurer when a loss occurs. It equals the gross payment less the premium P which is paid at the outset when the insurance agreement is arranged.

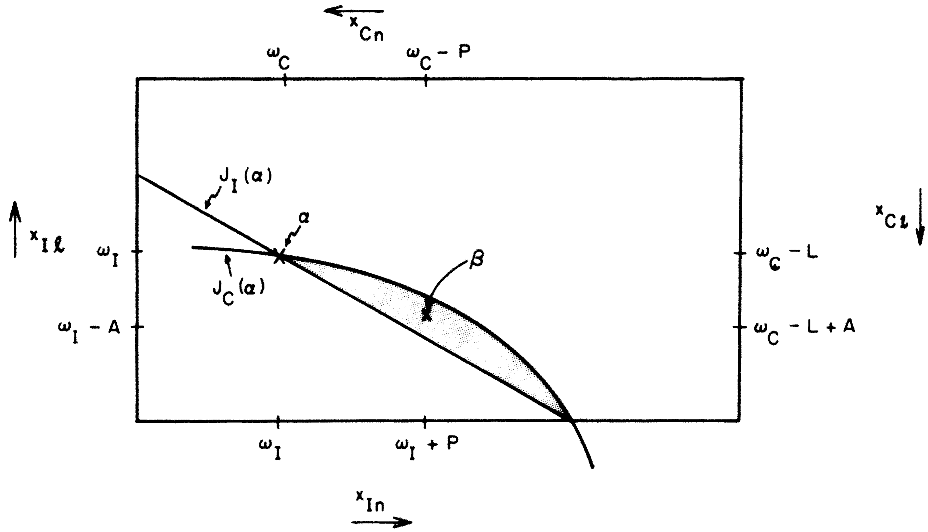

FIGURE 1

Since the insurer maximizes expected wealth, his or her indifference curve through α is

$$J_I(\alpha) = \{(x_{In}, x_{I\ell}) : \mu_\ell x_{I\ell} + \mu_n x_{In} = \omega_I\}$$

where μ_s is the *objective* probability of state s. That is, $J_I(\alpha)$ is the set of contingent claim vectors $(x_{In}, x_{I\ell})$ with expected value equal to ω_I. Note that, at every point on the indifference curve $J_I(\alpha)$, the marginal rate of substitution for the insurer is

$$MRS_I(x_{In}, x_{I\ell}) = \mu_n/\mu_\ell \tag{5}$$

the ratio of the probabilities. In fact, similar comments can be made about any of the insurer's indifference curves. That is, along any insurer indifference curve, the expected wealth of all contingent claim vectors is constant and, as a result, the MRS is given by (5).

The situation is different for the client since u_C, his or her utility function of wealth, is strictly concave. A number of authors have shown that the function $U_C(x_{Cn}, x_{C\ell})$ defined by

$$U_C(x_{Cn}, x_{C\ell}) \equiv \mu_n u_C(x_{Cn}) + \mu_\ell u_C(x_{C\ell}) \tag{6}$$

is strictly concave if and only if the function u_C is strictly concave. On this subject see, for example, Arrow [1963-4], Hirshleiffer [1965], Cox [1973], and Debreu and Koopmans [1978]. Because U_C is strictly concave, all of his or her indifference curves have the same shape as $J_C(\alpha)$ in Figure 1. $J_C(\alpha)$ is the client's indifference curve through his or her initial allocation $(\omega_C, \omega_C - L)$. Also notice that since the client's utility function U_C for contingent claims vectors $(x_{Cn}, x_{C\ell})$ is defined in (6), his or her marginal rate of substitution at $(x_{Cn}, x_{C\ell})$ is

$$MRS_C(x_{Cn}, x_{C\ell}) = \frac{\mu_n u_C'(x_{Cn})}{\mu_\ell u_C'(x_{C\ell})} , \tag{7}$$

if, as assumed here, u_C' exists.

Before deriving the bargaining game implicit in this situation, it is instructive to describe the Pareto optimal allocations of contingent claims and the core of this economy as well as the competitive equilibrium allocation that would result if there existed competitive markets for contingent claims to wealth in each state. Kihlstrom and Pauly [1971] have shown how the model of competitive contingent claims markets can be reinterpreted to yield a model of a competitive market for insurance contracts. Using this interpretation, the competitive equilibrium to be described below can be viewed as the equilibrium that would result if insurance contracts were competitively traded.

