Historically inflation targets change

US inflation target recently called into question
- Response to perceived decline in $r^*$
- Williams (2009), Blanchard et al. (2010), and Ball (2014)
- Creates uncertainty around the inflation target

Why could this uncertainty matter?
- Uncertainty in the future inflation target changes expected inflation
- Individual, firm, and central bank decisions respond to such changes
- Affects current outcomes regardless of whether the inflation target eventually changes
"So it’s that recognition that causes people to think we might be better off with a higher inflation objective. That is an important set, this is one of our most critical decisions and one we are attentive to evidence and outside thinking. It’s one that we will be reconsidering at some future time... But I would say that this is one of the most important questions facing monetary policy around the world in the future.”
This Paper

**Questions**
- How does $\pi^*$ uncertainty affect current inflation, output, and welfare?
- How should the central bank respond to inflation target uncertainty?
- Can inflation target uncertainty be a useful policy tool?

**Approach**
- Model inflation target uncertainty in a standard New Keynesian model
  - Policy rule with a regime specific inflation target
  - Exogenous Markov process determines regime
- Analytically solve the model without any additional uncertainty
- Numerically solve the stochastic model for full quantitative evaluation


This Paper

Questions

- How does $\pi^*$ uncertainty affect current inflation, output, and welfare?
  - Depends critically on current monetary policy profile
- How should the central bank respond to inflation target uncertainty?
  - Achieving the old inflation target requires changing the current inflation target
  - Optimal policy adjusts the current inflation target and potentially marginally adjusts other policy parameters
- Can inflation target uncertainty be a useful policy tool?
  - Yes, helps reduce variances on the transition path compared to a fully anticipated increase in the inflation target

Approach

- Model inflation target uncertainty in a standard New Keynesian model
  - Policy rule with a regime specific inflation target
  - Exogenous Markov process determines regime
- Analytically solve the model without any additional uncertainty
- Numerically solve the stochastic model for full quantitative evaluation
Regime switches

- DSGE estimation: Schorfheide (2005), Liu, Waggoner, Zha (2011), Bianchi and Melosi (2016), Bianchi (2012a,b), and Davig and Doh (2014)

Uncertainty shocks

- Bloom (2009), Baker et al. (2016), and Creal and Wu (2014), Ulrich (2012)
Outline

1 Introduction

2 Model
   - New Keynesian Model

3 Theoretical Analysis

4 Stochastic Model
Model: Representative Household

- Preferences

\[ u(C_t, N_t, \frac{M_t}{P_t}) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \frac{M_t^{1-\nu}}{1-\nu} \]

- Composite consumption good

\[ C_t = \left( \int_0^1 C_{it}^{\frac{et-1}{et}} \, di \right)^{\frac{et}{et-1}} \]

- Budget constraint

\[ \int_0^1 P_{it} C_{it} d_{it} + M_t + \frac{1}{1+i_t} B_t \leq M_{t-1} + B_{t-1} + W_t N_t + D_t \]
Model: Firms

- Continuum of monopolistically competitive firms producing differentiated goods
- Technology

\[ Y_{it} = A_t N_{it}^{1-\alpha} \]

- Calvo pricing: \( 1 - \omega \) adjust prices each period, others keep price constant
- Robustness: non-adjusters increase their previous price by the inflation target
Monetary Policy

- Interest rates are set according to a regime specific rule

\[ i(s_t) = \phi_{\pi,s}\pi_t + \phi_{\pi',s}E_t\pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1)\pi^*_s + \phi_{x,s}x_t + \mu^I_t \]

- Regime is determined by a time invariant Markov process with transition matrix

\[
\Pi = \begin{bmatrix}
p_{11} & p_{12} & \ldots & p_{1k} \\
p_{21} & p_{22} & \ldots & p_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
p_{k1} & p_{k2} & \ldots & p_{kk}
\end{bmatrix} \quad k = 2, 3
\]

- Central bank loss function

\[
L_{t_0} = \sum_{t=t_0}^{\infty} \beta^{t-t_0}(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)
\]

\[
EL = E(\pi_t^2 + \theta_x x_t^2 + \theta_i i_t^2)
\]
Log-Linearized Model

- Philips curve
  \[ \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + (1 - \beta) \bar{\pi} + \mu_t^S, \]  
  where \( \bar{\pi} \) is current regime’s inflation target

- IS curve
  \[ x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + \mu_t^D \]  

- Monetary policy
  \[ i(s_t) = \phi_{\pi,s} \pi_t + \phi_{\pi',s} E_t \pi_{t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi^*_s + \phi_{\chi,s} x_t + \mu_t^l \]

- Autoregressive shock processes
  \[ \mu_t^j = \rho_j \mu_{t-1}^j + \epsilon_t^j, \quad \epsilon_t^j \sim N(0, \sigma_j^2) \quad \forall j \]

