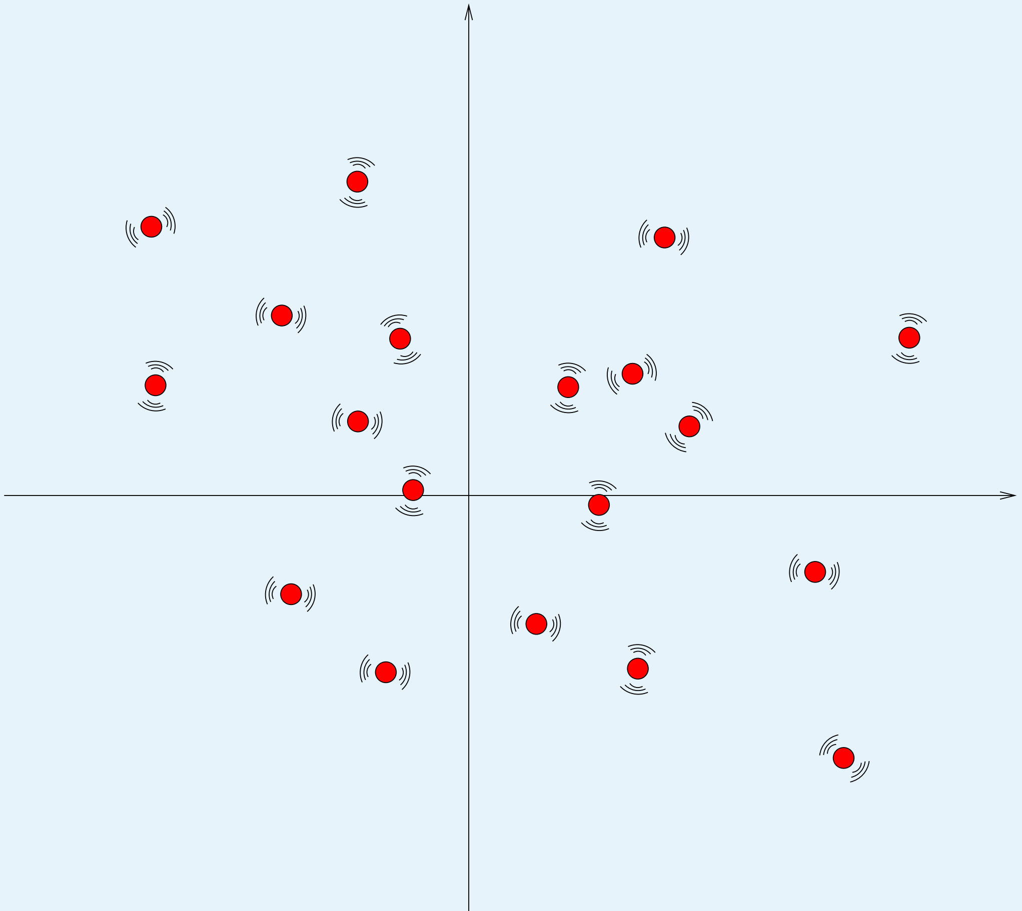
Stochastic Differential Equations



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Wiener Processes and Brownian Motions

Poisson Processes

The Tao of ODEs

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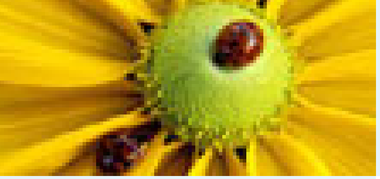

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Fix x_0

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Wiener Processes and Brownian Motions

You get told:

$$x \in \mathbb{R}^n,$$

Fix x_0 and a dynamical equation

$$\frac{dx}{dt} = f(x).$$

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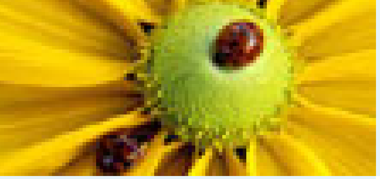

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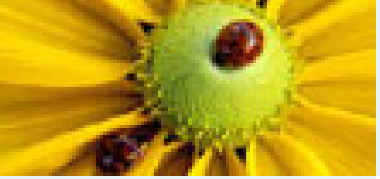

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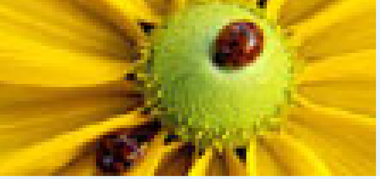

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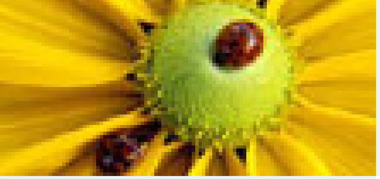
- The Chain rule.



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Underlying it all is calculus, with

- The Leibniz Rule.
- The Chain rule.
- The Fundamental Theorem of Calculus.

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Wiener Processes and Brownian Motions

It’s all very deterministic.

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Wiener Processes and Brownian Motions

It's all very deterministic. How do we put noise in?

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Wiener Processes and Brownian Motions

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Can we do something like:

$$dx = f(x)dt + Noise \tag{1}$$

And then follow the procedure from before?

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The GRAND PRINCIPLE: non-deterministic trajectories generated by statistical differential equations should be governed by a deterministic differential equation on the *probability density* of states.

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$$\frac{\partial \rho}{\partial t} = F(x, t, \text{noise coefficients}) \tag{2}$$

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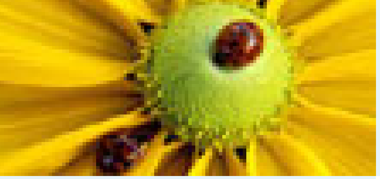

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Wiener Processes and Brownian Motions

We will approach the definition of noisy differential equations through two limiting procedures, one in space and one in time.

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We will approach the definition of noisy differential equations through two limiting procedures, one in space and one in time.

$N : \mathbb{N} \to \mathbb{N}$ given by

$$N_1(m) = N_1(m-1) + \begin{cases} 1 & \text{with probability } \lambda \\ 0 & \text{with probability } 1-\lambda \end{cases}$$

with $N(0) = 0$.

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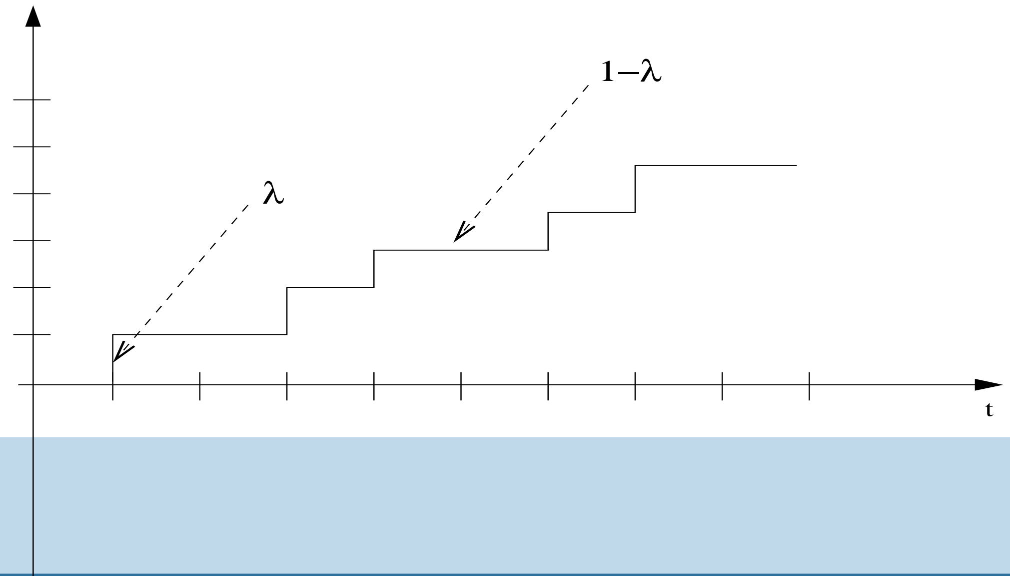

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It's a “Pascal process" (I think) because:

$$\rho(m,n) = \binom{m}{n} \lambda^n (1-\lambda)^{m-n}$$

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Wiener Processes and Brownian Motions

Define for each k

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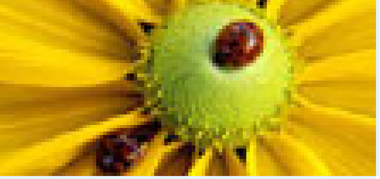
Definition 1 $N_\lambda : \mathbb{R} \to \mathbb{N}$

$$N_\lambda \stackrel{d}{=} \lim_{k\to\infty} N_k$$



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Wiener Processes and Brownian Motions

Define for each k

$$N_k\left(\frac{m}{k}\right) = N_k\left(\frac{m-1}{k}\right) + \begin{cases} 1 & \text{with probability } \lambda/k \\ 0 & \text{with probability } 1-\lambda/k \end{cases}$$

with $N(0) = 0$.