The derivation of the Pareto optimal allocations begins by noting that the concavity assumptions made about u_C and the risk neutrality of the insurer imply that an interior Pareto optimal allocation $(x_{In}, x_{I\ell}, x_{Cn}, x_{C\ell})$ satisfies the familar condition

$$MRS_C(x_{Cn}, x_{C\ell}) = MRS_I(x_{In}, x_{I\ell}) \tag{8}$$

Using (5) and (7), (8) simplifies to

$$\frac{\mu_n u_C'(x_{Cn})}{\mu_\ell u_C'(x_{C\ell})} = \mu_n/\mu_\ell . \tag{9}$$

Because the insurer and the client have been assumed to agree about the probability of a loss, the ratio μ_n/μ_l is the same on both sides of (9), which can therefore be reduced to

$$\frac{u_C'(x_{Cn})}{u_C'(x_{C\ell})} = 1 . \tag{10}$$

Since u_C is assumed to be strictly concave, equation (10) implies that in any interior Pareto optimal allocation the client must be completely insured in the sense that

$$x_{Cn} = x_{C\ell} ; \tag{11}$$

i.e., his or her wealth is subject to no random fluctuation. Since all losses must be covered, (11) in turn implies that

$$x_{I\ell} = x_{In} - L ; \tag{12}$$

i.e., the entire loss must be borne by the insurer. This result is not surprising in view of the fact that the insurer is risk neutral while the client is risk averse. Three interior Pareto optimal allocations γ, δ and ϵ are depicted in the Edgeworth Box of Figure 2. Clearly, the insurer prefers ϵ to δ and δ to γ, while the ranking is reversed for the client.

All of the interior optima lie on the line $x_{Cn} = x_{C\ell}$ shown in Figure 2. The noninterior Pareto optimal allocations are those that lie on the lower edge of the box between ξ and ζ. Notice that at a noninterior optimal allocation, say η, the client's losses are not completely covered; i.e., $x_{C\ell} < x_{Cn}$; and $x_{I\ell} = 0$. Uncovered losses can be optimal if the insurer does not have the financial

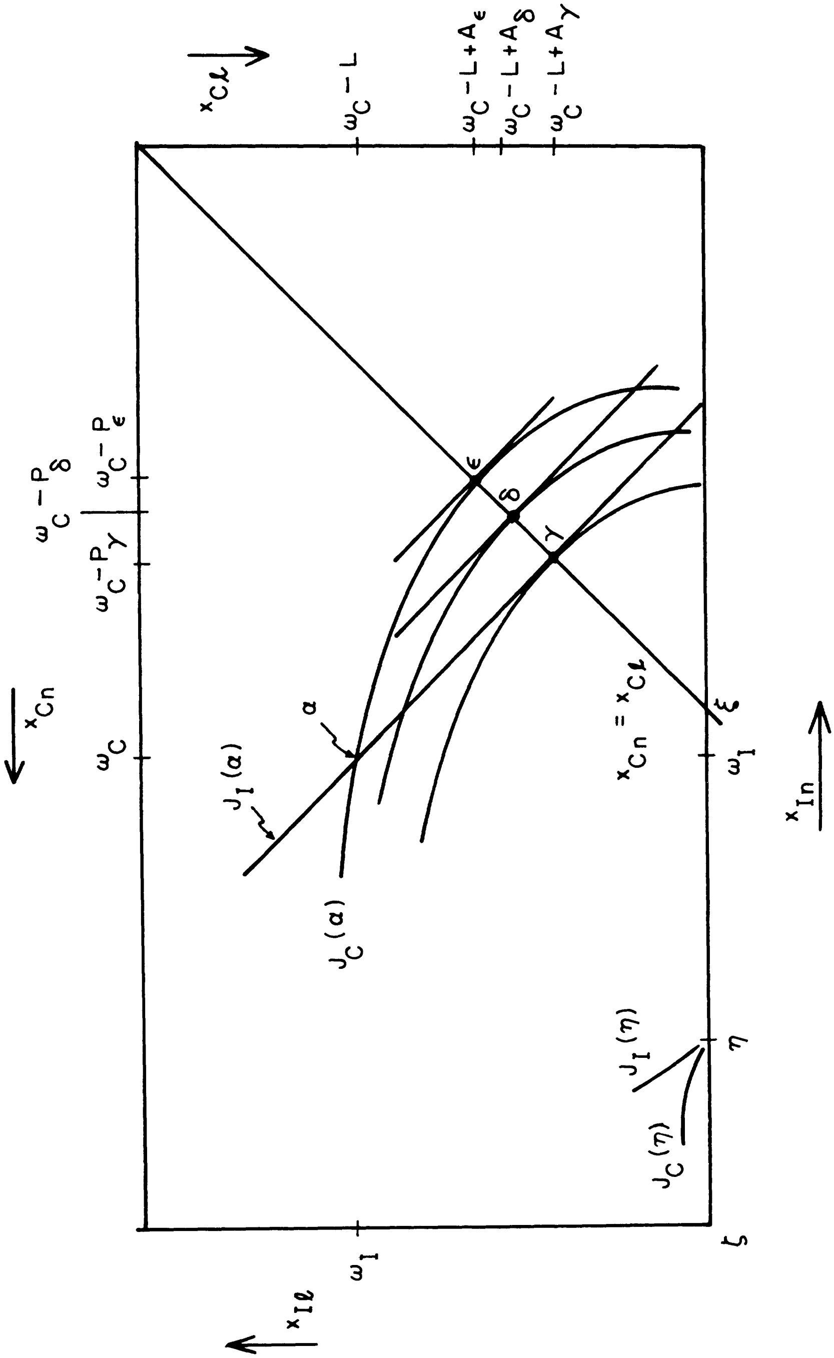
x_{Cℓ}
ω_C − L
ω_C − L + A_ε
ω_C − L + A_δ
ω_C − L + A_γ
x_{Cn}
ω_C
ω_C − P_γ
ω_C − P_δ
ω_C − P_ε
ε
δ
γ
α
J_I(α)
J_C(α)
x_{Cn} = x_{Cℓ}
ξ
ω_I
η
J_I(η)
J_C(η)
ζ
x_{Iℓ}
x_{In}

