

# **Mathematical Optimization in Machine Learning/Decision-Making**

**CUHKSZ, APRIL 6, 2023**

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**Stanford University and CUHKSZ (Sabbatical Leave)**

# Linear Programming and LP Giants won Nobel Prize...

$$\begin{aligned} & \max \quad \sum \pi_j x_j \\ & \text{s.t.} \quad \sum_j a_j x_j \leq b, \\ & \quad 0 \leq x_j \leq 1 \quad \forall j = 1, \dots, n \end{aligned}$$



# ChatGPT求解优化问题？

优化问题根据变量、约束和目标函数的特性，可以分为不同种类，如线性规划、非线性规划和整数规划等

- ▶ 小规模整数优化问题，可以通过穷举法求解
- ▶ 大规模整数规划问题一般由专业的整数规划软件求解（Gurobi, COPT）

## 一个简单的整数线性规划问题

假设有一个最多能承重5kg的背包和5个重量不同的物品，每个物品都具有一定的价值。将哪些物品装入背包，可以使得背包内的物品总价值最大？

物品的重量和价值如下表所示：


物品	1	2	3	4	5
重量 (kg)	2	1	2.5	3.5	1.5
价值 (元)	18	10	20	28	15



此问题的最优解是将物品2、物品3和物品5装入背包，总价值为45元。

# ChatGPT求解优化问题？

## 将上述背包问题向ChatGPT提问：

 假设有5件物品和一个最多能承重5kg的背包。第1件物品的重量是2kg，其价值为18元；第2件物品的重量是1kg，其价值为10元；第3件物品的重量是2.5kg，其价值为20元；第4件物品的重量是3.5kg，其价值为28元；第5件物品的重量是1.5kg，其价值为15元。请问将哪些物品装入背包里，物品的价值总和最大？请使用整数规划求解该问题。

 这是一个经典的背包问题，可以使用整数规划方法求解。

我们可以定义0/1变量 $x_i$ ，表示是否将第 $i$ 件物品放入背包中。同时，我们还需要定义一个非负整数变量 $z$ ，表示物品的总价值。

然后，我们可以将问题转化为以下的线性规划问题：

最大化  $z=18x_1+10x_2+20x_3+28x_4+15x_5$

满足约束条件：


$2x_1+x_2+2.5x_3+3.5x_4+1.5x_5 \leq 5$

$x_i \in \{0,1\}, i=1,2,3,4,5$

由于 $x_i$ 是整数变量，这是一个整数规划问题。可以使用整数规划求解器求解。

求解结果表明，将第1件、第3件、第5件物品装入背包中，可以使得物品的总价值最大，为 $18+20+15=53$ 元。

 ChatGPT 在不调用求解器的前提下，能否求解大规模优化问题？

 在不调用求解器的前提下，对于大规模的优化问题，通常也比较难直接求解得到最优解，因为现实中的大多数优化问题都非常复杂，存在着多个目标、多个约束、非线性、离散等多种问题，很容易遇到组合爆炸的问题。

## 向ChatGPT提问：

- ▶ ChatGPT 会尝试性给出解答，但是不能保证答案的正确性。
- ▶ ChatGPT会建议调用求解器求解优化问题。
- ▶ ChatGPT 并不是万能的，它是一个语言处理模型，回答复杂的计算问题还需要调用专业的软件

上述问答产生于2023年3月12日

# Today's Sharing

- 1. Online Linear Programming Algorithms and Applications**
- 2. Accelerated Second-Order Methods for Nonlinear Optimization and Applications**
- 3. Mixed Integer Linear Programming Solver and Applications**
- 4. Equitable Covering & Partition – Divide and Conquer and Applications**
- 5. Beyond Math: Love Sport and Have a Belief**

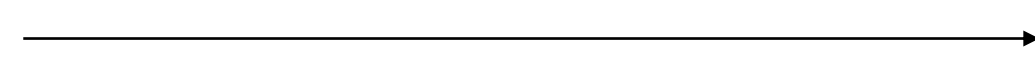
# Topic 1. Online Linear Programming

- 1、在线学习理论与算法研究 (Agrawal et al. 2010, 14, Li&Y 2022)

- What is OLP?

- 传统机器学习问题：有大量（训练）数据，找到最佳模型  
（例子：回归模型、树模型）

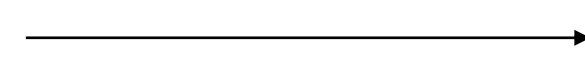
已有数据



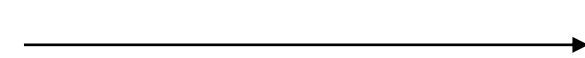
最佳模型

- **在线**学习：数据的生成和学习是同时发生的，由决策影响  
（例如多臂老虎机问题）

数据



决策



反馈

算法

潜在模型

需要一边学习，一边优化

# Online Auction Example

- There is a fixed selling period or number of buyers; and there is a fixed inventory of goods
- Customers come and require a bundle of goods and make a bid
- Decision: To sell or not to sell to each individual customer on the fly?
- Objective: Maximize the revenue.

<b>Bid #</b>	<b>\$100</b>	<b>\$30</b>	<b>....</b>	<b>...</b>	<b>...</b>	<b>Inventory</b>
<b>Decision</b>	<b>x1</b>	<b>x2</b>				
<b>Pants</b>	<b>1</b>	<b>0</b>	<b>....</b>	<b>...</b>	<b>...</b>	<b>100</b>
<b>Shoes</b>	<b>1</b>	<b>0</b>				<b>50</b>
<b>T-Shirts</b>	<b>0</b>	<b>1</b>				<b>500</b>
<b>Jackets</b>	<b>0</b>	<b>0</b>				<b>200</b>
<b>Hats</b>	<b>1</b>	<b>1</b>	<b>...</b>	<b>...</b>	<b>...</b>	<b>1000</b>

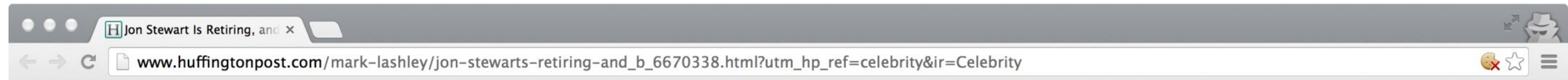
# Price Mechanism for Online Auction

- Learn and compute itemized optimal prices
- Use the prices to price each bid
- Accept if it is a over bid, and reject otherwise

<b>Bid #</b>	<b>\$100</b>	<b>\$30</b>	<b>....</b>	<b>...</b>	<b>...</b>	<b>Inventory</b>	<b>Price?</b>
<b>Decision</b>	<b>x1</b>	<b>x2</b>					
<b>Pants</b>	<b>1</b>	<b>0</b>	<b>....</b>	<b>...</b>	<b>...</b>	<b>100</b>	<b>45</b>
<b>Shoes</b>	<b>1</b>	<b>0</b>				<b>50</b>	<b>45</b>
<b>T-Shirts</b>	<b>0</b>	<b>1</b>				<b>500</b>	<b>10</b>
<b>Jackets</b>	<b>0</b>	<b>0</b>				<b>200</b>	<b>55</b>
<b>Hats</b>	<b>1</b>	<b>1</b>	<b>...</b>	<b>...</b>	<b>...</b>	<b>1000</b>	<b>15</b>



# App. I: Online Matching for Display Advertising



**Mark Lashley** Assistant Professor, La Salle University

## Jon Stewart Is Retiring, and it's Going to Be (Kind of) Okay

Posted: 02/13/2015 3:21 pm EST | Updated: 02/13/2015 3:59 pm EST



195 12 5 0 14

Like Share Tweet Pin it Comment

When the news broke Tuesday night that longtime *Daily Show* host Jon Stewart would be leaving his post in the coming months, the level of trauma on the internet was palpable. Some expected topics arose, within hours -- minutes, even -- of the announcement trickling out. Why would Stewart leave now? What's his plan? Who should replace him? Could the next *Daily Show* host be a woman? (Of course). Is this an elaborate ruse for Stewart to take over the *NBC Nightly News*? (Of course not).

The public conversation over the past two days has been so Stewart-centric that the retirement news effectively pushed NBC anchor Brian Williams's suspension off of social media's front pages. Part of that is the shock; we knew the other shoe was about to drop with (on?) Williams, but Stewart's departure was known only to Comedy Central brass before it was revealed to his studio audience. Part of it is how meme-worthy the parallels between the two hosts truly are ("fake newsman speaks truth, real newsman spins lies," some post on your Twitter timeline probably read). Breaking at

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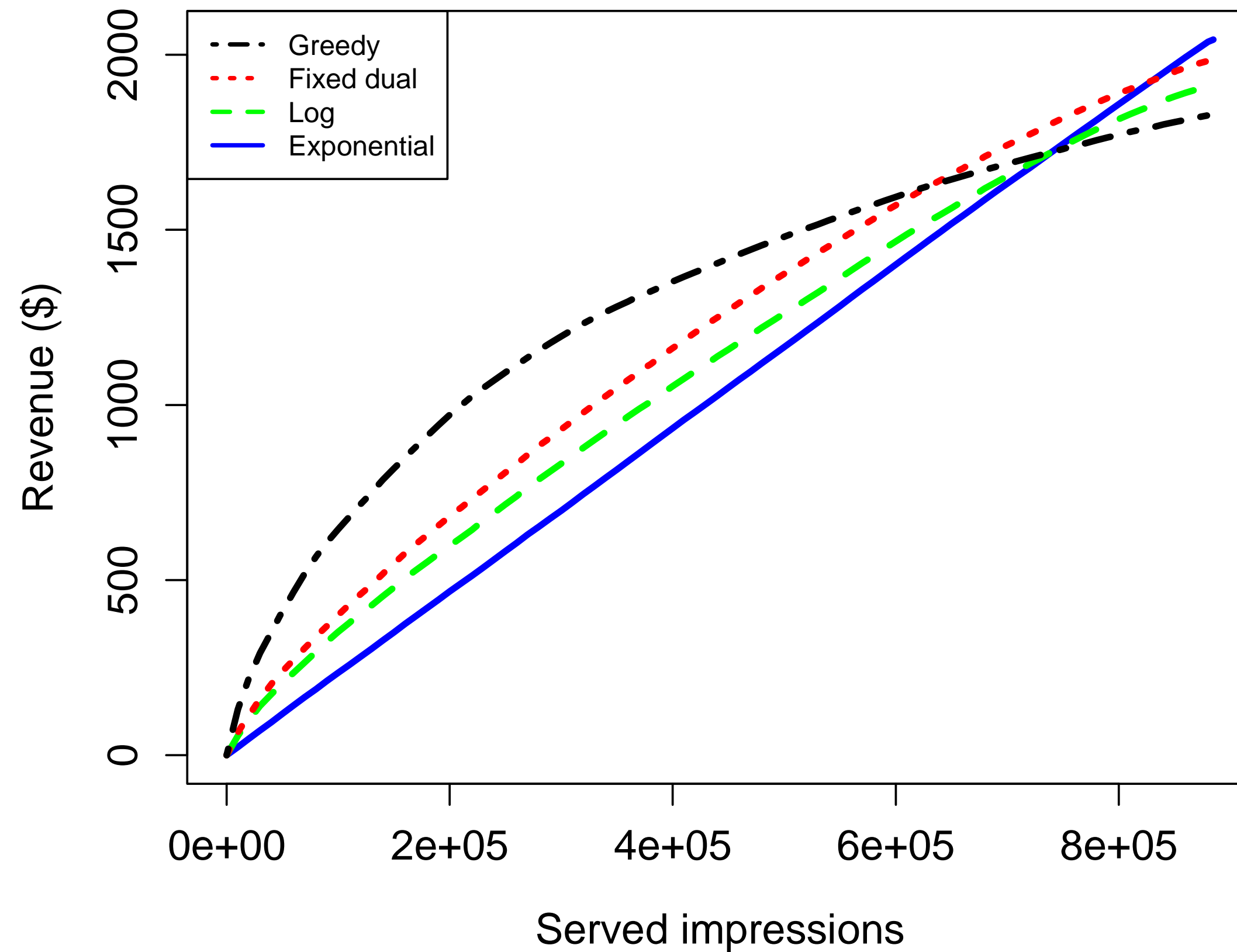


**Incredible Seal Vs Octopus Battle Caught On Camera**



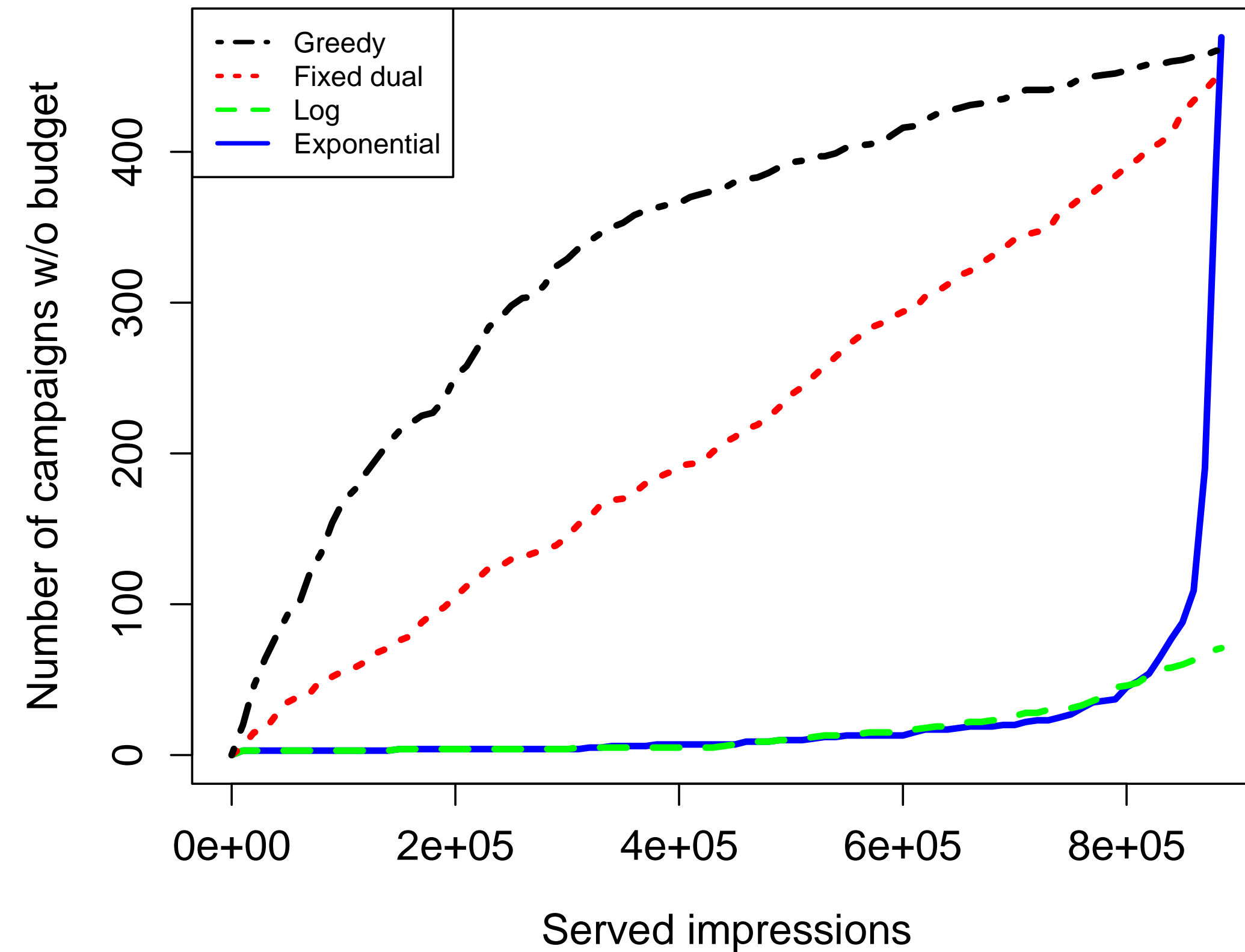
# Revenues generated by different methods

- Total Revenue for impressions in T2 by Greedy and OLP with different allocation risk functions



# # of Out-of-Budget Advertisers

- Greedy exhausts budget of many advertisers early.
- Log penalty keeps advertisers in budget but it is very conservative.
- Exponential penalty Keeps advertisers in budget until almost the end of the timeframe.



# 阿里巴巴在2019年云栖大会上提到在智能履行决策上使用OLP的算法

2018 杭州·云栖大会 Alibaba Group

## 智能履行决策

商家

杭州-上海 杭州-广州 杭州-北京 杭州-武汉 ...

YTO ZTO YUNDA

菜鸟智能发货引擎

时效	服务	成本	单量平衡	...
线路容量	网点容量	局部优化	全局优化	...

最优快递

智能决策 ML & Optimization

商家的履行是带有全局约束的序列执行决策

- Online assignment problem
- Control based method
- Online linear programming

Ref: Agrawal, Shipra, Zizhuo Wang, and Yinyu Ye. "A dynamic near-optimal algorithm for online linear programming." *Operations Research* 62.4 (2014): 876-890.

