Inflation Target Uncertainty and Monetary Policy

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- Uncertain optimal inflation rate
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US inflation target recently called into question
- Response to perceived decline in $r^*$
- Williams (2009), Blanchard et al. (2010), and Ball (2014)
- Creates uncertainty around the future inflation target
Inflation Target Uncertainty

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- US inflation target recently called into question
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  - Creates uncertainty around the future inflation target

- Why could this uncertainty matter?
  - Uncertainty in the future inflation target changes expected inflation
  - Individual, firm, and central bank decisions respond to such changes
  - Affects current outcomes regardless of the eventual resolution
"So it’s that recognition that causes people to think we might be better off with a higher inflation objective. That is an important set, this is one of our most critical decisions and one we are attentive to evidence and outside thinking. It’s one that we will be reconsidering at some future time... But I would say that this is one of the most important questions facing monetary policy around the world in the future."
This Paper

Questions

- How does $\pi^*$ uncertainty affect current inflation, output, and welfare?

- How should the central bank respond to inflation target uncertainty?

- What happens if the central bank commits to changing the inflation target?
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- How should the central bank respond to inflation target uncertainty?
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Approach
- Model inflation target uncertainty in a standard New Keynesian model
  - Policy rule with a regime specific inflation target
  - Exogenous Markov process determines regime
- Analytically solve the model without any additional uncertainty
- Numerically solve the stochastic model for full quantitative evaluation
This Paper

Questions
- How does $\pi^*$ uncertainty affect current inflation, output, and welfare?
  - A potential increase in $\pi^*$ usually generates stagflationary dynamics
  - But may qualitatively differ depending on the current monetary policy rule
- How should the central bank respond to inflation target uncertainty?
  - A trade off in levels between inflation and output
  - Optimal policy adjusts the current inflation target
- What happens if the central bank commits to changing the inflation target?
  - Anticipated change in $\pi^*$ usually results in cyclical dynamics
  - Under an optimal time varying policy rule inflation monotonically adjusts

Approach
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- Analytically solve the model without any additional uncertainty
- Numerically solve the stochastic model for full quantitative evaluation
Regime switches
- DSGE estimation: Schorfheide (2005), Liu, Waggoner, Zha (2011), Bianchi and Melosi (2016), Bianchi (2012a,b), and Davig and Doh (2014)

Uncertainty shocks
- Bloom (2009), Baker et al. (2016), and Creal and Wu (2014), Ulrich (2012)
Model: Representative Household

- **Preferences**

\[ u(C_t, N_t, \frac{M_t}{P_t}) = \frac{C_t^{1-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} + \frac{M_t^{1-\nu}}{1 - \nu} \]

- **Composite consumption good**

\[ C_t = \left( \int_0^1 C_{it}^{\frac{e_t - 1}{e_t}} di \right)^{\frac{e_t}{e_t - 1}} \]

- **Budget constraint**

\[ \int_0^1 P_{it} C_{it} d_{it} + M_t + \frac{1}{1 + i_t} B_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t \]
Model: Firms

- Continuum of monopolistically competitive firms producing differentiated goods
- Technology
  \[ Y_{it} = A_t N_{it}^{1-\alpha} \]
- Calvo pricing: \(1 - \omega\) adjust prices each period, others keep price constant
- Robustness: non-adjusters increase their previous price by the inflation target
Monetary Policy

- Interest rates are set according to a regime specific rule

\[ i(s_t) = \phi_{\pi,s} \pi_t + \phi_{\pi',s} E_t \pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi_s^* + \phi_{\pi,s} x_t + \mu_t \]

- Regime is determined by a time invariant Markov process with transition matrix

\[
\Pi = \begin{bmatrix}
p_{11} & p_{12} & \cdots & p_{1k} \\
p_{21} & p_{22} & \cdots & p_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
p_{k1} & p_{k2} & \cdots & p_{kk}
\end{bmatrix} \quad k = 2, 3
\]

- Central bank loss function

\[
L_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)
\]

\[
EL = E(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)
\]
Log-Linearized Model

- Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + (1 - \beta) \bar{\pi} + \mu^S_t, \]  
where \( \bar{\pi} \) is the current regime’s inflation target

- IS curve

\[ x_t = E_t x_{t+1} - \sigma^{-1} (i_t - E_t \pi_{t+1}) + \mu^D_t \]  

- Monetary policy

\[ i(s_t) = \phi_{\pi,s} \pi_t + \phi_{\pi',s} E_t \pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi_s^* + \phi_{x,s} x_t + \mu^l_t \]  