FIGURE 2

resources to provide complete coverage; i.e., if $x_{I\ell}$ becomes zero before $x_{C\ell}$ is raised to x_{Cn}.

The Pareto optimal allocations between γ and ϵ are the only ones that both the insurer and the client would voluntarily choose in preference to α at which no insurance is provided. These are the core allocations for this economy. At ϵ, all of the gains from insurance accrue to the insurer. The client is indifferent between α and ϵ, and the wealth, $x_{Cn} = x_{C\ell}$, he or she receives in this allocation is the certainty equivalent of the gamble faced without insurance. At γ, on the other hand, the client receives all of the gains from insurance. The insurer is indifferent between α and γ because he or she has the same expected wealth in both allocations.

Figure 2 assumes that all of the core allocations are interior. The remaining analysis will continue to assume that this property of Figure 2 holds; i.e., that all core allocations are interior allocations. This means that in any core allocation the insurer is financially able to provide complete coverage. Since

$$P_\gamma < P_\delta < P_\varepsilon \tag{13}$$

and

$$A_\gamma > A_\delta > A_\varepsilon \quad , \tag{14}$$

the policy (A_γ, P_γ) requires more finanical reserves from the insurer than any other core policy. Thus, if the insurer has sufficient wealth to provide policy (A_γ, P_γ), he or she will indeed have the resources to provide complete coverage in all other core allocations.

Inequalities (13) and (14) assert that although (P_δ, A_δ) is a more [less] expensive policy than (P_γ, A_γ) $[(P_\varepsilon, A_\varepsilon)]$, it provides less [more] insurance than (P_γ, A_γ) $[(P_\varepsilon, A_\varepsilon)]$. In fact, it is in general true that as the Pareto optimal allocation is moved from γ to ϵ, the client does worse in two ways: the net coverage, A, is reduced and the premium, P, is increased.

The allocation γ is the unique competitive equilibrium allocation in this economy and μ_n / μ_ℓ is the equilibrium relative price of contingent claims in state n in terms of claims to wealth in state ℓ. Thus the competitive equilibrium insurance policy is (P_γ, A_γ) at which point the client is receiving more coverage than at any other core policy and at which the price of this coverage is lower than at any other core policy.