- \( \pi \) is inflation, \( x \) is the output gap, \( i \) is the nominal interest rate
Solution Method

- Let $\Omega_t$ be the full information set and $\Omega_t^{-s}$ be the information set excluding the current regime, then

$$E_t \pi_{t+1} \equiv E[\pi_{t+1}|s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij} E[\pi_{jt+1}|\Omega_t^{-s}]$$

$$E_t x_{t+1} \equiv E[x_{t+1}|s_t = i, \Omega_t^{-s}] = \sum_{j=1}^{k} p_{ij} E[x_{jt+1}|\Omega_t^{-s}]$$

- The model in regime contingent notation:

$$x_{s,t} = \sum_{j=1}^{k} p_{sj} E_t x_{j,t+1} - \sigma^{-1}(i_{s,t} - \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1}) + \mu^D$$

$$\pi_{s,t} = \beta \sum_{j=1}^{k} p_{sj} E_t \pi_{j,t+1} + \kappa x_{s,t} + (1 - \beta) \bar{\pi}_s + \mu^S$$

$$i_{s,t} = \phi_{\pi,s} \pi_t + \phi_{\pi',s} \sum_{i=1}^{k} p_{sj} E_t \pi_{j,t+1} - (\phi_{\pi,s} + \phi_{\pi',s} - 1) \pi_{s}^\star + \phi_{x,s} x_t + \mu^I_t$$
## Calibration

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Outline

1 Introduction

2 Model

3 Theoretical Analysis
   - Perfect Foresight
   - Regime Switch

4 Stochastic Model
Perfect Foresight of a Future Inflation Target Increase

- At $t = 0$, a surprise announcement that at $t = T$ the inflation target will be permanently increased from 0 to $\pi^*$
- Deterministic equilibrium and $\phi_x = \phi_{\pi'} = 0$ (but $\phi_\pi$ arbitrary)
- What happens?
  - At $t = 0$ inflation and the output gap jump
  - For $t \geq T$, $\pi^*$ steady state
Perfect Foresight Transition Path

Key forces:

1. Higher $E\pi \rightarrow$ firms facing sticky prices raise prices
2. Real rates fall, rise, or remain constant depending on $\phi_{\pi} \rightarrow$ change in m.c.
3. Anticipation: at T-2 agents fully anticipate T-1 outcomes
Perfect Foresight Transition Path

Key forces:
1. Higher $E \pi \rightarrow$ firms facing sticky prices raise prices
2. Real rates fall, rise, or remain constant depending on $\phi_{\pi} \rightarrow$ change in m.c.
3. Anticipation: at T-2 agents fully anticipate T-1 outcomes

Define $\bar{\phi}_{\pi} = \frac{1}{4}\beta\left(\frac{(1-\beta)^2}{\sigma-1} + 2(1 - \beta) - \sigma^{-1}\kappa\right)$

1. $\phi_{\pi} < 1 \Rightarrow \pi_0$ and $x_0$ increase, $\pi_t$ and $x_t$ monotonically decline as $t \rightarrow T$
   - T-1: $E\pi \uparrow \Rightarrow \pi \uparrow$, $r \downarrow \Rightarrow$ savings $\downarrow \Rightarrow x$ and m.c. $\uparrow \Rightarrow \pi \uparrow$
   - T-2: Anticipating even higher $E\pi$ and $Ex \Rightarrow \pi$ and $x$ even larger

2. $\phi_{\pi} = 1 \Rightarrow \pi_0 = \pi_t = \pi_T$ and $x_0 = x_t = x_T$

3. $\frac{1}{\beta} \geq \phi_{\pi} > 1 \Rightarrow 0 < \pi_0 < \pi^*, 0 < x_0 < \frac{1-\beta}{\kappa} \pi^*$, and $\pi_t$ and $x_t$ monotonically increase as $t \rightarrow T$

4. $\bar{\phi}_{\pi} \geq \phi_{\pi} > \frac{1}{\beta} \Rightarrow$ may be a empty set, if not $\pi_t$ and $x_t$ monotonically increase for small $T$

5. $\phi_{\pi} > \bar{\phi}_{\pi}$, $\pi_t$ and $x_t$ exhibit cyclical dynamics that are magnified as $t \rightarrow T$
Perfect Foresight Transition Path to $\pi_{40}^* = 1$: Inflation
Perfect Foresight Transition Path $\pi^*_{40} = 1$: Output

![Graph showing the output gap over time for different values of $\phi_{\pi}$.]

