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David G. Luenberger • Yinyu Ye

# Linear and Nonlinear Programming

Fifth Edition

Springer

David G. Luenberger
Department of Management Science
and Engineering
Stanford University
Stanford, CA, USA

Yinyu Ye
Department of Management Science
and Engineering
Stanford University
Stanford, CA, USA

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*To Susan, Robert, Jill, and Jenna;*
*Daisun, Fei, Tim, Kaylee, and Rylee*

# Preface

This book is intended as a text covering the central concepts of practical optimization techniques. It is designed for either self-study by professionals or classroom work at the undergraduate or graduate level for students who have a technical background in engineering, mathematics, or science. Like the field of optimization itself, which involves many classical disciplines, the book should be useful to system analysts, operations researchers, numerical analysts, management scientists, and other specialists from the host of disciplines from which practical optimization applications are drawn. The prerequisites for convenient use of the book are relatively modest; the prime requirement being some familiarity with introductory elements of linear algebra. Certain sections and developments do assume some knowledge of more advanced concepts of linear algebra, such as eigenvector analysis, or some background in sets of real numbers, but the text is structured so that the mainstream of the development can be faithfully pursued without reliance on this more advanced background material.

Although the book covers primarily material that is now fairly standard, this edition emphasizes methods that are both state-of-the-art and popular in emerging fields such as Data Sciences, Machine Learning and Decision Analytics. One major insight is the connection between the purely analytical character of an optimization problem, expressed perhaps by properties of the optimality conditions, and the behavior of algorithms used to solve a problem. This was a major theme of the first edition of this book and the fifth edition further expands and illustrates this relationship.

As in the earlier editions, the material in this fifth edition is organized into three separate parts. Part I is a self-contained introduction to classical and conic linear programming, a key component of optimization theory. The presentation in this part is fairly conventional, covering the main elements of the underlying theory of linear programming, many of the most effective numerical algorithms, and many of its important special and emerging applications. Part II, which is independent of Part I, covers the theory of unconstrained optimization, including both derivations of the appropriate optimality conditions and an introduction to basic algorithms. This part of the book explores the general properties of algorithms and defines various notions of convergence. Part III extends the concepts developed in the second part to constrained optimization problems. Except for a few isolated sections, this part is also independent of Part I. It is possible to go directly into Parts II and III omitting Part I, and, in fact, the book has been used in this way in many universities. Each part of the book contains enough material to form the basis of a one-quarter course. In either classroom use or for self-study, it is important not to overlook the suggested exercises at the end of each chapter. The selections generally include exercises of a computational variety designed to test one's understanding of a particular algorithm, a theoretical variety designed to test one's understanding of a given theoretical development, or of the variety that extends the presentation of the chapter to new applications or theoretical areas. One should attempt at least four or five exercises from each chapter. In progressing through the book it would be unusual to read straight through from cover to cover. Generally, one will wish to skip around. In order to facilitate this mode, we have indicated sections of a specialized or digressive nature with an asterisk*.

New to this edition is, in Chap. 2, the introduction of quite a few problems in Machine Learning and Data Science that are closely related to linear programming. We added a section in Chap. 2 devoted to Farkas' Lemma and the Alternative-System theory. Consequently, we moved the Duality and Complementarity Chapter (Chap. 4) before the Simplex Method Chapter (Chap. 3). We restructured topics in Chap. 3 substantially, since linear programs are nowadays solved by computers rather than by hand. Therefore, we focus on introducing methods and algorithms most efficiently implementable by computer codes. Due to a recent breakthrough, we also add a section in (Chap. 3) on proving the efficiency of the Simplex method, which remains a dominate solver for linear programming.

As the field of optimization advances, researcher and practitioners face more challenges: addressing data-driven and dynamic programs, making decisions with uncertainty, developing online algorithms, and expanding the overall theory. We introduce modern optimization topics, such as Markov Decision Process, Reinforcement Learning, Distributionally Robust Stochastic Optimization and Online Optimization. In particular, we have added a section in Chap. 3 to illustrate online linear programming algorithms where the decisions need to be made "on the fly" in problem settings. One of the algorithms is related to the online Stochastic Gradient Decent method that is added in Chap. 8.

Another new topic is multiplicative descent-direction methods that exhibit good convergence properties in Chap. 8. We have included the affine-scaling and mirror-descent methods that are especially effective for optimization where decision variables are subject to nonnegativity constraints. We have also added a couple of globally convergent Newton's methods there.

We have added a section on Lagrangian duality for constrained nonlinear optimization in Chap. 11. The Lagrangian duality plays a fundamental role, as the duality does for linear optimization, in both theory and algorithm design. We introduce detailed rules on how to construct the dual explicitly for certain type of problems, such as the support vector machine problem.

Then we have added two sections into Chap. 12. The first is a "descent-first and feasible-second" steepest descent projection method for linear and nonlinear constrained optimization, which is simple and effective in practice. The second is an interior trust-region sequential quadratic optimization method which is suitable for computing a solution that meets the second-order optimality condition. The convergence analyses of the two methods are presented.

We have added a new section in Chap. 14 to introduce the randomized multi-block alternative direction method with multipliers, which are effective for optimization problems arising of both private and distributed data.

Finally, we have added two sections in Chap. 15 introducing the nonlinear monotone complementarity problem that includes the optimality condition problem as a special case. We also present the homogeneous model/algorithm that is a one-phase algorithm with capability to detect possible primal or dual infeasibility, which becomes an important task in nonlinear optimization.

In this revision, we have also removed a few sections where the methods and/or materials are not suitable for large-scale optimization and computer-coding in our modern computation age.

We wish to thank the many students and researchers who over the years have given us comments concerning the book and those who encouraged us to carry out this revision. We are especially thankful to Xiaocheng Li and Robert Luenberger for their careful readings and comments for this new revision.

Stanford, CA, USA D.G. Luenberger
Stanford, CA, USA Y. Ye
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