Definition 1 $N_\lambda : \mathbb{R} \to \mathbb{N}$

$$N_\lambda \stackrel{d}{=} \lim_{k\to\infty} N_k$$

N_λ has a well-defined *rate*.

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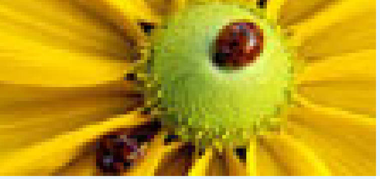
Definition 1 $N_\lambda : \mathbb{R} \to \mathbb{N}$

$$N_\lambda \stackrel{d}{=} \lim_{k\to\infty} N_k$$

N_λ has a well-defined *rate*. That is,

$$\lim_{\Delta\to 0}\left[\frac{Prob[N_\lambda(t+\Delta) = N_\lambda(t)+1]}{\Delta}\right]$$

is a constant function of time.



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Wiener Processes and Brownian Motions

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with $N(0) = 0$.

Definition 1 $N_\lambda : \mathbb{R} \to \mathbb{N}$

$$N_\lambda \stackrel{d}{=} \lim_{k\to\infty} N_k$$

In particular:

$$\lim_{\Delta\to 0}\left[\frac{Prob[N_\lambda(t+\Delta) = N_\lambda(t)+1]}{\Delta}\right] = \lambda$$

This is why N_λ is called a *poisson counter with rate* λ.

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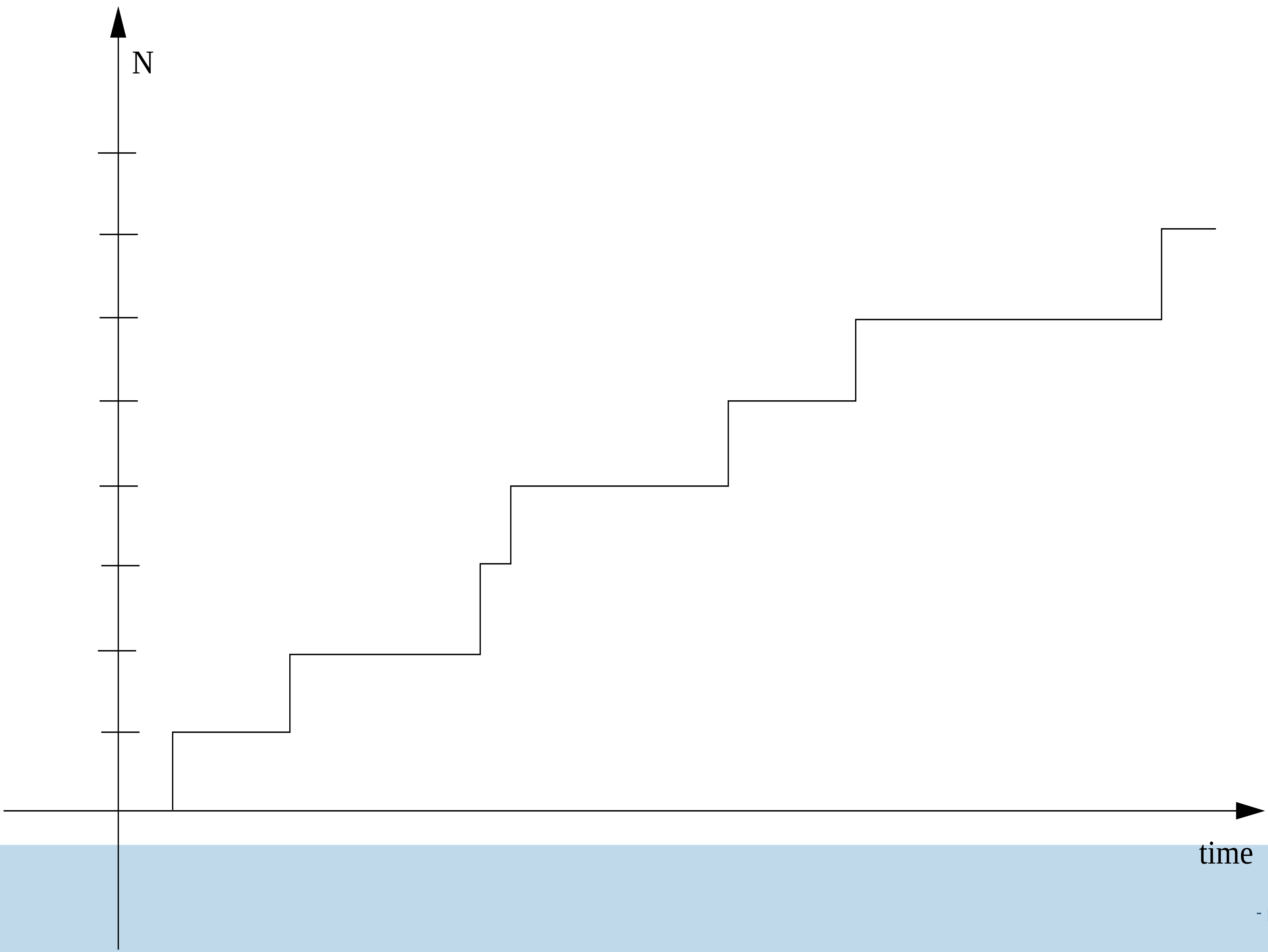
Wiener Processes and Brownian Motions



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Wiener Processes and Brownian Motions

We can derive the statistics of this process. Let

$$P_i(t) = Prob[N_\lambda(t) = i].$$

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Wiener Processes and Brownian Motions

We can derive the statistics of this process. Let

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What is $P_0(1)$?

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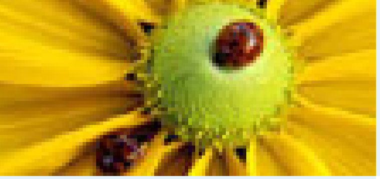
Wiener Processes and Brownian Motions

We can derive the statistics of this process. Let

$$P_i(t) = Prob[N_\lambda(t) = i].$$

It is

$$P_0(1) = \lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^n$$



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Wiener Processes and Brownian Motions

We can derive the statistics of this process. Let

$$P_i(t) = Prob[N_\lambda(t) = i].$$

But recall

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$$

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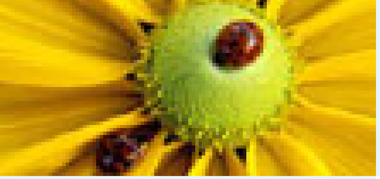
Wiener Processes and Brownian Motions

We can derive the statistics of this process. Let

$$P_i(t) = Prob[N_\lambda(t) = i].$$

So

$$P_0(1) = \lim_{n\to\infty}\left(1 - \frac{\lambda}{n}\right)^n = e^{-\lambda}$$



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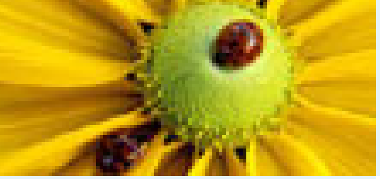
In general,

$$P_0(t) = e^{-\lambda t}$$

and

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$



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In general,

$$P_0(t) = e^{-\lambda t}$$

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$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$

Of course,

$$\sum_n P_n(t) = e^{-\lambda t} \sum_n \frac{(\lambda t)^n}{n!} = e^{-\lambda t} e^{\lambda t} = 1$$

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Wiener Processes and Brownian Motions

$$E[N_\lambda](t) = \sum_n n P_n(t) = \lambda e^{-\lambda t} \sum_n \frac{(\lambda t)^n}{n!} = \lambda t$$

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Wiener Processes and Brownian Motions

$$E[N_\lambda](t) = \sum_n n P_n(t) = \lambda e^{-\lambda t} \sum_n \frac{(\lambda t)^n}{n!} = \lambda t$$

Moreover, we can calculate higher moments as well:

$$E[N_\lambda^m](t) = e^{-\lambda t} \sum_n n^m (\lambda t)^n / n! \qquad (2)$$

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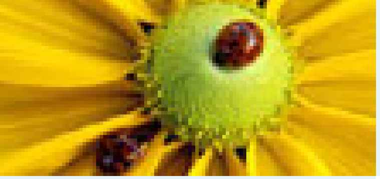
$$\begin{aligned} E[N_\lambda^m](t) &= e^{-\lambda t} \sum_n n^m (\lambda t)^n / n! \\ &= \lambda t e^{-\lambda t} \sum_j (j+1)^{m-1} (\lambda t)^j / j! \end{aligned} \tag{2}$$



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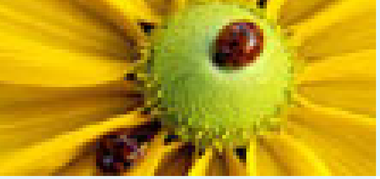

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This recursive calculation of moments is a hallmark of stochastic processes.