决策变量

$$C_{ij} = c1 * \text{成本} + c2 * \text{服务} + c3 * \text{时效}$$
$$\max_x \sum_{i=1}^n \sum_{j=1}^m C_{ij} x_{ij}$$

将订单 I 匹配给快递公司 j 与否

$$\text{s.t.} \sum_{j=1}^m x_{ij} \leq 1$$
$$\sum_{i=1}^n x_{ij} * a_j \leq u_j$$

商家发货CP总单量比例约束

$$\sum_{i=1}^n \sum_{j=1}^m x_{ij} b_{k,ij} \leq B_k$$

全局约束值, 比如总成本

阿里巴巴团队在2020年CIKM会议论文Online Electronic Coupon Allocation based on Real-Time User Intent Detection上提到他们设计的发红包的机制也使用了OLP的方法 [2]

### Spending Money Wisely: Online Electronic Coupon Allocation based on Real-Time User Intent Detection

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Weijun Ren  
afei@alibaba-inc.com  
Alibaba Group  
Hangzhou, Zhejiang

$$\begin{aligned} & \max \sum_{i=1}^M \sum_{j=1}^N v_{ij} x_{ij} \\ & \text{s.t.} \sum_{i=1}^M \sum_{j=1}^N c_j x_{ij} \leq B, \\ & \sum_j x_{ij} \leq 1, \quad \forall i \\ & x_{ij} \geq 0, \quad \forall i, j \end{aligned} \quad (5)$$

### 3.3 MCKP-Allocation

We adopt the primal-dual framework proposed by [2] to solve the problem defined in Equation 5. Let  $\alpha$  and  $\beta_j$  be the associated dual variables respectively. After obtaining the dual variables, we can solve the problem in an online fashion. Precisely, according to the principle of the primal-dual framework, we have the following allocation rule:

$$x_{ij} = \begin{cases} 1, & \text{where } j = \arg \max_i (v_{ij} - \alpha c_j) \\ 0, & \text{otherwise} \end{cases} \quad (9)$$



# Topic 2. Accelerated Second-Order Methods for Unconstrained Optimization and Applications

$\min f(x), x \in X \text{ in } \mathbb{R}^n,$

- where  $f$  is nonconvex and twice-differentiable,

$$g_k = \nabla f(x_k), H_k = \nabla^2 f(x_k)$$

- Goal: find  $x_k$  such that:

$$\|g_k\| \leq \epsilon \quad (\text{primary, first-order condition})$$

$$\lambda_{\min}(H_k) \geq -\sqrt{\epsilon} \quad (\text{secondary, second-order condition})$$

- First-order methods typically need  $O(n^2\epsilon^{-2})$  operations
- Second-order methods typically need  $O(n^3\epsilon^{-1.5})$  operations
- New? Yes, HSODM and DRSOM: a single-loop method with  $O(n^2\epsilon^{-1.75})$  operations  
(<https://arxiv.org/abs/2211.08212>)

# App. III: HSODM for Policy Optimization in Reinforcement Learning

- Consider policy optimization of linearized objective in reinforcement learning

$$\max_{\theta \in \mathbb{R}^d} L(\theta) := L(\pi_\theta),$$

$$\theta_{k+1} = \theta_k + \alpha_k \cdot M_k \nabla \eta(\theta_k),$$

- $M_k$  is usually a preconditioning matrix.

- The Natural Policy Gradient (NPG) method (Kakade, 2001) uses the Fisher information matrix where  $M_k$  is the inverse of

$$F_k(\theta) = \mathbb{E}_{\rho_{\theta_k}, \pi_{\theta_k}} \left[ \nabla \log \pi_{\theta_k}(s, a) \nabla \log \pi_{\theta_k}(s, a)^T \right]$$

- Based on KL divergence, TRPO (Schulman et al. 2015) uses KL divergence in the constraint:

$$\max_{\theta} \nabla L_{\theta_k}(\theta_k)^T (\theta - \theta_k)$$

$$\text{s.t. } \mathbb{E}_{s \sim \rho_{\theta_k}} [D_{KL}(\pi_{\theta_k}(\cdot | s); \pi_{\theta}(\cdot | s))] \leq \delta.$$

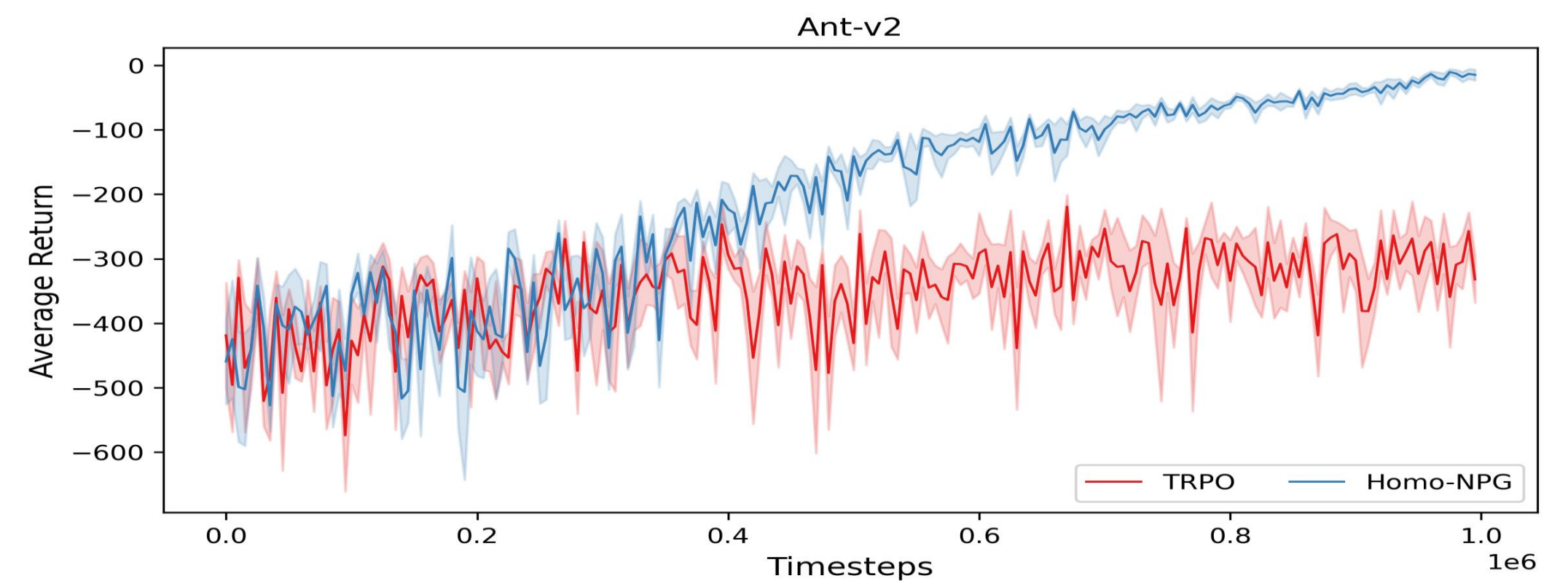
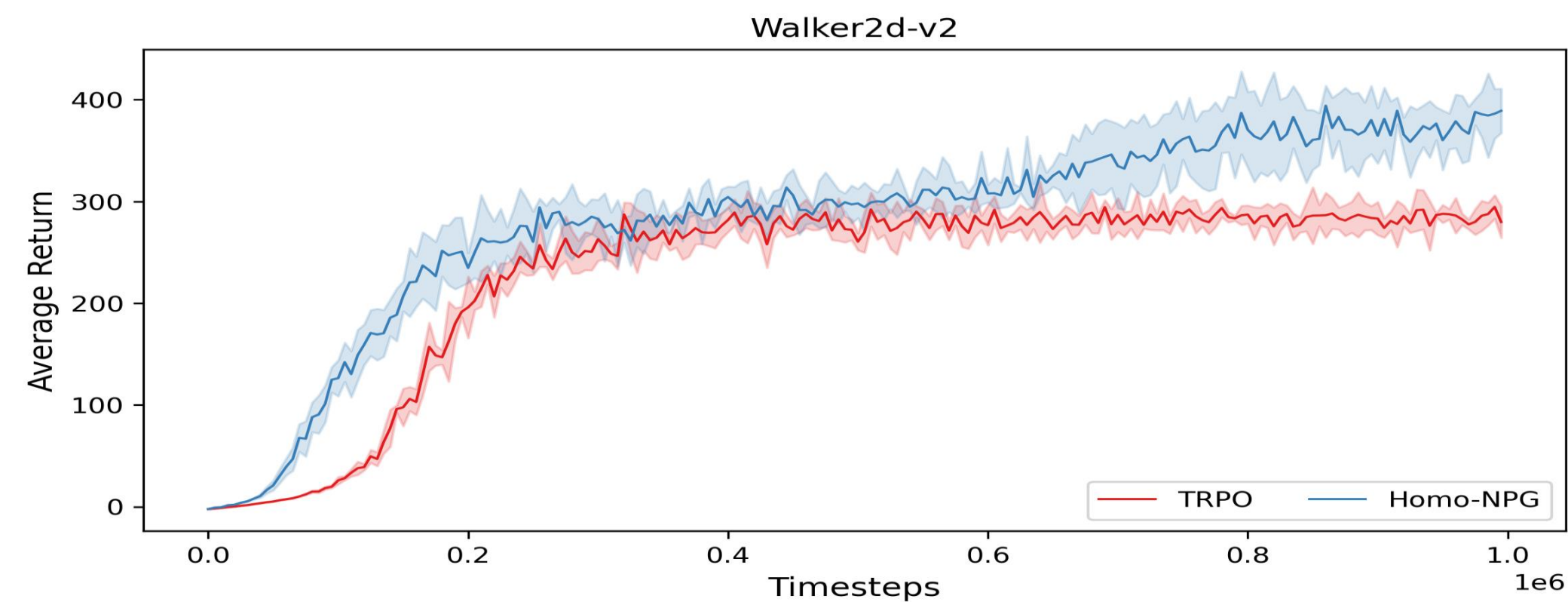
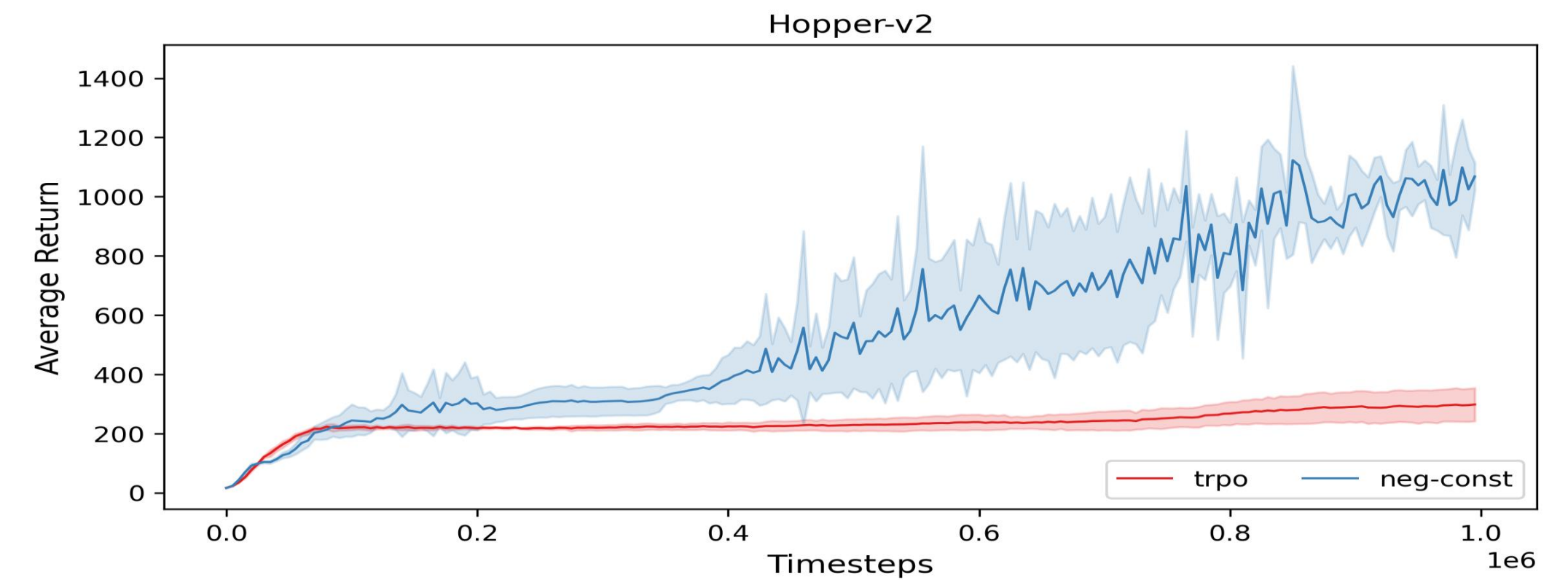
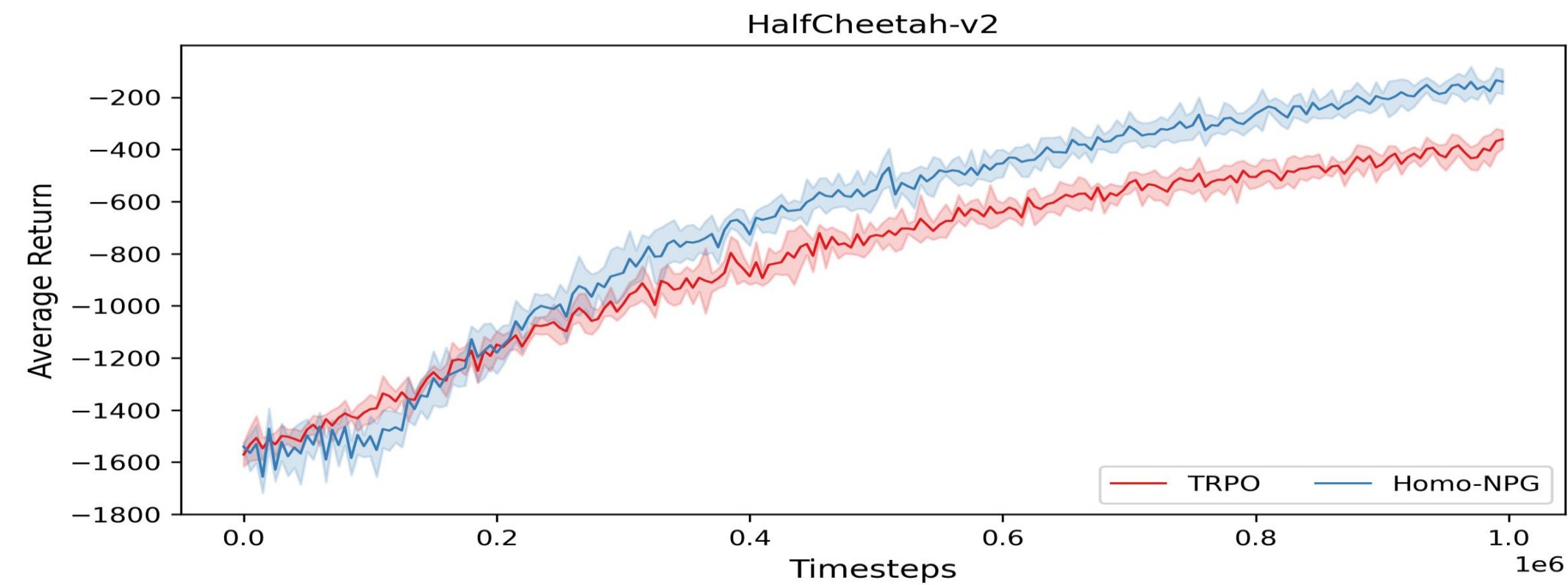


**Homogeneous NPG:  
Apply HSODM!**



# Preliminary Results: HSODM for Policy Optimization in RL

- A comparison of Homogeneous NPG and Trust-region Policy Optimization (Schultz, 2015)



- HSODM provides significant improvements over TRPO
- Ongoing: second-order information of  $L$ ?
- **Further reduce the computation cost per step**

# Dimension Reduced Second-Order Method (DRSOM)

- Motivation from Multi-Directional FOM and Subspace Method, such as CG and ADAM, DRSOM applies the trust-region method in low dimensional subspace.
- This results in a low-dimensional quadratic sub-minimization problem:
- Typically, DRSOM adopts two directions  $d = -\alpha^1 \nabla f(x_k) + \alpha^2 d_k$

$$\text{where } g_k = \nabla f(x_k), H_k = \nabla^2 f(x^k), d_k = x_k - x_{k-1}$$

- Then we solve a 2-d quadratic minimization problem:

$$\min m_k^\alpha(\alpha) := f(x_k) + (c_k)^T \alpha + \frac{1}{2} \alpha^T Q_k \alpha$$

