Separation of MR multiband images using complex independent component analysis

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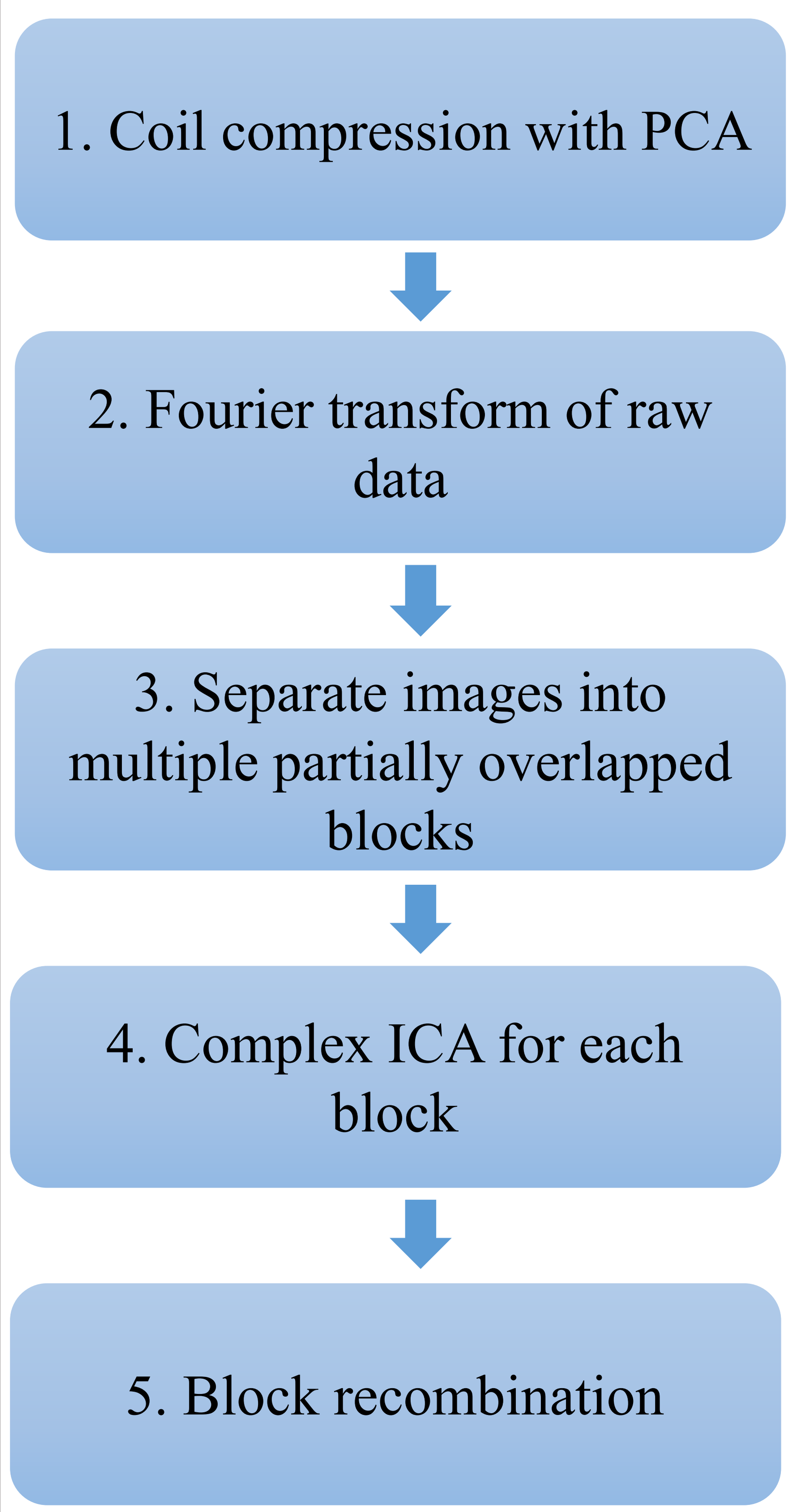
Background & Objective

- **Magnetic resonance imaging (MRI):**
  - Complex valued, Fourier transform of real space images
  - Multi-coil (recorders in the cocktail party problem) with different sensitivity maps (linear mixture matrix)A)9 varies over the images.
  - Long scan time, acceleration leads to mixing of images.
- **Objective**
  - Using the redundancy of the multi-coil and the complex-valued independent component analysis (cICA) to separate and reconstruct MR images.