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Wiener Processes and Brownian Motions

The mathematically inclined among you will be wondering: Is it OK to do what I did?

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Wiener Processes and Brownian Motions

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Well, yes:

Theorem 1 *Suppose* $X_n : \mathbb{N}[1/n] \longrightarrow \mathbb{N}$ *is a sequence of time-invariant random variables such that*

$$X = \lim_{n \to \infty} X_n : \mathbb{R} \to \mathbb{N}$$

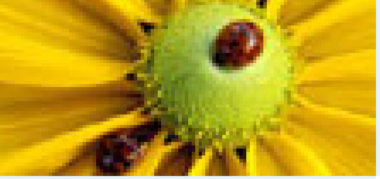
exists and satisfies

$$\lim_{\tau \to 0} \left[\frac{Prob[X(t+\tau) = X(t)+1]}{\tau} \right] = \lambda$$

and

$$\lim_{\tau \to 0} \left[\frac{Prob[X(t+\tau) = X(t)]}{\tau} \right] = 1 - \lambda.$$

Then $X \stackrel{d}{=} N_\lambda$.



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and

$$\lim_{\tau \longrightarrow 0} \left[\frac{Prob[X(t+\tau) = X(t)]}{\tau} \right] = 1 - \lambda.$$

Then $X \overset{d}{=} N_\lambda$.

Hence, N_λ is “the" poisson limit process with rate λ.

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Wiener Processes and Brownian Motions

Another way to think about N_λ is as that process which satisfies:

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Wiener Processes and Brownian Motions

Another way to think about N_λ is as that process which satisfies:

$$\frac{dP_i(t)}{dt} = -\lambda P_i(t) + \lambda P_{i-1}(t).$$

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Wiener Processes and Brownian Motions

Another way to think about N_λ is as that process which satisfies:

$$\frac{dP_i(t)}{dt} = -\lambda P_i(t) + \lambda P_{i-1}(t).$$

That is, the transition matrix is:

$$\dot{\mathbf{P}}(t) = \begin{bmatrix} -\lambda & 0 & 0 & 0 & 0 & \dots \\ \lambda & -\lambda & 0 & 0 & 0 & \dots \\ 0 & \lambda & -\lambda & 0 & 0 & \dots \\ \vdots & & & & & \vdots \end{bmatrix} \mathbf{P}(t)$$

where $\mathbf{P}(t) = (P_1(t), P_2(t), \dots)^T$.

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where $\mathbf{P}(t) = (P_1(t), P_2(t), \ldots)^T$.

This transition-matrix representation points to how poisson counters like N_λ can be really useful in representing probabilistic processes.

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Wiener Processes and Brownian Motions

Let us write the equation

$$dx = f(x,t)dt + g(x,t)dN_\lambda. \tag{3}$$

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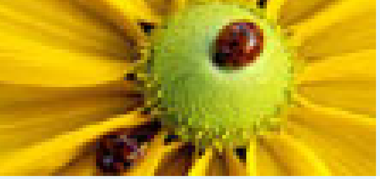
This does not define a single trajectory – instead, it defines a *set*, which possess a statistical distribution inherited from the distribution on the Poisson counters.



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So, given

$$dx = f(x,t)dt + \sum_i g_i(x,t)dN_i$$

what *are* the statistical properties of the solutions?

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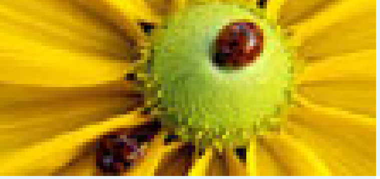

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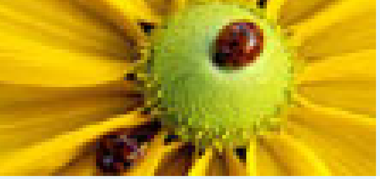

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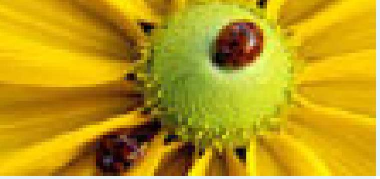
But this is a regular ODE!!



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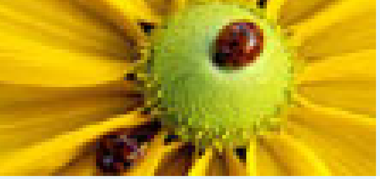
$$d(x^2) = 2x dx.$$



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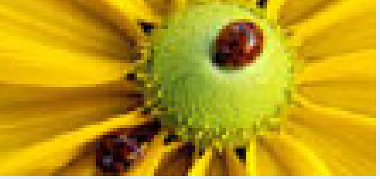

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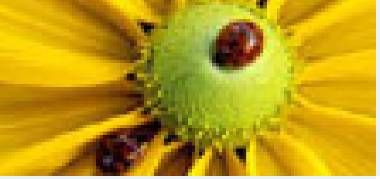

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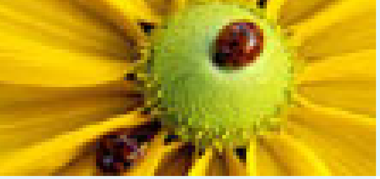
This is the “Ito Rule"; it is a combination of modified Leibniz and Chain-rule for stochastic calculus.



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Wiener Processes and Brownian Motions

Example 1 Suppose

$$dx(t) = -kx(t)dt + dN_1(t) - dN_2(t)$$

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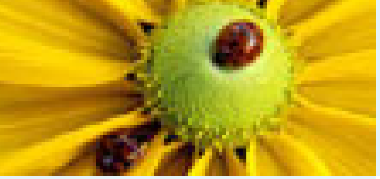

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$$dx^2 = -2kx^2(t)dt + [(x(t)+1)^2 - x^2(t)]dN_1 + [(x(t)-1)^2 - x^2(t)]dN_2 \tag{4}$$

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Again, recursive calculuation of moments.

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Wiener Processes and Brownian Motions

Suppose you’re given a finite-state transition scheme:

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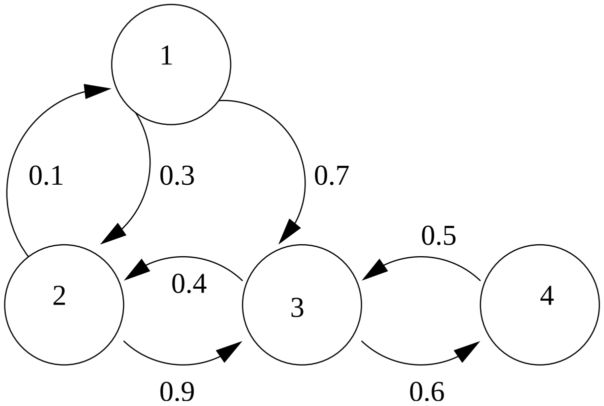

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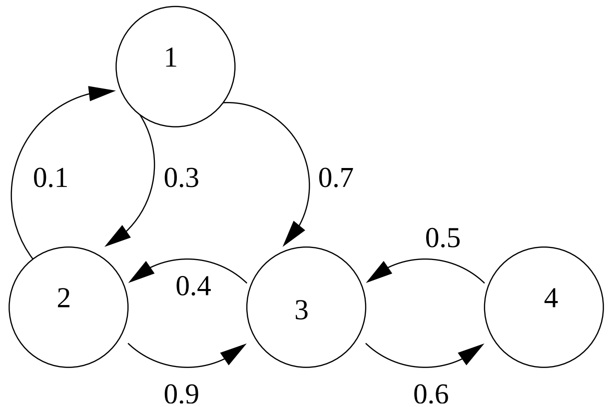

Definition 3 A state-transition equation

$$\dot{\mathbf{P}}(t) = A\mathbf{P}(t)$$

is called a *finite-state continuous time jump process* (FSCTJP), when A is a stochastic matrix, i.e. columns sum to 0 and (off-diagonal) entries are non-negative.

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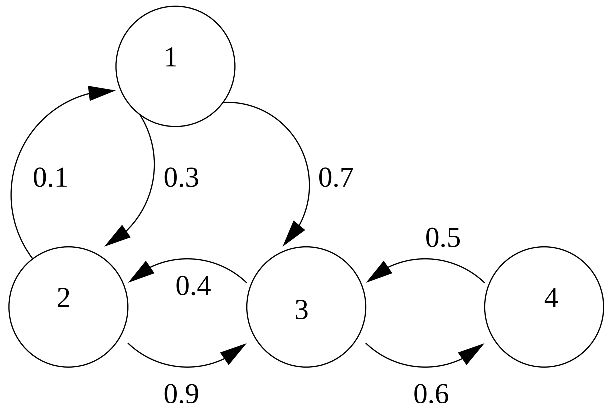

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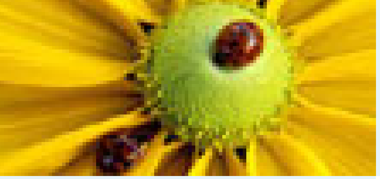
Such systems have obvious potential for being useful representations of scientific phenonmena.