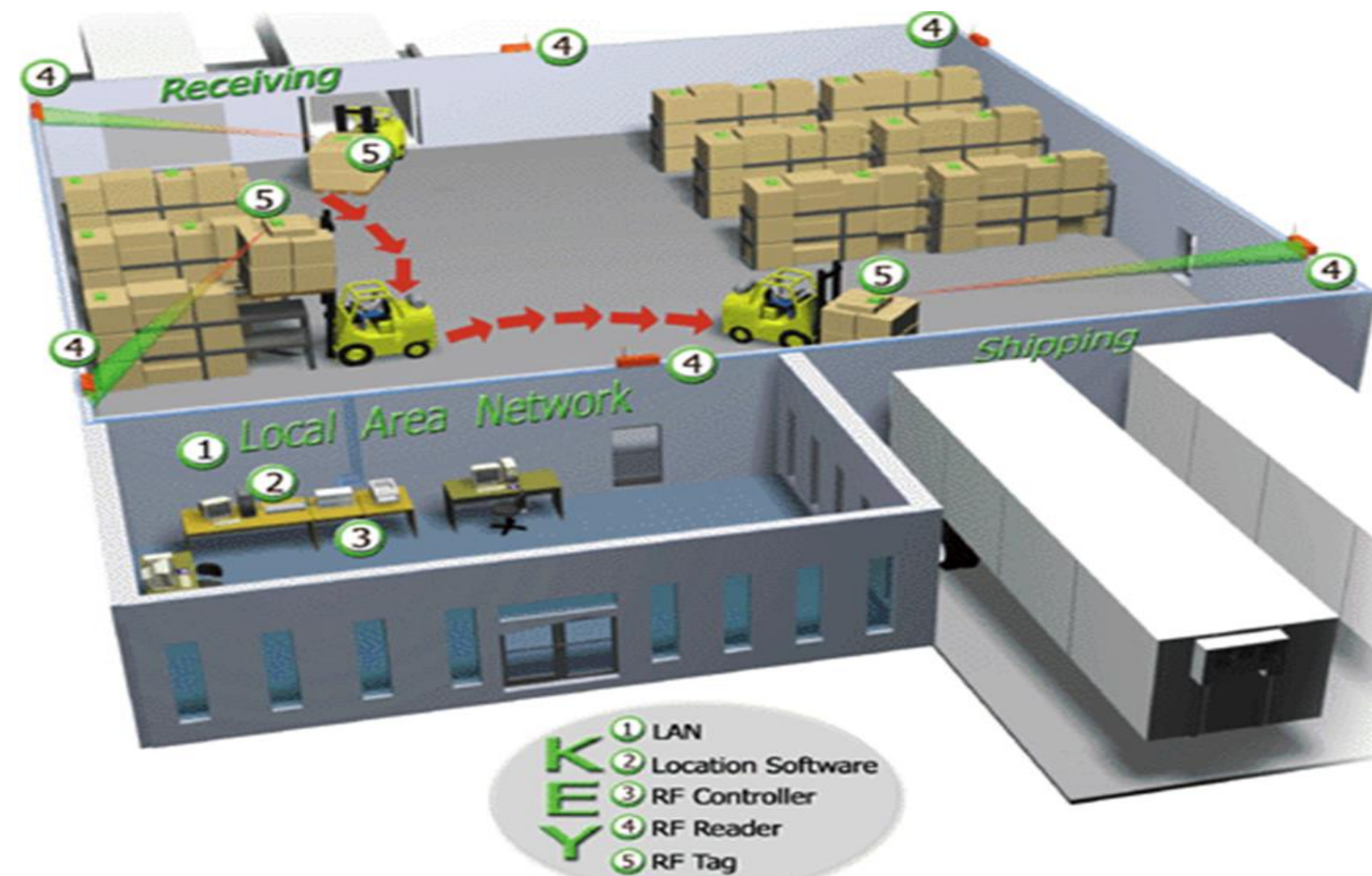
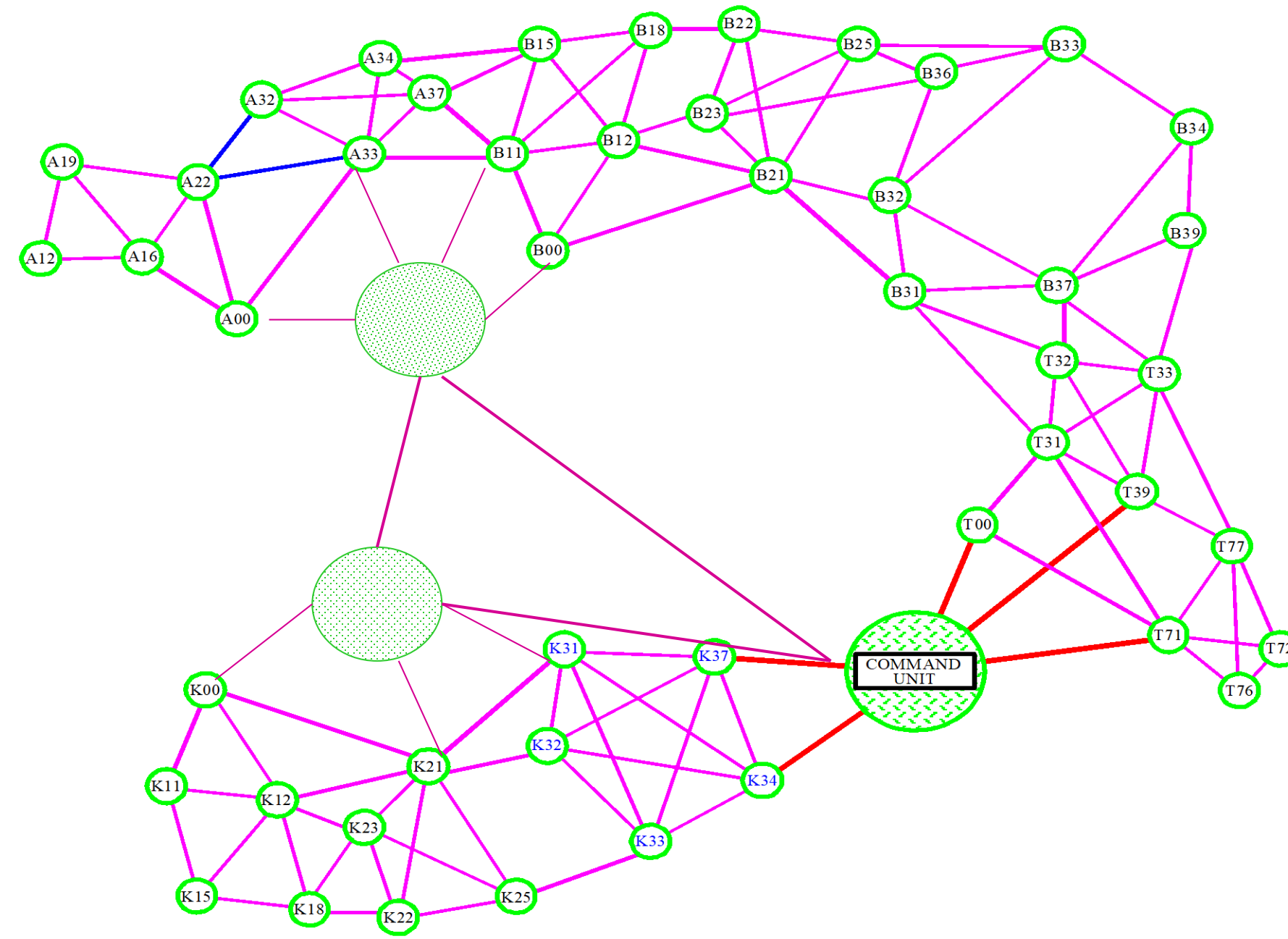
$$\|\alpha\|_{G_k} \leq \Delta_k$$
$$G_k = \begin{bmatrix} g_k^T g_k & -g_k^T d_k \\ -g_k^T d_k & d_k^T d_k \end{bmatrix}, Q_k = \begin{bmatrix} g_k^T H_k g_k & -g_k^T H_k d_k \\ -g_k^T H_k d_k & d_k^T H_k d_k \end{bmatrix}, c_k = \begin{bmatrix} -\|g_k\|^2 \\ g_k^T d_k \end{bmatrix}$$

# App. IV: Sensor Network Location (SNL)

- Localization

- Given partial pairwise measured distance values
- Given some anchors' positions
- Find locations of all other sensors that fit the measured distance values

This is also called graph realization on a fixed dimension Euclidean space



# Mathematical Formulation of Sensor Network Location (SNL)

- Consider Sensor Network Location (SNL)

$$N_x = \{(i, j) : \|x_i - x_j\| = d_{ij} \leq r_d\}, N_a = \{(i, k) : \|x_i - a_k\| = d_{ik} \leq r_d\}$$

where  $r_d$  is a fixed parameter known as the radio range. The SNL problem considers the following QCQP feasibility problem,

$$\|x_i - x_j\|^2 = d_{ij}^2, \forall (i, j) \in N_x$$

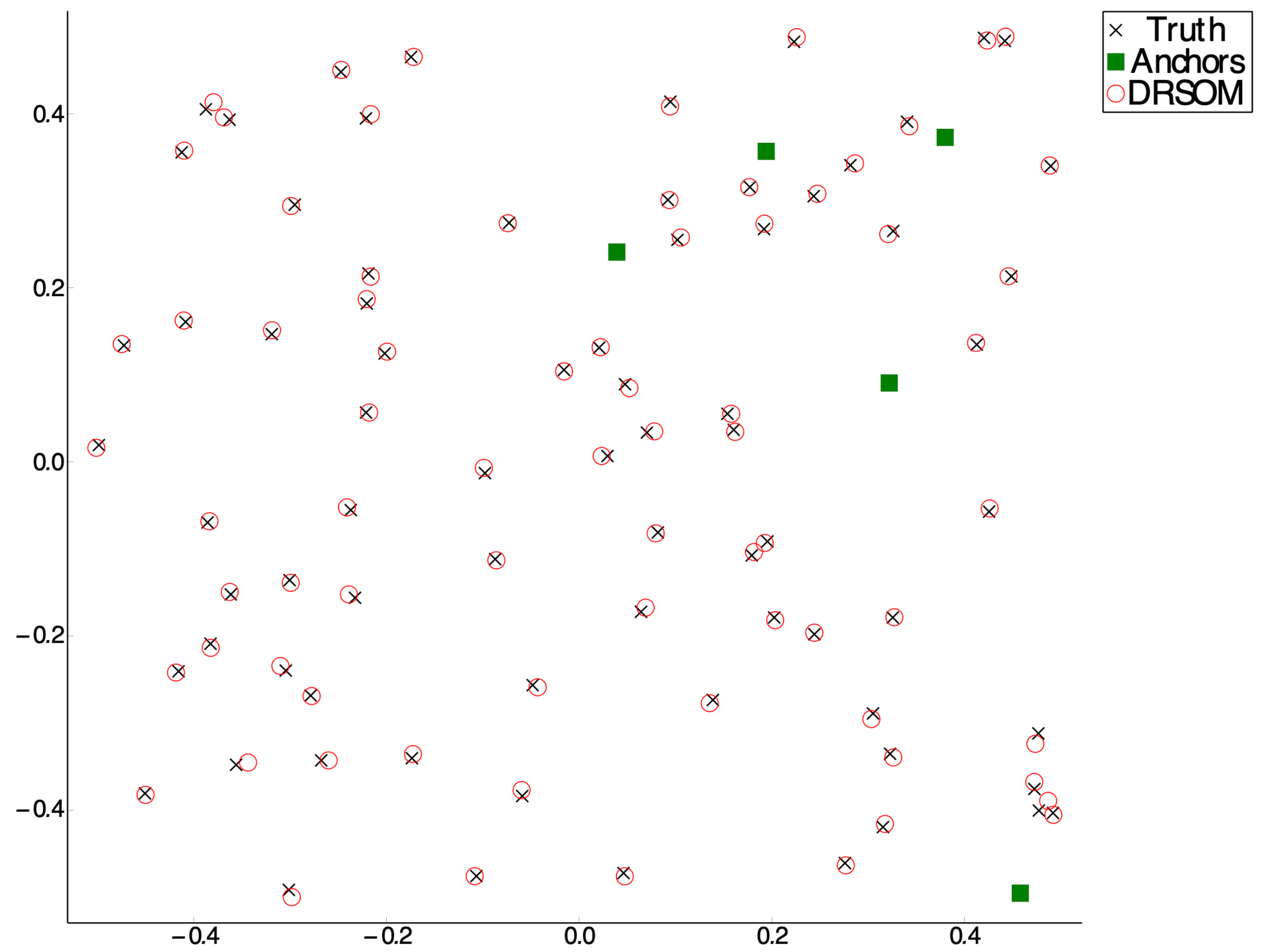
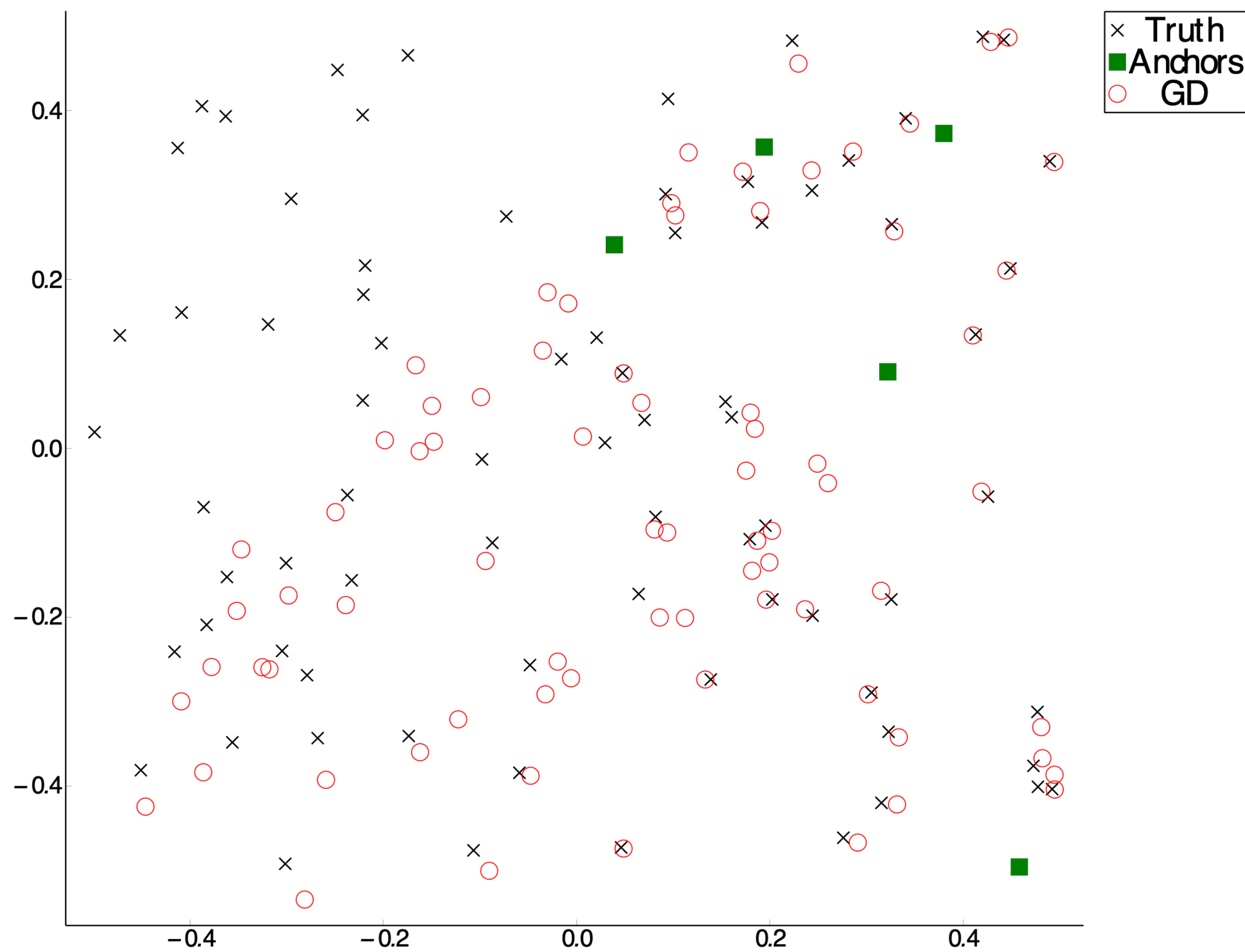
$$\|x_i - a_k\|^2 = \bar{d}_{ik}^2, \forall (i, k) \in N_a$$

- Alternatively, one can solve SNL by the nonconvex nonlinear least square (NLS) problem

$$\min_X \sum_{(i,j) \in N_x} (\|x_i - x_j\|^2 - d_{ij}^2)^2 + \sum_{(k,j) \in N_a} (\|a_k - x_j\|^2 - \bar{d}_{kj}^2)^2.$$