- Autoregressive shock processes

\[ \mu^j_t = \rho_j \mu^j_{t-1} + \epsilon^j_t, \quad \epsilon^j_t \sim N(0, \sigma^2_j) \quad \forall j \]  

- \( \pi \) is inflation, \( x \) is the output gap, \( i \) is the nominal interest rate
Solution Method

Let $\Omega_t$ be the full information set and $\Omega_t^{-s}$ be the information set excluding the current regime, then

$$E_t\pi_{t+1} \equiv E[\pi_{t+1}| s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij} E[\pi_{jt+1}| \Omega_t^{-s}]$$

$$E_tx_{t+1} \equiv E[x_{t+1}| s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij} E[x_{jt+1}| \Omega_t^{-s}]$$

The model in regime contingent notation:

$$x_{s,t} = \sum_{j=1}^{k} p_{sj} E_t x_{j,t+1} - \sigma^{-1}(i_{s,t} - \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1}) + \mu^D_t$$

$$\pi_{s,t} = \beta \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1} + \kappa x_{s,t} + (1 - \beta) \bar{\pi}_s + \mu^S_t$$

$$i_{s,t} = \phi_{\pi,s} \pi_t + \phi_{\pi',s} \sum_{i=1}^{k} p_{sj} E_t \pi_{j,t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi^*_s + \phi_{x,s} x_t + \mu^I_t$$
Quarterly Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>.99</td>
<td>( \rho_1, \rho_S, \rho_D )</td>
<td>.5</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>2</td>
<td>( \sigma_S )</td>
<td>1.5</td>
</tr>
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<td>( \varphi )</td>
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<td>( \sigma_D )</td>
<td>2</td>
</tr>
<tr>
<td>( E_{e_t} )</td>
<td>5</td>
<td>( \sigma_I )</td>
<td>2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>.33</td>
<td>( \theta_x )</td>
<td>.0408</td>
</tr>
<tr>
<td>( \omega )</td>
<td>.66</td>
<td>( \theta_i )</td>
<td>.25</td>
</tr>
</tbody>
</table>
Analytical Regime Switch Framework

- At $t = 0$, a surprise announcement that the central bank is considering permanently raising the inflation target to $\pi^*$ with probability $\lambda$ each period.
  - Results are symmetric for a potential decrease in the inflation target.
- If the inflation target is increased, it remains there forever.
- Formally, the Markov process switches from \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\] to \[
\begin{bmatrix}
1 - \lambda & \lambda \\
0 & 1
\end{bmatrix}
\]
- Assume monetary policy in regime two is determinate.
- Only shock is the realization of the Markov process.
  - Each regime is in a steady state.
  - Let $x_i$ denote the outcome in regime $i$. 

Regime Switch Outcomes

- Prior to the announcement, zero inflation steady state
  \[ x_t = \pi_t = 0 \]

- After the announcement (regime 1): level shifts in inflation and output gap
  - Qualitatively depends on policy rule parameters
  - Quantitatively depends on \( \pi^* \) and transition probability

- After inflation target changes (regime 2): \( \pi^* \) inflation steady state
Key Forces

- Higher expected inflation
  - A potential increase in $\pi^*$ increases expected inflation
  - Expected optimal price in future periods increases
  - Firms facing sticky prices raise prices immediately

- Different real interest rate
  - Higher expected inflation reduces real rates
  - Nominal rates adjust according to policy rule
  - Real rates rise if monetary policy responds enough to inflation

- A higher real interest rate reduces inflation and the output gap
  - Higher real interest rate incentivizes saving and reduces demand
  - Lower demand leads to lower consumption and output
  - Lower output reduces firm’s marginal costs leading to lower prices
Regime 1 Response to Inflation Target Uncertainty

- **Inflation**
  - Rises by more than $\pi^*$ if policy is passive
  - Rises by $\pi^*$ if interest rates move one for one with inflation
  - Rises but by less than $\pi^*$ if policy is active
  - Falls if $\phi_{\pi'}$ is large enough

- **Output gap**
  - Rises if policy is passive
  - Falls if policy is responsive enough to inflation

- **Policy rule coefficients**
  - A larger $\phi_{\pi}$ reduces the change in inflation and reduces output gap
  - A larger $\phi'_{\pi}$ reduces inflation and the output gap
  - A larger $\phi_{x}$ reduces inflation and the output gap if output gap is positive

Formal Version
Monetary policy response

- By changing $\pi_1^*$, any outcome along the regime 1 Phillips Curve

$$\pi_1 = \frac{\pi^* \beta \lambda + \kappa x_1}{1 - \beta (1 - \lambda)}$$

is achievable without affecting the volatility of inflation or output in a stochastic model

- A trade off in levels between inflation and output

- Reducing the constant in the policy rule (raising $\pi^*$) for the current regime will raise inflation and output in the current regime if monetary policy is active
Potential Future Increase in the Inflation Target

![Diagram showing the relationship between inflation and output gap with different scenarios of uncertainty.]

