The Big Picture
Announcements

- Problem Set 9 due right now. We'll release solutions right after lecture.
  - Congratulations - you're done with CS103 problem sets!

- Practice final exam is Monday, December 8 from 3:00PM – 6:00PM in Cemex Auditorium.

- Please evaluate this course on Axess! Your feedback really does make a difference.
The Big Picture
Imagine what it must have been like to discover all of the results in this class.
Cantor's Theorem: $|S| < |\mathcal{P}(S)|$

Corollary: Unsolvable problems exist.
What problems can be solved by computers?
First, we need to learn how to prove results with certainty.

Otherwise, how can we know for sure that we're right about anything?
Now, we need to learn how to prove things about processes that proceed step-by-step.

So let's learn induction.
We also should be sure we have some rules about reasoning itself. Let's add some logic into the mix.
Finally, let's study a few common discrete structures.

That way, we know how to model connected structures and relationships.
Okay! So now we're ready to go! What problems are unsolvable?
Well, first we need a definition of a computer!
Cool! Now we have a model of a computer!
We're not quite sure what we can solve at this point, but that's okay for now.

Let's call the languages we can capture this way the **regular languages**.
I wonder what other machines we can make?
Wow! Those new machines are way cooler than our old ones!
I wonder if they're more powerful?
Wow! I guess not. That's surprising!
So now we have a new way of modeling computers with finite memory!
I wonder how we can combine these machines together?
Cool! Since we can glue machines together, we can glue languages together as well.
How are we going to do that?
\( a^+(a^+)^*@a^+(a^+)^+ \)
Wow! We've got a new way of describing languages.
So what sorts of languages can we describe this way?
The image depicts a state transition diagram with four states labeled $q_s$, $q_1$, $q_2$, and $q_f$. The transitions are labeled with the following conditions:

- From $q_s$ to $q_1$: $\varepsilon R_{11}$
- From $q_1$ to $q_2$: $R_{22}$
- From $q_2$ to $q_f$: $R_{21}$
- From $q_s$ to $q_f$: $\varepsilon R_{11} * R_{12}$

The diagram also includes loops on $q_1$ and $q_2$ labeled with $\varepsilon$.
Awesome! We got back the exact same class of languages.
It seems like all our models give us the same power! Did we get every language?
\(xw \in L\)
\(yw \notin L\)
Wow, I guess not.
But we did learn something cool:

*We have just explored what problems can be solved with finite memory.*
So what else is out there?
Can we describe languages another way?
S → aX
X → b | C
C → Cc | ε
Awesome!
So, did we get every language yet?
Hmmm... guess not.
So what if we make our memory a little better?
\[ \square \rightarrow \square, R \]
\[ 0 \rightarrow 0, R \]
\[ 0 \rightarrow 0, L \]
\[ 1 \rightarrow 1, L \]
\[ 1 \rightarrow \square, L \]
\[ \square \rightarrow \square, R \]
\[ \square \rightarrow \square, R \]
\[ q_{\text{rej}} \]
\[ q_{\acc} \]
\[ \text{Check for 0} \]
\[ \text{Go to start} \]
\[ \text{Clear a 1} \]
\[ \text{Go to end} \]
\[ 0 \rightarrow 0, R \]
\[ 1 \rightarrow 1, R \]
Cool! Can we make these more powerful?
V = “On input ⟨w, T⟩, where T is a sequence of transitions:
- Run N on w, following transitions in the order specified in T.
- If any of the transitions in T are invalid or can't be followed, reject.
- If after following the transitions N accepts w, accept; otherwise reject.
Wow! Looks like we can't get any more powerful.

(The *Church-Turing thesis* says that this is not a coincidence!)
So why is that?
$U_{\text{TM}} = \text{"On input } \langle M, w \rangle \text{, where } M \text{ is a TM and } w \in \Sigma^*: \text{ Set up the initial configuration of } M \text{ running on } w. \text{ while (true) { \\
\hspace{1em} \text{If } M \text{ accepted } w, \text{ then } U_{\text{TM}} \text{ accepts } \langle M, w \rangle. \\
\hspace{1em} \text{If } M \text{ rejected } w, \text{ then } U_{\text{TM}} \text{ rejects } \langle M, w \rangle. \\
\hspace{1em} \text{Otherwise, simulate one more step of } M \text{ on } w. \\
}}$"
Wow! Our machines can simulate one another!
This is a theoretical justification for why all these models are equivalent to one another.
So... can we solve everything yet?
<table>
<thead>
<tr>
<th></th>
<th>\langle M_0 \rangle</th>
<th>\langle M_1 \rangle</th>
<th>\langle M_2 \rangle</th>
<th>\langle M_3 \rangle</th>
<th>\langle M_4 \rangle</th>
<th>\langle M_5 \rangle</th>
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<tbody>
<tr>
<td>M_0</td>
<td>Acc</td>
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<td>No</td>
<td>Acc</td>
<td>Acc</td>
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<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>...</td>
</tr>
<tr>
<td>M_2</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
<td>Acc</td>
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</tr>
<tr>
<td>M_3</td>
<td>No</td>
<td>Acc</td>
<td>Acc</td>
<td>No</td>
<td>Acc</td>
<td>Acc</td>
<td>...</td>
</tr>
<tr>
<td>M_4</td>
<td>Acc</td>
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</tr>
</tbody>
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No  No  No  Acc  No  Acc  ...
Oh great. Some problems are impossible to solve.
But why exactly is that?
#include <iostream>
#include <string>
#include <vector>
using namespace std;