- $\phi_{\pi} = 0.975$
- $\phi_{\pi} = 1$
- $\phi_{\pi} = 1.005$
- $\phi_{\pi} = 1.02$
- $\phi_{\pi} = 1.5$
Now introducing inflation target uncertainty

- Allow $\phi_x > 0$ and $\phi_{\pi'} > 0$
- At $t = 0$, a surprise announcement that the central bank is considering permanently raising the inflation target to $\pi^*$ with probability $\lambda$ each period
- If the inflation target is increased, it remains there forever

Formally, the Markov process switches from

$$
\begin{pmatrix}
1 & 0 \\
0 & 1 \\
\end{pmatrix}
$$

to

$$
\begin{pmatrix}
1 - \lambda & \lambda \\
0 & 1 \\
\end{pmatrix}
$$

Assume monetary policy in regime two is determinant

Only shock is the realization of the Markov process

- Each regime is in a steady state
- Let $x_i$ denote the outcome in regime $i$
Initial and Regime 2 Outcomes

- Prior to the announcement $x_t = \pi_t = 0$
- Regime 2 outcome:
  \[ x_2 = \pi_2 \frac{1 - \beta}{\kappa} \quad \text{and} \quad \pi_2 = \pi^* \frac{1 - \phi_\pi - \phi_\pi'}{1 - \phi_\pi - \phi_\pi' - \phi_x \frac{1 - \beta}{\kappa}} \]
- With price indexing $x_2 = 0$ and $\pi_2 = \pi^*$
Regime 1: Active Policy

Define: $C_1 = 1 + \frac{1 - \beta (1 - \lambda)}{\kappa \sigma - 1} + \frac{\beta \phi_x}{\kappa}$ and $C_2 = 1 + \frac{(1 - \beta)(1 - \beta (1 - \lambda))}{\kappa \sigma - 1}$

Monetary policy is active if $\phi_\pi + \phi_\pi' > 1$, then

1. $\pi_1$ is increasing in $\pi^*$ if $C_1 > \phi_\pi'$
   - $\phi_\pi'$ increases interest rates proportional to $(1 - \lambda) \pi_1 + \lambda \pi_2$
   - Large $\phi_\pi'$ can raise $i_1$ enough for current inflation to fall
   - Large $\phi_\pi$ can’t since if $\pi_1 < 0 \Rightarrow i_1 \downarrow$

2. $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_\pi'} < 0$

3. $\frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_\pi} < 0$ if $C_1 > \phi_\pi'$
Regime 1: Active Policy

Define: \( C_1 = 1 + \frac{1 - \beta(1 - \lambda)}{\kappa \sigma - 1} + \frac{\beta \phi_x}{\kappa} \) and \( C_2 = 1 + \frac{(1 - \beta)(1 - \beta(1 - \lambda))}{\kappa \sigma - 1} \)

Monetary policy is active if \( \phi_\pi + \phi_\pi' > 1 \), then

1. \( \pi_1 \) is increasing in \( \pi^* \) if \( C_1 > \phi_\pi' \)
   - \( \phi_\pi' \) increases interest rates proportional to \( (1 - \lambda)\pi_1 + \lambda \pi_2 \)
   - Large \( \phi_\pi' \) can raise \( i_1 \) enough for current inflation to fall
   - Large \( \phi_\pi \) can’t since if \( \pi_1 < 0 \Rightarrow i_1 \downarrow \)

2. \( \frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_\pi'} < 0 \)

3. \( \frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_\pi} < 0 \) if \( C_1 > \phi_\pi' \)

4. \( x_1 \) is decreasing in \( \pi^* \) if \( C_2 < \beta \phi_\pi + \phi_\pi' \)
   - Do interest rates rise enough to incentivize saving?

5. \( \frac{\partial^2 \pi_1}{\partial \pi^* \partial \phi_x} > 0 \) if \( C_2 < \beta \phi_\pi + \phi_\pi' + \frac{\beta(1 - \beta(1 - \lambda)^2)}{\kappa} \phi_x \)

6. If \( \phi_\pi, \phi_\pi', \phi_x \) are the same in both regimes, then \( \pi_1 < \pi_2 \leq \pi^* \) and \( x_1 < x_2 \)
Regime 1: Passive Policy

Define

$$\bar{\phi}_\pi = 1 - \lambda (1 + \frac{1 - \beta (1 - \lambda)}{\sigma^{-1} \kappa})$$

If monetary policy is passive ($\phi_\pi + \phi_\pi' \leq 1$) and $\phi_\pi' = \phi_x = 0$, then

1. If $\phi_\pi = 1$, then $\pi_1 = \pi_2$ and $x_1 = \frac{1 - \beta}{\kappa} \pi_2$
2. If $\phi_\pi \in (\bar{\phi}_\pi, 1)$, then $\pi_1 > \pi_2$ and $x_1 > \frac{1 - \beta}{\kappa} \pi_2$ and both approach infinity as $\phi_\pi$ is reduced to the lower bound
3. If $\phi_\pi < \bar{\phi}_\pi$, then stochastic equilibrium is indeterminate
Monetary policy response

- By changing $\pi_1^*$, any outcome along the regime 1 Phillips Curve

\[ \pi_1 = \frac{\pi^* \beta \lambda + \kappa x_1}{1 - \beta(1 - \lambda)} \]

is achievable without affecting the volatility of inflation or output in a stochastic model.