- **Initial PC**: Red solid line
- **Initial IS**: Blue solid line
- **PC with Uncertainty**: Red dashed line
- **IS with Uncertainty**: Blue dashed line

Points A and B illustrate different scenarios with and without uncertainty.
Central Bank Responds by Raising the Inflation Target

Theoretical Analysis

Output Gap

Inflation

- Initial PC
- Initial IS
- PC with Uncertainty
- IS with Uncertainty
- IS with decreased $\bar{i}$
- A
- B
- C
Zero Inflation and Optimal Policy

- To achieve no change in inflation the central bank must generate a recession of magnitude $\frac{\beta \lambda \pi^2}{\kappa}$.
- Optimal allocation is determined by loss function and can be anywhere on the Phillips Curve between:
  - Zero inflation and a recession
  - Zero output gap and an increase in inflation
- Can be achieved by adjusting the current inflation target.
Recall the central loss function

\[ L_{t_0} = \sum_{t = t_0}^{\infty} \beta^{t-t_0} (\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2) \]

Single regime optimal policy rule
- For \( \theta_x = 0.0408 \) and \( \theta_i = 0.25 \), \( i = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057 x_t \)

Multiple regime optimal policy rule may differ
- The optimal policy rule is an approximation of the Ramsey optimal policy
- Regime switches create additional degrees of freedom and allows for a closer approximation of the optimal policy
- For welfare evaluations under the optimal response, comparisons are to the single inflation target policy rule that uses the same Markov structure
Three regimes with $\phi_\pi$, $\phi_\pi'$, and $\phi_x$ at single regime optimal values
Initially in regime one, $\pi_1^* = 0$
Probability $p_2$ to transition to regime 2 which has inflation target of 2%
Probability $p_3$ to transition to regime 3 which has inflation target of 0%
Regimes 2 and 3 are absorbing regimes
For optimal policy considerations, comparison is to $\phi_{\pi,1}$, $\phi_{\pi',1}$, and $\phi_{x,1}$ that are optimal given a future regime shift to an absorbing regime
Next
1. Holding monetary policy fixed, vary parameters
2. Allow central bank to respond
Main Specification

Expected Outcomes in Regime 1 for Different Transition Probabilities

- Change in Losses
- $E_{\pi_1}$
- $E_{x_1}$
- $Ei_1$

Monetary policy causes an increase in real rates and a large recession.

More imminent resolution of the uncertainty magnifies the effects.
More Output Stabilizing Policy ($\theta_x = 1$)

Expected Outcomes in Regime 1 for Different Transition Probabilities

- **Change in Losses**
- $E_{\pi_1}$
- $E_{x_1}$
- $E_{i_1}$

Transition Probability ($p_2 = p_3$)

- Real rates barely rise leading to a large rise in inflation
- Nominal rates actually increase more
Current vs Expected Inflation

- Interest rates optimally respond only to $\pi_t$ (dashed)

A larger weight on output stability results in higher inflation
Interest rates optimally respond only $E\pi_{t+1}$ (solid)

Responding to $E\pi_{t+1}$ results in a larger recession and lower inflation.
Central Bank Sets $\pi_1^*$ s.t. $E\pi_1 = 0$

- Requires reducing $\pi_1^*$ more than one for one with the transition probabilities
Optimal Monetary Policy Response

The optimal response is a reduction in the current inflation target.

Results in higher real rates, a near zero inflation rate, and a large recession.
## Welfare Benefits of Optimal and Constrained Optimal Policy

<table>
<thead>
<tr>
<th>Welfare Weights</th>
<th>% of Inflation Target Uncertainty Losses Eliminated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimizing over $\phi_\pi, \phi_{\pi'}, \phi_x$</td>
</tr>
<tr>
<td>$\theta_x$</td>
<td>$\theta_i$</td>
</tr>
<tr>
<td>0.0000</td>
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<tr>
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<td>4.0000</td>
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<td>0.0408</td>
<td>0.05</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.50</td>
</tr>
</tbody>
</table>

$^1$ Base policy: All other parameters in regime one are at their single regime optima rather than the regime one optimal policy in the three regime model without uncertainty.