Method

Workflow Schematic



Step1: use principal component analysis to compress data from eight coils into two images[1].

Step3: sensitivity maps (A) are coil and position dependent. Group pixels into blocks and assume the sensitivity in one block is constant (A=$A(x,y)$). Use cICA to find W=A^(−1) and reconstruct source images.

Step5: correct the order, amplitude and phase of the blocks by comparing the overlapped regions and then combine them via region growing.

i^{th} $coil's\ image$:

$$I(x,y,z)_i = A(x,y,z) * I(x,y,z)$$

$one\ block\ of\ i^{th}\ coil's\ image$:

$$I_i \approx \sum_z A_{i,z} * I_z$$

Method

Complex ICA by mutual information rate minimization

Date sets:

$$x(t) = x_{real}(t) + i * x_{imaginary}(t) \in \mathbb{C}^T, \qquad 1 \le t \le T$$

Linear mixture model:

$$x(t) = [x_1(t), \dots\dots, x_N(t)] = A\, s(t), \qquad 1 \le t \le T$$
$$y(t) = [y_1(t), \dots\dots, y_N(t)] = W\, x(t), \qquad 1 \le t \le T$$

Cost function:

We use the ***Mutual Information Rate*** as the cost function in order to take both non-Gaussian and sample dependence into account[2]. The mutual information rate is defined as a measure of the the amount of information exchanged per unit time by two data sets[3]:

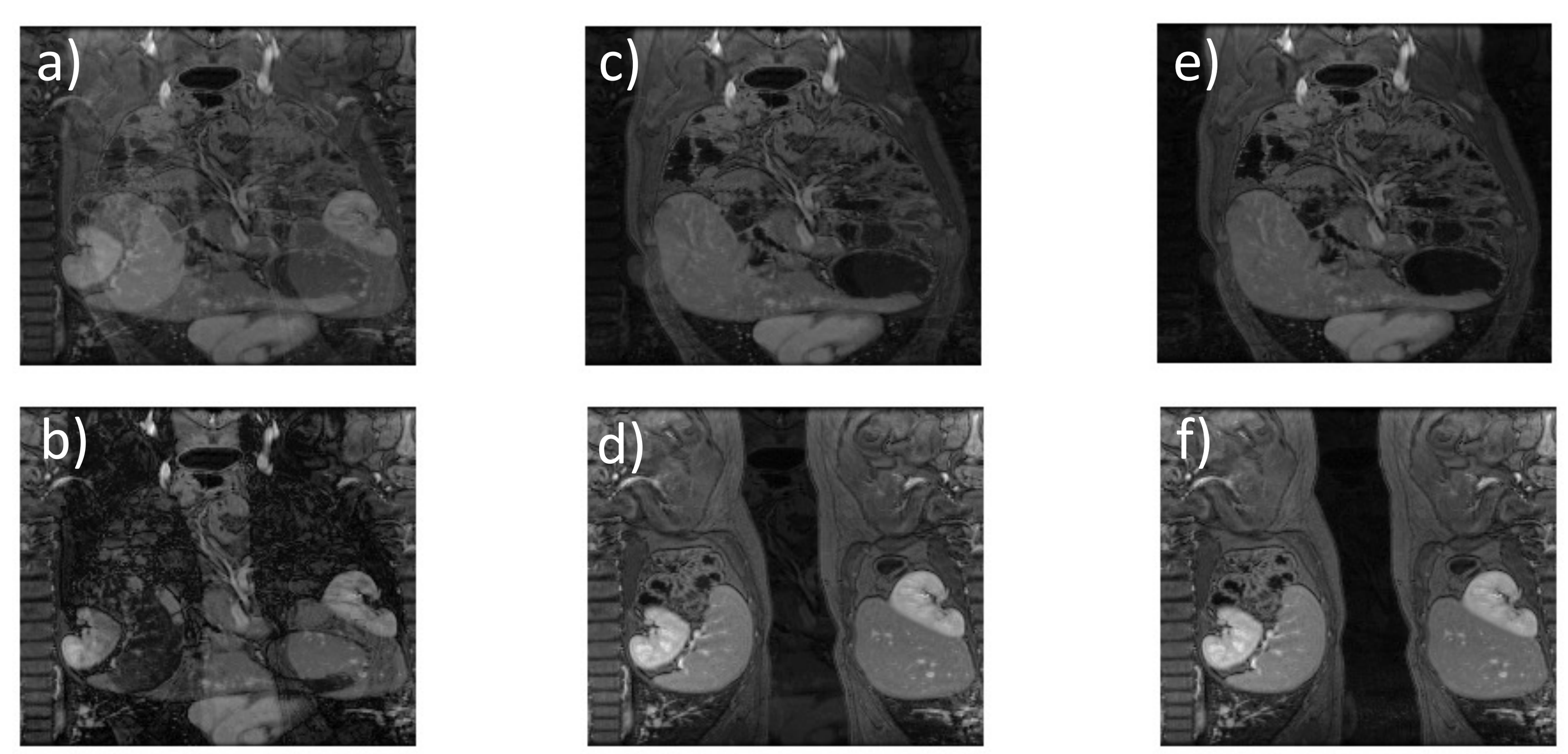
$$\mathcal{L}(W) = \sum_{i=1}^{N} H_{rate}(y_i) - 2Log(|\det(W)|),$$

