A Theory Approach to Local-to-Global Algorithms in Spatial Multi-Agent Systems

CS266, Fall 2007

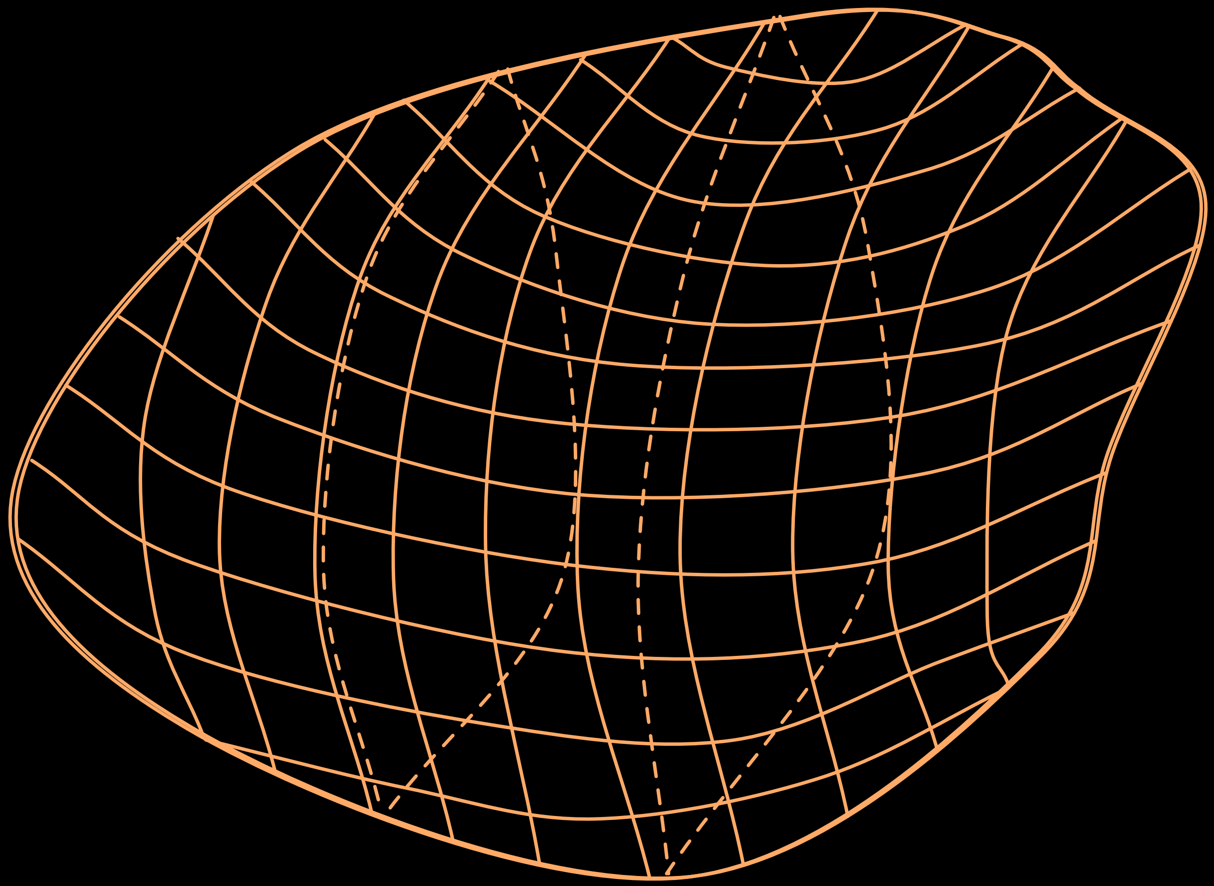
Dan Yamins

Session 1: 12.04.2007

Spatial Multi-agent Systems

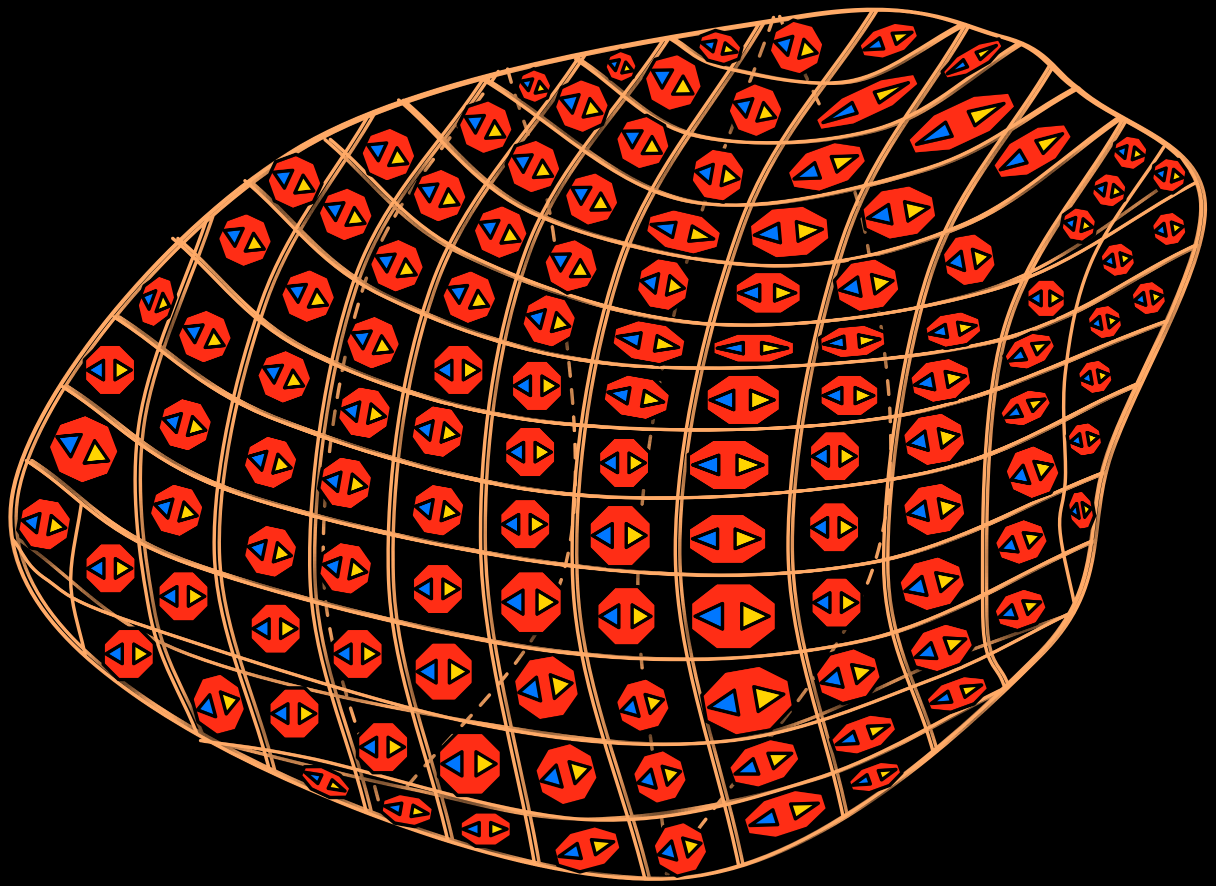
Spatial Multi-agent Systems

a space



Spatial Multi-agent Systems

a space with agents embedded in the space



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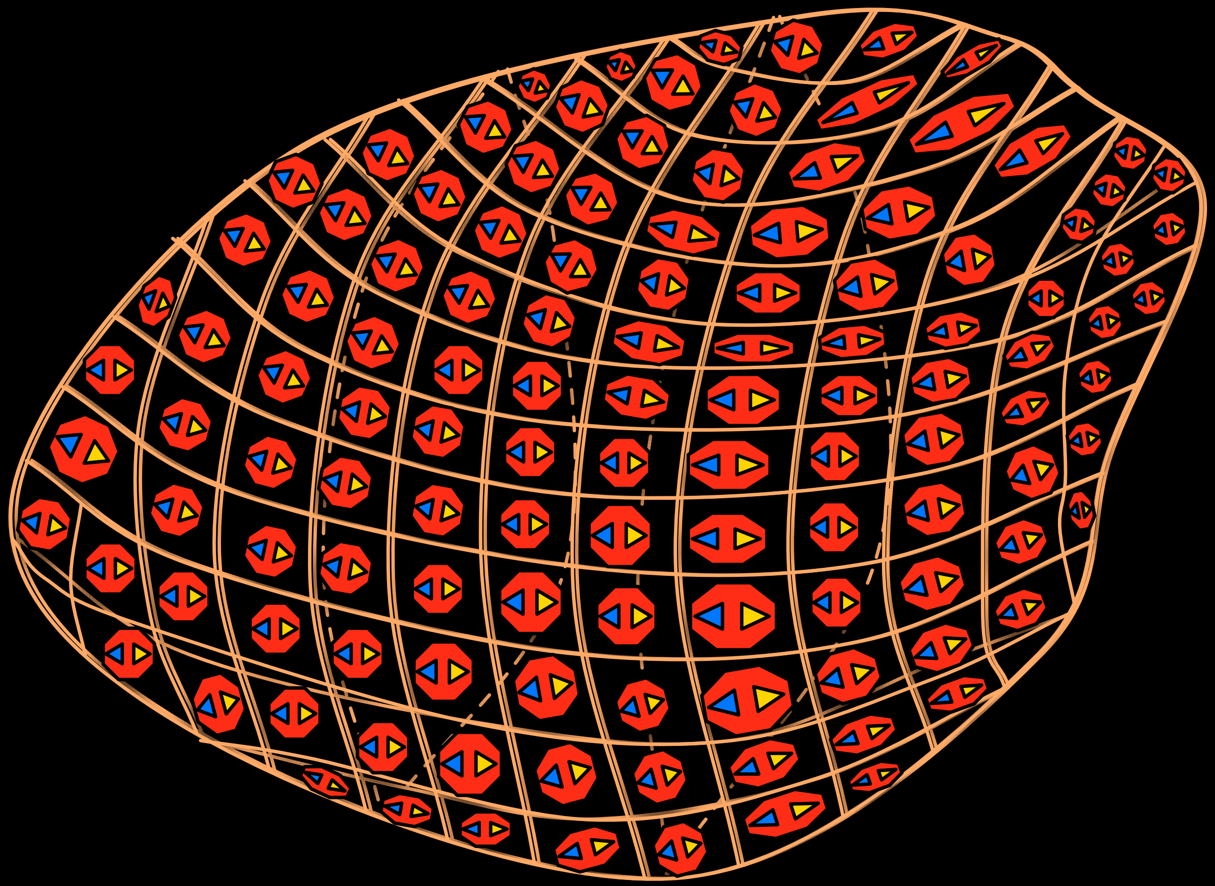

local information and processing

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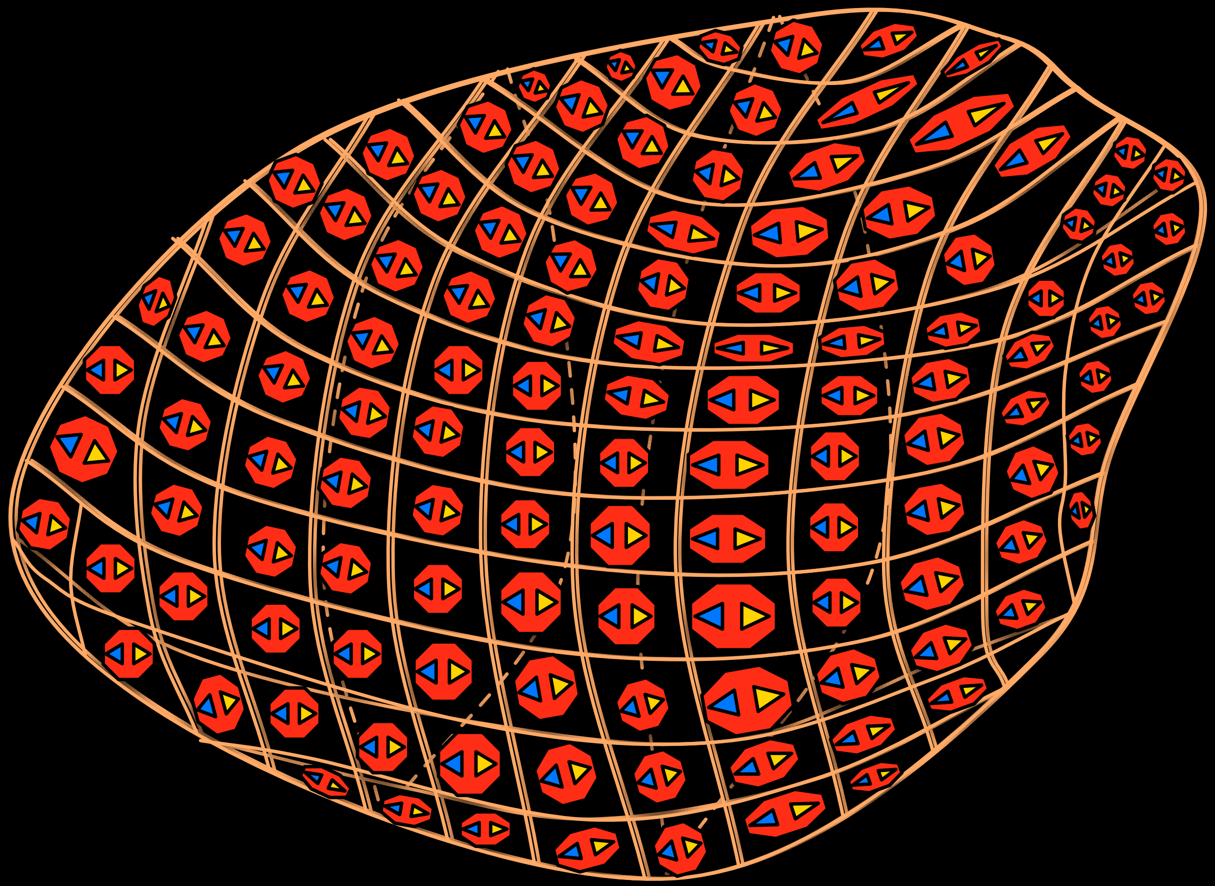

local information and processing

globally defined tasks

biology:

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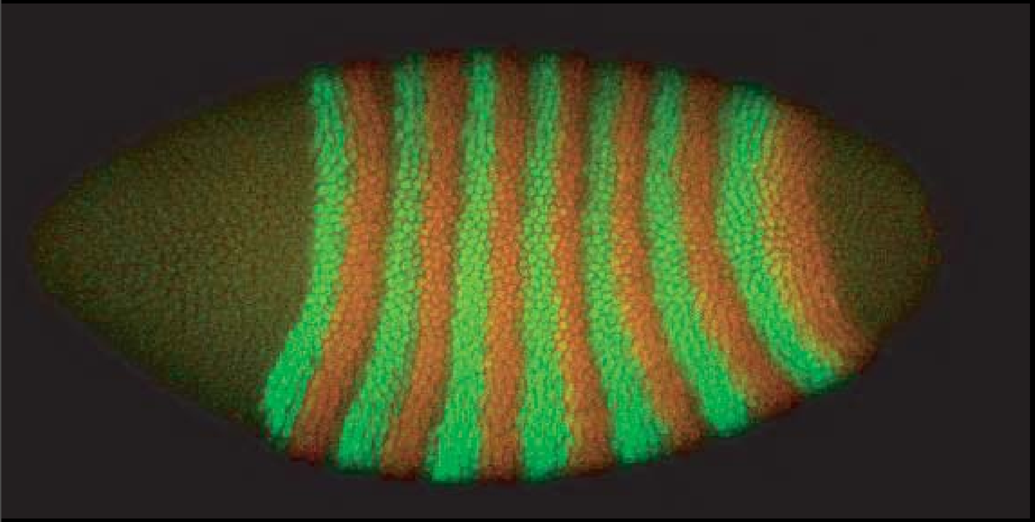

drosophila embryo

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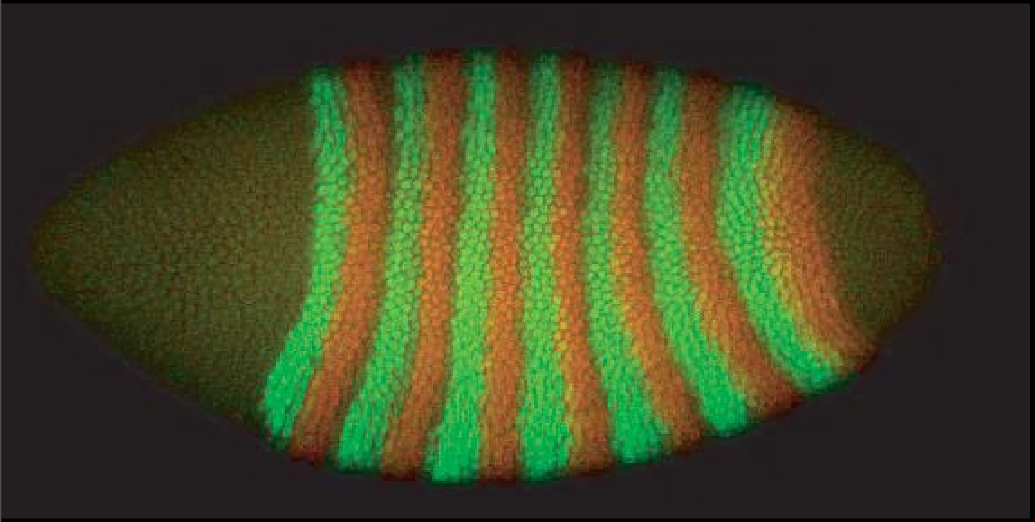

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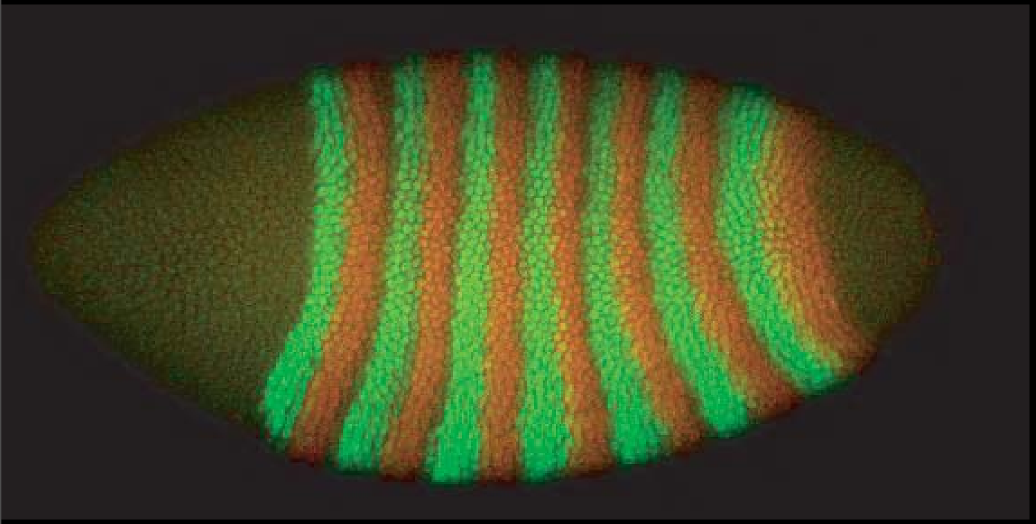

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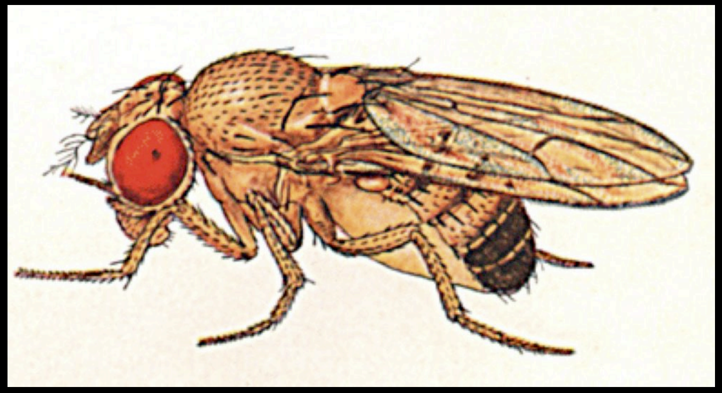

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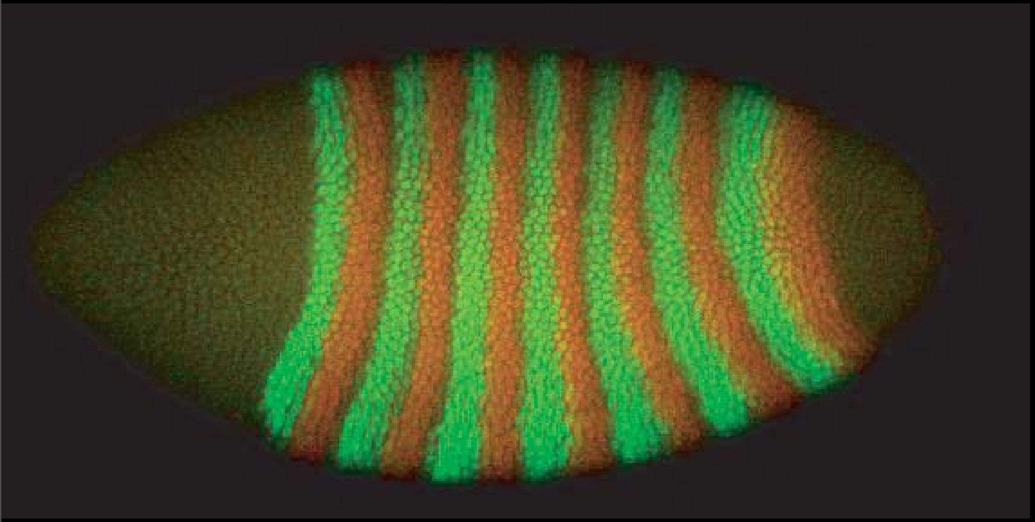

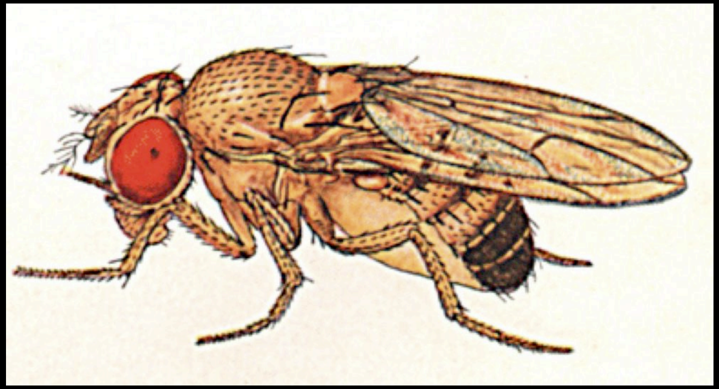

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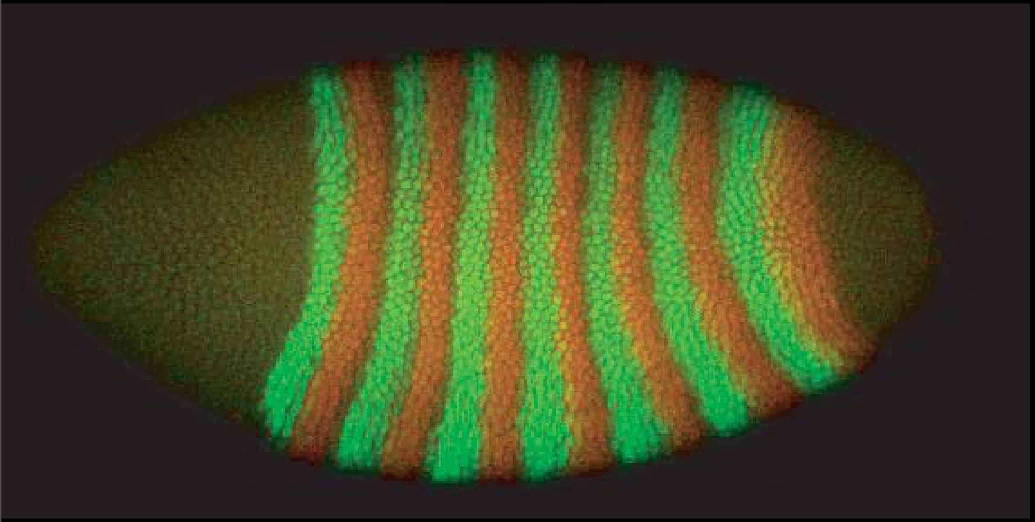

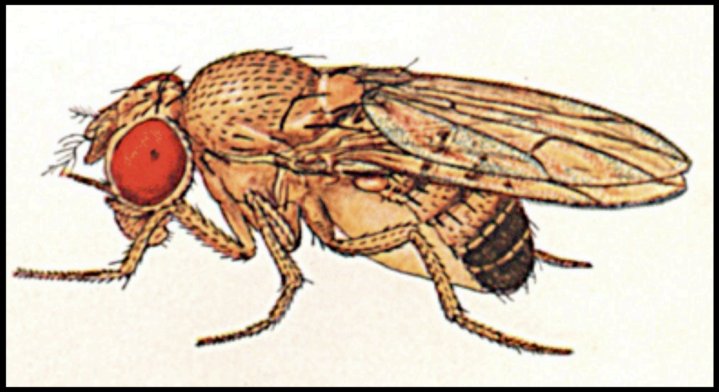

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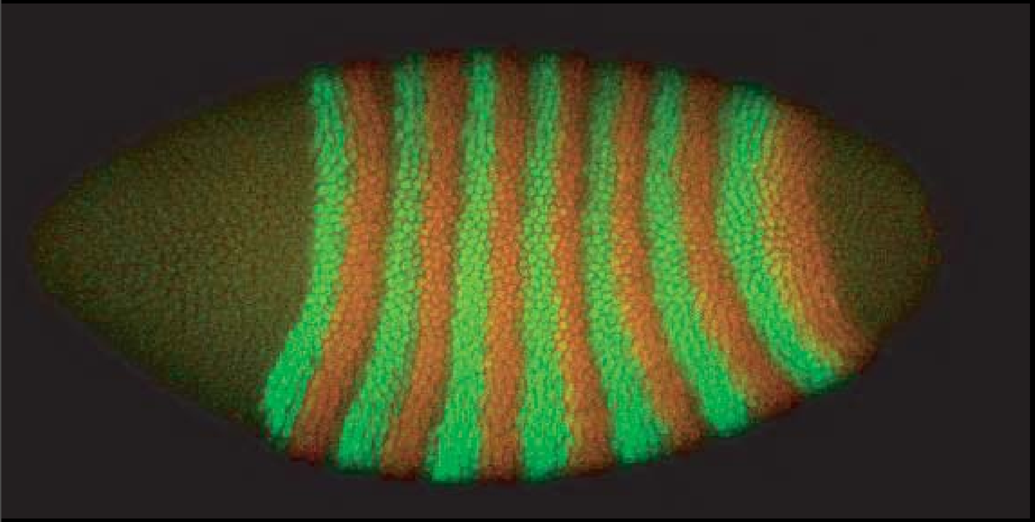

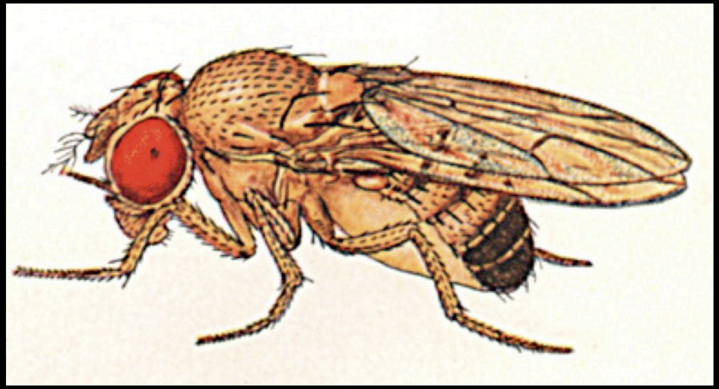

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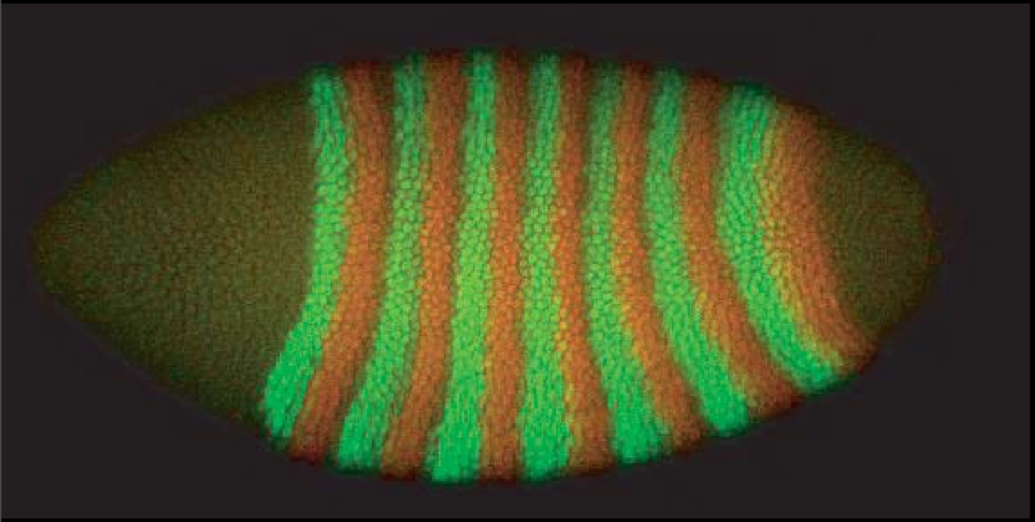