- Changing the inflation target is sufficient to minimize losses
- Optimal policy is most effective for large $\theta_i$ and small $\theta_x$
- Effective of only changing other parameters is diminishing in $\theta_x$
- With indexing similar optimal policy benefits, but require marginal adjustments of $\phi_\pi, \phi_{\pi'},$ and $\phi_x$
A potential resolution to the uncertainty is for the central bank to commit to changing the inflation target in the future. At \( t = 0 \), a surprise announcement that at \( t = T \) the inflation target will be permanently increased from 0 to \( \pi^* \).

Deterministic equilibrium and \( \phi_x = \phi_{\pi'} = 0 \) (but \( \phi_\pi \) arbitrary).

What happens?

- At \( t = 0 \) inflation and the output gap jump
- For \( t \geq T \), \( \pi^* \) steady state
Perfect Foresight Transition Path

Key forces:
1. Higher $E\pi \rightarrow$ firms facing sticky prices raise prices
2. Real rates fall, rise, or remain constant depending on $\phi_\pi \rightarrow$ change in m.c.
3. Anticipation: at T-2 agents fully anticipate T-1 outcomes
Perfect Foresight Transition Path

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Outcomes:
- If $\phi \pi < \phi^-\pi$, inflation and the output gap adjust monotonically
  - Multiple cases depending on whether $\phi \pi$ is $< 1$ or $> 1$
  - Analogous to regime switch cases
- If $\phi \pi > \phi^-\pi$, inflation and the output gap exhibit cyclical dynamics
  - Strong monetary policy response generates overshooting

where $\phi^-\pi = \frac{1}{4\beta} \left( \frac{(1-\beta)^2}{\sigma - 1}\kappa \right) + 2(1 - \beta) - \sigma^{-1}\kappa$
Perfect Foresight Transition Path to $\pi_{40}^* = 1$: Inflation

Inflation Target Uncertainty and Monetary Policy

Yevgeniy Teryoshin
Perfect Foresight Transition Path $\pi^*_{40} = 1$: Output
Perfect Foresight Transition Path with Optimal Policy

- Most active policy rules result in cyclical dynamics
  - $\hat{\phi}_\pi \approx 1.05$
  - Cyclical dynamics are suboptimal
- What is the optimal transition path?
  - Inflation monotonically adjusts to $\pi^*$
  - Output gap monotonically adjusts in the opposite direction
Perfect Foresight

Optimal Transition Path ($\pi^* = 1$, $\theta_x = 1$ and $\theta_i = 0$)
Perfect Foresight Transition Path with Optimal Policy

- Most active policy rules result in cyclical dynamics
  - $\bar{\phi}_\pi \approx 1.05$
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- What is the optimal transition path?
  - Inflation monotonically adjusts to $\pi^*$
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- Implement by announcing a path for the inflation target along the entire transition path
  - Large non-monotonic change in interest rates the period before the new inflation target is achieved
Perfect Foresight

Optimal Policy Along Transition Path ($\phi_\pi = 1.5$)

![Graph showing Optimal Policy Along Transition Path with $\bar{i}_t$ and $\pi_t^*$ lines over time.]

- $\bar{i}_t$ line
- $\pi_t^*$ line

Time: 0 to 20

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Inflation Target Uncertainty and Monetary Policy
Conclusion

- A potential future increase in the inflation target generates upward pressure on inflation.
- The monetary policy response generally raises real rates and leads to stagflation.
- Creates a level trade-off between inflation and output with the outcome determined by the current inflation target.
  - Approximately true even with indexing.
- Commitment to changing the inflation target at a certain point in time leads to cyclical fluctuations.
  - Fluctuations eliminated under the optimal policy which relies on an announced time varying path for the inflation target.
Regime 1: Active Policy

Define: $C_1 = 1 + \frac{1 - \beta (1 - \lambda)}{\kappa \sigma - 1} + \frac{\beta \phi_x}{\kappa}$ and $C_2 = 1 + \frac{(1 - \beta)(1 - \beta(1 - \lambda))}{\kappa \sigma - 1}$

Monetary policy is active if $\phi_{\pi} + \phi_{\pi'} > 1$, then

1. $\pi_1$ is increasing in $\pi^*$ if $C_1 > \phi_{\pi'}$
   - $\phi_{\pi'}$ increases interest rates proportional to $(1 - \lambda)\pi_1 + \lambda \pi_2$
   - Large $\phi_{\pi'}$ can raise $i_1$ enough for current inflation to fall
   - Large $\phi_{\pi}$ can’t since if $\pi_1 < 0 \Rightarrow i_1 \downarrow$

2. $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_{\pi'}} < 0$

3. $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_{\pi}} < 0$ if $C_1 > \phi_{\pi'}$

4. $x_1$ is decreasing in $\pi^*$ if $C_2 < \beta \phi_\pi + \phi_{\pi'}$
   - Do interest rates rise enough to incentivize saving?