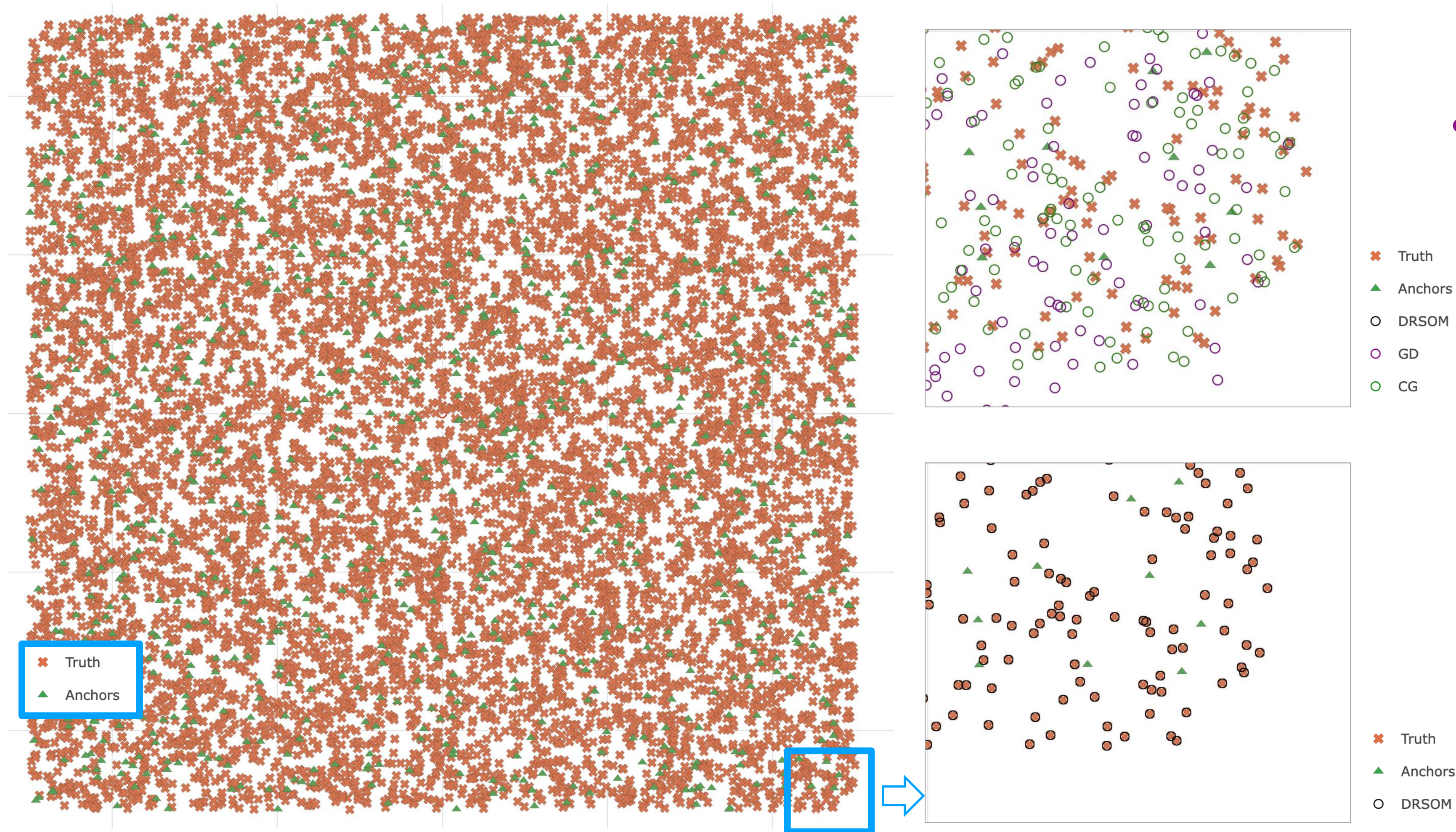
# Sensor Network Location (SNL)

- DRSOM vs the First-Order-Gradient-Descent Method



# Sensor Network Location, Large-Scale Instances

- Graphical results with 10,000 nodes and 1000 anchors (no noise) **within 3,000 seconds**



- GD with Line-search and Hager-Zhang CG both timeout**

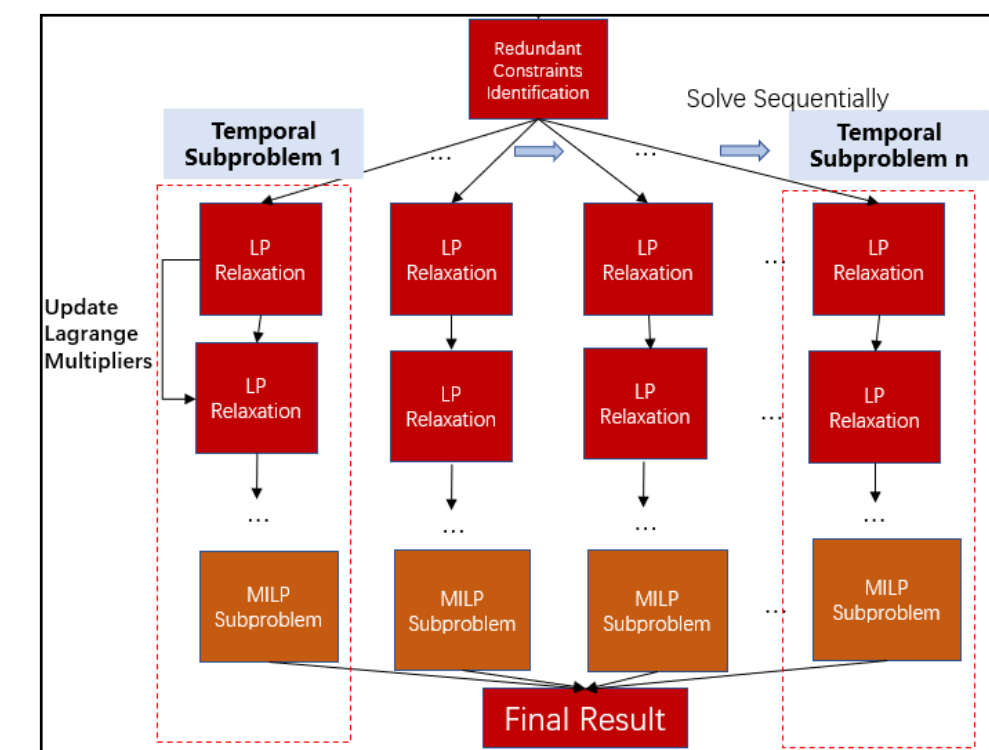
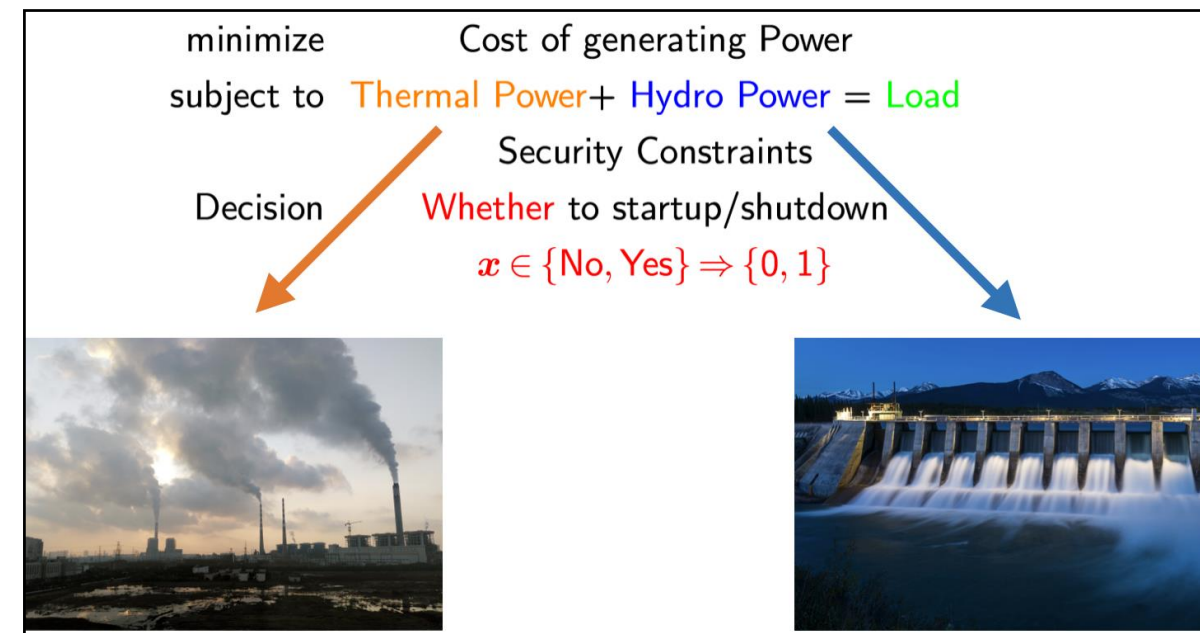
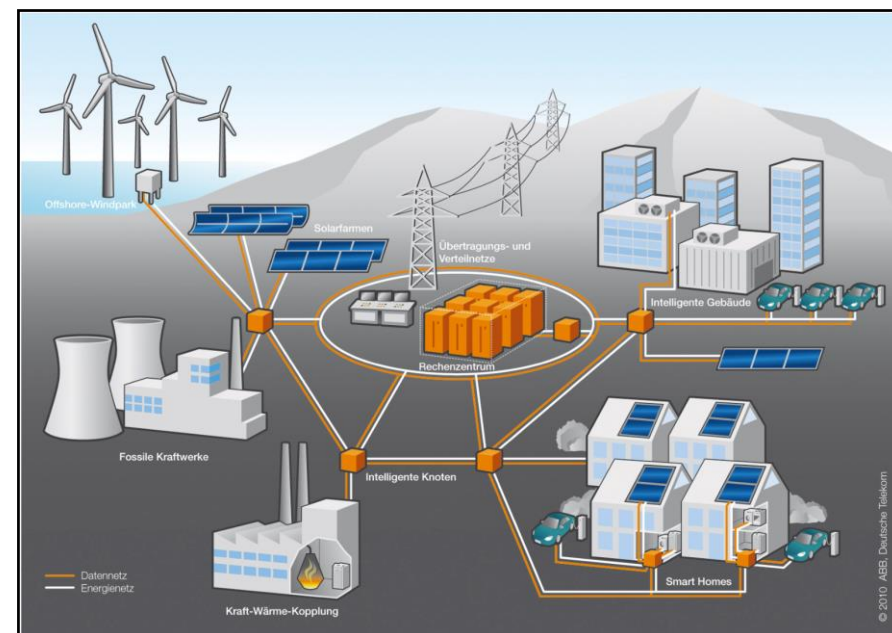
- DRSOM can converge to  $|g_k| \leq 1e^{-5}$  in 2,200s**

# Sensor Network Online Tracking, 2D and 3D

# Topic 3: Mixed Integer Linear Programming Solver

## Application V: Unit Commitment and Power Grid Optimization

### COPT, Cardinal Operations 2022





# Unit Commitment Problem

- Electricity is generated from units (**various** generators)
- Transmitted **safely** and **stably** through power grids
- Consumed at **minimum (reasonable)** price

*Optimization has its role to play*

minimize **Cost of electricity**  
subject to **Safety and Stability**  
Adaptivity to various units

Unit commitment problem dispatches the units **safely** and **stably** at **minimum** cost



# Case Study: Sichuan Thermal-Hydro Hybrid Model

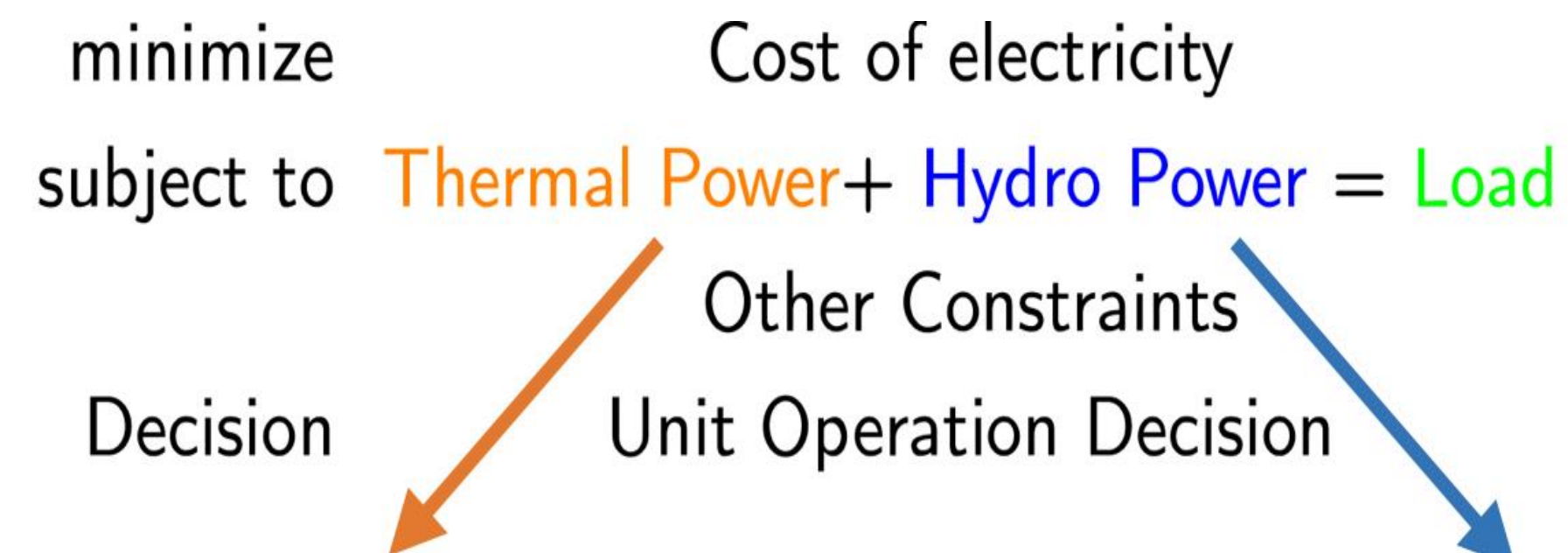
- A UC problem from real-life background (Sichuan Province)
- With 20 **thermal** and 230 **hydro** units
- Hydro units involve **no** decision (binary variables)



## Hardness

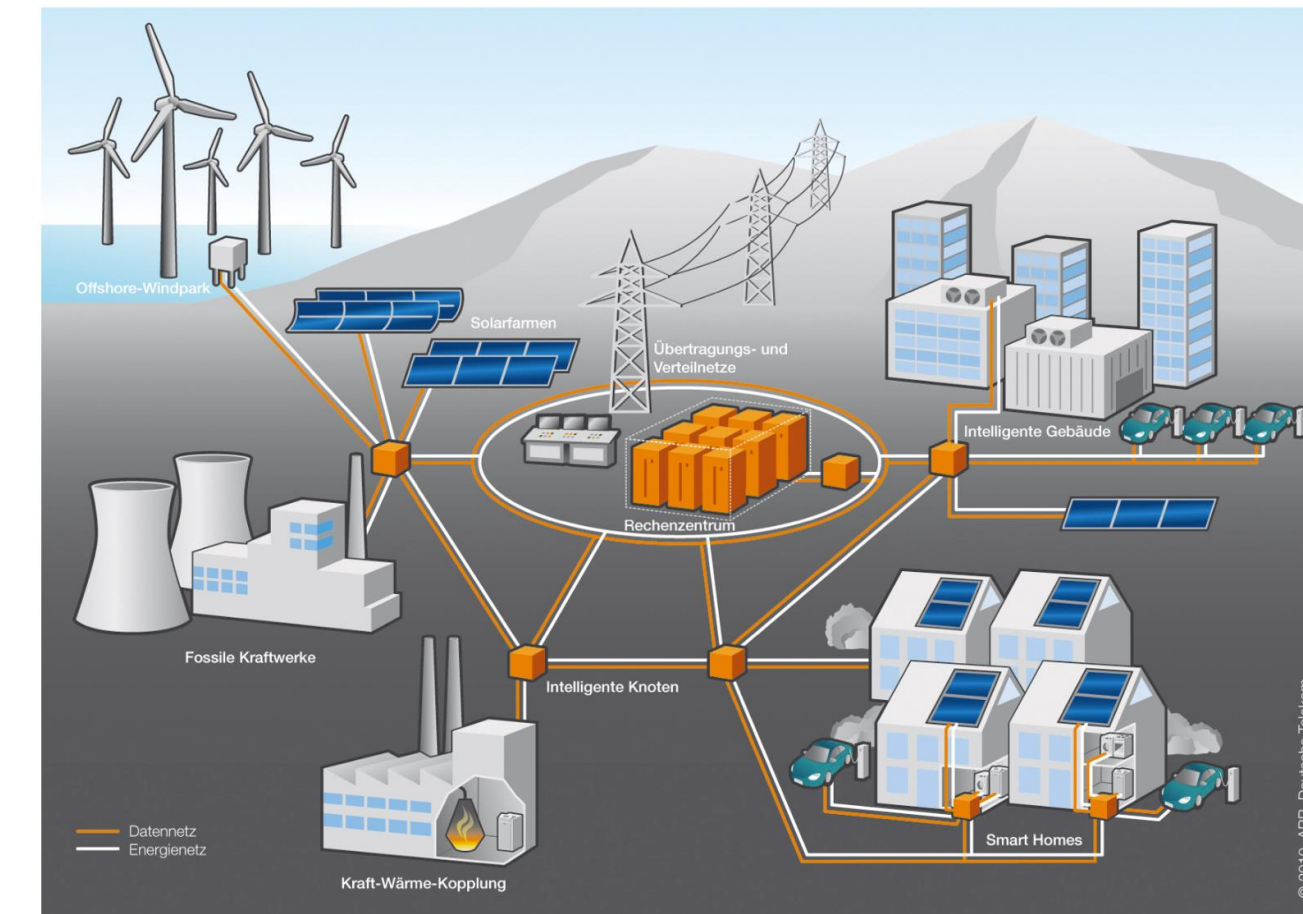
- Costs are piecewise in generated power
- All the units are coupled by the **Load balancing constraint**
- A much larger and harder MILP model, *but*