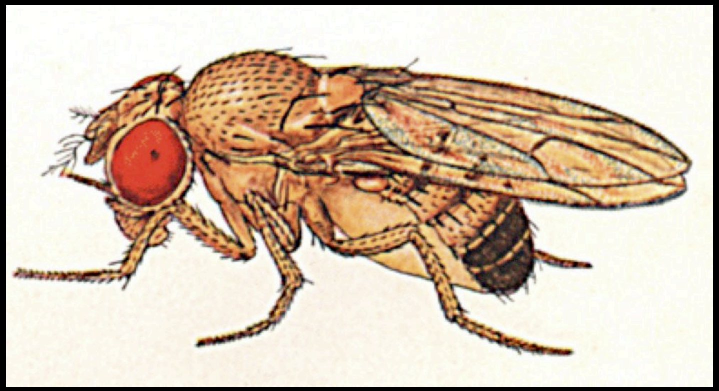

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white ibis flock

engineering:

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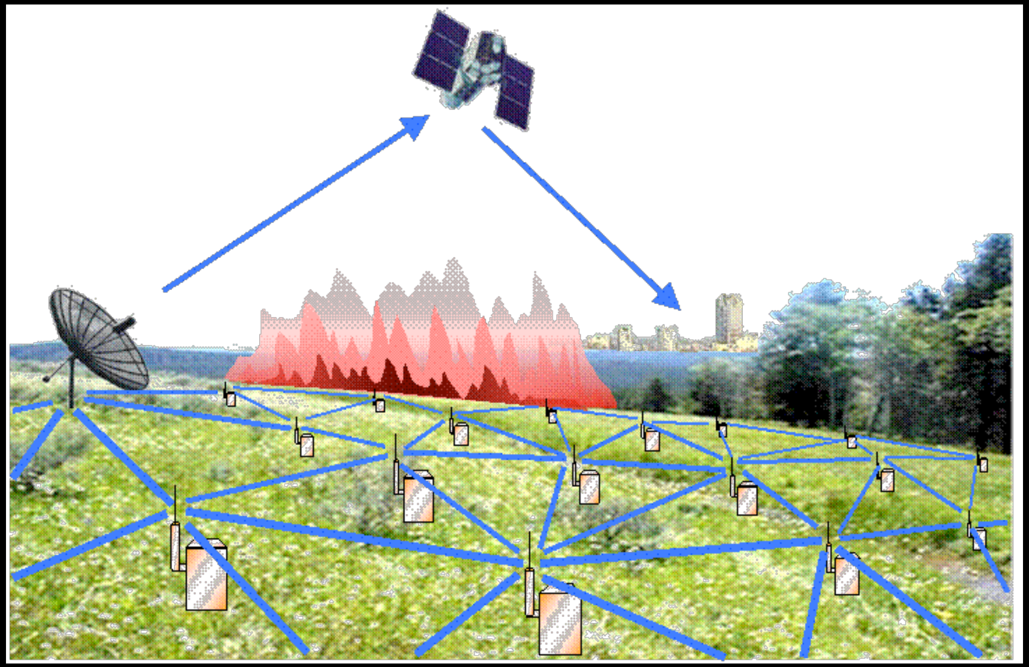

Sensor Networks

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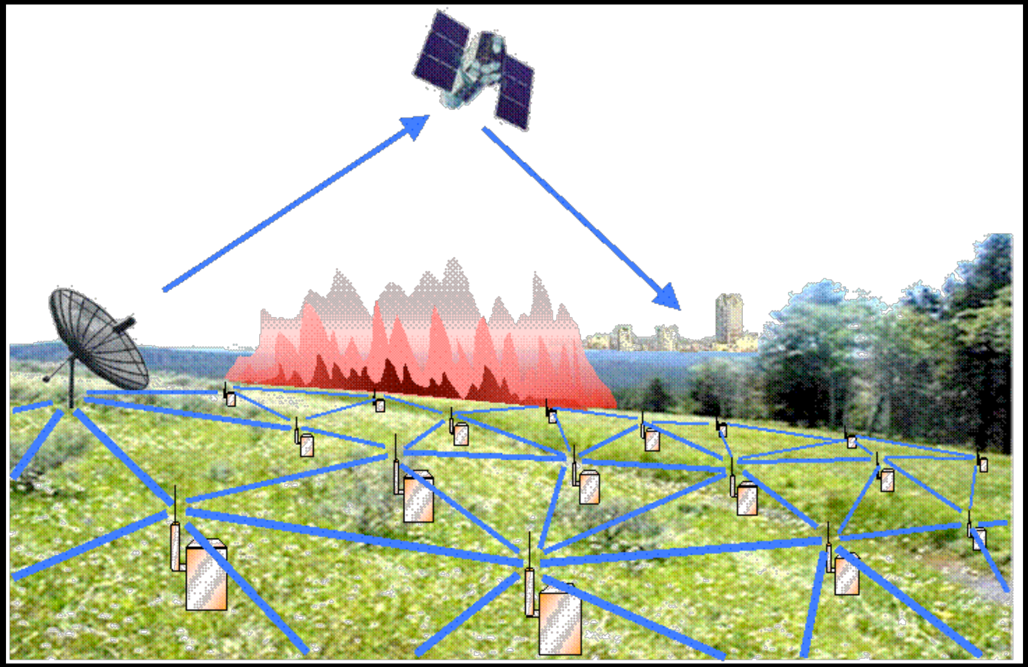

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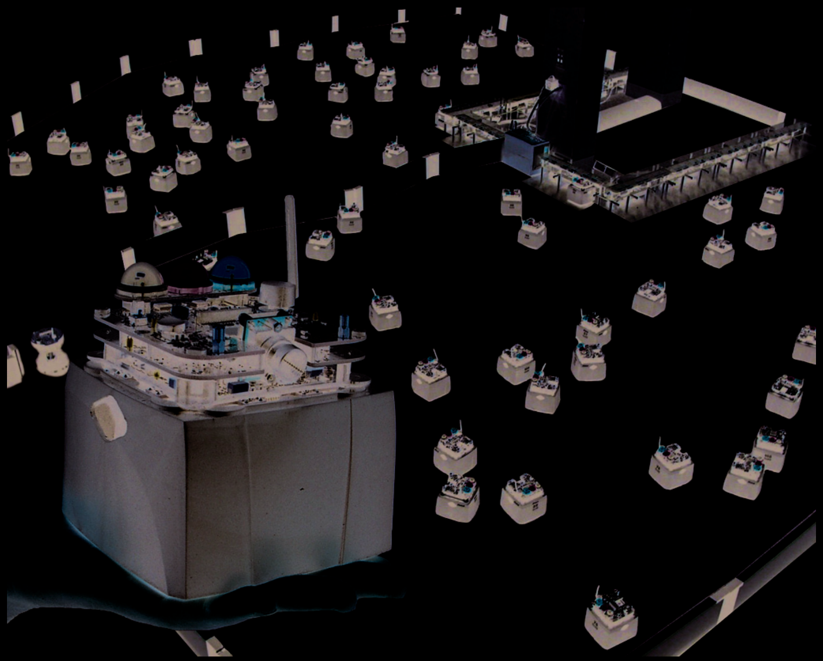

McLurkin iRobot Swarm

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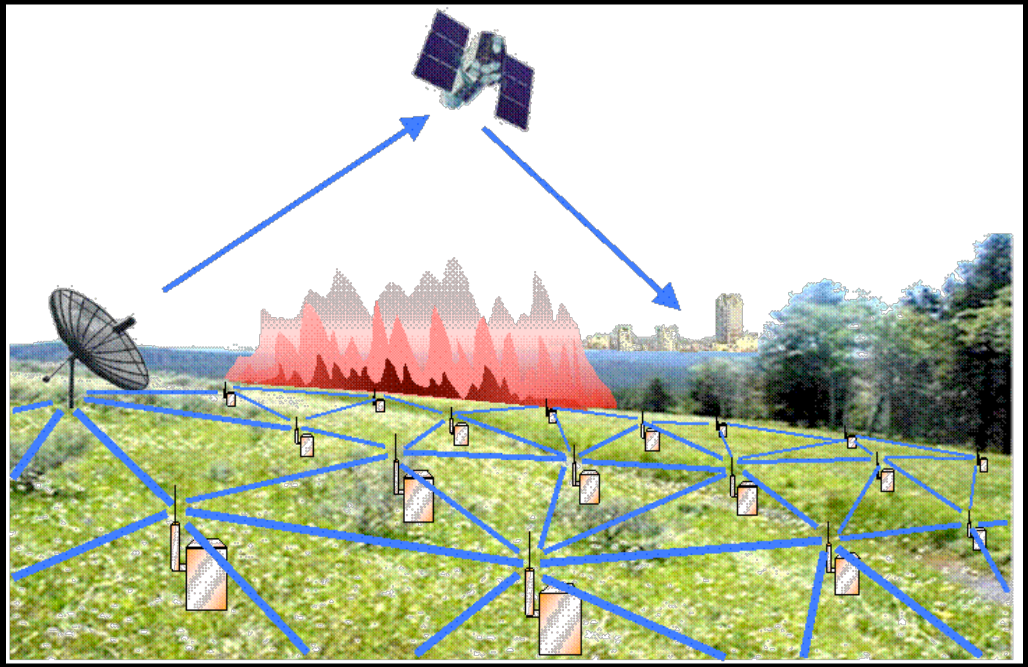

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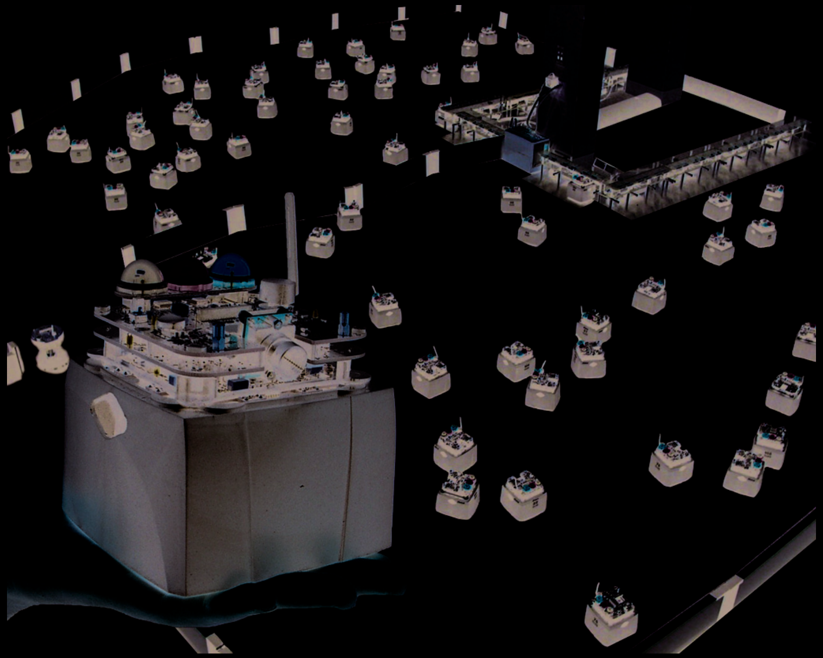

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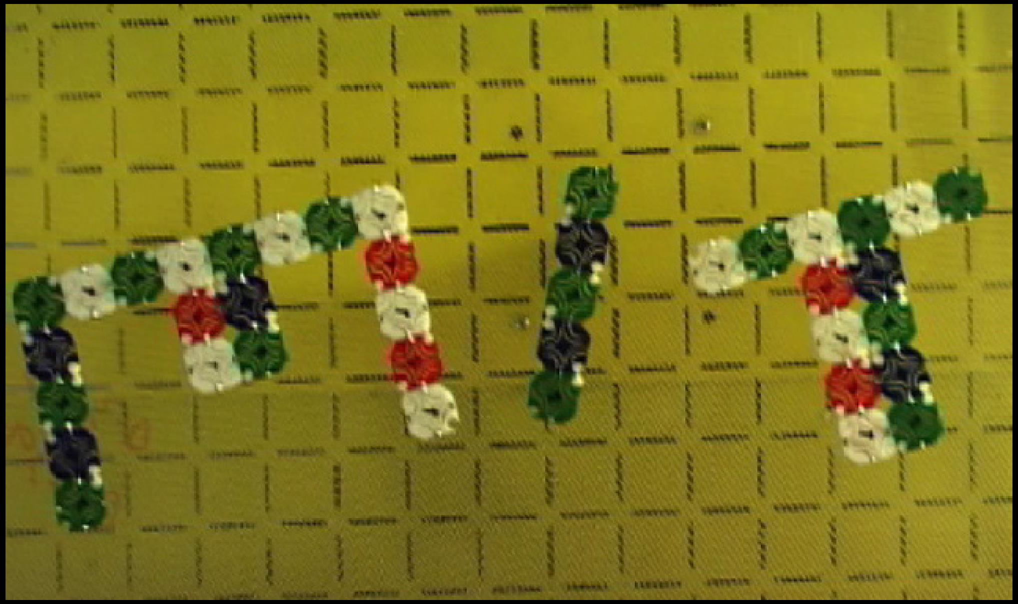