5. $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_x} > 0$ if $C_2 < \beta \phi_\pi + \phi_{\pi'} + \frac{\beta (1 - \beta(1 - \lambda)^2)}{\kappa} \phi_x$

6. If $\phi_{\pi}, \phi_{\pi'}, \phi_x$ are the same in both regimes, then $\pi_1 < \pi_2 \leq \pi^*$ and $x_1 < x_2$
Regime 1: Passive Policy

Define

\[ \bar{\phi}_\pi = 1 - \lambda (1 + \frac{1 - \beta (1 - \lambda)}{\sigma^{-1} \kappa}) \]

If monetary policy is passive (\( \phi_\pi + \phi_{\pi'} \leq 1 \)) and \( \phi_{\pi'} = \phi_x = 0 \), then

1. If \( \phi_\pi = 1 \), then \( \pi_1 = \pi_2 \) and \( x_1 = \frac{1 - \beta}{\kappa} \pi_2 \)

2. If \( \phi_\pi \in (\bar{\phi}_\pi, 1) \), then \( \pi_1 > \pi_2 \) and \( x_1 > \frac{1 - \beta}{\kappa} \pi_2 \) and both approach infinity as \( \phi_\pi \) is reduced to the lower bound

3. If \( \phi_\pi < \bar{\phi}_\pi \), then stochastic equilibrium is indeterminate
Proposition 4

Reducing the constant in the policy rule for the current regime, $\bar{i}_1$, will raise inflation and output in the current regime if

$$\phi_\pi + (1 - \lambda)\phi_{\pi'} + \phi_x \frac{1 - \beta(1 - \lambda)}{\kappa} > 1 - \lambda(1 + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1}\kappa}) < 1$$
Proposition 8

If

\[
\frac{(1 - \beta)^2}{\sigma^{-1} \kappa} + \sigma^{-1} \kappa \geq 2(\beta \phi_\pi - 1) + 2\beta(\phi_\pi - 1),
\]

then

\[
\lambda = 1 + \beta + \sigma^{-1} \kappa \pm \sqrt{(1 + \beta + \sigma^{-1} \kappa)^2 - 4\beta(1 + \sigma^{-1} \kappa \phi_\pi)}
\]

\[
2(1 + \sigma^{-1} \kappa \phi_\pi)
\]

is a real number, and for \( t \in [0, T] \)

\[
\pi_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1) - \lambda_1^{t+1}(1 - \lambda_2)}{\lambda_2 - \lambda_1}
\]

\[
x_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1)(\lambda_2 - \beta) - \lambda_1^{t+1}(1 - \lambda_2))(\lambda_1 - \beta)}{\kappa(\lambda_2 - \lambda_1)}
\]

Otherwise \( \lambda \) is a complex number, and for \( t \in [0, T] \)

\[
\pi_{T-t} = \frac{\pi^* (\phi_\pi - 1)\sigma^{-1} \kappa}{1 + \sigma^{-1} \kappa \phi_\pi} r^t \sum_{j=0}^{\infty} r^j \sin(\omega(t + 1 + j)) \sin(\omega) 
\]

where \( r = \sqrt{\frac{\beta}{1 + \sigma^{-1} \kappa \phi_\pi}} \), \( \omega = \cos^{-1} \left( \frac{1+\beta+\sigma^{-1} \kappa}{2\sqrt{\beta(1+\sigma^{-1} \kappa \phi_\pi)}} \right) \), and \( x_{T-t} = \frac{\pi_{T-t}-\beta \pi_{T-t+1}}{\kappa} \).
Proposition 9

If $\lambda$ is real and $\pi^* > 0$, then

1. If $\phi_\pi < 1$, then $\pi_{T-t} > \pi^*$, $x_{T-t} > \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically decreasing.

2. If $\phi_\pi = 1$, then $\pi_{T-t} = \pi^*$, $x_{T-t} = \frac{1-\beta}{\kappa}\pi^*$

3. If $\frac{1}{\beta} \geq \phi_\pi > 1$, then $0 < \pi_{T-t} < \pi^*$, $0 < x_{T-t} < \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically increasing.