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Wiener Processes and Brownian Motions

Theorem 2 ***[Basis Theorem]*** *Any FSCTJP is equivalent, in distribution, to*

$$dx = \sum_{i=1}^{m} f_i(x)dN_i$$

for some (nice) functions f_i and poisson counters N_i with rates $\lambda_i > 0$.

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$$\begin{bmatrix} \dot{p}_1(t) \\ \dot{p}_2(t) \\ \dot{p}_3(t) \end{bmatrix} = \begin{bmatrix} -3 & 0 & 8 \\ 3 & -2 & 0 \\ 0 & 2 & -8 \end{bmatrix} \begin{bmatrix} p_1(t) \\ p_2(t) \\ p_3(t) \end{bmatrix}$$

with $x(0) \in \{3, 7, 9\}$.

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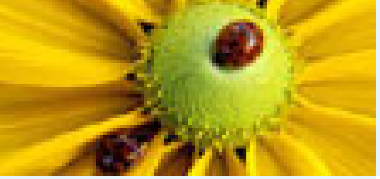

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Then

$$dx = \frac{(x-9)(x-7)}{6}\, dN_3 + \frac{(x-3)(x-9)}{4}\, dN_2 + \frac{(3-x)(x-7)}{2}\, dN_8$$

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Let ψ be any smooth function with $\psi = 0$ for large $|x|$. Then of course

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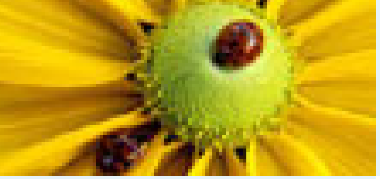

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So

$$\frac{d}{dt} E[\psi(x)](t) = E\left[\left\langle \frac{d\phi}{dx}, f(x) \right\rangle \right] + \sum_{i=1}^{n} \lambda_i E[\psi(x + g_i(x)) - \psi(x)].$$

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Wiener Processes and Brownian Motions

Let ψ be any smooth function with $\psi = 0$ for large $|x|$. Then of course

$$d\psi = \left\langle \frac{d\phi}{dx}, f(x) \right\rangle dt + \sum_{i=1}^{n} [\psi(x + g_i(x)) - \psi(x)] dN_i.$$

If $\rho(x,t)$ exists and is smooth then:

$$\frac{d}{dt} E[\psi(x)](t) = \int \left\langle \frac{d\phi}{dx}, f(x) \right\rangle \rho(x,t) dx + \sum_{i=1}^{n} \lambda_i \int (\psi(x+g_i(x)) - \psi(x)) \rho$$

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And

$$E[\psi(x)](t) = \int \psi(x) \rho(x,t) dx$$

just by definition.

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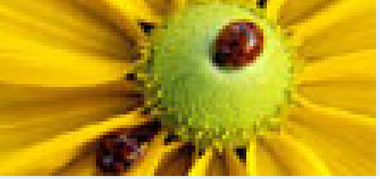
Now, differentiating w.r.t t and comparing gives:

$$\int \psi(x) \frac{d\rho(x,t)}{dt} = \int \left[\frac{d\psi}{dx} + \sum_i \lambda_i (\psi(x + g_i(x)) - \psi(x) \right] \rho(x,t) dx$$



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$$\int \left(-\psi(x) \frac{\partial (f\rho)}{\partial x} - \sum_i \lambda_i \psi \rho \right) dx + \sum_i \lambda_i \int \psi(x + g_i(x)) \rho(t,x) dx$$

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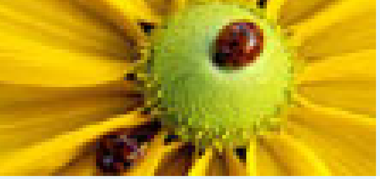
Wiener Processes and Brownian Motions

Let $h_i(x) = x + g_i(x)$. Assume that h_i is finite-to-one. Change variables $x \rightarrow z$ so that $dz = |det(I + dg)|dx$.



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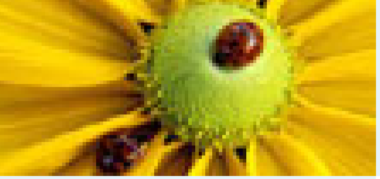

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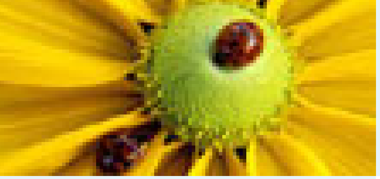
This can be collected as $\int \psi(x)[\text{stuff}] = 0$.



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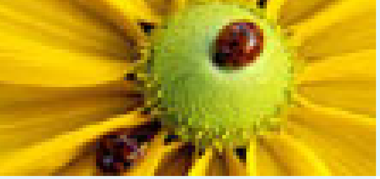
This can be collected as $\int \psi(x)[\text{stuff}] = 0$. But $\psi(x)$ was chosen arbitrarily!



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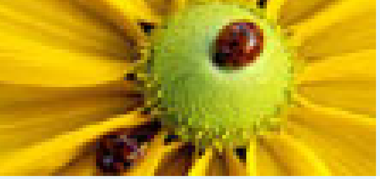
Wiener Processes and Brownian Motions

Hence, stuff $= 0$.



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$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(f\rho) + \sum_i \lambda_i [\rho(h_i^{-1}(x), t)|det(I + dg)|^{-1} - \rho(x, t)].$$

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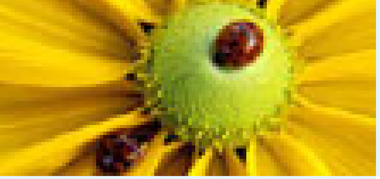
This is a deterministic PDE for the distribution. We've achieved the Grand Principle.



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It's very hard to solve. But: things can be done (including solve for steady states).

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Wiener Processes and Brownian Motions

Various things can now be done:

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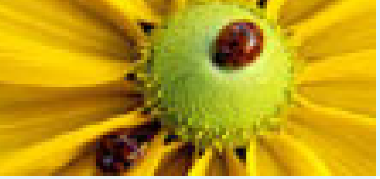
Wiener Processes and Brownian Motions

Various things can now be done:

- Construct non-constant-rate poisson counters, i.e. let $\lambda = \lambda(t)$. And then, generalize results.



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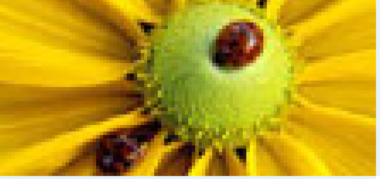

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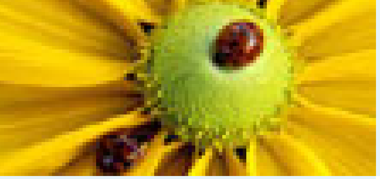
- **Take away discretization in space, going from jump processes to continuous processes.**



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- Construct non-deterministic-rate poisson counters, i.e. given $\lambda(t)$ by distribution. And generalize results.
- **Take away discretization in space, going from jump processes to continuous processes.**

The first two can be done, and are interesting, but the third is really where it's at.

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Wiener Processes and Brownian Motions

Spatial Continuization

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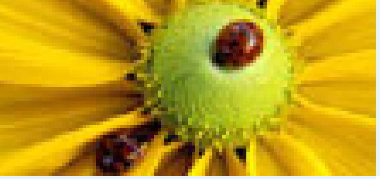
How should we continu-ize in space, as a limit of poisson counters?



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Let w_λ be given by $dw_\lambda = \frac{1}{\sqrt{\lambda}}(dN^+_{\lambda/2} - dN^-_{\lambda/2})$ where $N^+_{\lambda/2}, N^-_{\lambda/2}$ are independent poisson counters of rate $\lambda/2$, $w_\lambda(0) = 0$.