Saul Griffith’s self-folding structures

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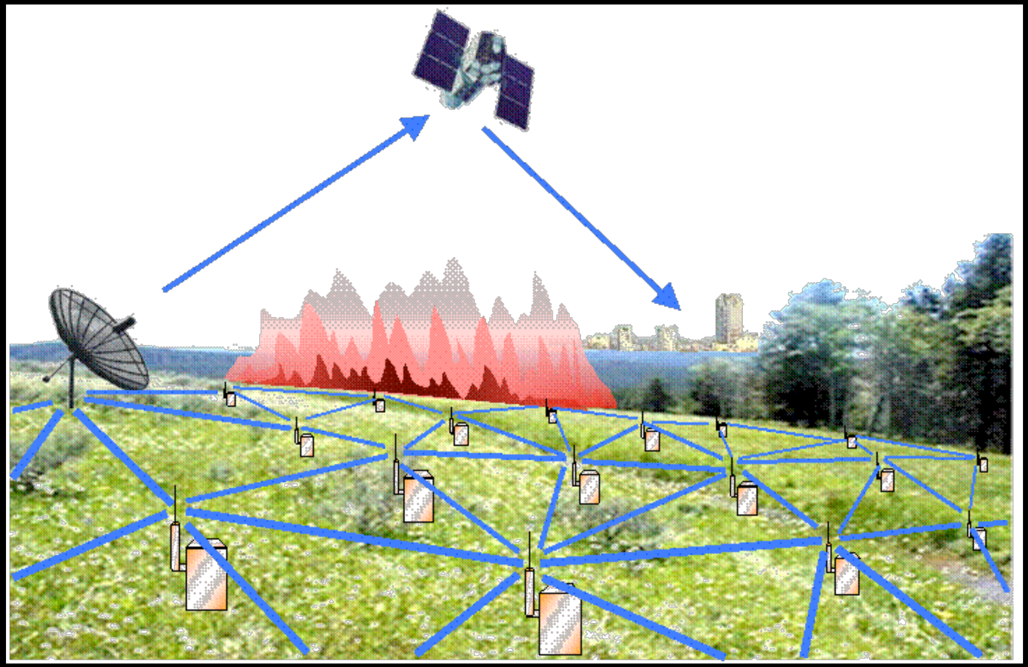

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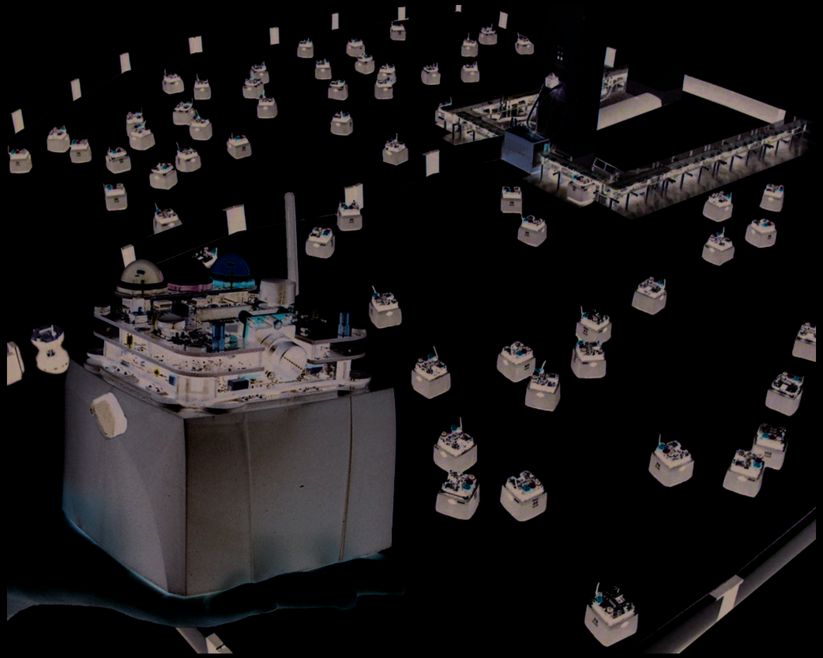

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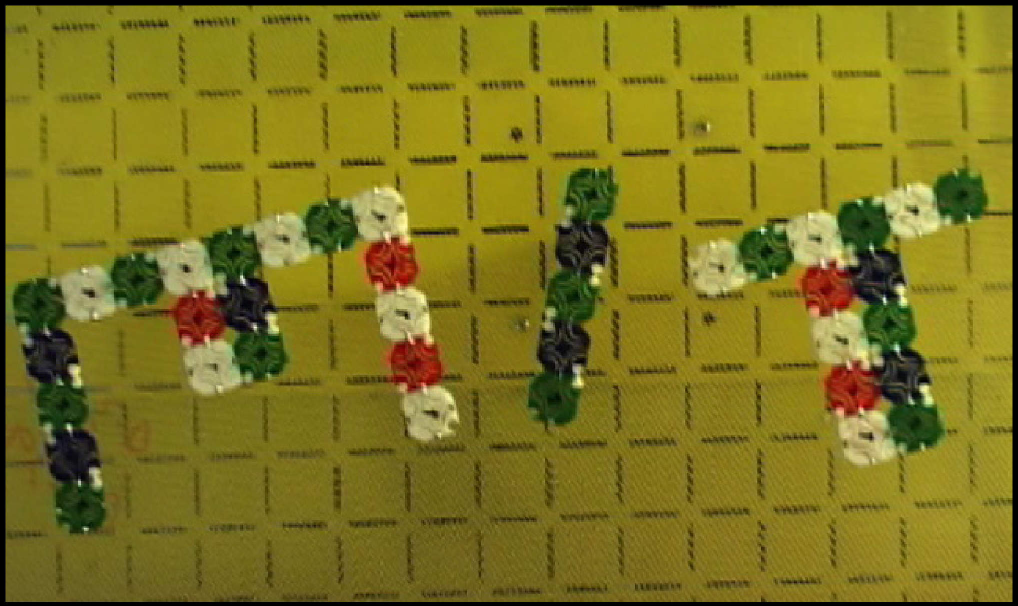

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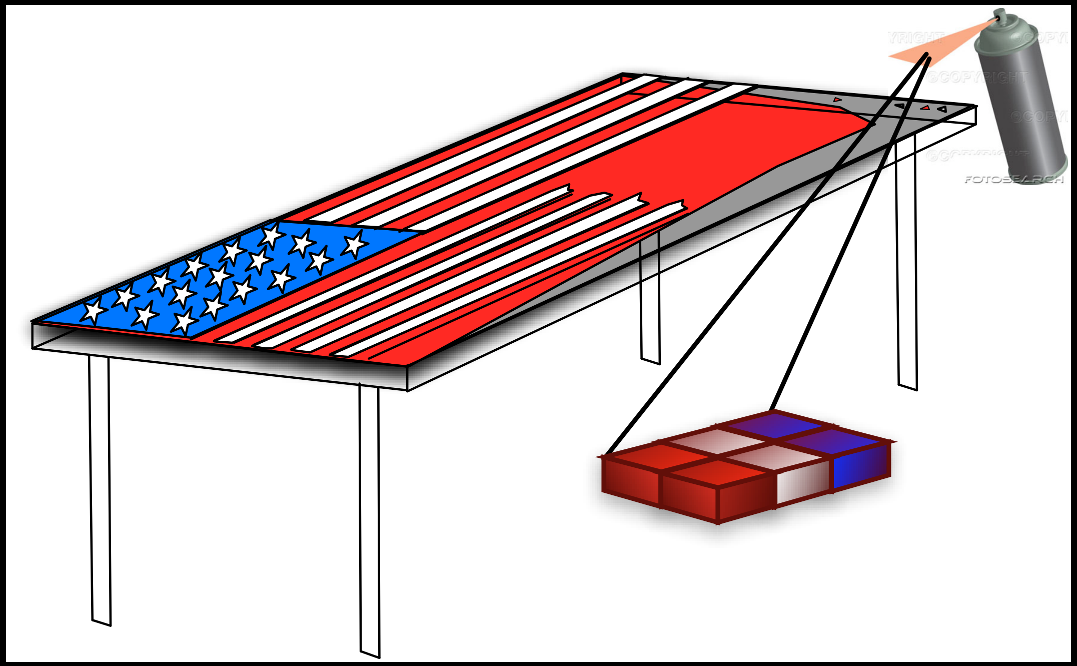

Butera's "paintable computer" concept

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... the Need for A Theory

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... Global-to-Local compilation.

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The Model: Static Configurations

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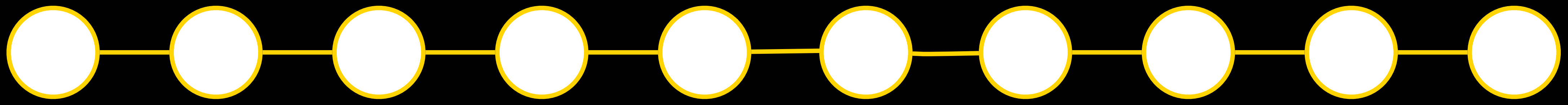

Undirected Lines

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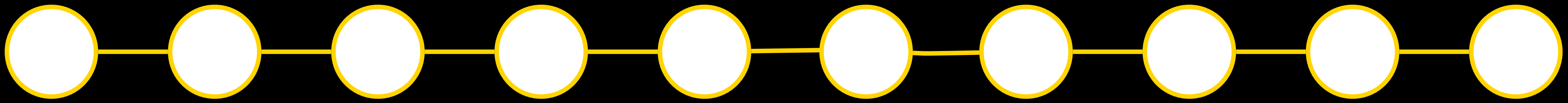

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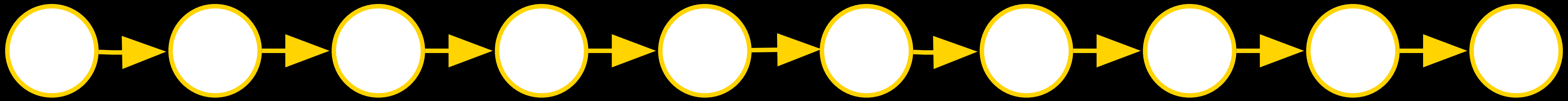

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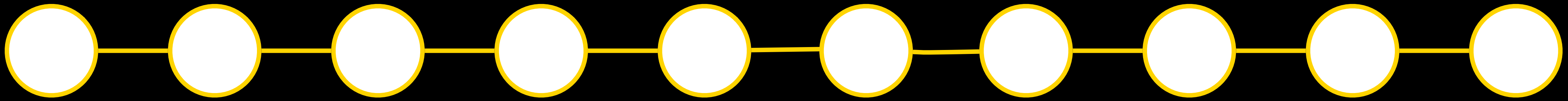

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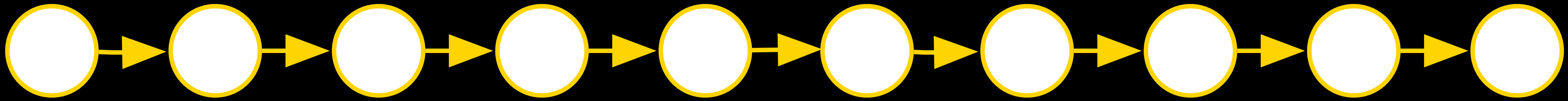

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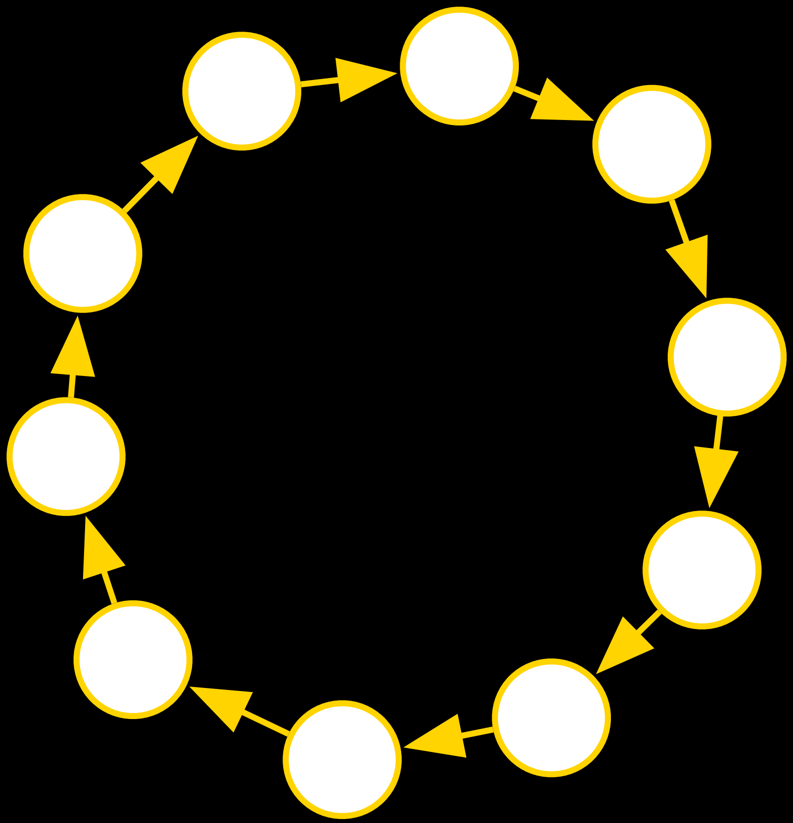