- Reducing the constant in the policy rule for the current regime (increasing $\pi_1^*$ if policy is active) will raise inflation and output in the current regime if the monetary response is strong enough:

\[ \phi_\pi + (1 - \lambda)\phi_\pi' + \phi_x \frac{1 - \beta(1 - \lambda)}{\kappa} > 1 - \lambda \left(1 + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1}\kappa}\right) > 1 \]
To achieve $\pi_1 = 0$, requires a recession of magnitude $\frac{\beta \lambda \pi_2}{\kappa}$ which can be accomplished by setting

$$\pi_1^* = \frac{\lambda \pi_2}{1 - \phi_\pi - \phi_{\pi'}} \left( \frac{1}{1 - \phi_\pi - \phi_{\pi'}}(1 - \phi_{\pi'} + \frac{\beta \phi_x}{\kappa} + \frac{1 - \beta(1 - \lambda)}{\sigma^{-1} \kappa}) \right)$$

The optimal commitment policy in regime one will set the inflation target in regime one such that

$$\chi_1 = \frac{-\pi_2 \lambda \beta}{\kappa + \kappa^{-1}(1 - \beta(1 - \lambda))^2 \theta_x} \in \left[-\frac{\beta \lambda \pi_2}{\kappa}, 0\right]$$

$$\pi_1 = \frac{\pi_2 \lambda \beta}{1 - \beta(1 - \lambda)} \frac{\kappa^{-1}(1 - \beta(1 - \lambda))^2 \theta_x}{\kappa + \kappa^{-1}(1 - \beta(1 - \lambda))^2 \theta_x} \in \left[0, \frac{\pi_2 \lambda \beta}{1 - \beta(1 - \lambda)}\right]$$
Outline

1. Introduction
2. Model
3. Theoretical Analysis
4. Stochastic Model
Stochastic Model

- Single regime optimal policy
  - For $\theta_x = .0408$ and $\theta_i = .25$, $i = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$

- Multiple regime optimal policy may differ
  - Let the Markov process be $\begin{bmatrix} 1 - \lambda & \lambda \\ 0 & 1 \end{bmatrix}$
  - Minimizes losses by choosing $\phi_{\pi,2}, \phi_{\pi',2}, \pi^*_x, \pi^*_2$ same as single regime optimal policy
  - $\phi_{\pi,1}, \phi_{\pi',1}, \phi_{\pi,1}$ are different
  - Need to differentiate optimal policy response to $\pi^*_1$ uncertainty from Markov structure
Inflation Target Uncertainty

- Three regimes with $\phi_\pi$, $\phi'_{\pi}$, and $\phi_x$ at single regime optimal values
- Initially in regime one, $\pi^*_1 = 0$
- Probability $p_2$ to transition to regime 2 which has inflation target of 1%
- Probability $p_3$ to transition to regime 3 which has inflation target of 0%
- Regimes 2 and 3 are absorbing regimes
- For optimal policy considerations, comparison is to $\phi_{\pi,1}$, $\phi'_{\pi,1}$, and $\phi_{x,1}$ that are optimal given a future regime shift to an absorbing regime

Next
  1. Holding monetary policy fixed, vary parameters
  2. Allow central bank to respond
Outcomes with Constant Monetary Policy: $\theta_x = .0408$ and $\theta_i = .25$

- Output drops substantially more than inflation rises

**Chart**

- Change in Losses
- $E_{\pi_1}$
- $E_{x_1}$
- $E_{i_1}$

**Graph**

Transition Probability ($p_2 = p_3$) vs. Expected Outcomes in Regime 1 for Different Transition Probabilities.
Outcomes with Constant Monetary Policy: $\theta_x = 1$ and $\theta_i = .25$

Expected Outcomes in Regime 1 for Different Transition Probabilities

- $E\pi_1$
- $E\chi_1$
- $Ei_1$

- Inflation rises substantially more than output drops
Current vs Expected Inflation

- Interest rates optimally respond only to $\pi_t$ (dashed) and $E\pi_{t+1}$ (solid)
- Vary $\theta_x$, set $\theta_i = .25$

Responding to $E\pi_{t+1}$ results in a larger recession and lower inflation
Central Bank Sets $\pi_1^*$ s.t. $E\pi_1 = 0$