Better Modeling + Algorithm Makes it **Easier!**



# Successively Implemented in a Much Larger Region

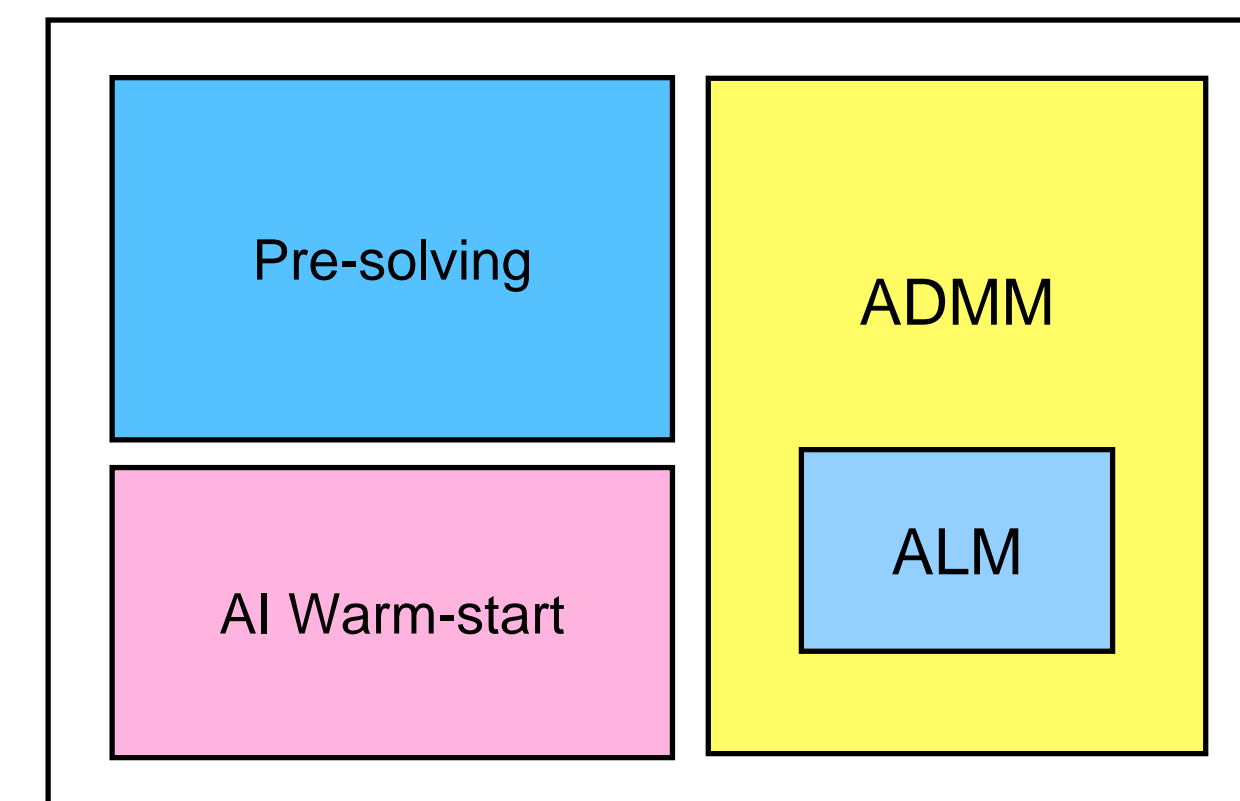
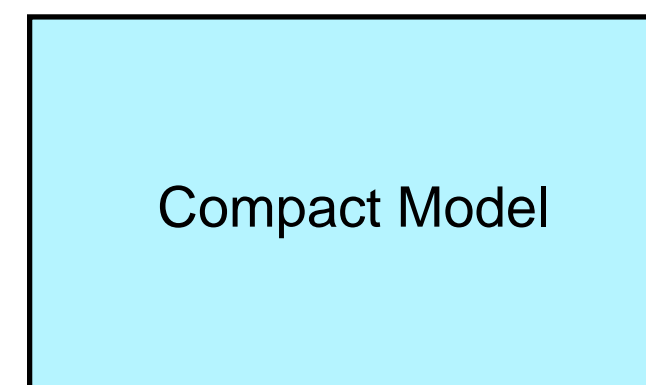
- A much larger UC problem with **security constraint**
- With much more (**millions of**) constraints and variables
- More than 1000 units of Thermal, Hydro and New energy
- Consider interaction between regions and time periods



Huge size + Various business logic + Complicated coupling constraints

- Intractable without exploring structure
- Accurate and succinct model helps
- Domain specific algorithms matter a lot
- ML/AI has a big role to play

Model, Algorithm and ML/AI together make it tractable



# App. VI: Beijing Public Transport Intelligent Urban Bus Operations Management with Mixed Fleet Types and Charging Schedule



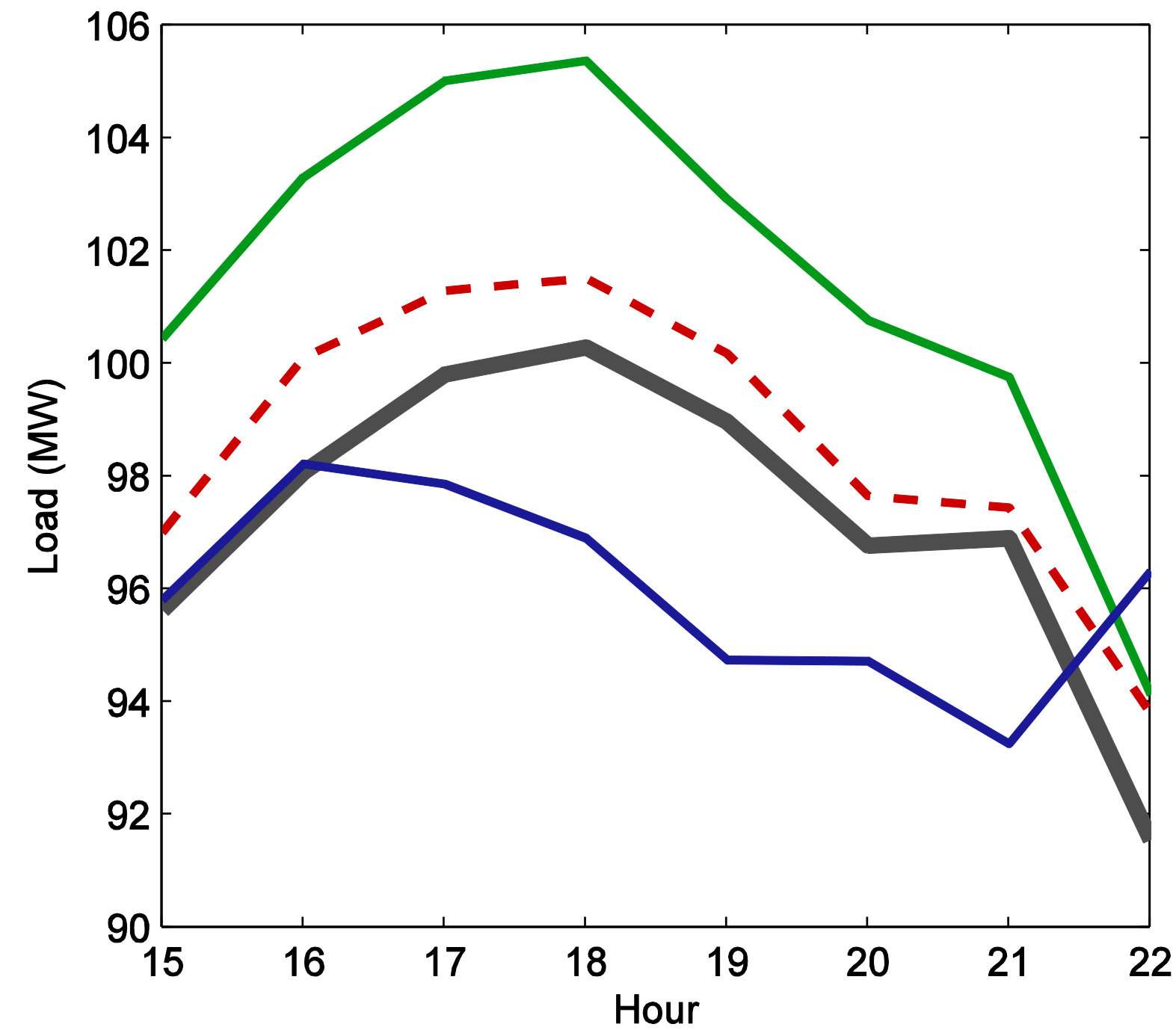
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**Kickoff 2022.8**



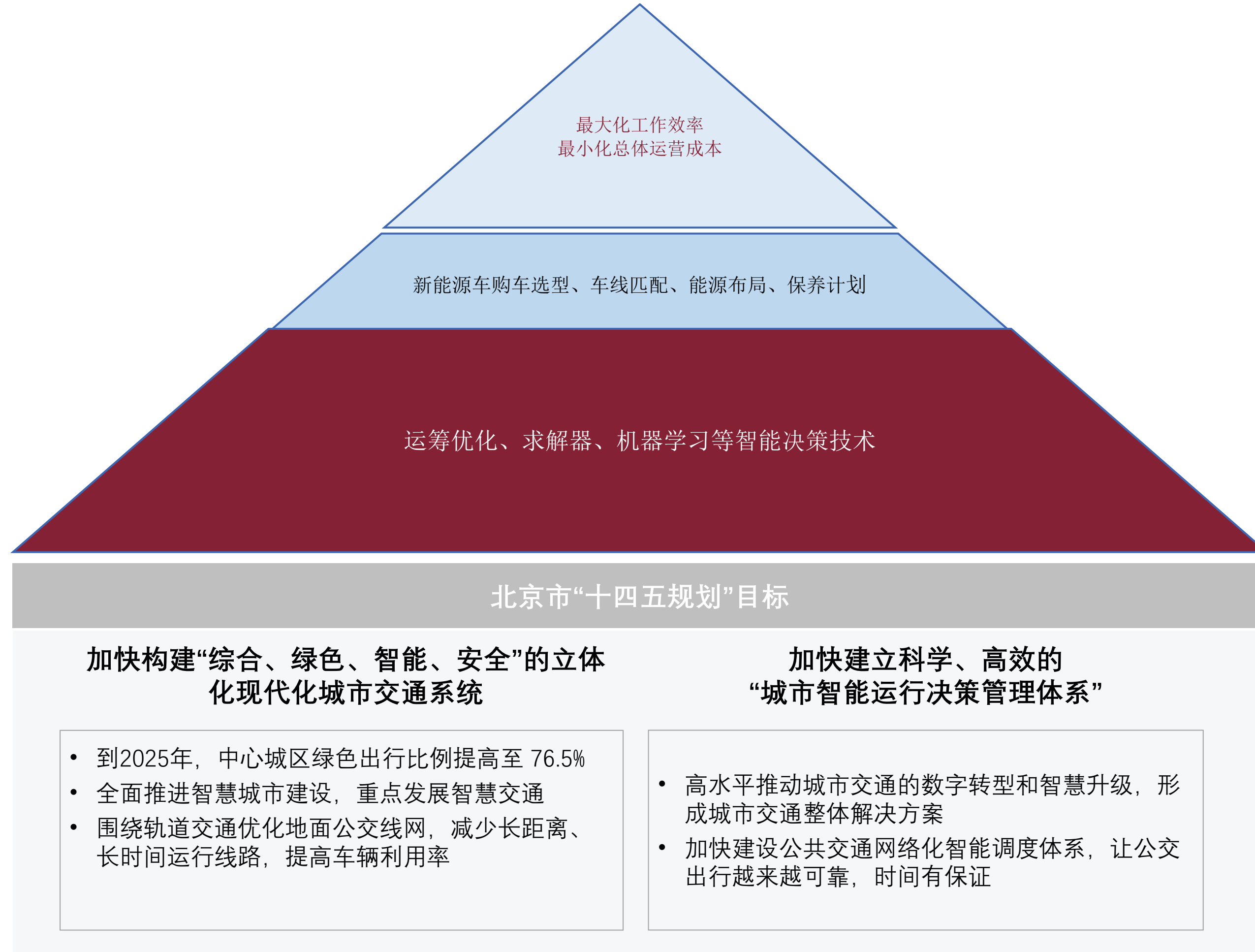


# Peak Reduction due to Smart Charging and Discharging



	<b>Standard</b>	<b>Low PGE</b>	<b>Linear Progr.</b>
Total Fleet (\$)	97,678	83,695	65,349
Mean Cost / Mile	0.068	0.044	0.0054
Increase in Peak	5.1%	1.4%	-0.25%

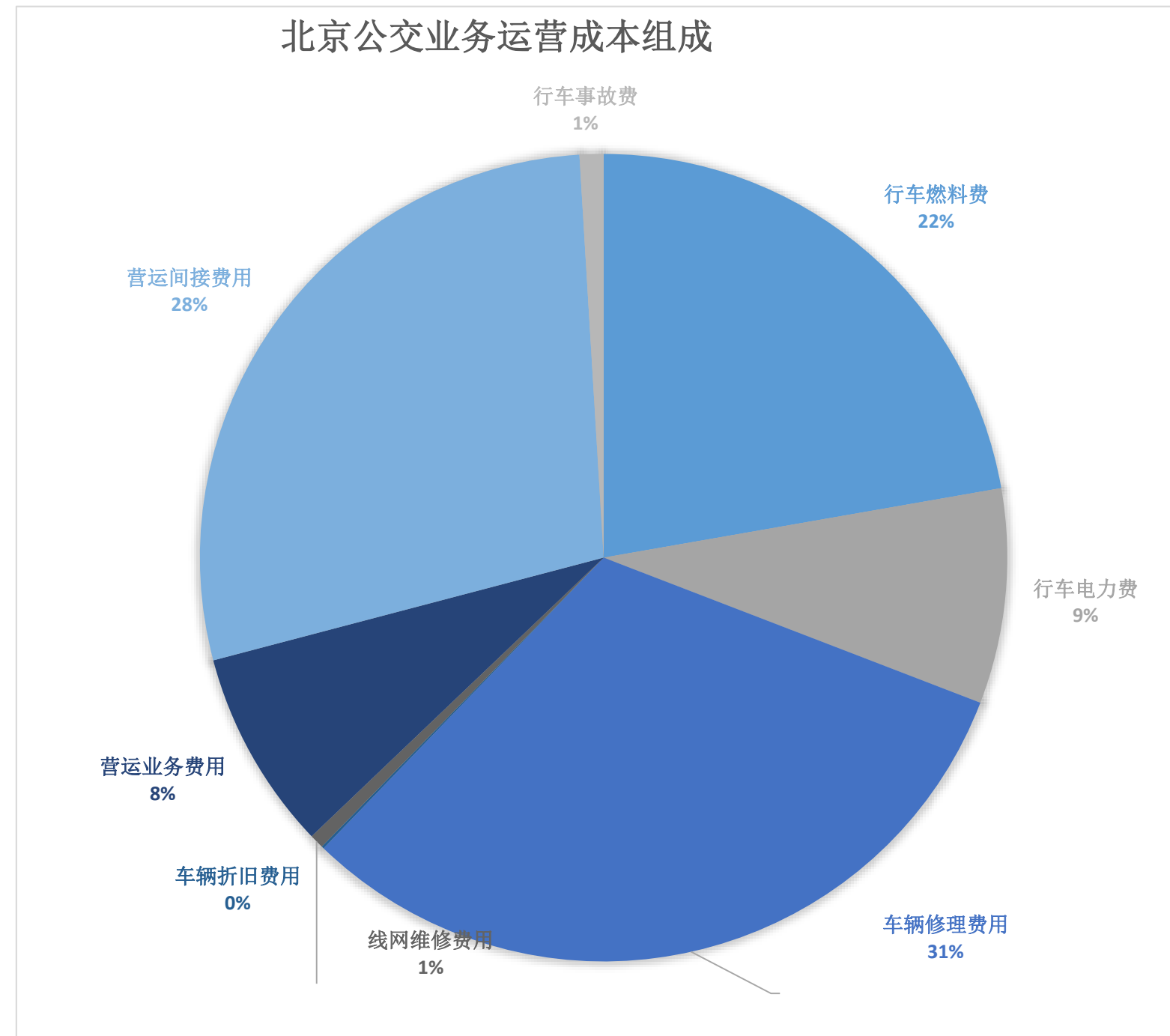
# Background: Decision Intelligence in the case of Beijing Public Transport



More **efficient and intelligent** decision-making to **achieve 14th Five-Year Plan goals**

Beijing Public Transport Line 7 is selected as the **Key Pilot Unit** of the intelligent transformation of Beijing Public Transport