Having described the Pareto optimal, core and competitive allocations, the paper now asks how an increase in risk aversion changes these allocations. These changes are described in Figure 3, in which the indifference curves of a client whose utility function is $u_C[\bar{u}_C]$ are denoted by $J_C[\bar{J}_C]$. The utility function u_C is assumed to be less risk averse than $\bar{u}_C$ in the Arrow-Pratt sense. The indifference curves of the less [more] risk averse client are thus depicted as solid [dotted] curves. Observe that the increase in risk aversion from u_C to $\bar{u}_C$ does not alter the set of Pareto optimal allocations. For a client with utility function $\bar{u}_C$, as for a client with utility function u_C, the interior optima continue to satisfy (11) and (12) and lie on the line $x_{Cn} = C_{C\ell}$ in Figure 3. Similarly for clients with either utility function, the noninterior optima are

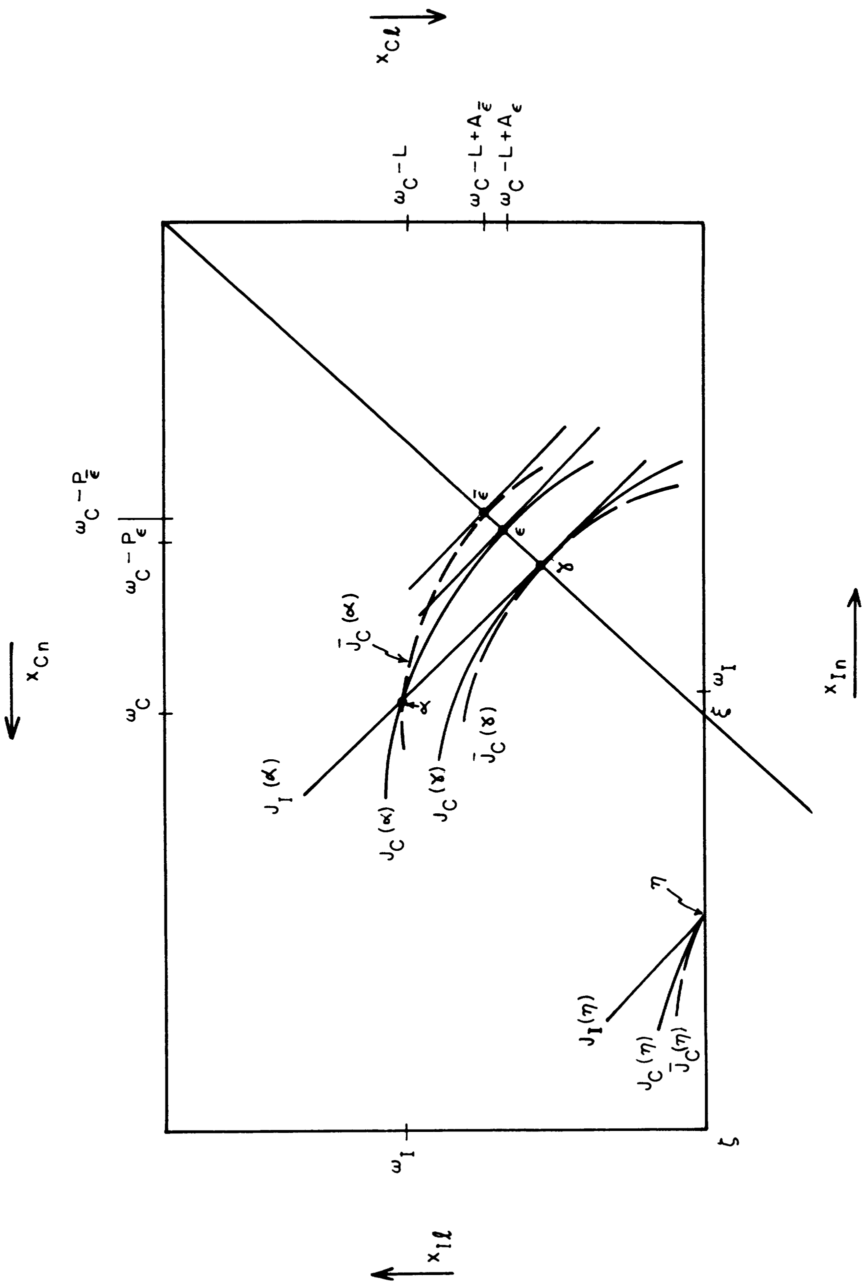
x_Cℓ
ω_C − L
ω_C − L + A_ε̄
ω_C − L + A_ε
ω_C − P_ε̄
ω_C − P_ε
ω_C
x_Cn
ω_I
x_In
ω_I
x_Iℓ
J_I(α)
J_C(α)
J̄_C(α)
J_C(γ)
J̄_C(γ)
J_I(η)
J_C(η)
J̄_C(η)
α
γ
ε
ε̄
ξ
ζ
η

FIGURE 3

the allocations between ξ and ζ on the lower edge of the Edgeworth Box in Figure 3.