- Requires reducing $\pi_1^*$ more than one for one with the transition probabilities
Stochastic Model

Optimal Monetary Policy Response: \( \theta_x = 0.0408 \) and \( \theta_i = 0.25 \)

Expected Outcomes in Regime 1

- Near zero inflation and large recession

Graph showing expected outcomes with transition probabilities and welfare improvements.
Welfare Benefits of Optimal and Constrained Optimal Policy

<table>
<thead>
<tr>
<th>Welfare Weights</th>
<th>% of Inflation Target Uncertainty Losses Eliminated</th>
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</tr>
<tr>
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<td>0.50</td>
</tr>
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</table>

$^1$ Base policy: All other parameters in regime one are at their single regime optimums rather than the regime one optimal policy in the three regime model without uncertainty.

- Changing the inflation target is sufficient to minimize losses
- Optimal policy is most effective for large $\theta_i$ and small $\theta_x$
- Effective of only changing other parameters is diminishing in $\theta_x$
- With indexing similar optimal policy benefits, but require marginal adjustments of $\phi_\pi, \phi_\pi', \phi_x$
A fully anticipated increase in $\pi^*$ results in high variance transition paths
Can introducing uncertainty in the period of the $\pi^*$ change be beneficial?
Compare the transition path for
- A certain $T$ period in advance change in the inflation target
- A regime shift to the higher $\pi^*$ with an expected duration of the current regime equal to $T-1$

Turn off shocks
Possible Transition Path to $\pi_{40}^* = 1$
Anticipated Future 1% Increase in the Inflation Target

\[ i_t = 1.5\pi_t - 0.5\pi_t^* \]

Regime switch framework results in:

- Lower expected losses
- Lower per period losses prior to the inflation target change
- Larger proportion of losses incurred after the inflation target change
- Expected values are similar
- Variance is lower (none)
**Conclusion**

- Effects of inflation target uncertainty depend on the current policy rule coefficients
  - \( \phi_{\pi'} \) large \( \Rightarrow \pi \downarrow \), \( \phi_{\pi'} + \phi_{\pi} < 1 \) \( \Rightarrow \pi > \pi^*_2 \)

- The optimal policy response to inflation target uncertainty can be achieved by changing the current inflation target
  - Approximately true even with indexing

- Introducing uncertainty in the date the inflation target changes reduces the variance of inflation and output on the transition path
Proposition 2

If

\[
\frac{(1 - \beta)^2}{\sigma^{-1} \kappa} + \sigma^{-1} \kappa \geq 2(\beta \phi_{\pi} - 1) + 2\beta(\phi_{\pi} - 1),
\]
then

\[
\lambda = 1 + \beta + \sigma^{-1} \kappa \pm \sqrt{(1 + \beta + \sigma^{-1} \kappa)^2 - 4\beta(1 + \sigma^{-1} \kappa \phi_{\pi})}
\]

\[
\frac{2(1 + \sigma^{-1} \kappa \phi_{\pi})}{2(1 + \sigma^{-1} \kappa \phi_{\pi})}
\]
is a real number, and for \( t \in [0, T] \)

\[
\pi_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1) - \lambda_1^{t+1}(1 - \lambda_2)}{\lambda_2 - \lambda_1}
\]

\[
\chi_{T-t} = \pi^* \frac{\lambda_2^{t+1}(1 - \lambda_1)(\lambda_2 - \beta) - \lambda_1^{t+1}(1 - \lambda_2))(\lambda_1 - \beta)}{\kappa(\lambda_2 - \lambda_1)}
\]

Otherwise \( \lambda \) is a complex number, and for \( t \in [0, T] \)

\[
\pi_{T-t} = \pi^* \frac{(\phi_{\pi} - 1)\sigma^{-1} \kappa}{1 + \sigma^{-1} \kappa \phi_{\pi}} \frac{r^t}{t} \sum_{j=0}^{\infty} r^j \frac{\sin(\omega(t + 1 + j))}{\sin(\omega)},
\]

where \( r = \sqrt{\frac{\beta}{1 + \sigma^{-1} \kappa \phi_{\pi}}} \), \( \omega = \cos^{-1} \left( \frac{1 + \beta + \sigma^{-1} \kappa}{2\sqrt{\beta(1 + \sigma^{-1} \kappa \phi_{\pi})}} \right) \), and \( \chi_{T-t} = \frac{\pi_{T-t} - \beta \pi_{T-t+1}}{\kappa} \).
Solving the full stochastic model it can be shown that the solutions for output and inflation are of the form

\[ \pi_{j,t} = g_S e_{j,1} \mu_t^S + g_I e_{j,2} \mu_t^I + g_D e_{j,3} \mu_t^D + g_1 e_{j,4} a_j \]

\[ x_{1,t} = f_{1,S} \mu_t^S + f_{2,I} \mu_t^I + f_{1,D} u_t^D + f_{1,1} a_1 + f_{1,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_1 \]

\[ x_{2,t} = f_{2,S} \mu_t^S + f_{2,I} \mu_t^I + f_{2,D} u_t^D + f_{2,1} a_1 + f_{2,2} a_2 - (1 - \beta) \kappa^{-1} \bar{\pi}_2 \]