# Intelligent Transformation Empowered by Cardinal Operations



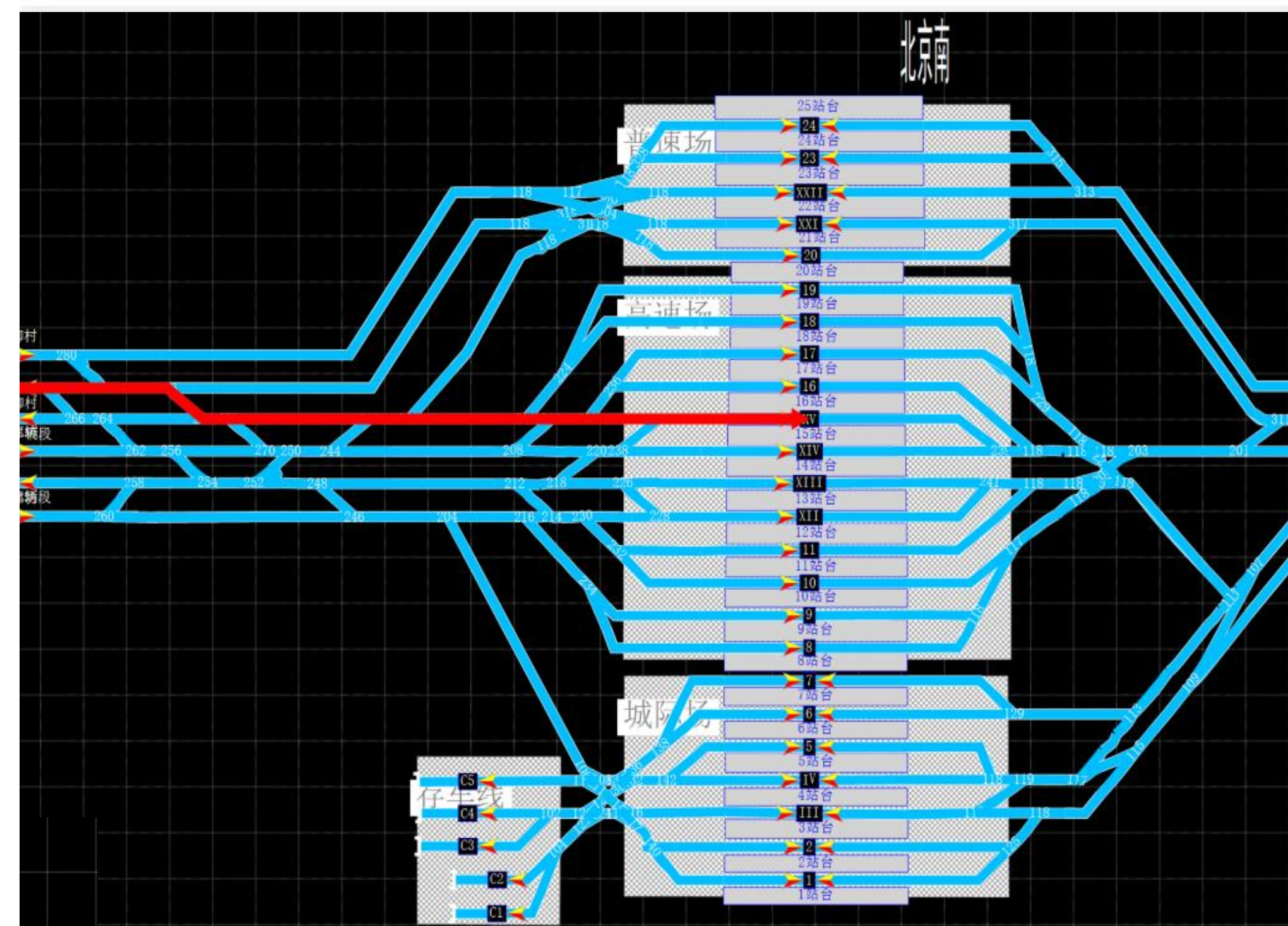
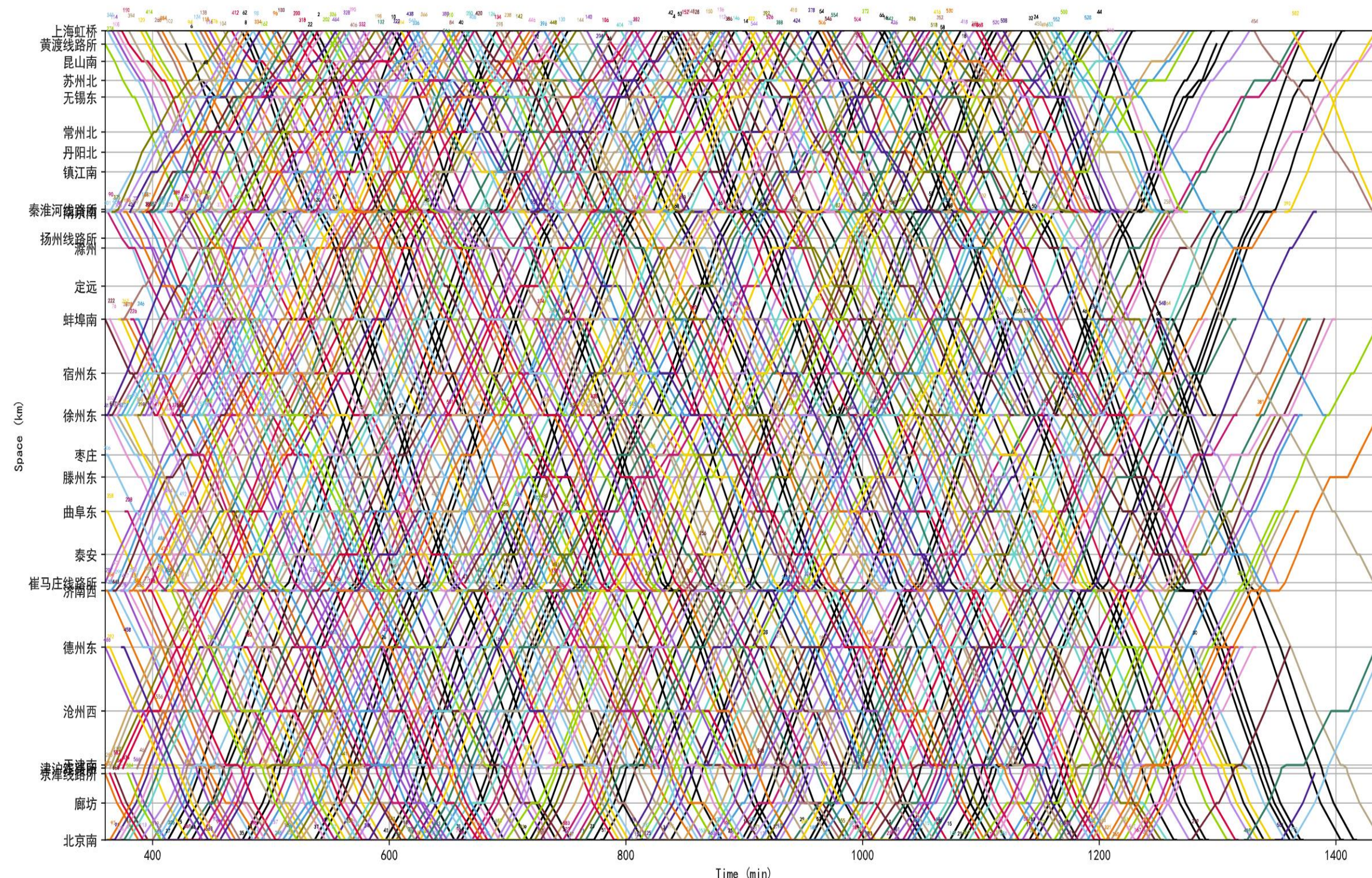
Beijing Public Transport's total operational costs reached **6.65 billion** Yuan in 2020, of which **fuels, electricity, maintenance, repair and other indirect costs** accounted for **over 90%**. Preliminary analysis shows various potential use cases for optimization in cost reduction.



Beijing Public Transport, in partner with Cardinal Operations, aims to build **an innovative integrated system for smart operations** in urban public transportation operations, and explore larger markets in the future.

# App. VII: Beijing-Shanghai High-speed Railway Scheduling Optimization

## COPT, Cardinal Operations 2022



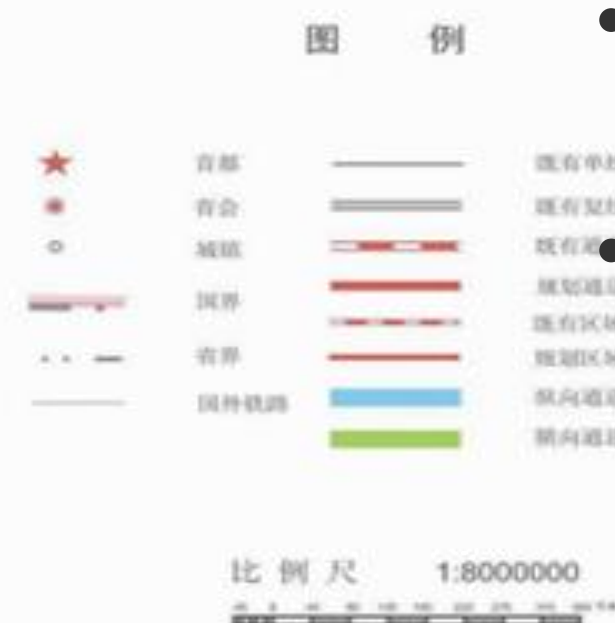


# Background

- China High-speed Railway has been committed to providing high-quality transportation services to passengers, and the formulation of train scheduling is a key link in the operation.
- **At present, train scheduling is based on human experience**, which becomes increasingly difficult to handle the growing network. Therefore, both industry and academia are seeking ways to **automate train scheduling**.
- The train scheduling problem can be divided into **Train Timetabling Problem (TTP)** and **Train Platforming Problem (TPP)**.

## Optimization Model:

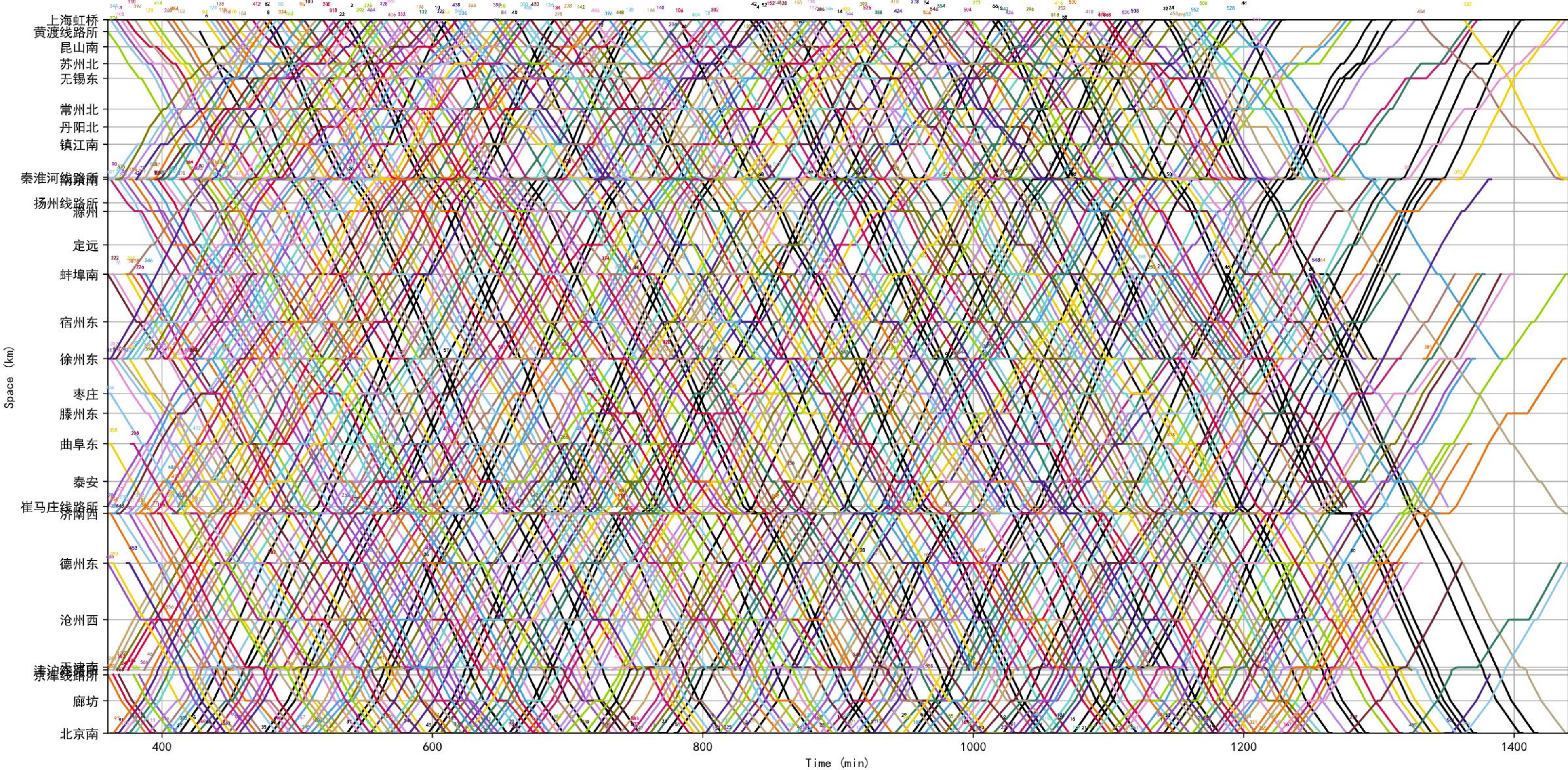
- **Objective:** maximize the number of trains placed in the train scheduling, thereby maximizing operating revenue;
- **Constraints:** describe the running behavior of trains and prevent train collisions;
- The project mainly solves **TTP for Beijing-Shanghai High-speed Railway** and **TPP at Beijingnan Railway Station**.
  - **Beijing-Shanghai High-speed Railway** is the busiest high-speed railway with the largest number of passengers in China. It is 1,318 km in total and passes 29 stations.
  - **Beijingnan Railway Station** is the largest railway station in Beijing, with the largest area and the largest number of trains.
- Both problems are challenging scheduling tasks, which can be formulated as Mixed Integer Programming (MIP).



# Numerical Results: TTP for Beijing-Shanghai



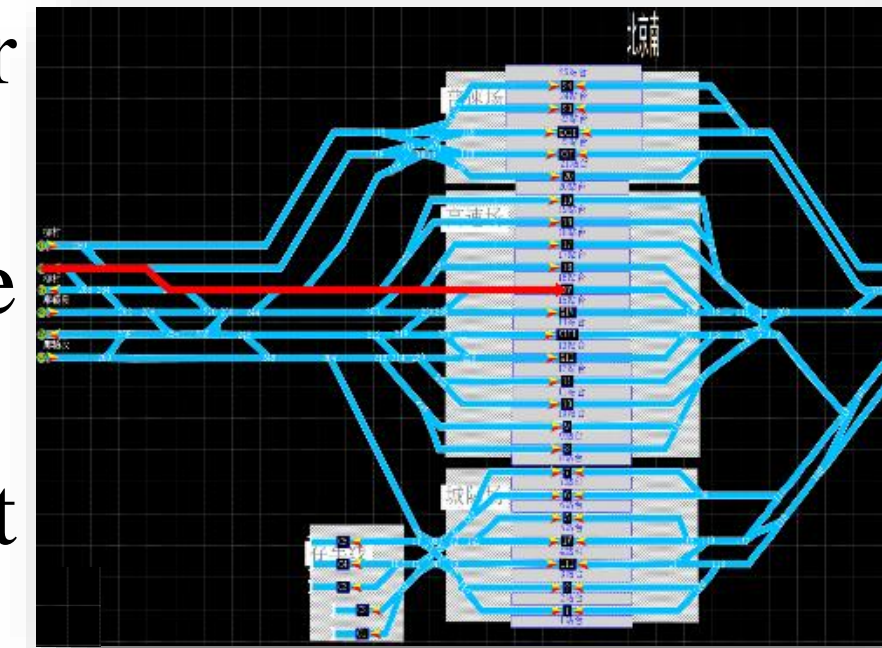
- We solve the TTP for Beijing-Shanghai high-speed railway using Cardinal Optimizer (COPT).
- COPT is the first fully independently developed mathematical programming solver in China with strong solving ability of MIP problem. It also has excellent performance in solving this problem.
- The result is presented in the following figure. We only need about **1000 seconds** to schedule 584 train in two directions.



# Numerical Results: TPP at Beijingnan Station



- We solve the TPP at Beijingnan Railway Station using Cardinal Optimizer (COPT).
- Considering the connection pairs and ensuring the feasibility, we solve the model within **2 hours**, which is much less than manual scheduling.
- The result is presented in the following table, including time nodes about occupation at boundaries and tracks.