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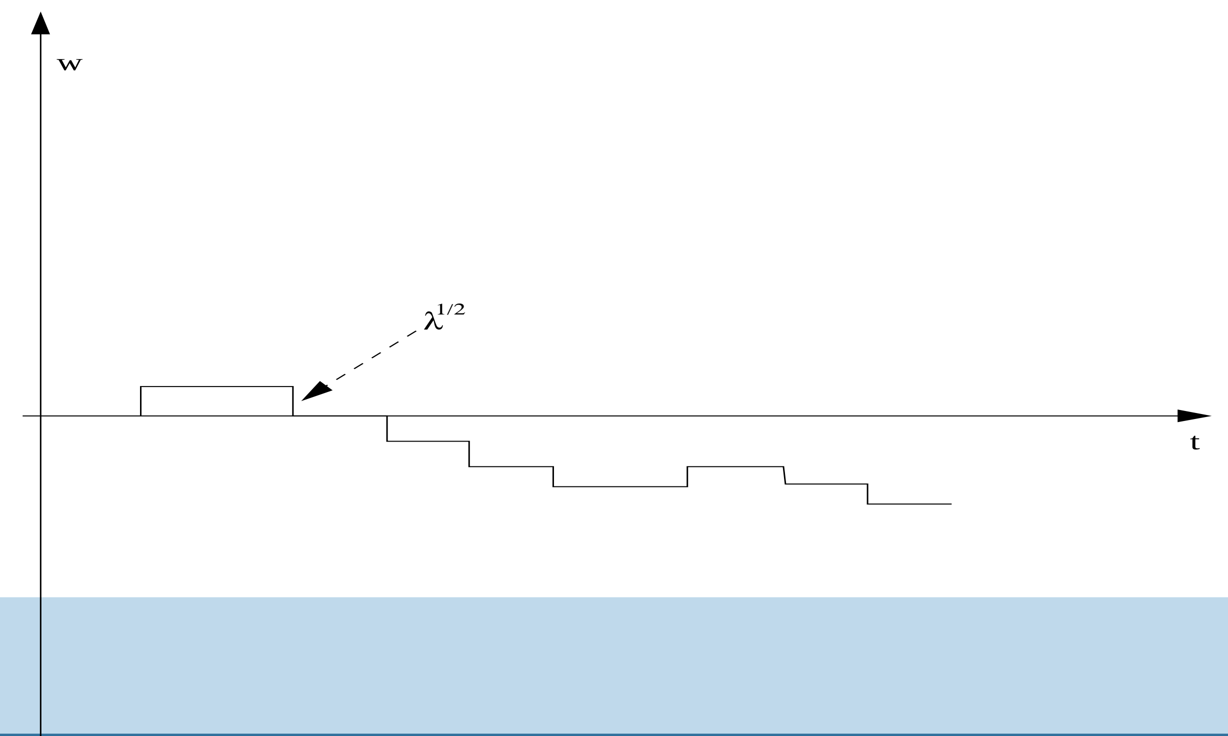
Spatial Continuization

How should we continu-ize in space, as a limit of poisson counters? Take a limit in which the rate $\lambda \to 0$.

Let w_λ be given by $dw_\lambda = \frac{1}{\sqrt{\lambda}}(dN^+_{\lambda/2} - dN^-_{\lambda/2})$ where $N^+_{\lambda/2}, N^-_{\lambda/2}$ are independent poisson counters of rate $\lambda/2$, $w_\lambda(0) = 0$.



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dw_λ is:

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dw_λ is:

- Zero-mean.

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What are its statistics?

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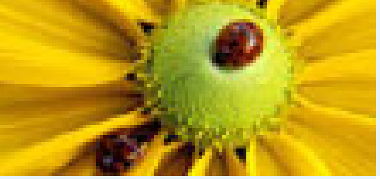
What are its statistics? Ito's rule says:

$$dw_\lambda^m = \left(\left(w_\lambda + \frac{1}{\sqrt{\lambda}}\right)^m - w_\lambda^m\right) dN^+ + \left(\left(w_\lambda - \frac{1}{\sqrt{\lambda}}\right)^m - w_\lambda^m\right) dN^-.$$



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Thus (using the binomial expansion)

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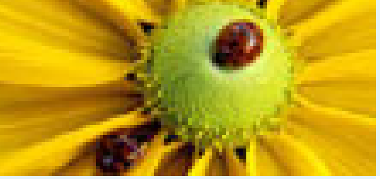
Thus (using the binomial expansion)

$$\frac{d}{dt}E[w_\lambda^m] = \begin{cases} 0 & \text{if } m \text{ is odd} \\ \sum_{i=1}^{m/2-1} \frac{1}{\lambda^{i-1}} \binom{m}{2i} E[w_\lambda^{m-2i}] & \text{if } m \text{ if even} \end{cases}.$$



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Again, notice the recursive calculation of moments.

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Definition 4

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dw defines a continuous process $w : \mathbb{R} \to \mathbb{R}$ with Gaussian statistics.
dw is called a *Brownian motion* (if the limit exists, which it does).

Brownian Motion

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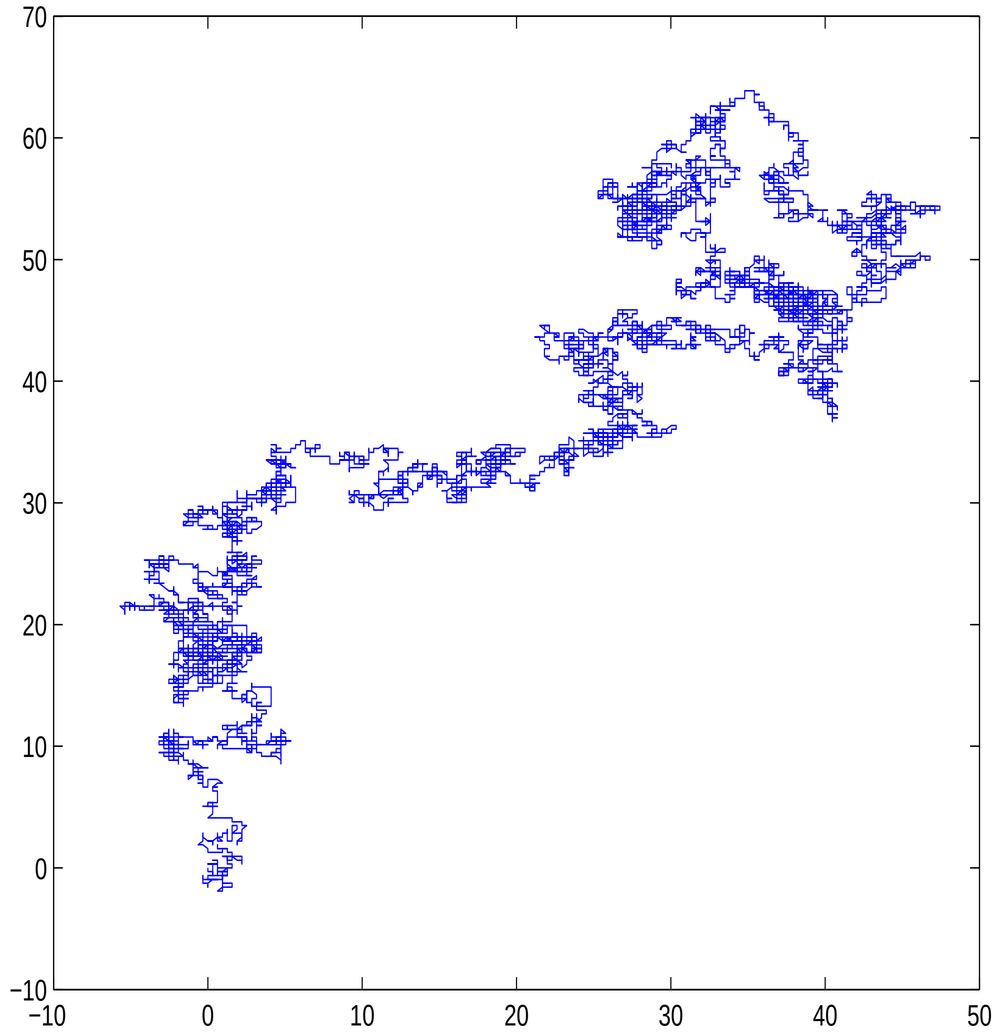

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Some properties of Brownian motion are:

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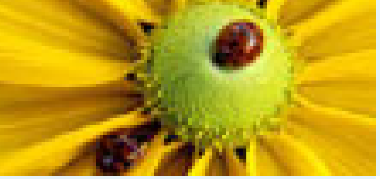

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$$\begin{bmatrix} dx \\ dy \end{bmatrix} = k \begin{bmatrix} dw_1 \\ dw_2 \end{bmatrix}$$

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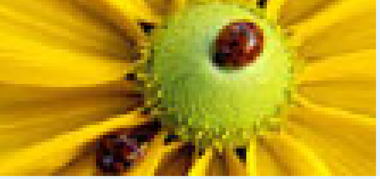

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- Einstein, Smoluchowski, etc, figured this out (three dimensions), and also how to find k as a phyical constant.
- dw is self-similar. That is, given $a > 0,\ \exists\ b$, such that

$$w(at) \stackrel{d}{=} bw(t) \ \ \forall t.$$

  In fact $b = a^{1/2}$.