Where the entropy rate $H_{rate}(y_i) = \frac{H(y_i)}{T} = -\frac{\sum_{j=1}^{T} p_j(y_i) Log(p_j(y_i))}{T}$.

Gradient update rule to minimized cost function $\mathcal{L}(W)$.

Results

Comparison of results by using mutual information and mutual information rate as cost function. (Simulation data with linear mixture matrix A being constant over the entire image).



Linear mixture — mutual information as cost function — mutual information rate as cost function

Figure a) and b) shows the linear mixture of true MRI scanning data. It clearly shows that the mutual information rate cost function does a better job to extract the independent components.

Results

Comparison of results without and with grouping pixels into blocks

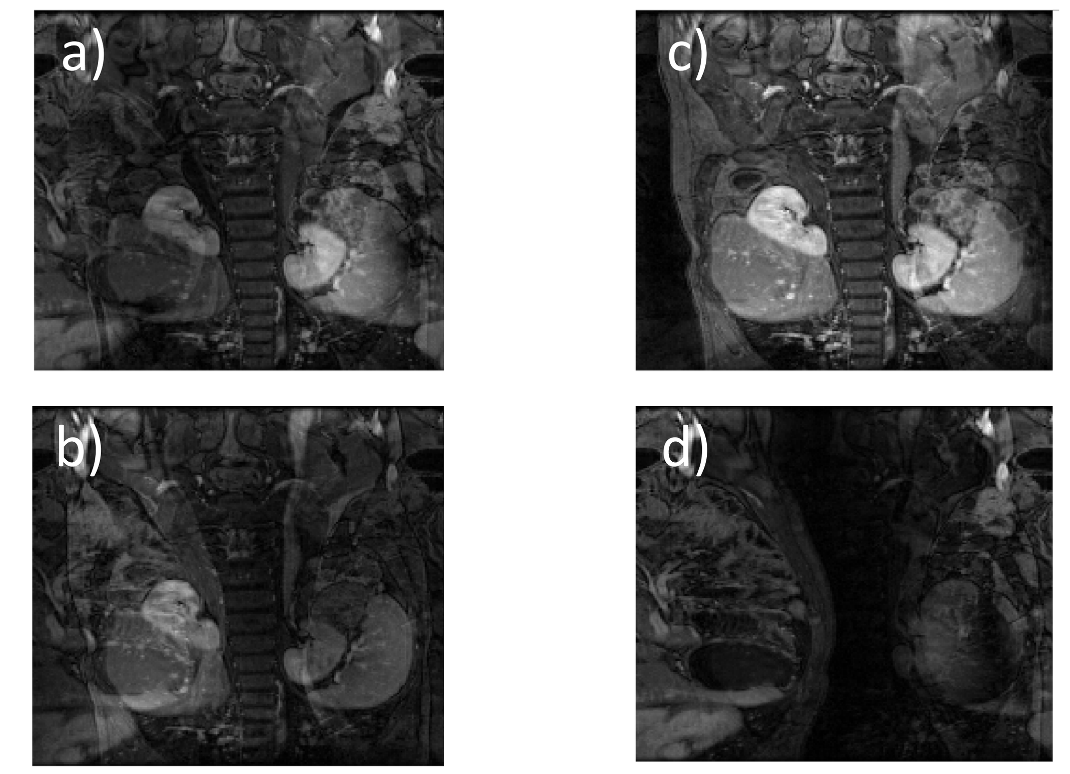
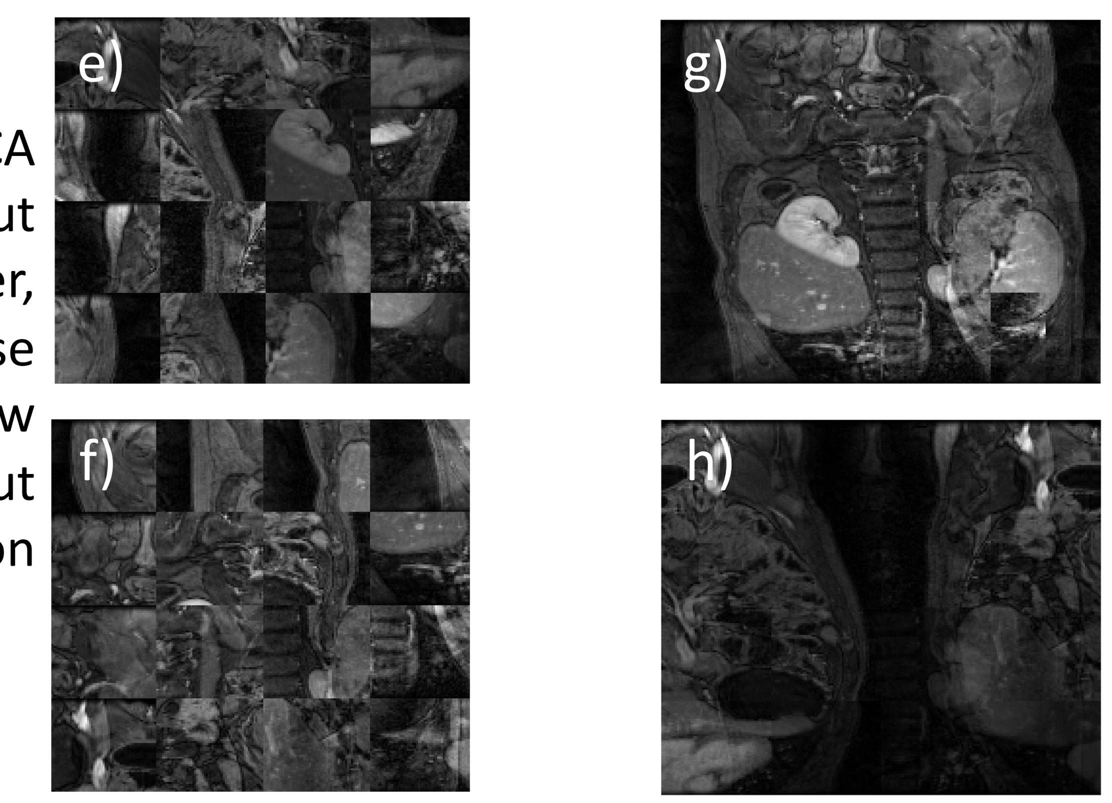


Figure a) and b) shows the mixed images of two coils and c), d) show the unmixed images from cICA without dividing them into blocks.



e) and f) show the cICA results using divided input but without no order, amplitude and phase corrections. g) and h) show the final results put together by our region growing method.

Comparing the resulting separated images, we conclude that for most part of the unmixed images, grouping pixels into blocks before applying cICA did improve the overall performance.

Discussion & Future work

- **Current problems**
  - cICA is not always perfect, especially for regions where one of the unmixed image has meaningful data while the other only has Gaussian noise.
  - Error accumulation and amplification due to the region growing method, need to be corrected manually in amplitude.
- **Future work**
  - Achieve more accurate amplitude/phase correction.
  - The estimated weighting matrix A is actually the sensitivity map which can be used in traditional MR image reconstruction method.

References: [1] Zhang, Tao, et al. "Coil compression for accelerated imaging with Cartesian sampling." *Magnetic resonance in medicine* 69.2 (2013): 571-582.
[2] Fu, Geng-Shen, et al. "Complex Independent Component Analysis Using Three Types of Diversity: Non-Gaussianity, Nonwhiteness, and Noncircularity." *IEEE Transactions on Signal Processing* 63.3 (2015): 794-805.
[3] Baptista, Murilo S., et al. "Mutual information rate and bounds for it." *PLoS One* 7.10 (2012): e46745.