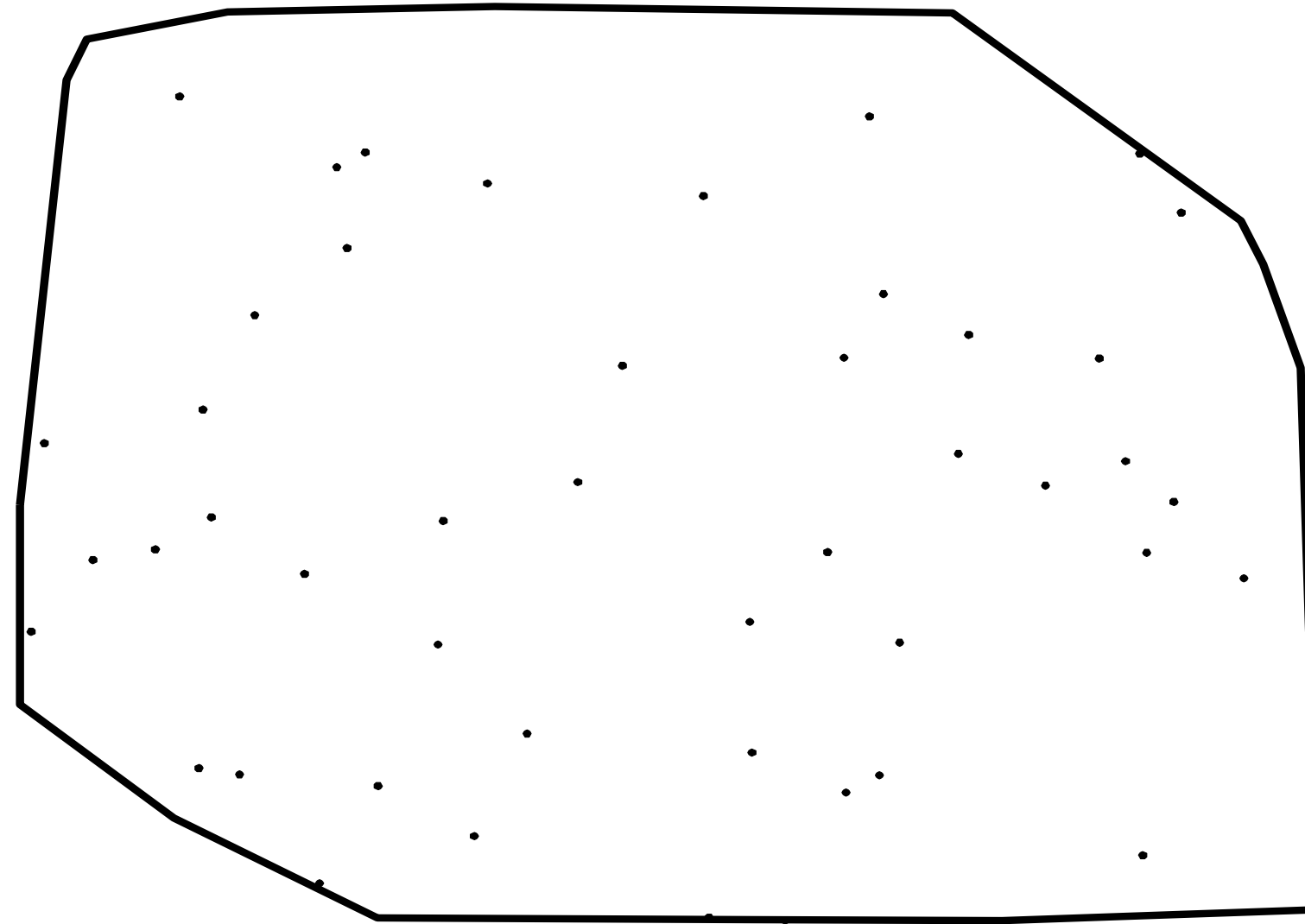


列车编号	前序车站	进入站界	进站路径	停靠站线	离开站界	出站路径	后序车站	进入站界时间	进入站线时间	离开站线时间	离开站界时间
361		站界:B10		站线:XIV	站界:B9	站线:10:XIV	廊坊		12:00:00	12:06:00	12:10:00
74	廊坊	站界:B8	站线:16:8	站线:8	站界:B7			11:57:00	12:02:00	12:17:00	
125		站界:B10		站线:11	站界:B9	站线:13:11	廊坊		12:06:00	12:13:00	12:17:00
114	廊坊	站界:B8	站线:7:17	站线:17	站界:B7			12:10:00	12:14:00	12:29:00	
251		站界:B10		站线:8	站界:B9	站线:16:8	廊坊		12:17:00	12:27:00	12:32:00
20	廊坊	站界:B8	站线:7:17	站线:17	站界:B7	站线:7:17		12:19:00	12:23:00	12:25:00	12:29:00
96	廊坊	站界:B8	站线:13:11	站线:11	站界:B7			12:25:00	12:29:00	12:44:00	
223		站界:B10		站线:17	站界:B9	站线:7:17	廊坊		12:29:00	12:44:00	12:48:00
8	廊坊	站界:B8	站线:8:16	站线:16	站界:B7			12:33:00	12:37:00	12:42:00	
23		站界:B10		站线:16	站界:B9	站线:8:16	廊坊		12:42:00	12:57:00	13:01:00
127		站界:B10		站线:11	站界:B9	站线:13:11	廊坊		12:44:00	12:49:00	12:53:00
572	廊坊	站界:B8	站线:5:19	站线:19	站界:B7			12:43:00	12:48:00	13:03:00	
124	廊坊	站界:B8	站线:6:18	站线:18	站界:B7			12:47:00	12:52:00	12:57:00	
102	廊坊	站界:B8	站线:15:9	站线:9	站界:B7			12:51:00	12:56:00	13:07:00	
225		站界:B10		站线:18	站界:B9	站线:6:18	廊坊		12:57:00	13:12:00	13:17:00
51		站界:B10		站线:17	站界:B9	站线:7:17	廊坊		12:59:00	13:01:00	13:05:00
116	廊坊	站界:B8	站线:13:11	站线:11	站界:B7			12:56:00	13:00:00	13:15:00	
169		站界:B10		站线:19	站界:B9	站线:5:19	廊坊		13:03:00	13:18:00	13:23:00
133		站界:B10		站线:9	站界:B9	站线:15:9	廊坊		13:07:00	13:22:00	13:27:00
161		站界:B10		站线:11	站界:B9	站线:13:11	廊坊		13:15:00	13:26:00	13:30:00
138	廊坊	站界:B8	站线:5:19	站线:19	站界:B7			13:13:00	13:18:00	13:33:00	
118	廊坊	站界:B8	站线:8:16	站线:16	站界:B7			13:27:00	13:31:00	13:36:00	
109		站界:B10		站线:19	站界:B9	站线:5:19	廊坊		13:33:00	13:41:00	13:46:00
100	廊坊	站界:B8	站线:8:16	站线:16	站界:B7			13:31:00	13:35:00	13:40:00	
229		站界:B10		站线:16	站界:B9	站线:8:16	廊坊		13:36:00	13:51:00	13:55:00
2	廊坊	站界:B8	站线:16:8	站线:8	站界:B7			13:34:00	13:39:00	13:47:00	
131		站界:B10		站线:16	站界:B9	站线:8:16	廊坊		13:40:00	13:55:00	13:59:00
3		站界:B10		站线:8	站界:B9	站线:16:8	廊坊		13:47:00	14:02:00	14:07:00
98	廊坊	站界:B8	站线:10:XIV	站线:XIV	站界:B7			13:43:00	13:47:00	14:02:00	
108	廊坊	站界:B8	站线:13:11	站线:11	站界:B7			13:47:00	13:51:00	14:06:00	



**Topic 4: Equitable Covering & Partition –  
Divide and Conquer (Carlsson et al. 2009)**

# Problem Statement: Divide-Conquer

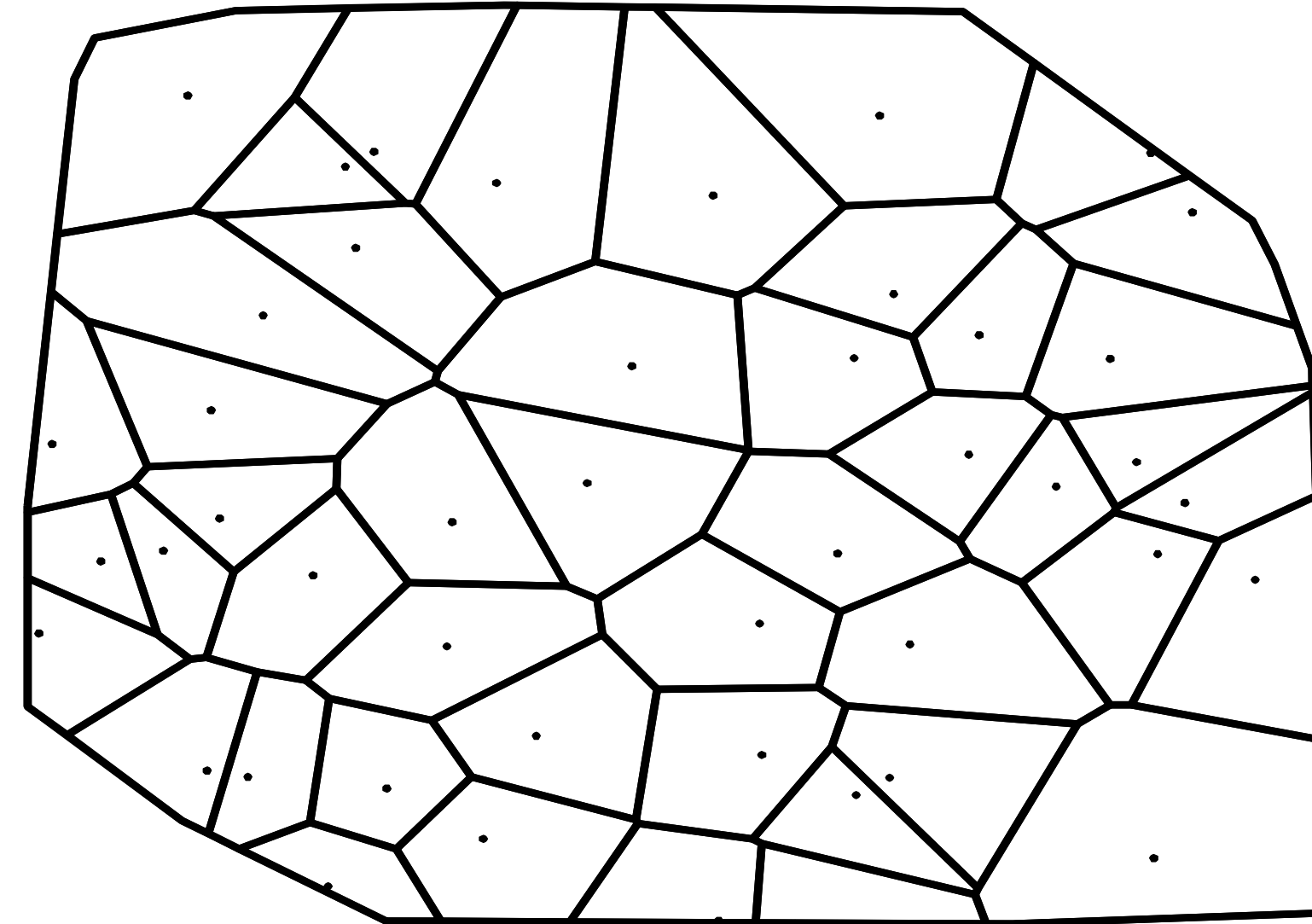


$n$  points are scattered inside a convex polygon  $P$  (in 2D) with  $m$  vertices.

Does there exist a partition of  $P$  into  $n$  sub-regions satisfying the following:

- Each sub-region is a convex polygon
- Each sub-region contains one point
- All sub-regions have equal area

# Related ML Problem: Voronoi Diagram

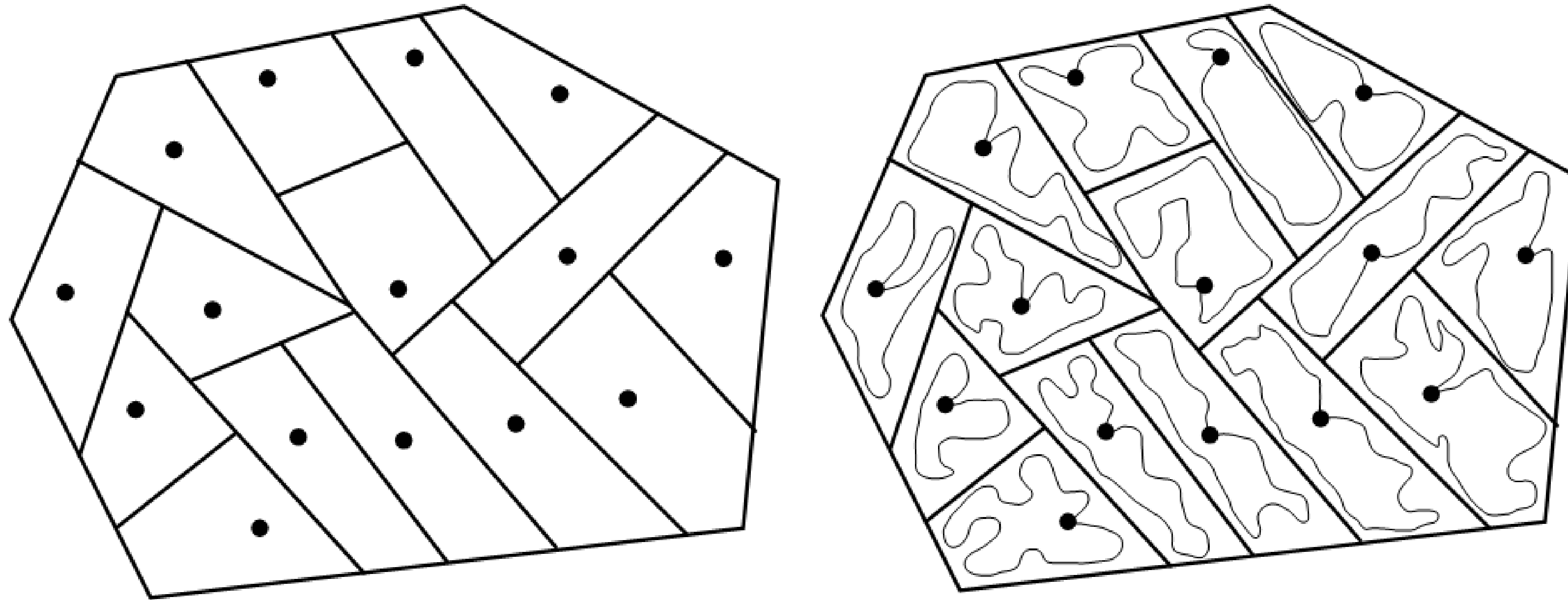


In the *Voronoi Diagram*, we satisfy the first two properties (each sub-region is convex and contains one point), but the sub-regions have different areas.

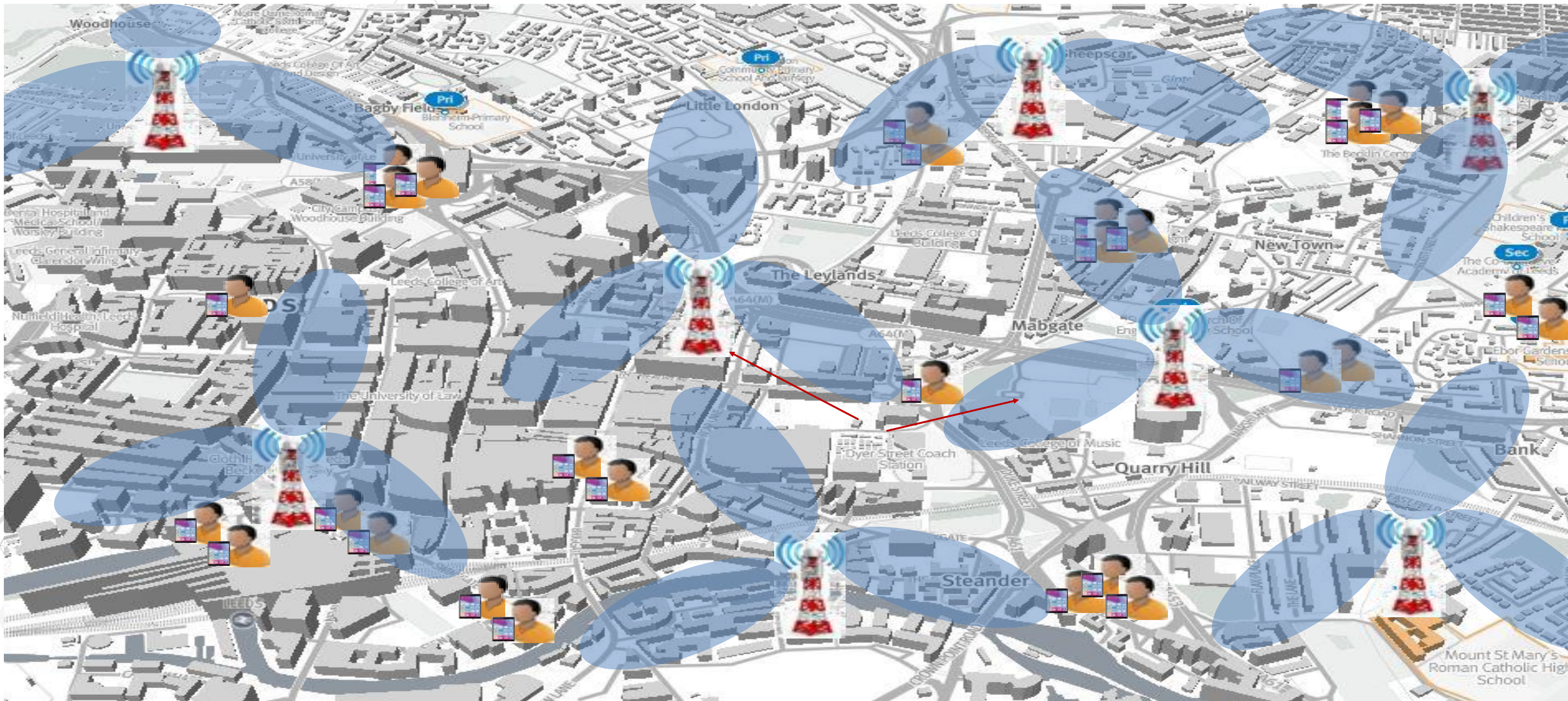


# Our Result

Not only such an equitable partition always exists, but also we can find it exactly in running time  $O(Nn \log N)$ , where  $N = m + n$ .