Then

\[ \text{Var}(\pi_{j,t}) = g_S^2 e_{j,1}^2 \frac{\text{Var}(\epsilon^S_t)}{1 - \rho_S^2} + g_I^2 e_{j,2}^2 \frac{\text{Var}(\epsilon^I_t)}{1 - \rho_I^2} + g_D^2 e_{j,3}^2 \frac{\text{Var}(\epsilon^D_t)}{1 - \rho_D^2} \]

\[ \text{Var}(x_{j,t}) = f_{j,S}^2 \frac{\text{Var}(\epsilon^S_t)}{1 - \rho_S^2} + f_{j,I}^2 \frac{\text{Var}(\epsilon^I_t)}{1 - \rho_I^2} + f_{j,D}^2 \frac{\text{Var}(\epsilon^D_t)}{1 - \rho_D^2} \]

But the inflation targets only appear in the \( a_1 \) and \( a_2 \) terms.
## Single Regime Optimal Policy

### Table: Optimal Policy Rule Under a Single Regime

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<tr>
<th>$\theta_x$</th>
<th>$\theta_i$</th>
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<th>$\phi_i$</th>
<th>$\phi_x$</th>
<th>$E\pi^2$</th>
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<td>0.5000</td>
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<td>1.8255</td>
<td>1.1628</td>
<td>3.3829</td>
<td>8.6727</td>
<td>2.6147</td>
<td>22.8060</td>
<td>15.6816</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.2500</td>
<td>1.8256</td>
<td>1.1628</td>
<td>6.5161</td>
<td>9.7410</td>
<td>1.0113</td>
<td>22.9697</td>
<td>16.4947</td>
</tr>
<tr>
<td>2.0000</td>
<td>0.2500</td>
<td>1.8254</td>
<td>1.1627</td>
<td>12.7805</td>
<td>10.5714</td>
<td>0.3279</td>
<td>23.3732</td>
<td>17.0705</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.0500</td>
<td>9.9279</td>
<td>5.2140</td>
<td>1.5283</td>
<td>1.4561</td>
<td>31.6754</td>
<td>55.1211</td>
<td>5.5045</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.5000</td>
<td>0.8128</td>
<td>0.6564</td>
<td>0.3778</td>
<td>9.9605</td>
<td>11.7571</td>
<td>14.6360</td>
<td>17.7582</td>
</tr>
</tbody>
</table>
Monetary Policy Uncertainty

Table: Regime One and Two Outcomes with \( i_1 = 1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t \) and \( p_{11} = p_{22} = 0.9 \)

<table>
<thead>
<tr>
<th>( i_2 )</th>
<th>( E\pi^2_1 )</th>
<th>( E\pi^2_2 )</th>
<th>( EI^2_1 )</th>
<th>( E\pi^2_1 )</th>
<th>( E\pi^2_2 )</th>
<th>( EI^2_2 )</th>
<th>( EL_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9633</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
</tr>
<tr>
<td>2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t</td>
<td>5.8953</td>
<td>13.4296</td>
<td>25.1804</td>
<td>12.7385</td>
<td>5.1295</td>
<td>15.4225</td>
<td>30.1017</td>
</tr>
<tr>
<td>1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t</td>
<td>6.0968</td>
<td>13.3725</td>
<td>26.3523</td>
<td>13.2306</td>
<td>7.0804</td>
<td>11.6451</td>
<td>21.0922</td>
</tr>
<tr>
<td>1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t</td>
<td>5.9394</td>
<td>13.4170</td>
<td>25.4357</td>
<td>12.8459</td>
<td>5.5321</td>
<td>14.3956</td>
<td>27.9493</td>
</tr>
<tr>
<td>1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t</td>
<td>6.0396</td>
<td>13.3886</td>
<td>26.0181</td>
<td>13.0905</td>
<td>6.4977</td>
<td>12.4814</td>
<td>23.4373</td>
</tr>
<tr>
<td>1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t</td>
<td>6.0481</td>
<td>13.3330</td>
<td>25.9876</td>
<td>13.0902</td>
<td>6.6329</td>
<td>9.0575</td>
<td>24.0142</td>
</tr>
<tr>
<td>1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t</td>
<td>5.8944</td>
<td>13.5128</td>
<td>25.3164</td>
<td>12.7750</td>
<td>5.2176</td>
<td>21.8323</td>
<td>29.8090</td>
</tr>
</tbody>
</table>