Ring Lattices

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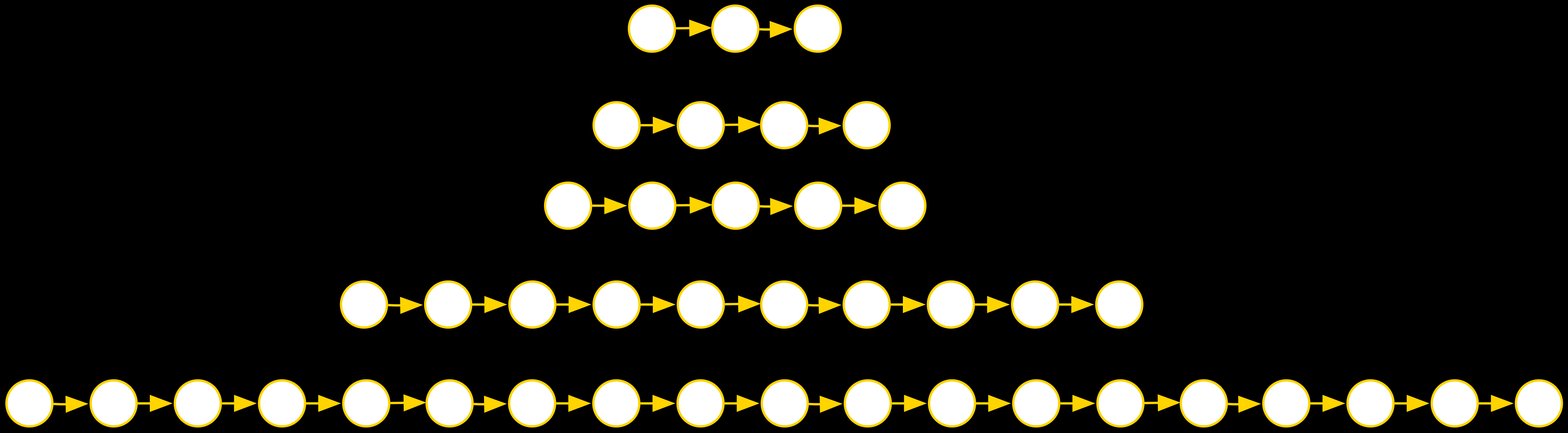

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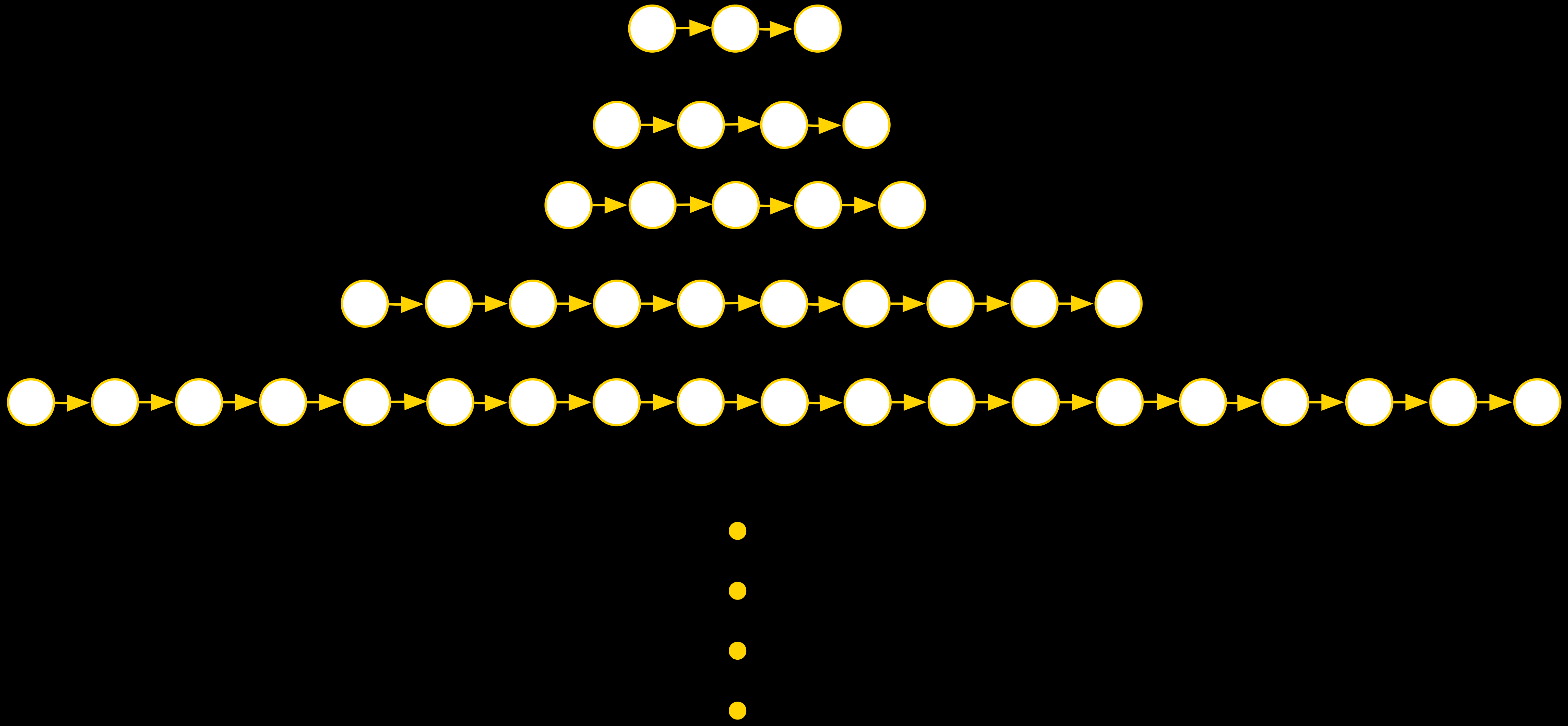
Usually is a *large or infinite* set of configurations



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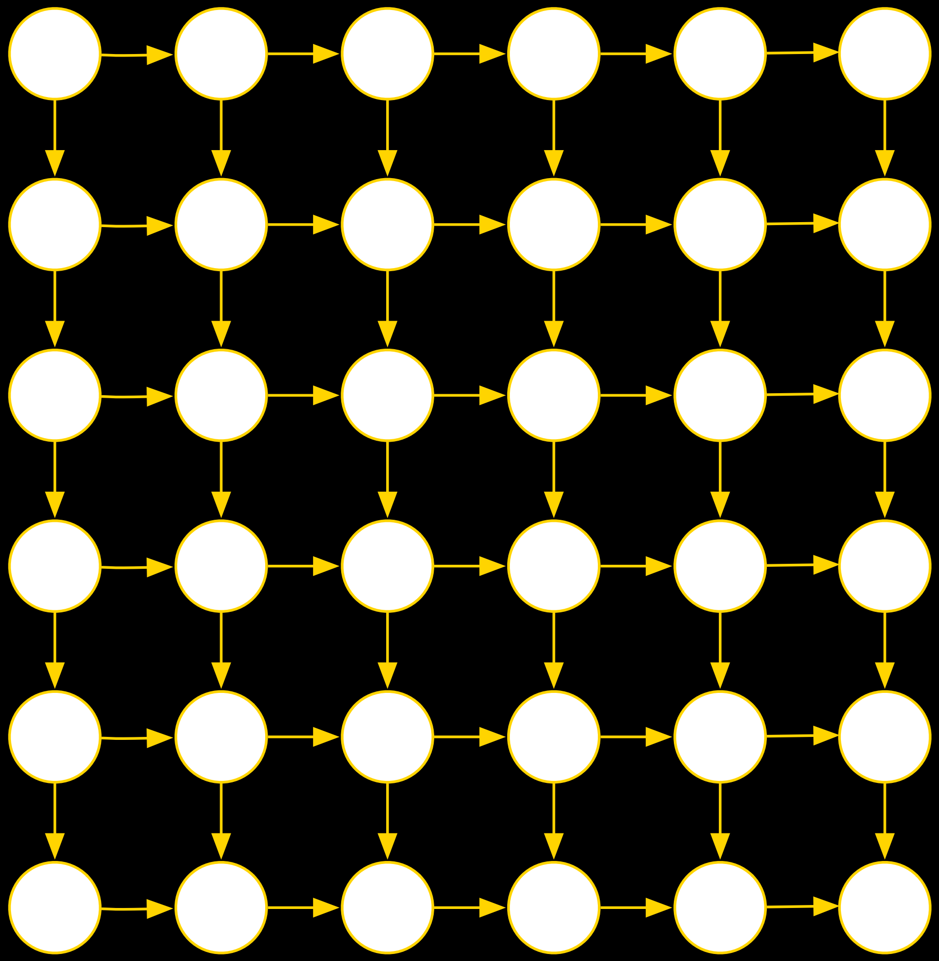

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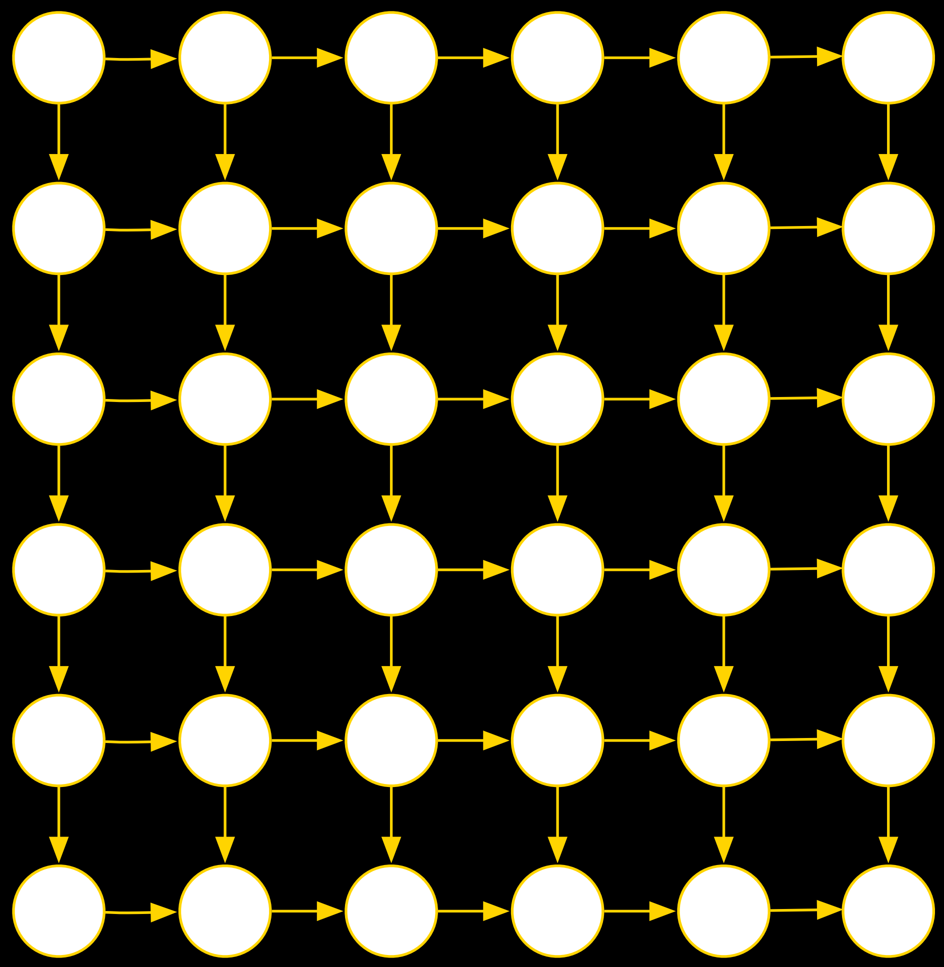