const vector<string> kToPrint = {
    /* ... */
};

void printProgramInQuotes() {
    for (string line: kToPrint) {
        cout << "  " "\"
        for (char ch: line) {
            if (ch == '"') cout << "\\"
            else if (ch == '\') cout << "\\\\n            else cout << ch;
        }
        cout << ""," << endl;
    }
}

int main() {
    for (string line: kToPrint) {
        if (line == "@") printProgramInQuotes();
        else cout << line << endl;
    }
}
Weird! Programs can gain access to their own source code!
Why does that matter?
$M = “$On input $w$:

- Have $M$ get its own description, $\langle M \rangle$.
- Decide whether $M$ will accept $w$.
- If $M$ will accept $w$, choose to reject $w$.
- If $M$ will not accept $w$, choose to accept $w$."


Okay... maybe we can't decide or recognize everything.

Can we at least verify or refute everything?
\[ EQ_{TM} \notin RE \]

\[ EQ_{TM} \notin co\text{-}RE \]
Wow. That's pretty deep.
So... what can we do efficiently?
So... how are you two related again?
No clue.
But what do we know about them?
Congratulations on making it this far!
What's next in CS theory?
What problems can be solved by computers?

- Regular languages
- Context-Free Languages
  - R, RE, and co-RE
  - P and NP

- DFAs
- NFAs
- Regular Expressions
- Context-Free Grammars
  - Recognizers
  - Deciders
  - Verifiers
  - NTMs
  - Corecognizers
  - Poly-time TMs/NTMs/Verifiers
What problems can be solved by computers?

Function problems (CS254)
Counting problems (CS254)

Interactive proof systems (CS254)
Approximation algorithms (CS261/361)
Average-case efficiency (CS264)
Randomized algorithms (CS265/254)
Parameterized complexity (CS266)
Communication complexity (CS369E)

Oracle machines (CS154)
Space-Bounded TMs (CS154/254)
Machines with Advice (CS254/354)
Streaming algorithms (CS263)
μ-Recursive functions (CS258)
Quantum computers (CS259Q)
Circuit complexity (CS354)
How do we actually get the computer to effectively solve problems?

DFA design intuitions
Guess-and-check
Massive parallelism
Myhill-Nerode lower bounds
Verification
Polynomial-time reductions
How do we actually get the computer to effectively solve problems?

<table>
<thead>
<tr>
<th>Course Title</th>
<th>Code</th>
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<td>Efficient data structures</td>
<td>CS166</td>
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<td>Modern algorithmic techniques</td>
<td>CS168</td>
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<tr>
<td>Approximation algorithms</td>
<td>CS261</td>
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<td>Average-case efficient algorithms</td>
<td>CS264</td>
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<tr>
<td>Randomized algorithms</td>
<td>CS265</td>
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<tr>
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<td>CS266</td>
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<td>Geometric algorithms</td>
<td>CS268</td>
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<td>Game-theoretic algorithms</td>
<td>CS364A/B</td>
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</table>
Where does CS theory meet CS practice?

Finite state machines
Regular expressions
CFGs and programming languages
Password-checking
Autograding
“This program is not responding”
Polynomial-time reducibility
\textbf{NP}-hardness and \textbf{NP}-completeness
Where does CS theory meet CS practice?

- Compilers (CS143)
- Computational logic (CS157)
- Program optimization (CS243)
- Data mining (CS246)
- Cryptography (CS255)
- Programming languages (CS258)
- Network protocol analysis (CS259)
- Techniques in big data (CS263)
- Graph algorithms (CS267)
- Computational geometry (CS268)
- Algorithmic game theory (CS364)
A Whole World of Theory Awaits!
What's being done here at Stanford?
Hardness results for easy problems  
(Virginia Williams)
Algorithms \cap Game theory
(Tim Roughgarden)
Learning patterns in randomness
(Greg Valiant)
Optimizing programs... randomly
(Alex Aiken)
Computing on encrypted data
(Dan Boneh)
Interpreting structure from shape
(Leonidas Guibas)
Lower bounds from upper bounds
(Ryan Williams)
So many options – what to do next?
Interested in trying out CS?  
**Continue on to CS109!**
Really enjoyed this class?

Give CS154 a try!
Want to see this material come to life?

Check out CS143!
Want to just go write code? Take CS107!
Keep on exploring! There's so much more to learn!
A Final “Your Questions”
CS theory is all about asking what's possible in computer science.
There are more problems to solve than there are programs capable of solving them.
There is so much more to explore and so many big questions to ask – many of which haven't been asked yet!
What We've Covered

- Sets
- Proof Techniques
- Induction
- Graphs
- Logic
- Pigeonhole Principle
- Functions
- Relations
- DFAs
- NFAs
- Regular Expressions
- Closure Properties
- Nonregular Languages
- CFGs
- Turing Machines
- R, RE, and co-RE
- The Recursion Theorem
- NTMs and Verifiers
- Unsolvable Problems
- Reductions
- Time Complexity
- P
- NP
- NP-Completeness
Final Thoughts