Return
## Monetary Policy Uncertainty

**Table:** Expected Outcomes with $i_1 = 1.8255 \pi_t + 1.1628 E \pi_{t+1} + 0.5057 x_t$ and $p_{11} = p_{22} = .9$

<table>
<thead>
<tr>
<th>$i_2$</th>
<th>$E \pi^2$</th>
<th>$Ex^2$</th>
<th>$Ei^2$</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8255(\pi_t + 1.1628 E \pi_{t+1} + 0.5057 x_t)</td>
<td>5.9875</td>
<td>13.4033</td>
<td>25.7152</td>
<td>12.9633</td>
</tr>
<tr>
<td>2.3255(\pi_t + 1.1628 E \pi_{t+1} + 0.5057 x_t)</td>
<td>5.5124</td>
<td>14.4261</td>
<td>27.6411</td>
<td>13.0114</td>
</tr>
<tr>
<td>1.3255(\pi_t + 1.1628 E \pi_{t+1} + 0.5057 x_t)</td>
<td>6.5886</td>
<td>12.5088</td>
<td>23.7222</td>
<td>13.0296</td>
</tr>
<tr>
<td>1.8255(\pi_t + 1.6628 E \pi_{t+1} + 0.5057 x_t)</td>
<td>5.7357</td>
<td>13.9063</td>
<td>26.6925</td>
<td>12.9764</td>
</tr>
<tr>
<td>1.8255(\pi_t + 0.6628 E \pi_{t+1} + 0.5057 x_t)</td>
<td>6.2686</td>
<td>12.9350</td>
<td>24.7277</td>
<td>12.9784</td>
</tr>
<tr>
<td>1.8255(\pi_t + 1.1628 E \pi_{t+1} + 1.0057 x_t)</td>
<td>6.3410</td>
<td>11.1952</td>
<td>25.0009</td>
<td>13.0482</td>
</tr>
<tr>
<td>1.8255(\pi_t + 1.1628 E \pi_{t+1} + 0.0057 x_t)</td>
<td>5.5560</td>
<td>17.6725</td>
<td>27.5627</td>
<td>13.1679</td>
</tr>
</tbody>
</table>
Table: Optimal Regime 1 Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

<table>
<thead>
<tr>
<th>$i_1^*$</th>
<th>$i_2$</th>
<th>$E \pi^2$</th>
<th>$E x^2$</th>
<th>$E l^2$</th>
<th>Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.8255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>$1.8255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>5.9876</td>
<td>13.4029</td>
<td>25.7150</td>
<td>12.9633</td>
</tr>
<tr>
<td>$1.85927 \pi_t + 1.1630 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>$2.3255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>5.4778</td>
<td>14.4912</td>
<td>27.7677</td>
<td>13.0111</td>
</tr>
<tr>
<td>$1.8054 \pi_t + 1.1439 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>$1.3255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>6.6216</td>
<td>12.4503</td>
<td>23.5990</td>
<td>13.0294</td>
</tr>
<tr>
<td>$1.8454 \pi_t + 1.1579 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>$1.8255 \pi_t + 1.6628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>5.7177</td>
<td>13.9397</td>
<td>26.7590</td>
<td>12.9763</td>
</tr>
<tr>
<td>$1.8183 \pi_t + 1.1483 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>$1.8255 \pi_t + 0.6628 \bar{E}<em>t \pi</em>{t+1} + 0.5057 x_t$</td>
<td>6.2845</td>
<td>12.9061</td>
<td>24.6687</td>
<td>12.9784</td>
</tr>
<tr>
<td>$1.8298 \pi_t + 1.1542 \bar{E}<em>t \pi</em>{t+1} + 0.5336 x_t$</td>
<td>$1.8255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 1.0057 x_t$</td>
<td>6.3637</td>
<td>11.0348</td>
<td>24.9350</td>
<td>13.0478</td>
</tr>
<tr>
<td>$1.8322 \pi_t + 1.1495 \bar{E}<em>t \pi</em>{t+1} + 0.4680 x_t$</td>
<td>$1.8255 \pi_t + 1.1628 \bar{E}<em>t \pi</em>{t+1} + 0.0057 x_t$</td>
<td>5.5276</td>
<td>17.9038</td>
<td>27.6354</td>
<td>13.1672</td>
</tr>
</tbody>
</table>

There are alternative policy rules for regime one that result in the same losses in the two regime model.
Optimal Policy Response

Table: Expected Per Period Regime 1 Loss Minimizing Response to Exogenous Regime 2 with $p_{11} = p_{22} = .9$