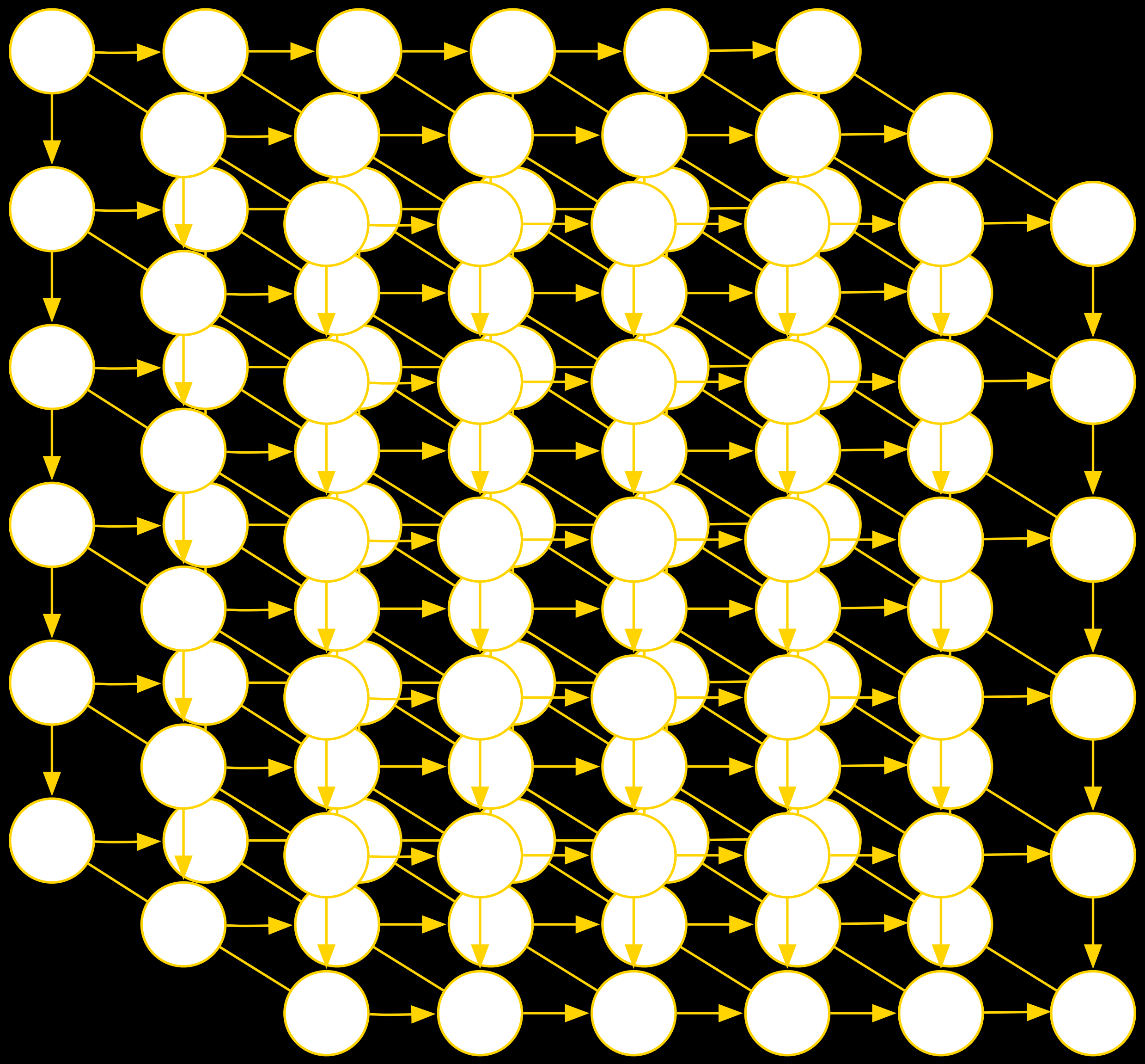

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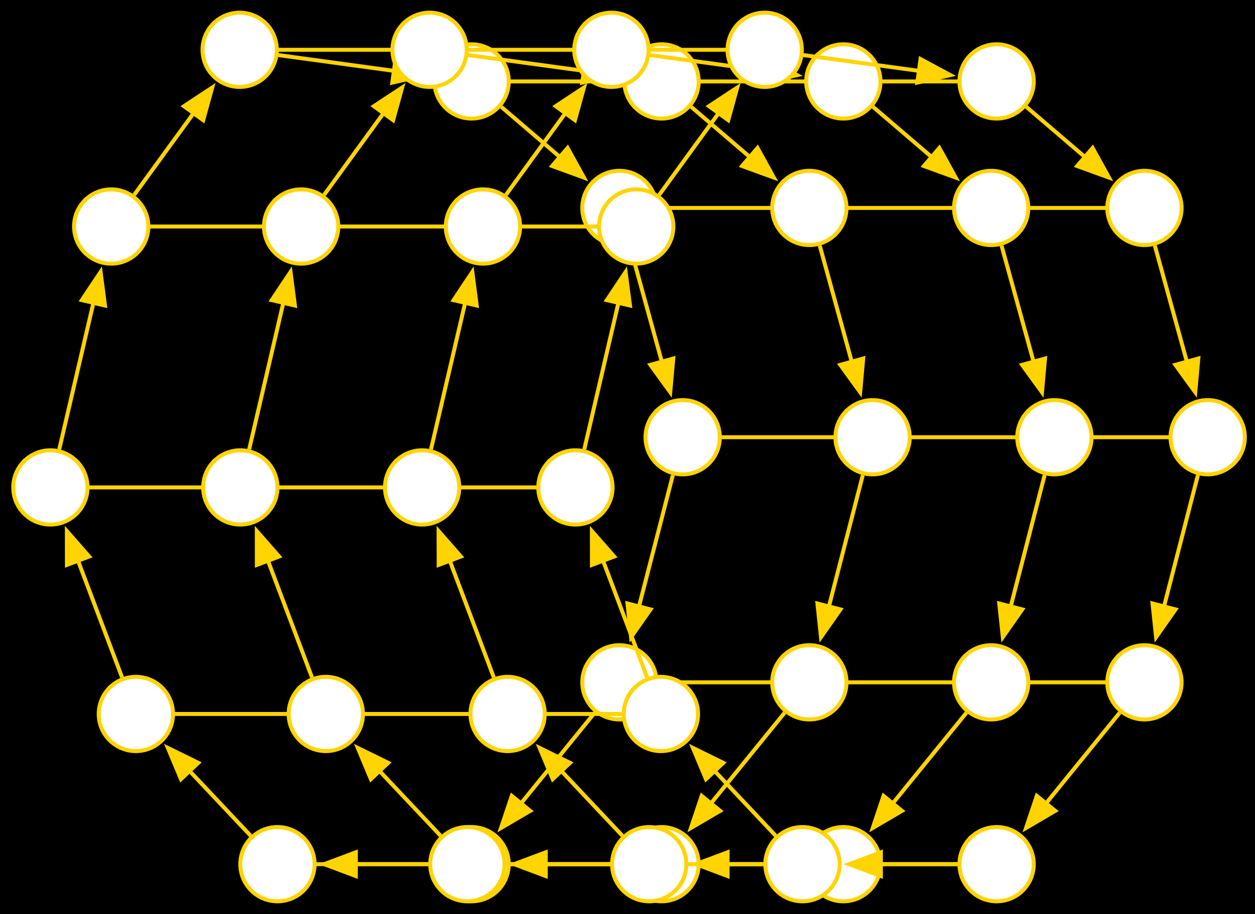

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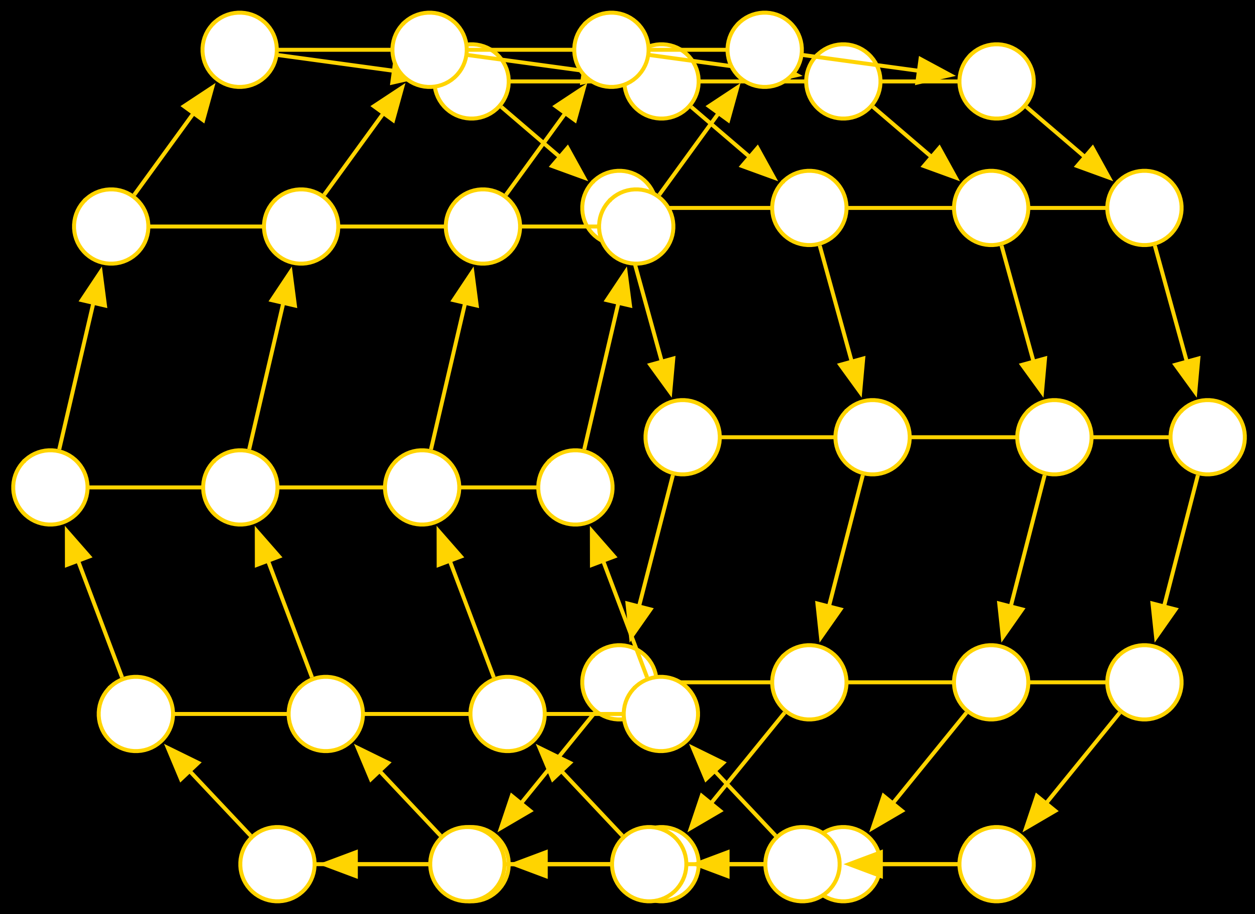

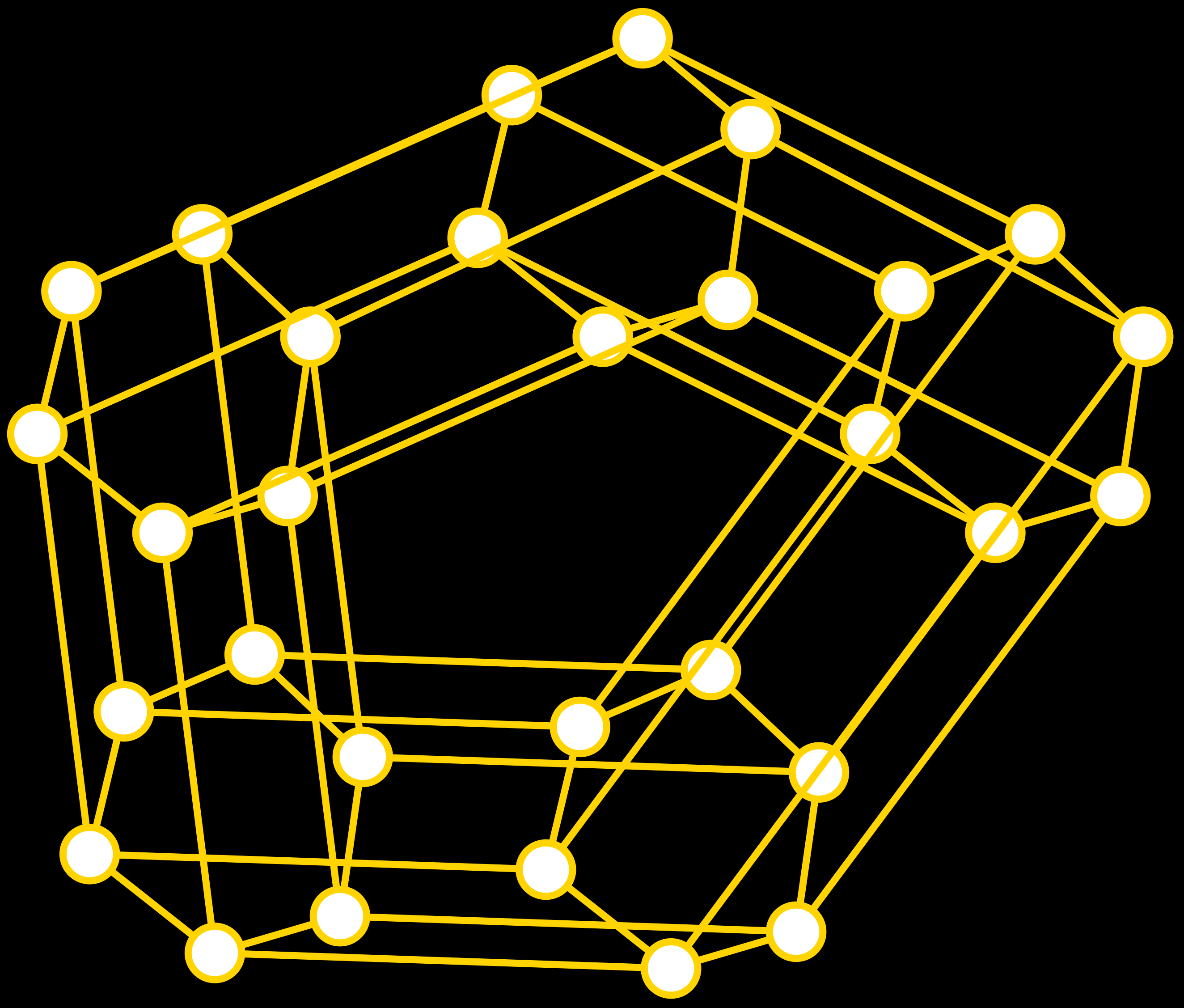

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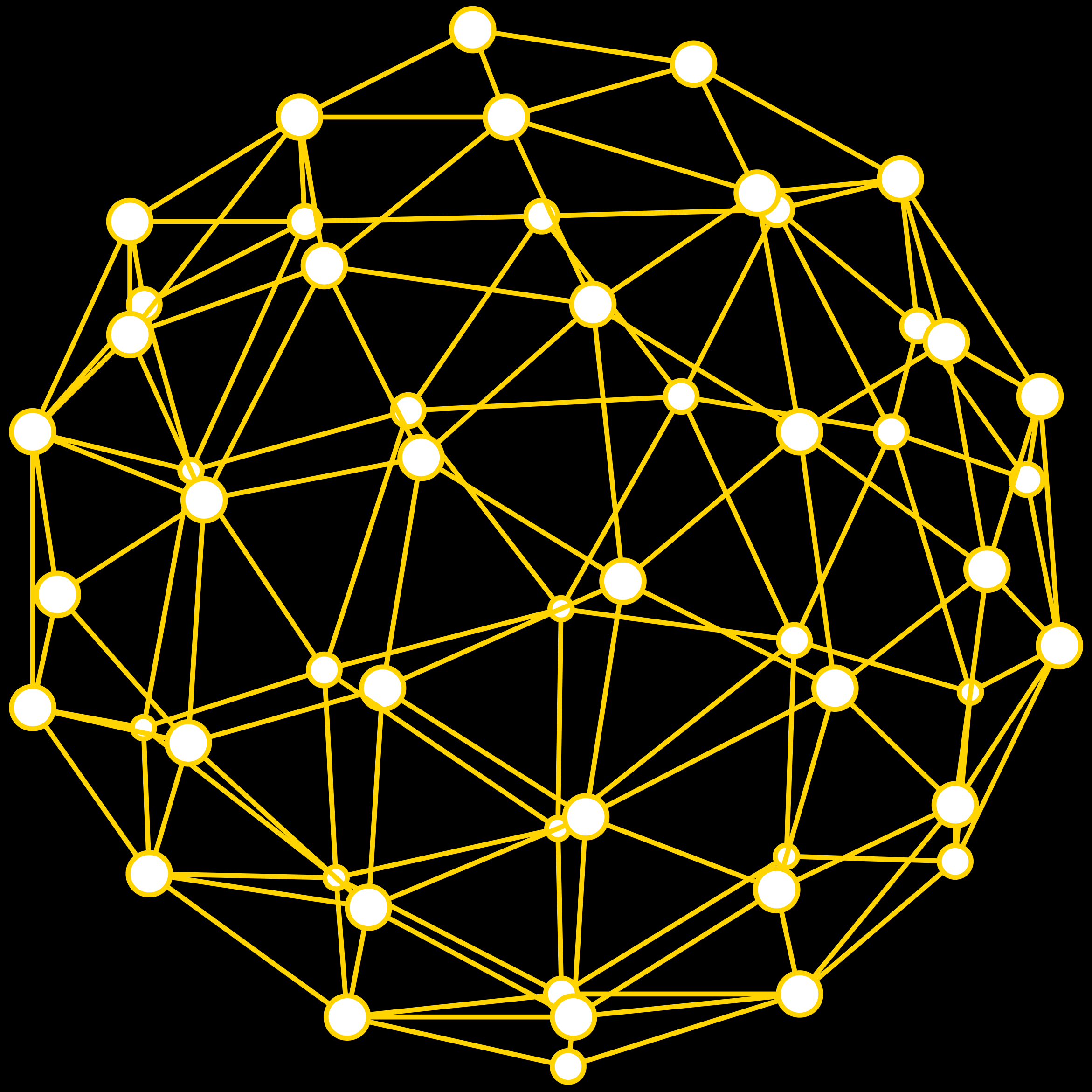