Stochastic Differential Equations

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Definition 5 Let

$$dx = f(x)dt + \sum_i g_i(x)dw_i$$

be interpreted using the limit procedure from above; that is, its solutions are limits of Ito solutions to

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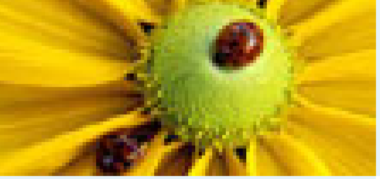

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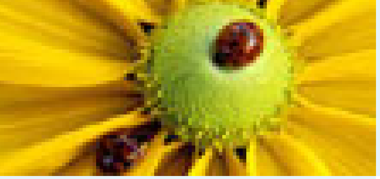
$$dx = vdt; \;\; dv = -\alpha(\gamma - v)dt + \sigma dw$$



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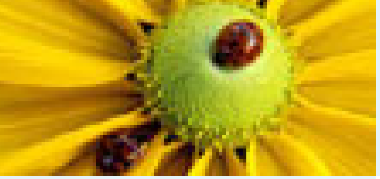

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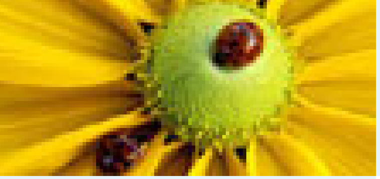

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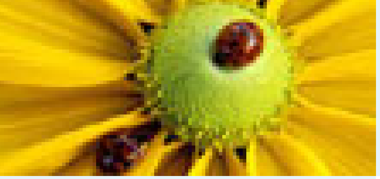
- α is the “pressure to revert to the mean"



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- γ is the long-term mean interest rate.
- α is the “pressure to revert to the mean"
- σ is the financial volatility.

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Let ϕ be a function $\mathbb{R}^n \to \mathbb{R}$, and suppose x is governed by a Wiener process SDE as above. $\phi(x)$ is itself a Winer process, but what is its SDE?

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Let ϕ be a function $\mathbb{R}^n \to \mathbb{R}$, and suppose x is governed by a Wiener process SDE as above. $\phi(x)$ is itself a Winer process, but what is its SDE?
In other words, what is the Ito calculus for Wiener processes?

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It's easy to derive as a limiting version of Poisson version.

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Let's start by introducing the process y_λ given by:

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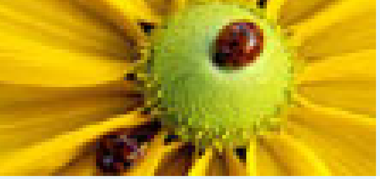

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Using Ito calculus, one finds $E[y_\lambda](t) = t + E[y_\lambda](0)$ and $E[y_\lambda^2](t) = t^2 + t/\lambda$. Hence

$$Var[y_\lambda](t) = E[(y_\lambda(t) - E[y_\lambda](t))^2] = t/\lambda.$$

But thus:

$$y(t) \stackrel{d}{=} \lim_{\lambda\to\infty} y_\lambda = t,$$

a simple deterministic process!

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So let ϕ be a twice-differential function $\mathbb{R}^n \to \mathbb{R}$, and suppose x is governed by a Wiener process SDE as above.

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Using the Ito Rule for ϕ on the process

$$dx_\lambda = f(x)dt + \sum_i \frac{g_i(x)}{\sqrt{\lambda}}(dN^+_{\lambda/2} - dN^-_{\lambda/2}),$$

we get

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$$\begin{aligned} d\phi = \left\langle \frac{d\phi}{dx}, f(x) \right\rangle dt + \sum_i \left[\phi(x + \frac{g_i(x)}{\sqrt{\lambda}}) - \phi(x) \right] dN^+_{\lambda,i} \\ + \sum_i \left[\phi(x - \frac{g_i(x)}{\sqrt{\lambda}}) - \phi(x) \right] dN^-_{\lambda,i} \end{aligned} \tag{6}$$

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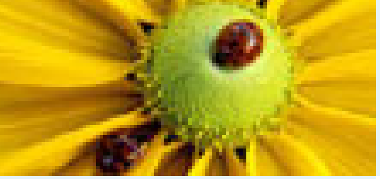
So let ϕ be a twice-differential function $\mathbb{R}^n \to \mathbb{R}$, and suppose x is governed by a Wiener process SDE as above.

$$\begin{aligned} d\phi = \left\langle \frac{d\phi}{dx}, f(x) \right\rangle dt + \sum_i \left[\phi(x + \frac{g_i(x)}{\sqrt{\lambda}}) - \phi(x) \right] dN^+_{\lambda,i} \\ + \sum_i \left[\phi(x - \frac{g_i(x)}{\sqrt{\lambda}}) - \phi(x) \right] dN^-_{\lambda,i} \end{aligned} \tag{6}$$

Now, let's expand ϕ in a Taylor series in x, which gives us



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d\phi = & \left\langle \frac{d\phi}{dx}, f(x) \right\rangle dt + \sum_i \left\langle \frac{d\phi}{dx}, g_i(x) \right\rangle dw_\lambda \\
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\end{aligned} \tag{9}
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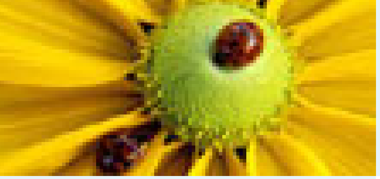

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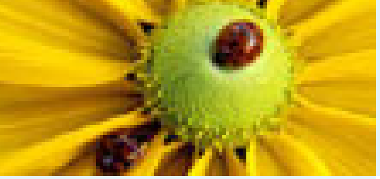
\end{aligned} \tag{9}
$$

So now let's take the limit $\lambda \to \infty$, replacing dw_λ with dw, dy_λ with dt, and higher terms vanish.



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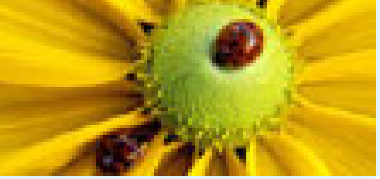
This is the Ito rule for SDEs.



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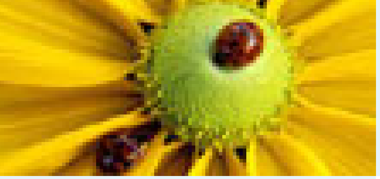
It can used to do all sorts of things, even just changing variables.



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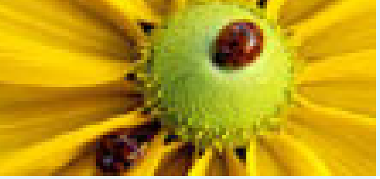
Now, we have to look into taking expectations.



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But recall that $E[dw] = 0$.

Calculating Moments

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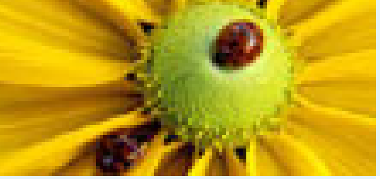
Hence



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Hence

$$dE[\phi] = E\left[\left\langle \frac{d\phi}{dx}, f(x) \right\rangle\right] dt + \frac{1}{2} \sum_i E\left[\left\langle g_i(x), g_i(x) \frac{\partial^2 \phi}{\partial x^2} \right\rangle\right] dt,$$

a deterministic ODE, just like before.

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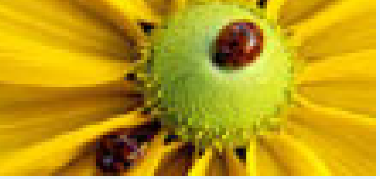
Example 4 Suppose x is given by

$$dx = -xdt + xdw.$$

What is the second moment of this process?



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$$dE[x^2](t) = E[2x(-xdt + xdw)]dt + E[x^2]dt = E[-x^2dt + 2x^2dw] = -E[x$$

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$$E[x^2](t) = e^{-t}E[x^2](0).$$

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The Langevin equation – simplest stochastic version of Newton's equations:

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$$\frac{d\vec{r}}{dt} = \vec{v}; \quad \frac{d\vec{v}}{dt} = -\zeta\vec{v} + C dw$$

where ζ is the hydrodynamic friction and C is a constant to be determined.

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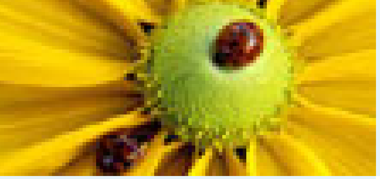
$$\zeta = 6\pi\eta a/m$$

where η is viscosity, a is particle radius, and m mass.