<table>
<thead>
<tr>
<th>$i_1^*$</th>
<th>$i_2$</th>
<th>$E\pi_{1t}^2$</th>
<th>$E\pi_{1t+1}^2$</th>
<th>$Ei_{1t}^2$</th>
<th>$EL_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.4114\pi_t + 0.9439E\pi_{t+1} + 0.4702x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.0968</td>
<td>11.9718</td>
<td>20.9683</td>
<td>12.8275</td>
</tr>
<tr>
<td>$1.4160\pi_t + 0.9439E\pi_{t+1} + 0.4699x_t$</td>
<td>$2.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>6.9723</td>
<td>12.0217</td>
<td>20.5802</td>
<td>12.6080</td>
</tr>
<tr>
<td>$1.4062\pi_t + 0.9438E\pi_{t+1} + 0.4706x_t$</td>
<td>$1.3255\pi_t + 1.1628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.2444</td>
<td>11.9145</td>
<td>21.4314</td>
<td>13.0885</td>
</tr>
<tr>
<td>$1.4125\pi_t + 0.9432E\pi_{t+1} + 0.4700x_t$</td>
<td>$1.8255\pi_t + 1.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.0351</td>
<td>11.9943</td>
<td>20.7491</td>
<td>12.7119</td>
</tr>
<tr>
<td>$1.4102\pi_t + 0.9445E\pi_{t+1} + 0.4705x_t$</td>
<td>$1.8255\pi_t + 0.6628E\pi_{t+1} + 0.5057x_t$</td>
<td>7.1634</td>
<td>11.9480</td>
<td>21.2063</td>
<td>12.9526</td>
</tr>
<tr>
<td>$1.4082\pi_t + 0.9444E\pi_{t+1} + 0.4778x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 1.0057x_t$</td>
<td>7.1865</td>
<td>11.7591</td>
<td>21.1317</td>
<td>12.9494</td>
</tr>
<tr>
<td>$1.4164\pi_t + 0.9429E\pi_{t+1} + 0.4586x_t$</td>
<td>$1.8255\pi_t + 1.1628E\pi_{t+1} + 0.0057x_t$</td>
<td>6.9620</td>
<td>12.3057</td>
<td>20.7275</td>
<td>12.6461</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Welfare Weights</th>
<th>% of Inflation Target Uncertainty Losses Eliminated</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_x$</td>
<td>$\theta_i$</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.2500</td>
</tr>
<tr>
<td>1.0000</td>
<td>0.2500</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.2500</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.0500</td>
</tr>
<tr>
<td>0.0408</td>
<td>0.5000</td>
</tr>
<tr>
<td>Optimal Policy</td>
<td>$\phi_1, \phi_{1'}, \phi_x$</td>
</tr>
<tr>
<td>0.8202</td>
<td>0.8082</td>
</tr>
<tr>
<td>0.7733</td>
<td>0.7263</td>
</tr>
<tr>
<td>0.7699</td>
<td>0.3853</td>
</tr>
<tr>
<td>0.8478</td>
<td>0.1837</td>
</tr>
<tr>
<td>0.1287</td>
<td>0.1134</td>
</tr>
<tr>
<td>0.9962</td>
<td>0.8443</td>
</tr>
<tr>
<td>Optimizing over</td>
<td>$\pi_1^*$</td>
</tr>
<tr>
<td>0.8165</td>
<td>0.7645</td>
</tr>
<tr>
<td>0.7218</td>
<td>0.7218</td>
</tr>
<tr>
<td>0.7383</td>
<td>0.7218</td>
</tr>
<tr>
<td>0.1198</td>
<td>0.1198</td>
</tr>
<tr>
<td>Optimizing over</td>
<td>$\pi_1^*$ and Base Policy$^1$</td>
</tr>
<tr>
<td>0.6878</td>
<td>0.6367</td>
</tr>
<tr>
<td>0.6490</td>
<td>0.6490</td>
</tr>
<tr>
<td>0.6384</td>
<td>0.6490</td>
</tr>
<tr>
<td>0.0616</td>
<td>0.0616</td>
</tr>
<tr>
<td>0.9177</td>
<td>0.9177</td>
</tr>
</tbody>
</table>

$^1$ Base policy: All other parameters in regime one are at their single regime optimums rather than the regime one optimal policy in the three regime model without uncertainty.

- Optimal response requires a change in both $\pi_1^*$ and $\phi_1, \phi_{1'}, \phi_x$
- Only changing $\pi_1^*$ captures most of the benefit of the optimal response
Anticipated Future 1% Decrease in the Inflation Target

\[ i_t = 1.5 \pi_t - 0.5 \pi^* \]

Regime switch framework results in:

- Stabilizing inflation at initial target increases losses
- Similar or lower expected total and per period losses
- Expected values are similar
- Variance is lower (none)