Triangulated Sphere

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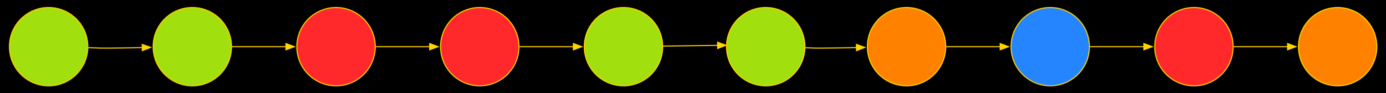

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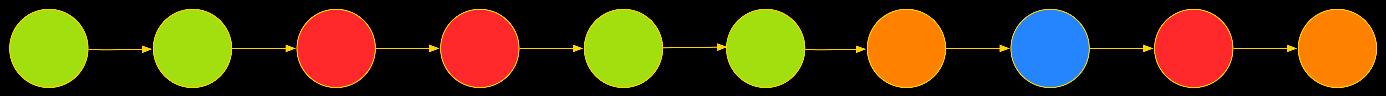

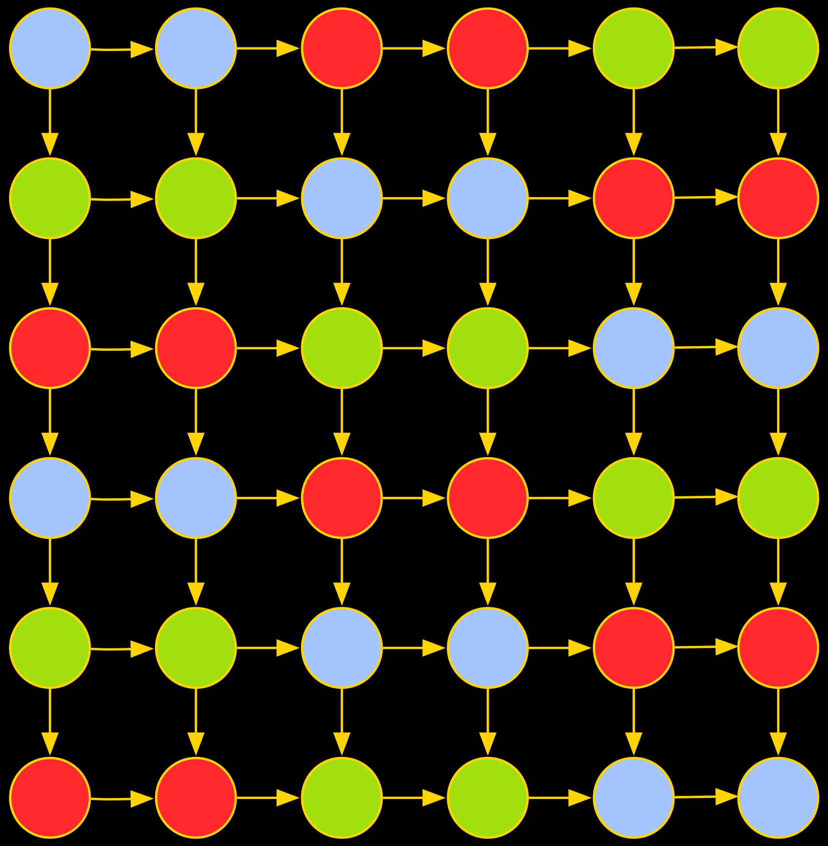

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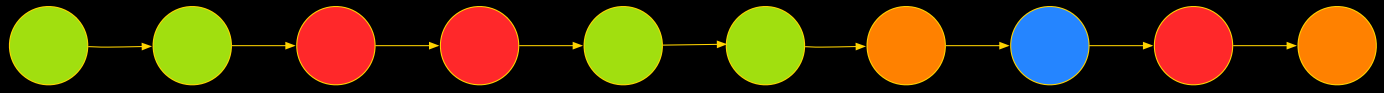

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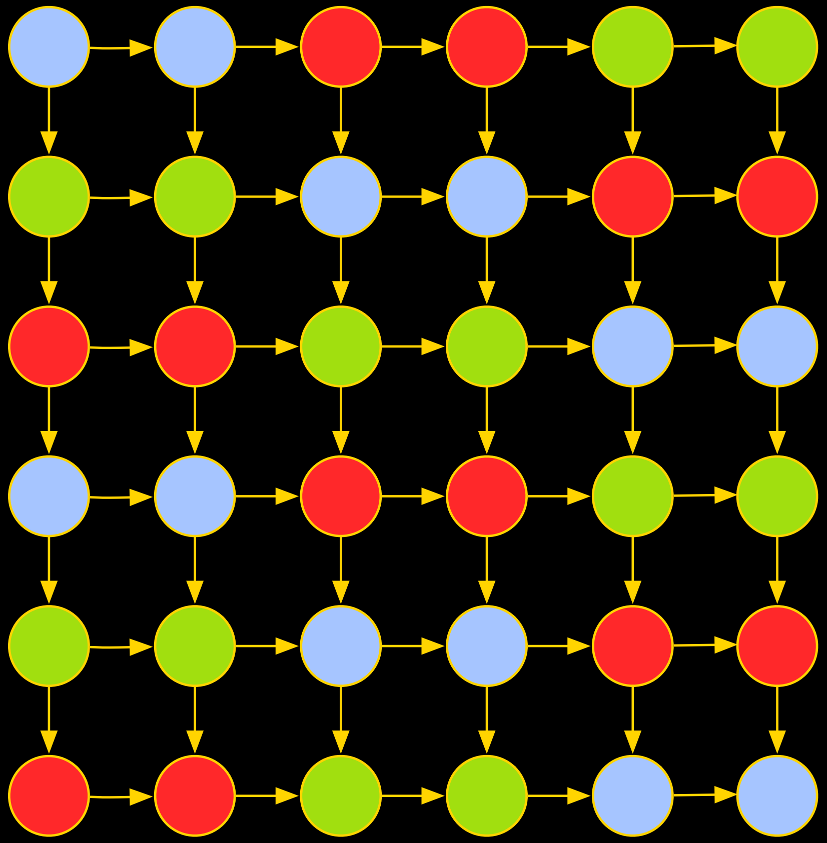

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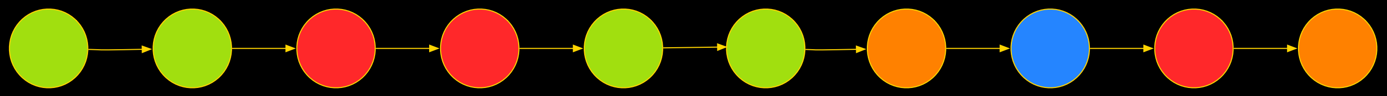

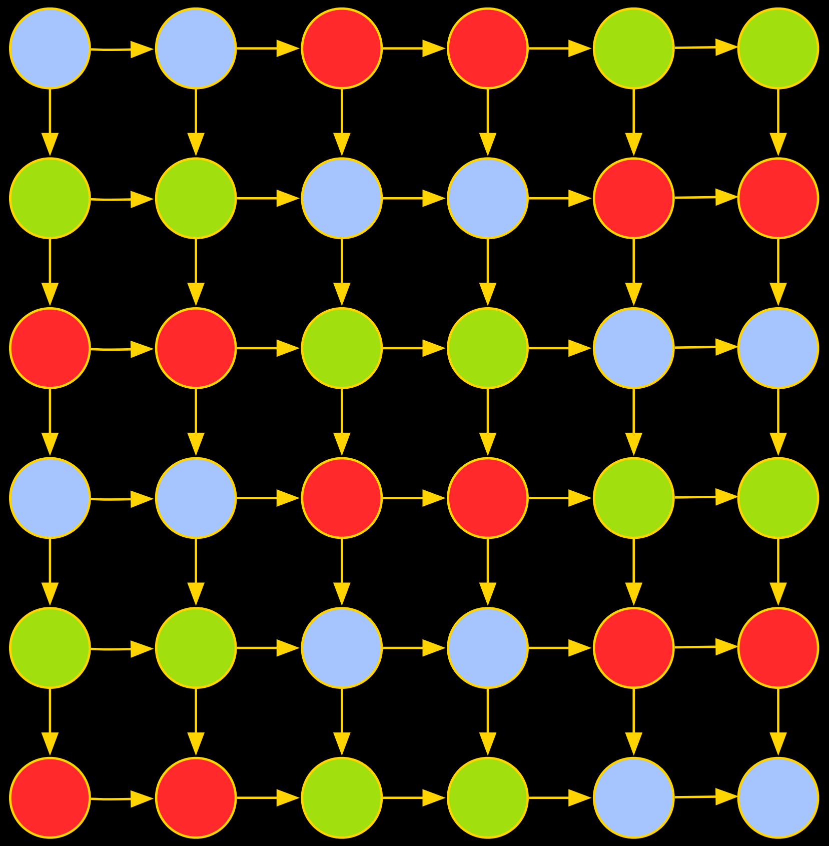

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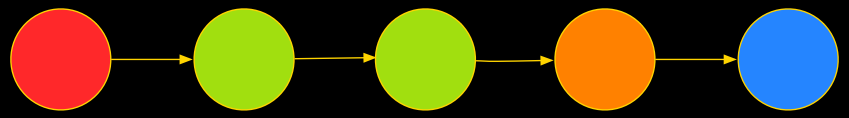

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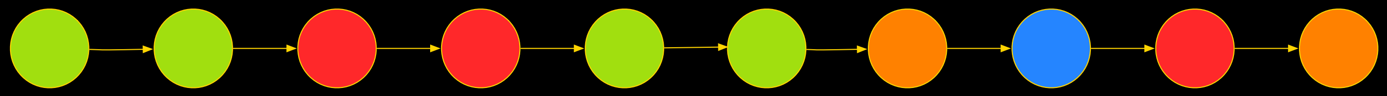

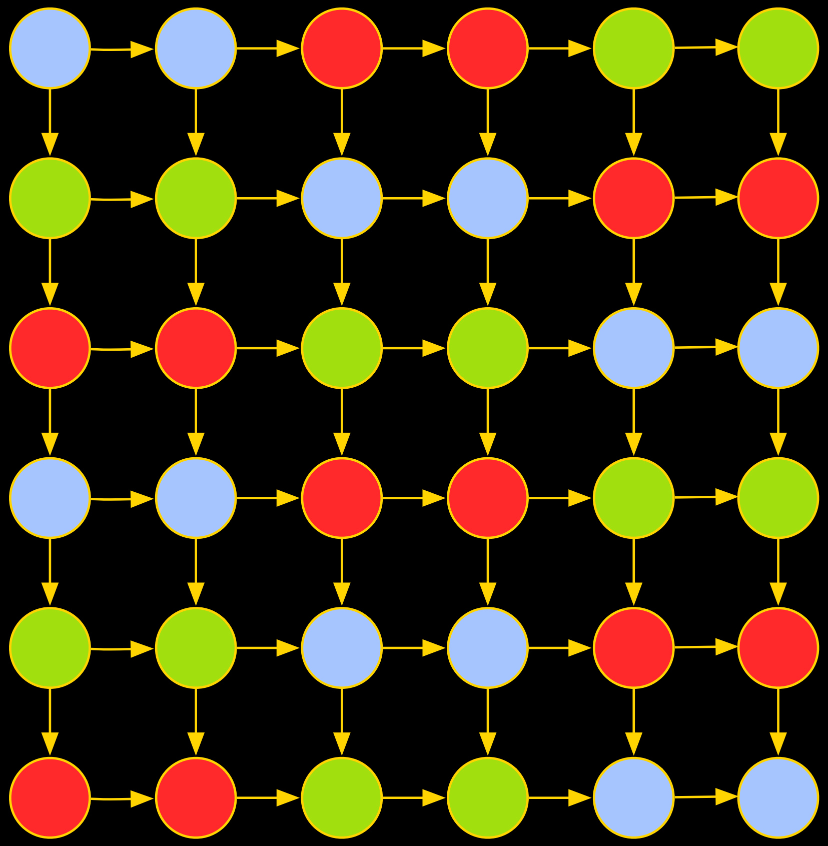

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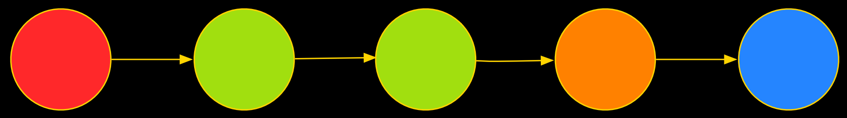 ← $B_2(6, X)$ for X = 1-D configuration shown above

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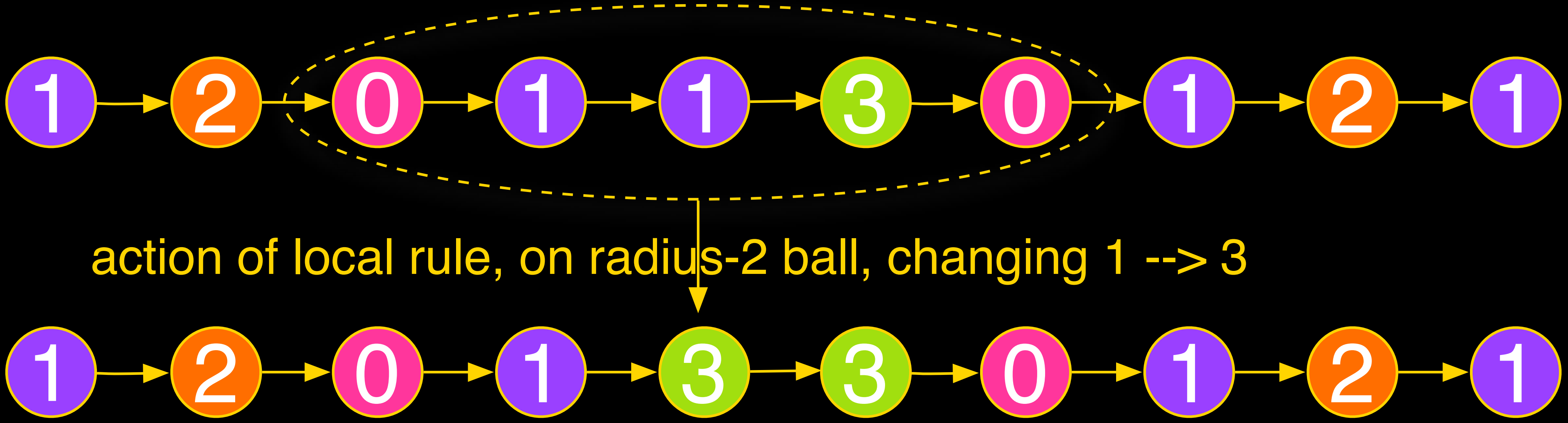