Although the set of Pareto optimal allocations is unaffected when the client becomes more risk averse, the core is enlarged by this change. Specifically, when the utility function u_c is replaced by $\bar{u}_C$, the certainty equivalent of the gamble faced by the uninsured client is reduced. Thus ϵ is replaced by $\bar{\varepsilon}$. The Pareto optimal allocation that is indifferent to α for the insurer continues to be γ even after $\bar{u}_C$ replaces u_C. In Figure 3, the change in the utility function from u_C to $\bar{u}_C$ changes the core from the points between γ and ϵ to those between γ and $\bar{\varepsilon}$.

Finally, the competitive equilibrium allocation is unchanged by the increase in risk aversion. In Figure 3, γ is the competitive allocation whether the client's utility function is u_C or $\bar{u}_C$.

The following section considers insurance contracts arrived at through bargaining between the client and the insurer, using Nash's model of bargaining. It will show that, in contrast to the situation just described in which a change in the client's risk aversion has no impact on the competitive contract, the client's risk aversion does influence the contract arrived at as the Nash solution to the bargaining game played by the insurer and client.

Bargaining Over the Insurance Contract

Let the insurer be Player 1 and the client Player 2, in a bargaining game (S,d) such that S is the set of expected utility payoffs to the players resulting from feasible insurance contracts and d is the pair of utility payoffs corresponding to the initial allocation α. That is, $d_1 = U_I(\alpha) = u_I(\omega_I) = \omega_I$, $d_2 = U_C(\alpha) = U_C(\omega_C, \omega_C - L) = \mu_n u_C(\omega_C) + \mu_\ell u_C(\omega_C - L)$, and if (y_1, y_2) is a point in S, there exists an insurance contract (A,P) such that

$$y_1 = \mu_n u_I(\omega_I + P) + \mu_\ell u_I(\omega_I - A) = \mu_n(\omega_I + P) + \mu_\ell(\omega_I - A)$$

$$y_2 = \mu_n u_C(\omega_C - P) + \mu_\ell u_C(\omega_C - L + A)$$

That is, each (y_1, y_2) in S is the utility payoff vector to the players that corresponds to some feasible allocation of contingent claims (x_{In}, $x_{I\ell}$, x_{Cn}, $x_{C\ell}$); i.e., $y_1 = U_I(x_{In}, x_{I\ell})$ and $y_2 = U_C(x_{Cn}, x_{C\ell})$. Then S is a compact set, and the concavity of U_C insures that S is convex.

The set of Pareto optimal utility payoffs in S corresponds to the set of contingent claims allocations that satisfy equations (11) and (12). So if (z_1, z_2) is Pareto optimal in S, $z_1 = x_{In} - \mu_\ell L$ (since $x_{I\ell} = x_{In} - L$) and $z_2 = u_C(\omega_C + \omega_I - x_{In})$, since $x_{C\ell} = x_{Cn} = \omega_C + \omega_I - x_{In}$. So for any Pareto optimal point (z_1, z_2), $z_2 = \phi(z_1)$ where ϕ is the decreasing concave function defined by $\phi(z_1) = u_C(\omega_C + \omega_I - (z_1 + \mu_\ell L))$. The function ϕ can be thought of as determining the Pareto optimal subset P(S), since all Pareto optimal utility payoffs are of the form $(z_1, \phi(z_1))$.

Consider two potential insurance clients with utility functions for money w and $\hat{w}$, such that $\hat{w}$ is more risk averse than w. Then $\hat{w} = k(w)$, where k is an increasing concave function. Suppose that the game (S,d) is the bargaining game that results when client w bargains with the insurer over the set of feasible insurance contracts; i.e., (S,d) is the game in which $u_c = w$. Let $(\hat{S},\hat{d})$ be the game that results when, instead, client $\hat{w}$ bargains with the insurer; i.e., $(\hat{S},\hat{d})$ is the game in which $u_C = \hat{w} = k(w)$. Note that the set of insurance contracts corresponding to Pareto optimal utility payoffs is unaffected by the change from w to $\hat{w}$. So if ϕ and $\hat{\phi}$ are the functions that define the Pareto optimal subsets of S and $\hat{S}$ respectively, $\hat{\phi} = k(\phi)$. This observation will permit a proof of the following result.