4. If $\phi_\pi > \frac{1}{\beta}$, then for small $t$; $\pi_{T-t} < \pi^*$, $x_{T-t} < \frac{1-\beta}{\kappa}\pi^*$, and both are monotonically increasing. For large enough $t$, either or both $\pi_{T-t}$ and $x_{T-t}$ may be monotonically decreasing.
Stochastic Solution

Solving the full stochastic model it can be shown that the solutions for output and inflation are of the form

\[ \pi_{j,t} = g_S e_{j,1} \mu^S_t + g_I e_{j,2} \mu^I_t + g_D e_{j,3} \mu^D_t + g_1 e_{j,4} a_j \]

\[ x_{1,t} = f_{1,S} \mu^S_t + f_{2,I} \mu^I_t + f_{1,D} u^D_t + f_{1,1} a_1 + f_{1,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_1 \]

\[ x_{2,t} = f_{2,S} \mu^S_t + f_{2,I} \mu^I_t + f_{2,D} u^D_t + f_{2,1} a_1 + f_{2,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_2 \]

Then

\[ \text{Var}(\pi_{j,t}) = g_S^2 e_{j,1}^2 \frac{\text{Var}(\epsilon^S_t)}{1 - \rho_S^2} + g_I^2 e_{j,2}^2 \frac{\text{Var}(\epsilon^I_t)}{1 - \rho_I^2} + g_D^2 e_{j,3}^2 \frac{\text{Var}(\epsilon^D_t)}{1 - \rho_D^2} \]

\[ \text{Var}(x_{j,t}) = f_{j,S}^2 \frac{\text{Var}(\epsilon^S_t)}{1 - \rho_S^2} + f_{j,I}^2 \frac{\text{Var}(\epsilon^I_t)}{1 - \rho_I^2} + f_{j,D}^2 \frac{\text{Var}(\epsilon^D_t)}{1 - \rho_D^2} \]

But the inflation targets only appear in the \( a_1 \) and \( a_2 \) terms.
Zero Inflation and Optimal Policy

- To achieve $\pi_1 = 0$, requires a recession of magnitude $\frac{\beta \lambda \pi_2}{\kappa}$ which can be accomplished by setting

$$
\pi^*_1 = \lambda \pi_2 \left( \frac{1}{1 - \phi_\pi - \phi_\pi'} \left( 1 - \phi_\pi' + \frac{\beta \phi_x}{\kappa} + \frac{1 - \beta (1 - \lambda)}{\sigma^{-1} \kappa} \right) \right)
$$

- The optimal commitment policy in regime one will set the inflation target in regime one such that

$$
x_1 = \frac{-\pi_2 \lambda \beta}{\kappa + \kappa^{-1} (1 - \beta (1 - \lambda))^2 \theta_x} \in \left[ -\frac{\beta \lambda \pi_2}{\kappa}, 0 \right]
$$

$$
\pi_1 = \frac{\pi_2 \lambda \beta}{1 - \beta (1 - \lambda)} \frac{\kappa^{-1} (1 - \beta (1 - \lambda))^2 \theta_x}{\kappa + \kappa^{-1} (1 - \beta (1 - \lambda))^2 \theta_x} \in \left[ 0, \frac{\pi_2 \lambda \beta}{1 - \beta (1 - \lambda)} \right]
$$
# Single Regime Optimal Policy

**Table:** Optimal Policy Rule Under a Single Regime

<table>
<thead>
<tr>
<th>$\theta_x$</th>
<th>$\theta_i$</th>
<th>$\phi_\pi$</th>
<th>$\phi_\pi'$</th>
<th>$\phi_x$</th>
<th>$E\pi^2$</th>
<th>$Ex^2$</th>
<th>$Ei^2$</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0408</td>
<td>0.2500</td>
<td>1.8255</td>
<td>1.1628</td>
<td>0.5057</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9632</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.2500</td>
<td>1.8256</td>
<td>1.1628</td>
<td>0.2500</td>
<td>5.5686</td>
<td>17.0096</td>
<td>27.1140</td>
<td>12.3471</td>
</tr>
<tr>
<td>0.5000</td>
<td>0.2500</td>
<td>1.8255</td>
<td>1.1628</td>
<td>3.3829</td>
<td>8.6727</td>
<td>2.6147</td>
<td>22.8060</td>
<td>15.6816</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.2500</td>
<td>1.8256</td>
<td>1.1628</td>
<td>6.5161</td>
<td>9.7410</td>
<td>1.0113</td>
<td>22.9697</td>
<td>16.4947</td>
</tr>
<tr>
<td>2.0000</td>
<td>0.2500</td>
<td>1.8254</td>
<td>1.1627</td>
<td>12.7805</td>
<td>10.5714</td>
<td>0.3279</td>
<td>23.3732</td>
<td>17.0705</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.0500</td>
<td>9.9279</td>
<td>5.2140</td>
<td>1.5283</td>
<td>1.4561</td>
<td>31.6754</td>
<td>55.1211</td>
<td>5.5045</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.5000</td>
<td>0.8128</td>
<td>0.6564</td>
<td>0.3778</td>
<td>9.9605</td>
<td>11.7571</td>
<td>14.6360</td>
<td>17.7582</td>
</tr>
</tbody>
</table>
Monetary Policy Uncertainty