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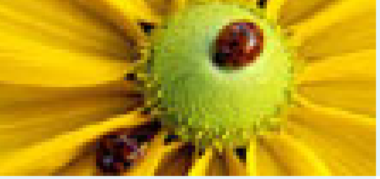
$$E[v^2](t) = v_0^2 e^{-2\zeta t} + \frac{C}{2\zeta}(1 - e^{-2\zeta t}).$$



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But stat. mech. tells us that in equilibrium

$$E[v^2] = \frac{3kT}{m}.$$

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Now, we can also use Ito rule to find $Var[r](t)$ – the “mean square displacement":

$$E[(r(t)-E[r](t))^2] = \frac{E[v^2](0)}{\zeta}(1-e^{-\zeta t})^2 + \frac{3kT}{m\zeta^2}(2\zeta t - 3 + 4e^{-\zeta t} - e^{-2\zeta t})$$

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At equilibrium this becomes

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At equilibrium this becomes

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But this is Einstein’s result:

$$D = \frac{kT}{m\zeta} = \frac{kT}{6\pi\eta a}.$$

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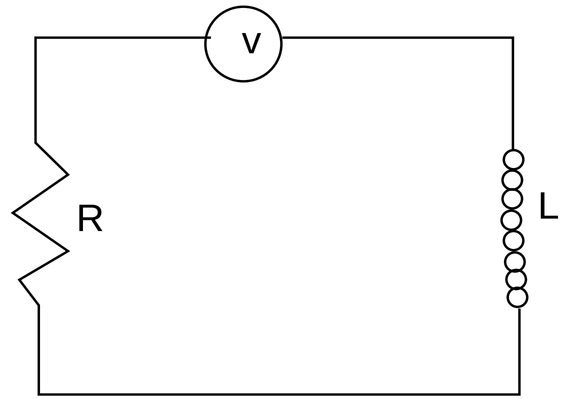
Consider the resistor-inductor:



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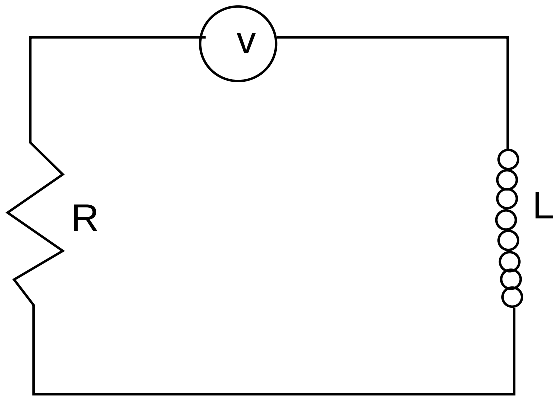

What is the expected energy at steady state?

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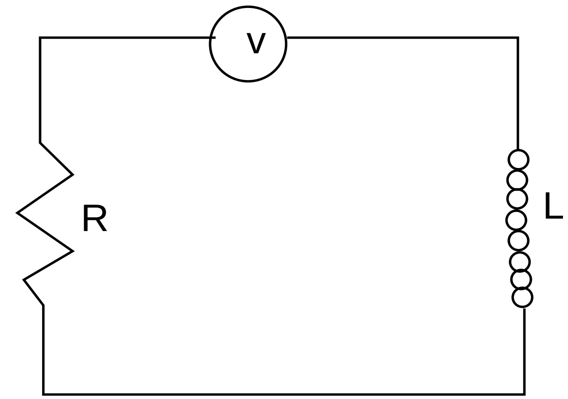

What is the expected energy at steady state?

Well, energy in a circuit is $\frac{1}{2}Li^2$

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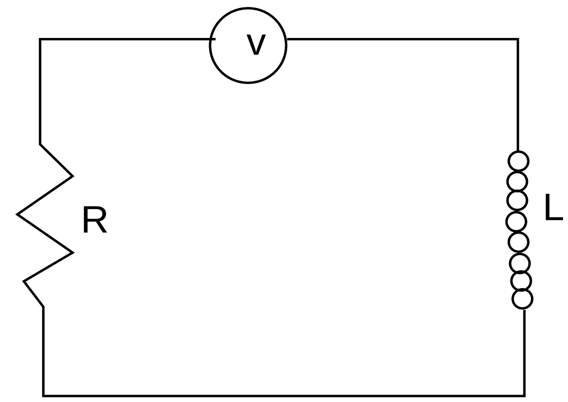

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The Nyquist-Johnson model of current flow is

$$Ldi = -Ridt + \sqrt{2kRT}dw$$

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Let’s apply the Ito rule.

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Let's apply the Ito rule.

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$$di^2(t) = -\frac{2R}{L} i^2 dt + 2\frac{\sqrt{2kRT}}{L} i dw + \frac{2kRT}{L^2} dt.$$

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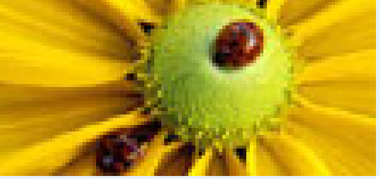
$$E[i^2] = \frac{kT}{L}.$$

Thus



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Consider the system:

$$dx = (S - GG^T)xdt + \sqrt{\epsilon}Gdw$$

where $S = -S^T$ and

$$rank(G|SG|\ldots|S^{n-1}G) = n.$$

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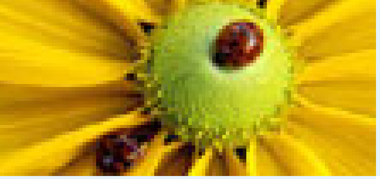

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The condition on rank means that each mode is correctly couple; S being antisymmetric means the system isn't losing energy; and dw is standard n-dim brownian motion. ϵ is the strength of the coupling.

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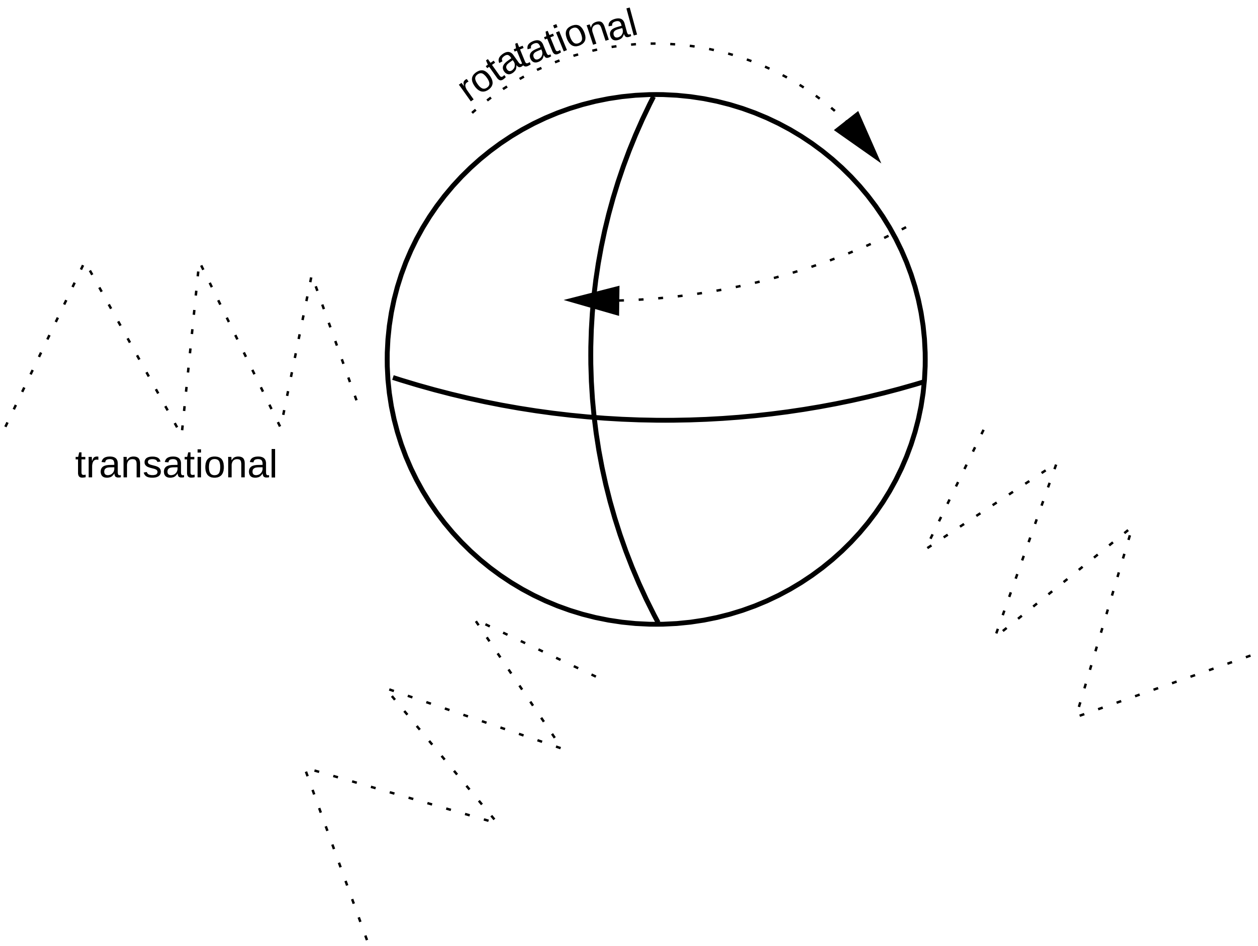

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The system again:

$$dx = (S - \epsilon GG^T)xdt + \sqrt{\epsilon}Gdw$$

where $S = -S^T$ and

$$rank(G|SG|\dots|S^{n-1}G) = n.$$

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Theorem 3 ***[Equipartition Thm.]*** *At thermal equilibrium, every mode possesses the same amount of energy.*

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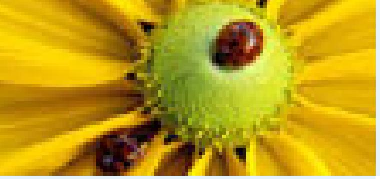
This is a simple result of stochastic calculus.