Theorem 4.1: Let $(\hat{S},\hat{d})$ be a bargaining game over insurance contracts, derived from (S,d) by replacing Player 2 (the client with utility function w) with a more risk averse client (whose utility function is $\hat{w} = k(w)$).

1) Then Nash's solution predicts that Player 1 will gain a higher utility when bargaining with the more risk averse client; i.e., $F_1(\hat{S},\hat{d}) > F_1(S,d)$.
2) If (A,P) and $(\hat{A},\hat{P})$ are the insurance contracts predicted by Nash's solution in the games (s,d) and $(\hat{S},\hat{d})$, respectively, $\hat{A} < A$ and $\hat{P} > P$. That is, a more risk averse client pays a higher premium and receives less coverage of his or her potential loss.

Roth [1978] has shown that Nash's solution can be interpreted as the utility function for bargaining in a given game for certain kinds of risk neutral players.[5] Interpreted in this way, Theorem 4.1 means that a risk neutral insurer prefers to bargain with a more risk averse client than with a less risk averse client.

Since bargaining over insurance contracts involves risky events, the results of Kihlstrom, Roth and Schmeidler [1979] cannot be directly applied to prove Theorem 4.1. Instead, the proof will proceed via the following lemma.

Lemma 4.1: Let (S,d) be a bargaining game whose Pareto optimal points are of the form $(y_1, \phi(y_1))$, and let $(\hat{S},\hat{d})$ be a game whose Pareto optimal points are of the form $(y_1, \hat{\phi}(y_1))$, where ϕ and $\hat{\phi}$ are decreasing concave functions. Then, if $\hat{\phi} = k(\phi)$ where k is an increasing (strictly/ concave function, it follows that $F_1(\hat{S},\hat{d}) > F_1(S,d)$.

Proof of the lemma: Since Nash's solution F is independent of equivalent utility representations, it will be sufficient to prove the lemma for the case when $d = \hat{d} = \bar{0}$, where $\bar{0}$ denotes the origin (i.e., $\bar{0} = (0,0)$). So let $z = F(S,\bar{0})$ and $\hat{z} = F(\hat{S},\bar{0})$; it is necessary to show that $\hat{z}_1 > z_1$. Since Nash's solution selects the point in S that maximizes the geometric average of the gains, it will be sufficient to show that the geometric average $A(y_1) = k(\phi(y_1))y_1$ as positive first derivative at z_1. But

$$A'(z_1) = k'(\phi(z_1))\phi'(z_1)z_1 + k(\phi(z_1)) ,$$

and by Lemma 2.1, $\phi'(z_1)z_1 = -\phi(z_1)$, so

$$A'(z_1) = -k'(\phi(z_1))\phi(z_1) + k(\phi(z_1))$$

$$= -k'(z_2)z_2 + k(z_2) = z_2[-k'(z_2) + (k(z_2)/z_2)] .$$

[5] Specifically, the Nash solution represents the utility of bargaining for a player who is neutral both to ordinary (probabilistic) risk and to strategic risk (cf. Roth [1978]).

Because the (strict) concavity of the function k imples that $(k(z_2)/z_2) > k'(z_2)$, while the (strict) individual rationality of Nash's solution implies $z_2 > 0$, $A'(z_1) > 0$, as required.

Proof of Theorem 4.1: Part (1) of the theorem will follow from Lemma 4.1 once it can be shown that $\hat{\phi} = k(\phi)$, where $\hat{\phi}$ and ϕ are the functions defining the Pareto sets of $\hat{S}$ and S, respectively. But if $(y_1, \phi(y_1))$ is a Pareto optimal point in S, $y_1 = x_{In} - \mu_\ell L$ and $\phi(y_1) = w(\omega_C + \omega_I - x_{In})$, and the point $(y_1, k(\phi(y_1)))$ is Pareto optimal in $\hat{S}$, since $\hat{w}(\omega_C + \omega_I - x_{In}) = k(w(\omega_C + \omega_I - x_{In}))$. So $\hat{\phi} = k(\phi)$, and so $F_1(\hat{S},\hat{d}) > F_1(S,d)$, as required.