Table: Regime One and Two Outcomes with \( i_1 = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t \) and \( p_{11} = p_{22} = .9 \)

<table>
<thead>
<tr>
<th>( i_2 )</th>
<th>( E\pi_1^2 )</th>
<th>( Ex^2_1 )</th>
<th>( Ei^2_1 )</th>
<th>( EL_1 )</th>
<th>( E\pi_2^2 )</th>
<th>( Ex^2_2 )</th>
<th>( Ei^2_2 )</th>
<th>( EL_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9633</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9633</td>
</tr>
<tr>
<td>( 2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.8953</td>
<td>13.4296</td>
<td>25.1804</td>
<td>12.7385</td>
<td>5.1295</td>
<td>15.4225</td>
<td>30.1017</td>
<td>13.2843</td>
</tr>
<tr>
<td>( 1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>6.0968</td>
<td>13.3725</td>
<td>26.3523</td>
<td>13.2306</td>
<td>7.0804</td>
<td>11.6451</td>
<td>21.0922</td>
<td>12.8287</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.9394</td>
<td>13.4170</td>
<td>25.4357</td>
<td>12.8459</td>
<td>5.5321</td>
<td>14.3956</td>
<td>27.9493</td>
<td>13.1069</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t )</td>
<td>6.0396</td>
<td>13.3886</td>
<td>26.0181</td>
<td>13.0905</td>
<td>6.4977</td>
<td>12.4814</td>
<td>23.4373</td>
<td>12.8664</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t )</td>
<td>6.0491</td>
<td>13.3330</td>
<td>25.9876</td>
<td>13.0902</td>
<td>6.6329</td>
<td>9.0575</td>
<td>24.0142</td>
<td>13.0062</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t )</td>
<td>5.8944</td>
<td>13.5128</td>
<td>25.3164</td>
<td>12.7750</td>
<td>5.2176</td>
<td>21.8323</td>
<td>29.8090</td>
<td>13.5608</td>
</tr>
</tbody>
</table>
Monetary Policy Uncertainty

Table: Expected Outcomes with \( i_1 = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t \) and \( p_{11} = p_{22} = .9 \)

<table>
<thead>
<tr>
<th>( i_2 )</th>
<th>( E\pi^2 )</th>
<th>( Ex^2 )</th>
<th>( Ei^2 )</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9633</td>
</tr>
<tr>
<td>( 2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.5124</td>
<td>14.4261</td>
<td>27.6411</td>
<td>13.0114</td>
</tr>
<tr>
<td>( 1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t )</td>
<td>6.5886</td>
<td>12.5088</td>
<td>23.7222</td>
<td>13.0296</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t )</td>
<td>5.7357</td>
<td>13.9063</td>
<td>26.6925</td>
<td>12.9764</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t )</td>
<td>6.2686</td>
<td>12.9350</td>
<td>24.7277</td>
<td>12.9784</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t )</td>
<td>6.3410</td>
<td>11.1952</td>
<td>25.0009</td>
<td>13.0482</td>
</tr>
<tr>
<td>( 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t )</td>
<td>5.5560</td>
<td>17.6725</td>
<td>27.5627</td>
<td>13.1679</td>
</tr>
</tbody>
</table>
Optimal Policy Response