# App. VIII: Wireless Tower – Dynamic Resource Allocation

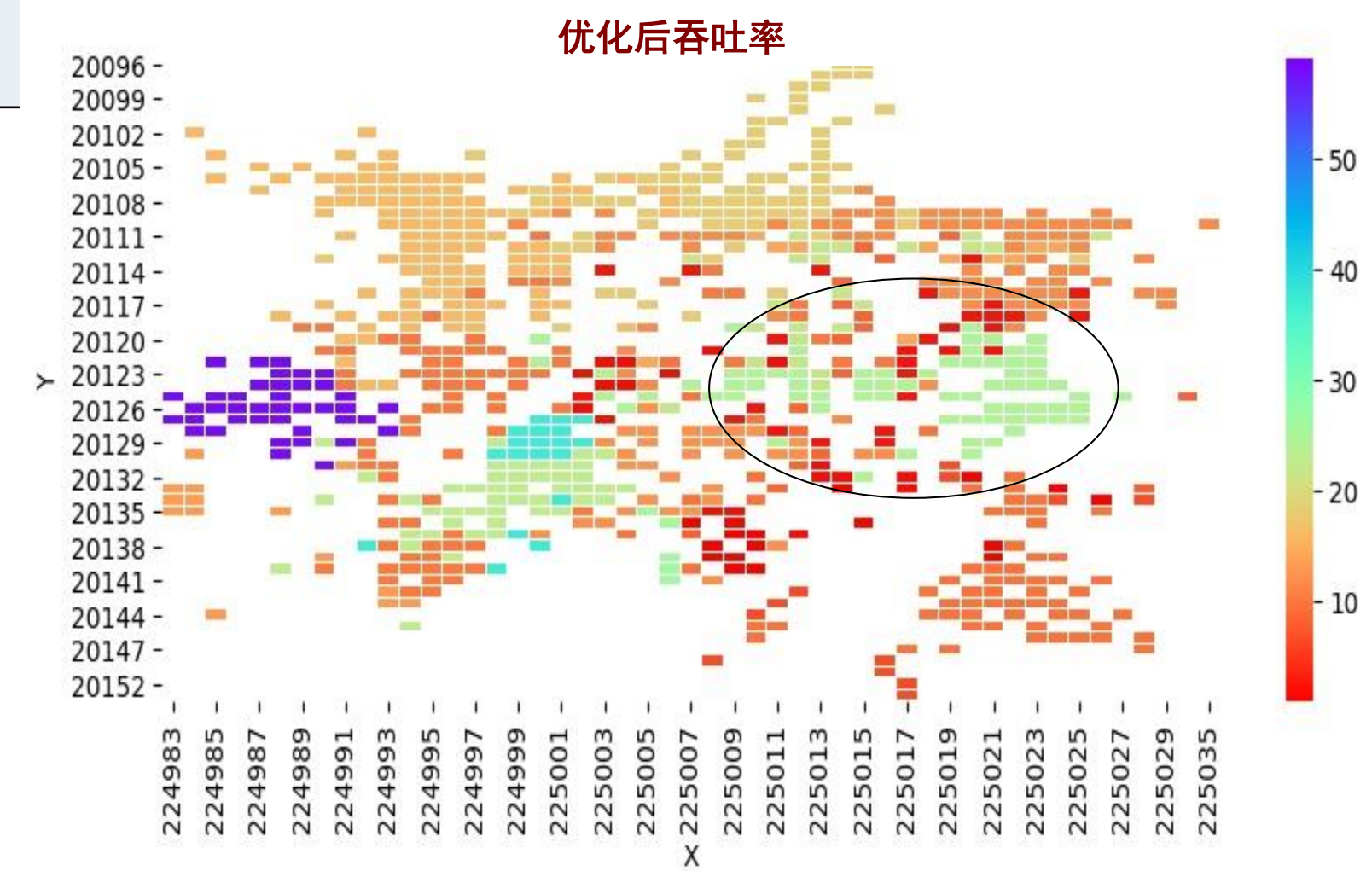
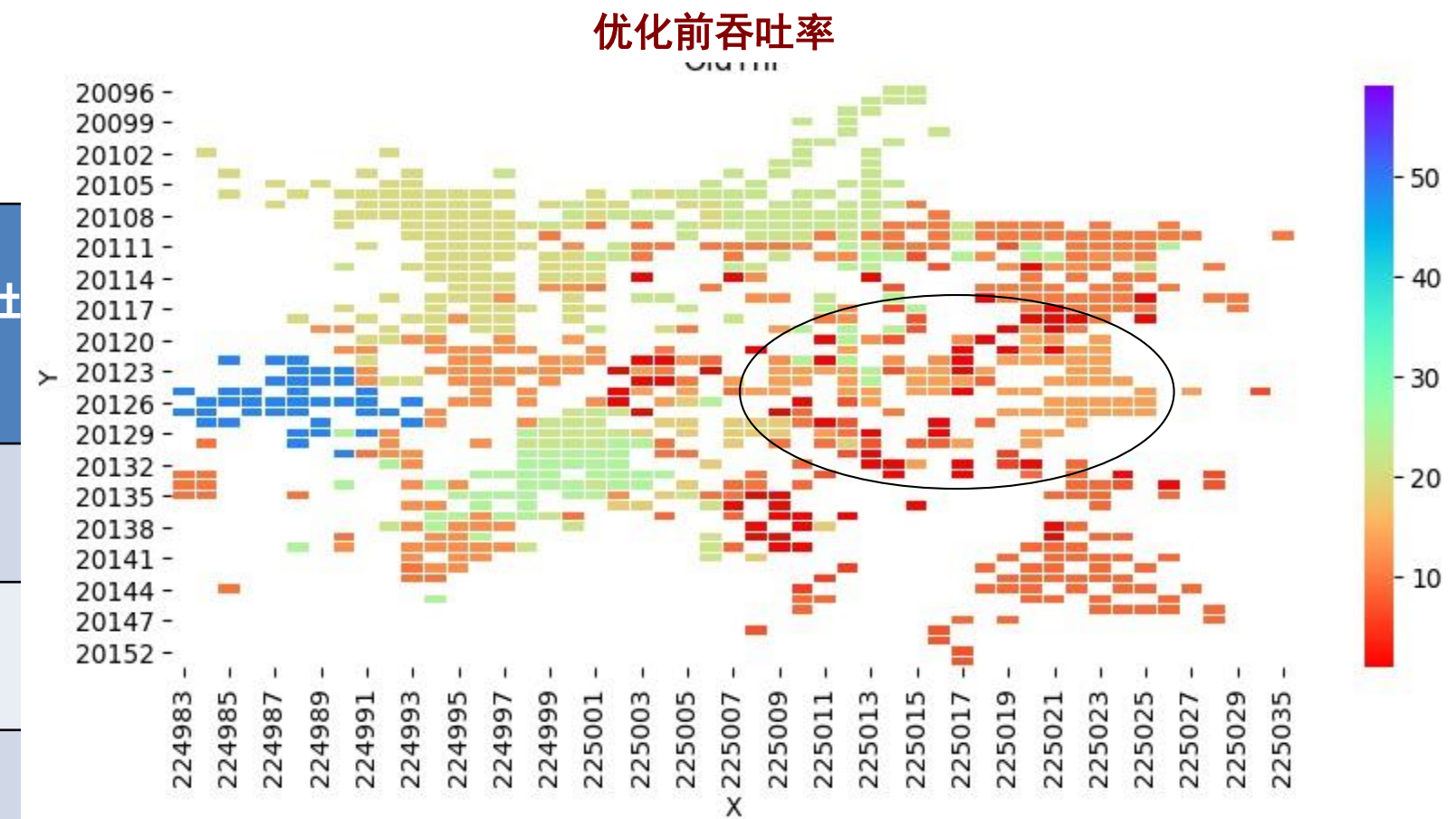
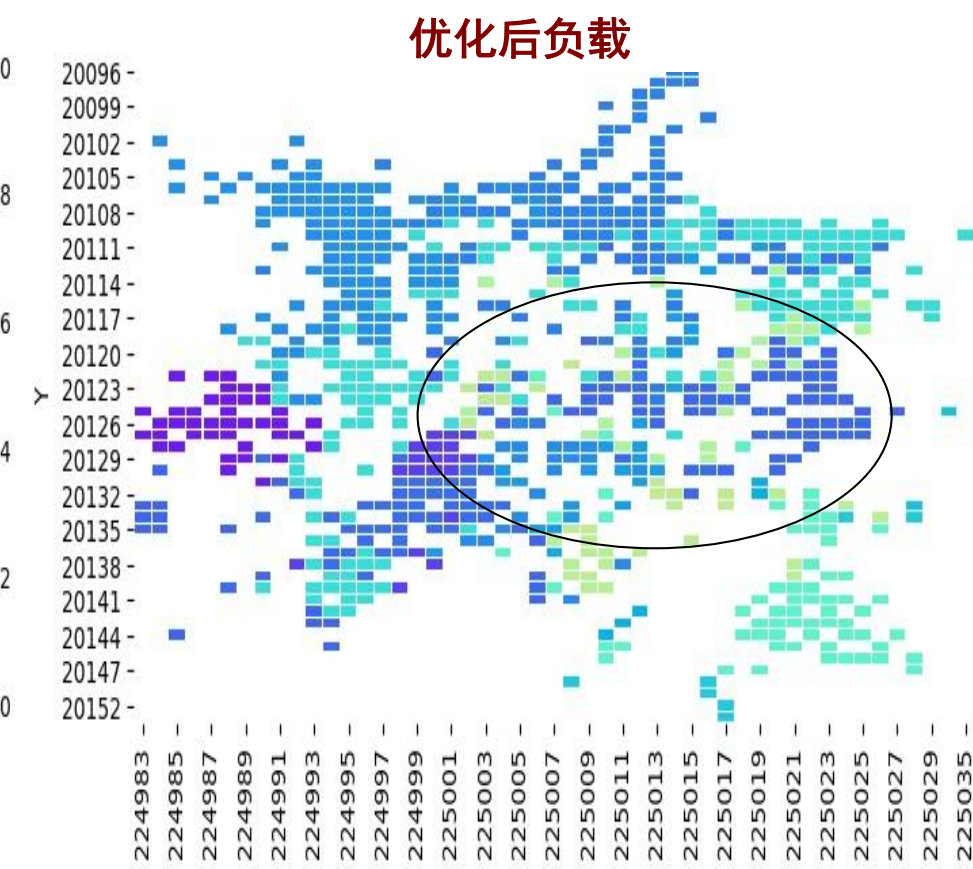
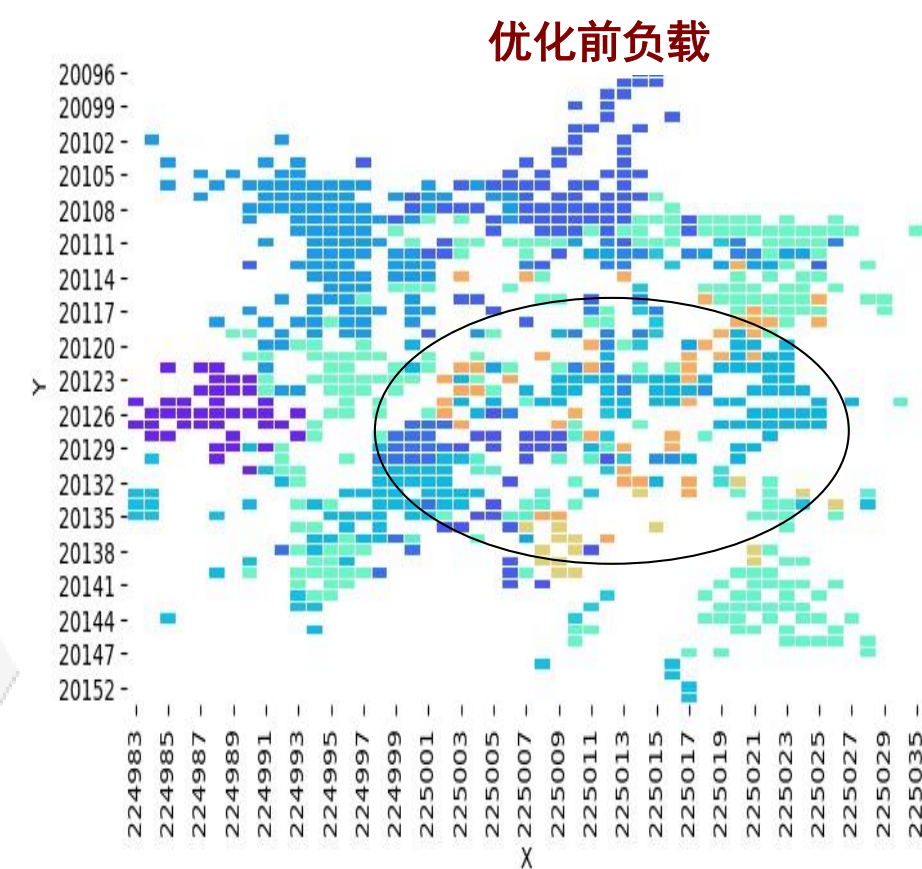




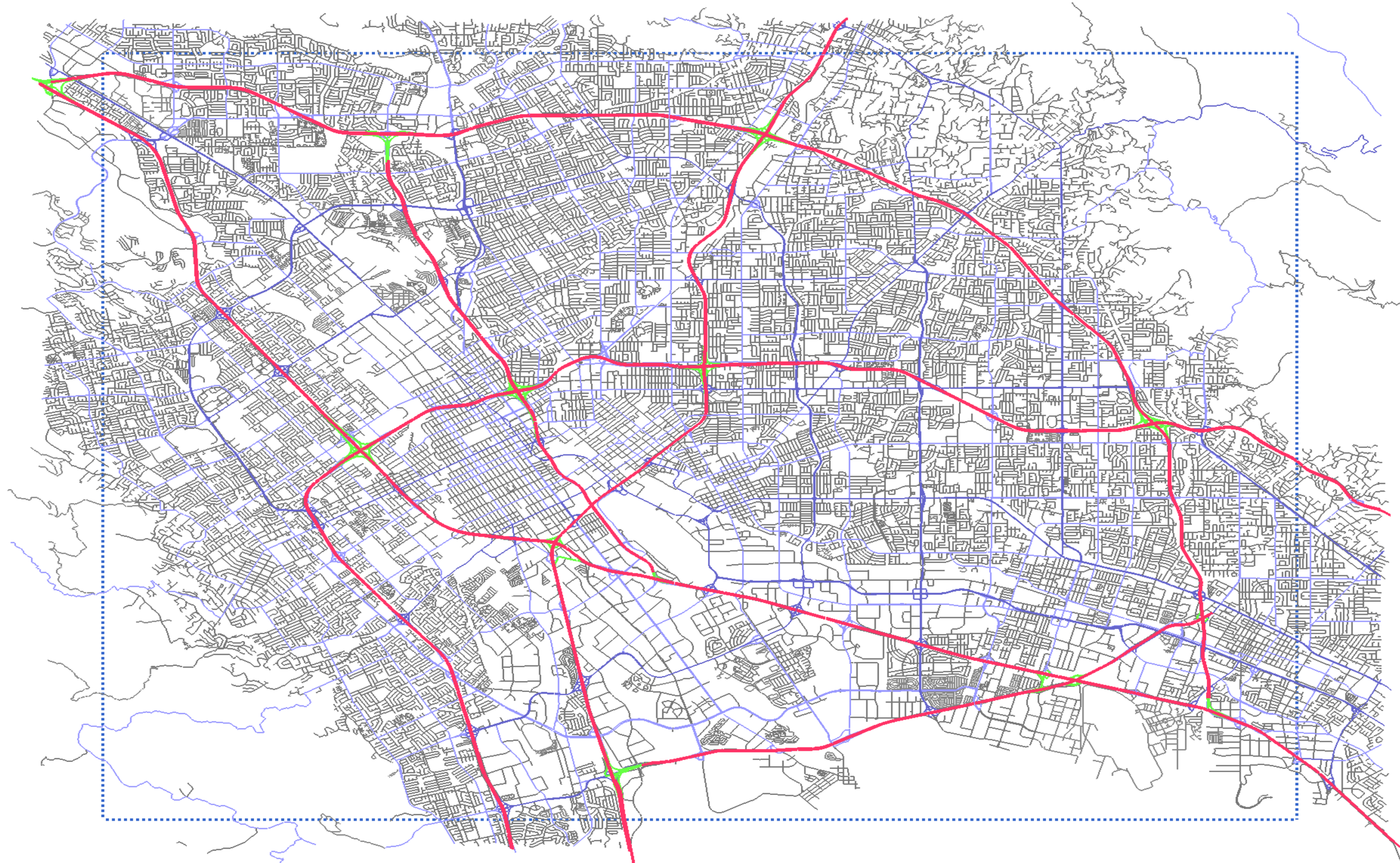
# Preliminary Test Result—Effectiveness

基于真实商用网络进行模型优化效果的测试验证验证统计结果：

	小区数	时段	区域平均负载	区域平均吞吐率 (Mb/S)	高负载小区负载	高负载小区吞吐率 (Mb/S)
优化前	27	中午及晚共6小时	31%	5.3	68%	2.3
优化后			30%	6.12(提升15%)	66%	2.8(提升22%)
优化前		晚7时话务高峰	37%	3.9	77%	1.6
优化后			33%	5.2(提升33%)	68%	2.1(提升32%)



# App. IX: Street View Application Map-Making



# Topic 5: Beyond Math - Love Sport and Have a Belief



# Sport Changed my Life I



<https://msande.stanford.edu/news/yinyu-ye-sports-led-me-rice-fields-stanford>

# Sport Changed my Life II



Competitive spirit  
Training hard  
Team work  
Take a loss  
Play by rules



# **Beyond Academics**

**Be Grateful and Hopeful: no envy and nor self-doubt**

**Be Kind and Tolerant: love others and love yourself too**

**Have a Hobby: find something to relax**

**Have a Faith: find something to believe**

# Overall Takeaways

It is possible to make online decisions for quantitative decision models with performance guarantees close to that of the offline decision-making with complete information

Second-Order Derivative information matters and better to integrate FOM and SOM for nonlinear optimization!

Mixed Integer LP solvers benefit real economy

Decomposition (Divide and Conquer) helps solving large-scale optimization problems

Life is bigger and richer than what you think – treasure and enjoy it at CUHKSZ

- THANK YOU