To prove Part (2) of the theorem, note that if $x = (x_{In}, x_{I\ell}, x_{Cn}, x_{C\ell})$ and $\hat{x} = (\hat{x}_{In}, \hat{x}_{I\ell}, \hat{x}_{Cn}, \hat{x}_{C\ell})$ are the contingent claims allocations corresponding to the utility pairs $z = F(S,d)$ and $\hat{z} = F(\hat{S},\hat{d})$, $F_1(\hat{S},\hat{d}) > F_1(S,d)$ implies that $\hat{x}_{In} > x_{In}$, (since x and $\hat{x}$ satisfy equations (11) and (12)). Consequently

$$\hat{x}_{Cn} = \hat{x}_{C\ell} = \omega_I + \omega_C - \hat{x}_{In} < \omega_I + \omega_C - x_{In} = x_{Cn} = x_{C\ell}.$$

By equation (2), $\hat{x}_{In} > x_{In}$ implies that $\hat{P} > P$, and by equation (3) $\hat{x}_{C\ell} < x_{C\ell}$ imples $\hat{A} < A$ where (A,P) and $(\hat{A},\hat{P})$ are the contracts that give rise to the contingent claims allocations x and $\hat{x}$, respectively. This completes the proof of the theorem.

The lemma can also be quickly demonstrated via the following informal, graphical representation. In figure 4 below, the solid curve ϕ represents the Pareto set of S. Then Lemma 2.1 states that the angles α and β are equal, where α is the angle with the x_1 axis formed by the line joining d to z = F(S,d), and β is the angle with the x_1 axis formed by the tangent to ϕ at F(S,d).

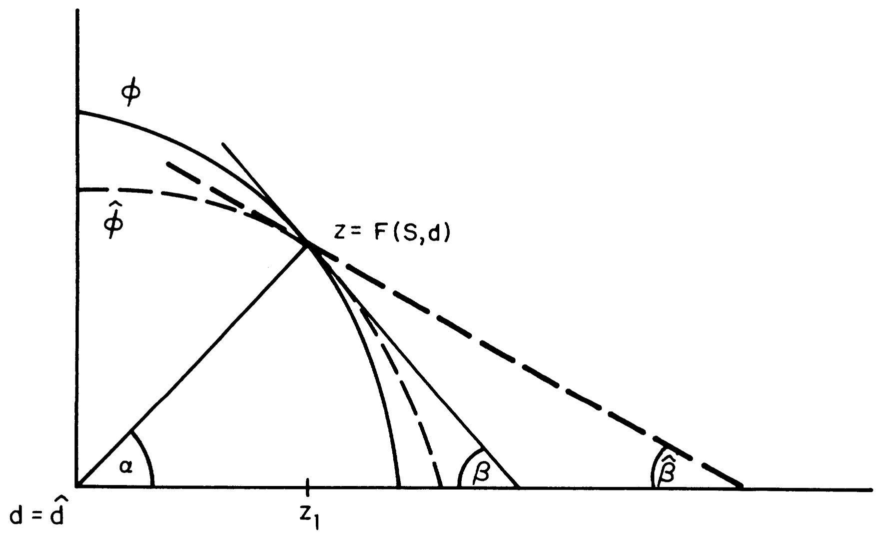

FIGURE 4

The dotted curve $\hat{\phi} = k(\phi)$ represents the Pareto set of $\hat{S}$. The angle $\hat{\beta}$ made by the tangent to $\hat{\phi}$ at z is smaller than the angle β. Therefore $F(\hat{S},\hat{d}) = \hat{z}$ must fall further to the right than z_1 so the angle $\hat{a}$ made by the line from $\hat{d}$ to $\hat{Z}$ will equal $\hat{\beta}$, as required by Lemma 2.1.

The results presented above can be generalized to bargaining solutions other than Nash's solution. In particular, Kihlstrom, Roth and Schmeidler [1981] showed that several solutions found in the literature possess essentially the same risk sensitivity property as does Nash's solution, and it is straightforward to generalize those results to the insurance problem considered here.

It is less clear how the results presented here can be generalized to insurance problems in which both the insurer and client are risk averse. Roth and Rothblum [1982] characterize how Nash's solution responds to changes in the risk aversion of the bargainers in a general setting where bargaining may involve lotteries, but where the disagreement point is riskless. Their results show that there are situations in which risk aversion can be advantageous, so that the results of Kihlstrom, Roth and Schmeidler [1981] do not generalize in a straightforward manner to all bargaining situations. The behavior of negotiated insurance contracts for more general insurance problems thus remains an open question.

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