**Table:** Optimal Regime 1 Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

<table>
<thead>
<tr>
<th>$i_1^*$</th>
<th>$i_2$</th>
<th>$E\pi^2$</th>
<th>$Ex^2$</th>
<th>$Ei^2$</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>5.9876</td>
<td>13.4029</td>
<td>25.7150</td>
<td>12.9633</td>
</tr>
<tr>
<td>$1.85927\pi_t + 1.1630E\pi_{t+1} + 0.5057x_t$</td>
<td>$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>5.4778</td>
<td>14.4912</td>
<td>27.7677</td>
<td>13.0111</td>
</tr>
<tr>
<td>$1.8054\pi_t + 1.1439E\pi_{t+1} + 0.5057x_t$</td>
<td>$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>6.6216</td>
<td>12.4503</td>
<td>23.5990</td>
<td>13.0294</td>
</tr>
<tr>
<td>$1.8454\pi_t + 1.1579E\pi_{t+1} + 0.5057x_t$</td>
<td>$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>5.7177</td>
<td>13.9397</td>
<td>26.7590</td>
<td>12.9763</td>
</tr>
<tr>
<td>$1.8183\pi_t + 1.1483E\pi_{t+1} + 0.5057x_t$</td>
<td>$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>6.2845</td>
<td>12.9061</td>
<td>24.6687</td>
<td>12.9784</td>
</tr>
<tr>
<td>$1.8298\pi_t + 1.1542E\pi_{t+1} + 0.5336x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$</td>
<td>6.3637</td>
<td>11.0348</td>
<td>24.9350</td>
<td>13.0478</td>
</tr>
<tr>
<td>$1.8322\pi_t + 1.1495E\pi_{t+1} + 0.4680x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$</td>
<td>5.5276</td>
<td>17.9038</td>
<td>27.6354</td>
<td>13.1672</td>
</tr>
</tbody>
</table>

There are alternative policy rules for regime one that result in the same losses in the two regime model.
Table: Expected Per Period Regime 1 Loss Minimizing Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

<table>
<thead>
<tr>
<th>$i_1^*$</th>
<th>$i_2$</th>
<th>$\pi_t^2$</th>
<th>$\pi_{t+1}^2$</th>
<th>$\pi_t^2$</th>
<th>$\pi_{t+1}^2$</th>
<th>$\pi_t^2$</th>
<th>$\pi_{t+1}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4114$\pi_t + 0.9439E\pi_{t+1} + 0.4702x_t$</td>
<td>1.8255$\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.0968</td>
<td>11.9718</td>
<td>20.9683</td>
<td>12.8275</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4160$\pi_t + 0.9439E\pi_{t+1} + 0.4699x_t$</td>
<td>2.3255$\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>6.9723</td>
<td>12.0217</td>
<td>20.5802</td>
<td>12.6080</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4062$\pi_t + 0.9438E\pi_{t+1} + 0.4706x_t$</td>
<td>1.3255$\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.2444</td>
<td>11.9145</td>
<td>21.4314</td>
<td>13.0885</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4125$\pi_t + 0.9432E\pi_{t+1} + 0.4700x_t$</td>
<td>1.8255$\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.0351</td>
<td>11.9943</td>
<td>20.7491</td>
<td>12.7119</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4102$\pi_t + 0.9445E\pi_{t+1} + 0.4705x_t$</td>
<td>1.8255$\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.1634</td>
<td>11.9480</td>
<td>21.2063</td>
<td>12.9526</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4082$\pi_t + 0.9444E\pi_{t+1} + 0.4778x_t$</td>
<td>1.8255$\pi_t + 1.628E\pi_{t+1} + 1.0057x_t$</td>
<td>7.1865</td>
<td>11.7591</td>
<td>21.1317</td>
<td>12.9494</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4164$\pi_t + 0.9429E\pi_{t+1} + 0.4586x_t$</td>
<td>1.8255$\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$</td>
<td>6.9620</td>
<td>12.3057</td>
<td>20.7275</td>
<td>12.6461</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Welfare Weights</th>
<th>% of Inflation Target Uncertainty Losses Eliminated</th>
<th>Optimizing over</th>
<th>Optimizing over</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>$\theta_i$</td>
<td>Optimal Policy</td>
<td>$\phi_{\pi}, \phi_{\pi'}, \phi_x$</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.2500</td>
<td>0.8202</td>
<td>0.8082</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.2500</td>
<td>0.7733</td>
<td>0.7263</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.2500</td>
<td>0.7699</td>
<td>0.3853</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.2500</td>
<td>0.8478</td>
<td>0.1837</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.0500</td>
<td>0.1287</td>
<td>0.1134</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.5000</td>
<td>0.9962</td>
<td>0.8443</td>
</tr>
</tbody>
</table>

\(^1\) Base policy: All other parameters in regime one are at their single regime optimums rather than the regime one optimal policy in the three regime model without uncertainty.

- Optimal response requires a change in both $\pi^*_1$ and $\phi_{\pi}, \phi_{\pi'}, \phi_x$
- Only changing $\pi^*_1$ captures most of the benefit of the optimal response
Appendix

Anticipated Future 1% Decrease in the Inflation Target

\[ i_t = 1.5\pi_t - 0.5\pi^* \]

Regime switch framework results in:

- Stabilizing inflation at initial target increases losses
- Similar or lower expected total and per period losses
- Expected values are similar
- Variance is lower (none)