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Theorem 3 ***[Equipartition Thm.]*** *At thermal equilibrium, every mode possesses the same amount of energy.*

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Let's write the Ito equation for $E[xx^T]$.

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For the system

$$dx = Axdt + Bdw$$

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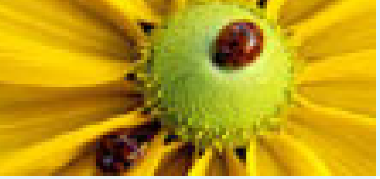
For the system

$$dx = Axdt + Bdw$$

the Ito equation for $\phi(x) = xx^T$ is



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For the system

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$$d(xx^T) = [Axx^T + xx^T A^T]dt + \text{stuff}dw + BB^T dt.$$

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$$dE[xx^T] = AE[xx^T]dt + E[xx^T]A^T dt + BB^T dt.$$

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With $A = S - GG^T$ and $B = \sqrt{\epsilon}G$:

$$\frac{dE[\Sigma]}{dt} = (S - \epsilon GG^T)E[\Sigma] + E[\Sigma](S - \epsilon GG^T)^T + \epsilon GG^T.$$

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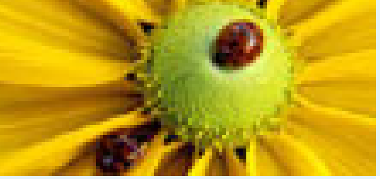

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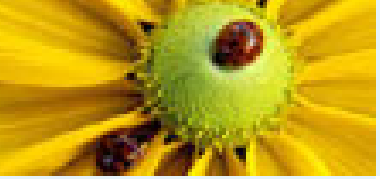

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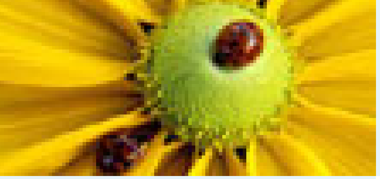

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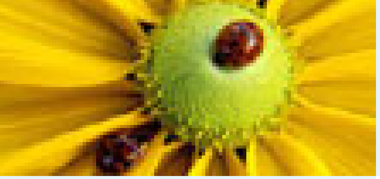
Aside from uniqueness, this is it!



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Aside from uniqueness, this is it!

We’ve shown that all modes get precisely $I/2$ fraction of total enegy;

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Recall the GRAND PRINCIPLE: non-deterministic trajectories generated by statistical differential equations should be governed by a deterministic PDE on the probability density of states.

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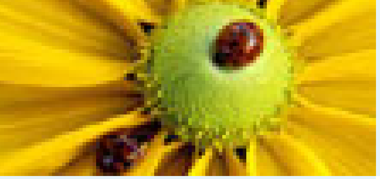
Now, let's let $\rho(x, t)$ be the PDF of x at time t.



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ASSUME: twice differentiable.

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Recall the GRAND PRINCIPLE: non-deterministic trajectories generated by statistical differential equations should be governed by a deterministic PDE on the probability density of states.

$$dx = f(x)dt + \sum_i g_i(x)dw_i$$

Now, let's let $\rho(x, t)$ be the PDF of x at time t.

ASSUME: twice differentiable. ERGODICITY UNDERLIES THIS ASSUMPTION.

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Do the test-function trick: ϕ smooth and with $\phi(x) = 0$ for large $|x|$.

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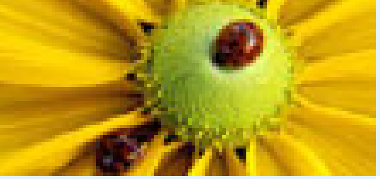

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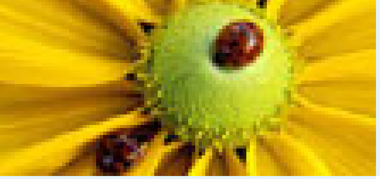

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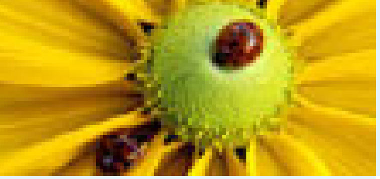

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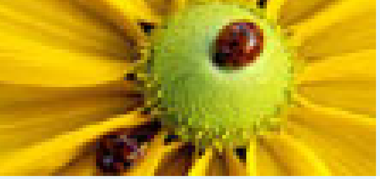

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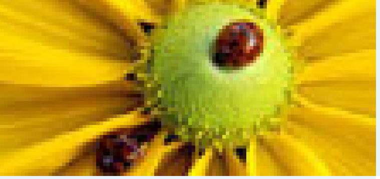

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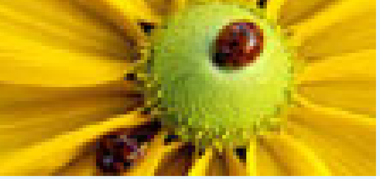

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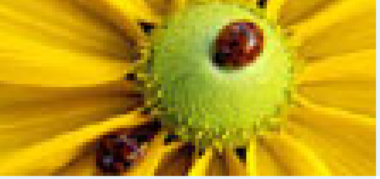

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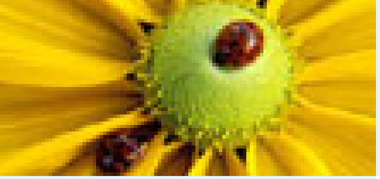

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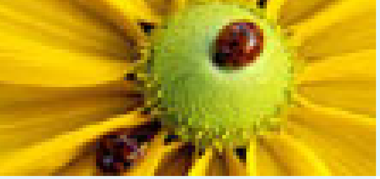

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Example 5 Suppose $dx = dw$.

Then

$$\frac{\partial \rho}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}\rho(x,t).$$

This is the diffusion equation!

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Evidently,

$$\rho(x, t) = \frac{1}{\sqrt{2\pi t}} \int e^{-(x-z)^2/2t} \rho(z, 0) dz$$

where $\rho(x, 0)$ is the initial distribution – if it is twice differentiable.

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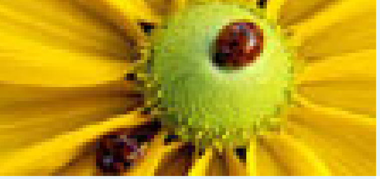
Example 6 Suppose $dx = -xdt + dw$, i.e there's a drift term.



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In this case,

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{\partial (x\rho)}{\partial x} + \frac{1}{2}\frac{\partial^2 \rho}{\partial x^2}.$$

Diffusion

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where $s(t) = \frac{1}{2}(1-e^{-2t})$.

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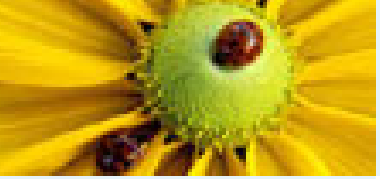

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But this is solvable (using trig fns) to get

$$\rho(x,t) = \sum_n p_n(t) cos(nx)$$

where $\dot{p}_n = (1 - n^2 - 1/n)p_n$.

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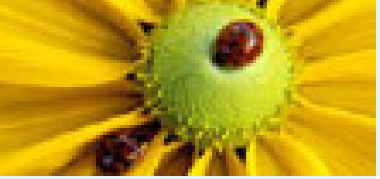

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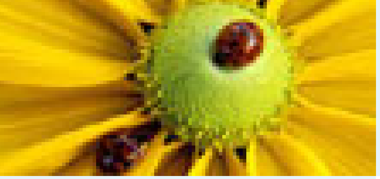

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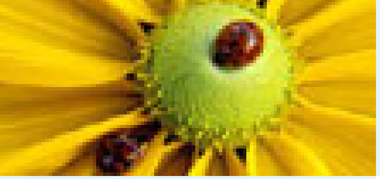

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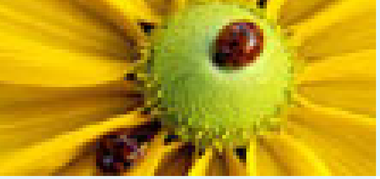

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