Game Theory

Kevin: So according to game theory, it is not advantageous for either of us to work on this lecture

Tom: I didn’t need game theory to tell me that I’m lazy; I already knew that
What is Game Theory?

• Game theory is a mathy formalization of strategic situations
• A strategic situation is any situation where your success depends on what other people do
• This includes
Chess
Baseball – ball or strike
Baseball – pitching in general
Asking someone on a date

*Likely not important in this class
Production levels

Diagram I: 
- Vertical axis: Price
- Horizontal axis: Quantity
- Graph showing the intersection of demand (D) and supply (S) curves.

Diagram II: 
- Vertical axis: Price
- Horizontal axis: Quantity
- Graph showing the intersection of demand (D) and supply (S') curves.
NOT - Solitaire
NOT – iTunes choices
Breakdown of the class

• 3 examples, 3 topics
• Let’s go
Example #1: Star

is the loneliest number...
Star Format

- 5 players, each playing a different color
- Each player trying to defeat enemy colors before anyone else
  - G player tries to destroy B and U
- Interesting format because of ally pairings
- Friend of an friend is an enemy?
  - Friend of an enemy is a frenemy?
Example situation – Star Format
Example situation – Star Format

It’s my turn
Do I attack?
I Swing, Akroma sits
Akroma swings, I sit
Both Swing

[Image of Magic: The Gathering card: Chugo, the Glacierspike Guardian]

Flying, first strike, vigilance, trample, haste, protection from red
"No net. No mercy. No matter what."
Neither swing
What should I do?

Assume ceteris paribus because it makes life easy
A Better Way to Look at it
If Akroma Swings...

(15,15)

(20,5)
We should sit
If Akroma sits...

(15,15)  (5,20)

(20,5)  (10,10)
We should sit

(5,20)

(10,10)
We have a winner!

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Our dominant strategy!

This is easy; so why is it known as the Prisoner’s Dilemma?
It’s worse.

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How can we fix this?

• Why doesn’t communication help?
• What did the 19th century railroad tycoons do?
Why don’t we play Magic like this?

• It’s a repeated game
• Nothing is ceteris paribus
• Akroma knows what your choice is
• Life totals don’t necessarily represent whether you’re going to win
• More generally, you don’t know what your opponents’ goals are
  – This is why playing against Tom in a multiplayer match always gets me angry
Example #2: Kithkin

can be as bad as 1

* if 2 is Hilary Clinton
A Different Situation
Deck Lists

You – Kithkin - Florian Michalec

Lands (23)
16 Plains
3 Rustic Clachan
4 Windbrisk Heights

Creatures (20)
2 Burrenton Forge-Tender
3 Cloudgoat Ranger
4 Figure of Destiny
4 Goldmeadow Stalwart
4 Knight of the White Orchid
3 Ranger of Eos

Spells (17)
2 Ajani Goldmane
3 Harm's Way
4 Honor of the Pure
4 Path to Exile
4 Spectral Procession

Him – Jund Cascade – Pierre Simonnin

Lands (24)
It’s irrelevant

Creatures (22)
4 Bloodbraid Elf
4 Boggart Ram-Gang
4 Demigod of Revenge
4 Kitchen Finks
4 Putrid Leech
2 Sygg, River Cutthroat

Other Spells (14)
2 Doom Blade
3 Jund Charm
4 Lightning Bolt
4 Maelstrom Pulse
1 Volcanic Fallout
So What Are Our Options?
1) Just Swing
2) HotP
3) Tom
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What did we learn from the last game?

Can we do better?
How can we think of this problem differently?
Analysis

• There is no dominant strategy
• Instead, we should choose based on our certainty
• If we’re 50% sure they will bolt, what should we do?
• If we’re 80% sure they will bolt, what should we do?
Best Response Graph

HotP
Swing
Nothing
Some Math...

• $4x + 5(1 - x) = 3x + 7(1 - x)$
• $5 - x = 7 - 4x$
• $3x = 2$
• $x = 2/3$
Example #3: Drafting*

* also a good way to catch up in MarioKart
Drafting

- Drafting is just as much of a game as Magic itself
- 8 people sit in a circle, each opens a pack
- Takes 1, passes 14 left, takes 1, passes 13 left...
- Pack 2, pass right
- Pack 3, pass left again
- 40 card deck, usually ~17 lands
- Sideboard is everything else
How Do I Draft?

• BREAD
• Bombs
• Removal
• Evasion
• Acceleration / Aggressive
• Dumb stuff? It kind of breaks down
• So what colors do I pick?
• Volunteers?
Drafting buddies

• Drafting the same color as someone else is bad
• You both like white
  – That’s common knowledge
• Both open good white and red cards
• What do you choose?
You knew it was coming...

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Make your choice...

So are there any dominated strategies?

What does best response tell you?
Nash Equilibrium

When everyone is playing a best response to everyone else’s strategy

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**Nash Equilibrium**

When everyone is playing a best response to everyone else's strategy

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What can you do to get your best payoff?

Coordinate!
Signaling in booster draft

- Avoid over-drafting a color
  - passing information is good
- 1\textsuperscript{st} pack: doom blade, assassinate, pacifism
  - What do you take?
- Just as important to read signals
Also Not Original

• Better known as the “Battle of the Sexes” based on picking a movie to see
  – Unsurprisingly, I’ve never had a problem like this
• Different than Prisoner’s Dilemma because it was just a coordination game
  – In Prisoner’s Dilemma, communication doesn’t matter
  – In this case, communication can help by playing a particular Nash Equilibrium
• Self-enforcing Nash Equilibrium
How are you a better player?

• Do not play dominated strategies
  — Play close endgames like you get perfect topdecks
• Consider the expected payoff based on certainty
• Put yourself in their shoes
• Pay attention to signals and send signals
• Think about your games in terms of payoffs and strategies
Want more?

• Mixed strategies use randomizations of pure strategies
  – Maybe a way to determine whether you should bluff attacks or not

• Iterative deletion towards NE in metagame shift
Where Can You Learn More?

• Econ 160 (not offered; life sucks)
• Ben Polak’s lectures on Game Theory at Yale
• Intro to Game Theory – Frank Karsten
• Theory Games – Frank Karsten