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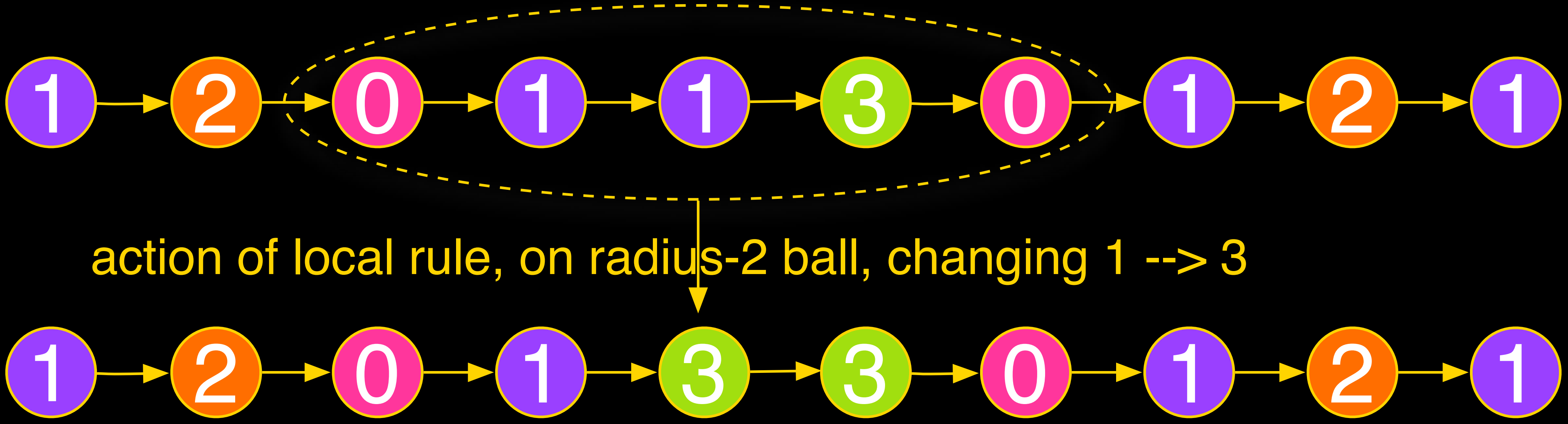
Definition. *A* radius *R* local rule *is a look-up table that maps R-ball configurations to "updated state" choices. Formally,*

$$F : \mathcal{B}_{r,S} \longrightarrow S\,.$$



action of local rule, on radius-2 ball, changing 1 --> 3

Thought of as "agent-based local programs".

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Definition. *A* size-*n call sequence is a call sequence acting on an n-agent configuration. A* timing model *is a set of call sequences for each size n.*

The Model: Local Rule Dynamics

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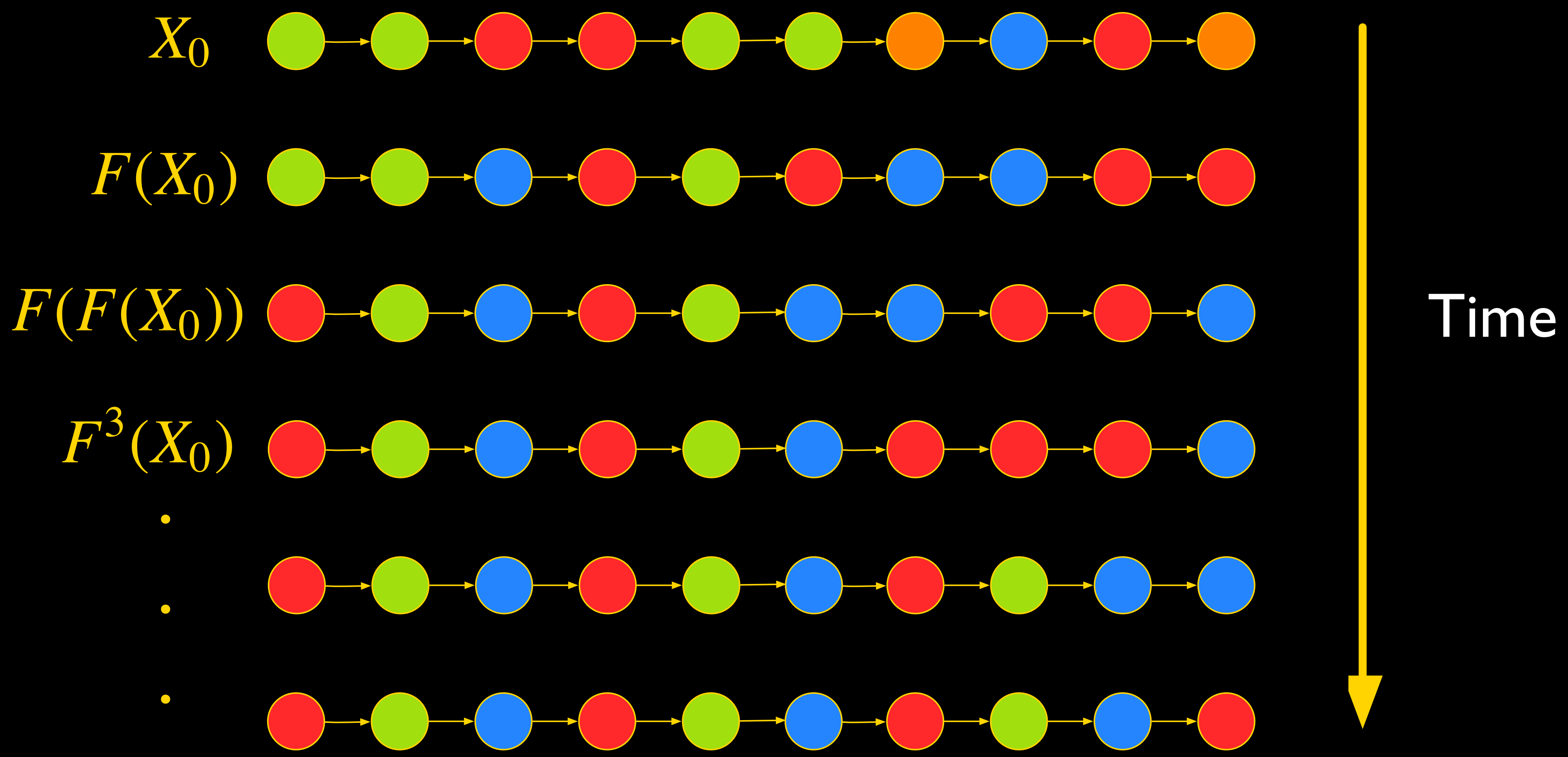

The Model: Patterns

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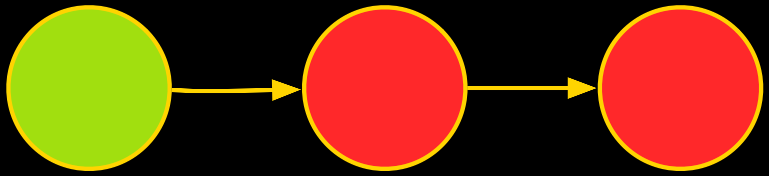

Repeat Patterns

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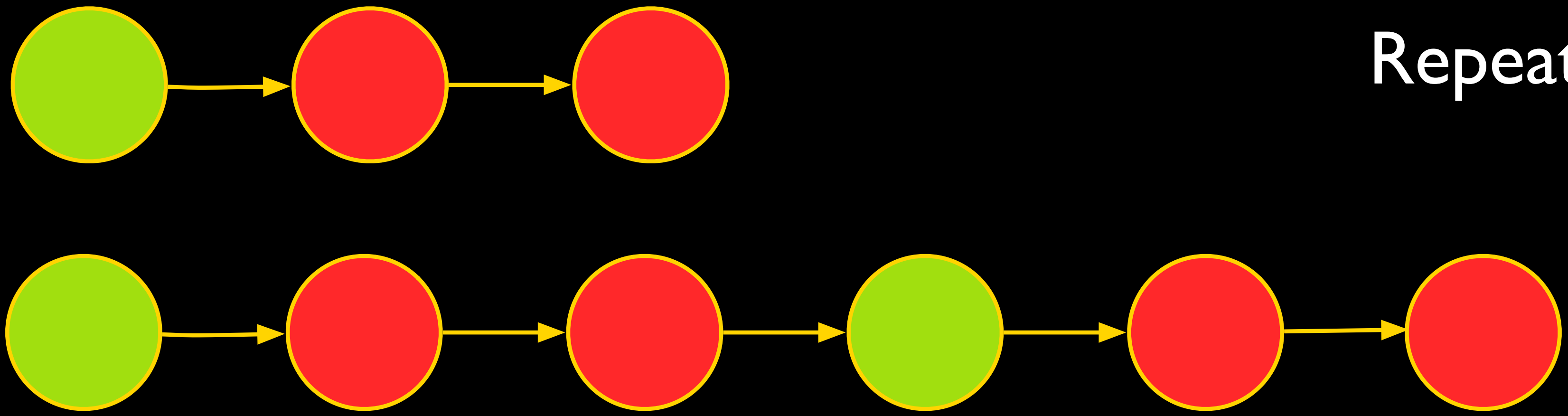

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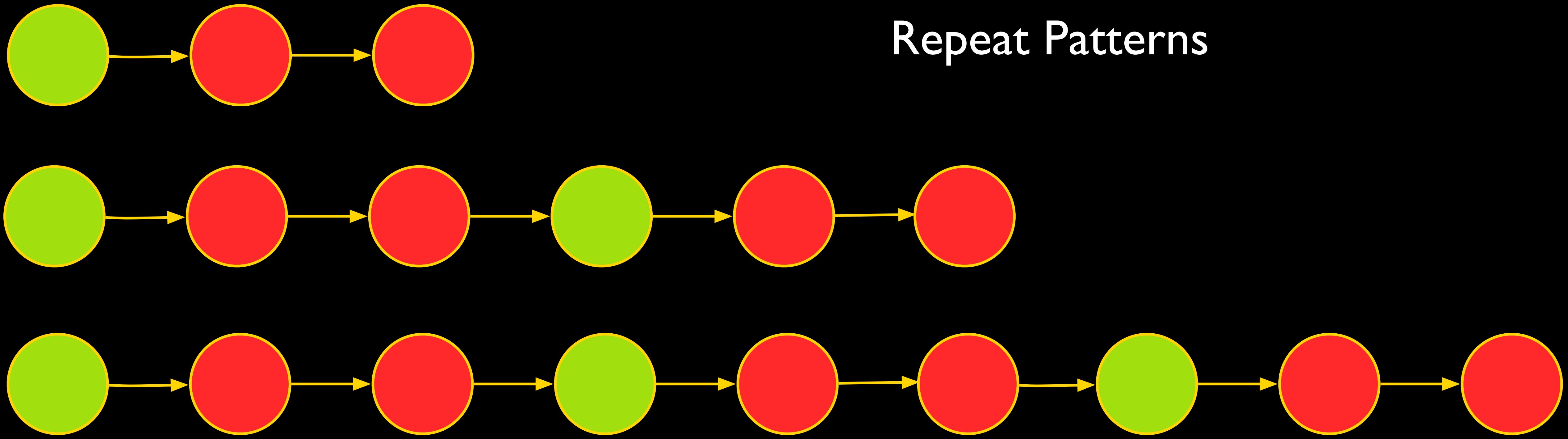

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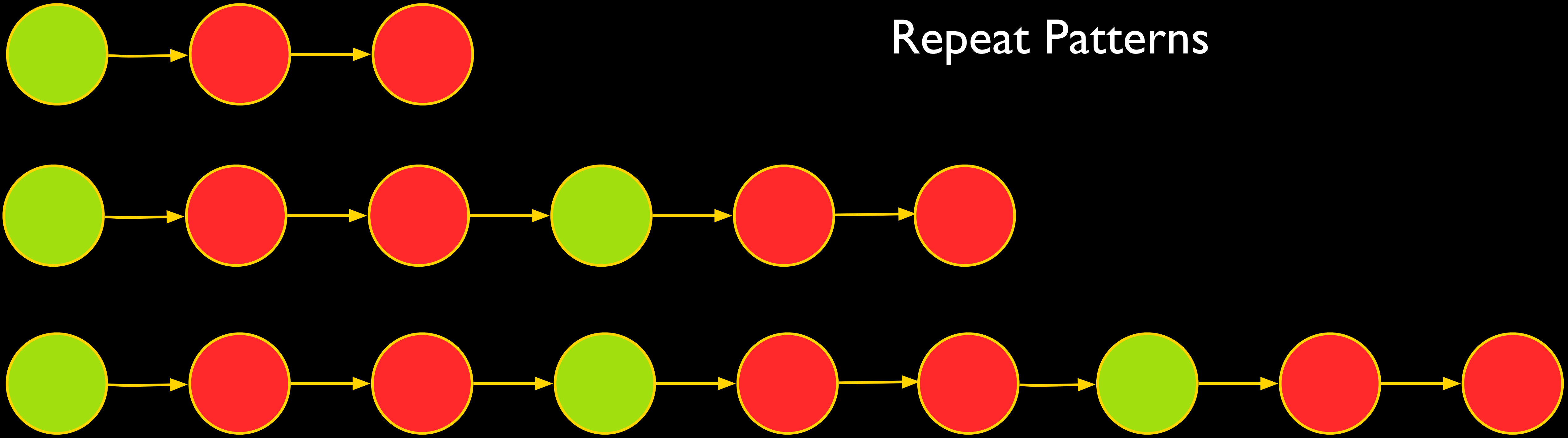

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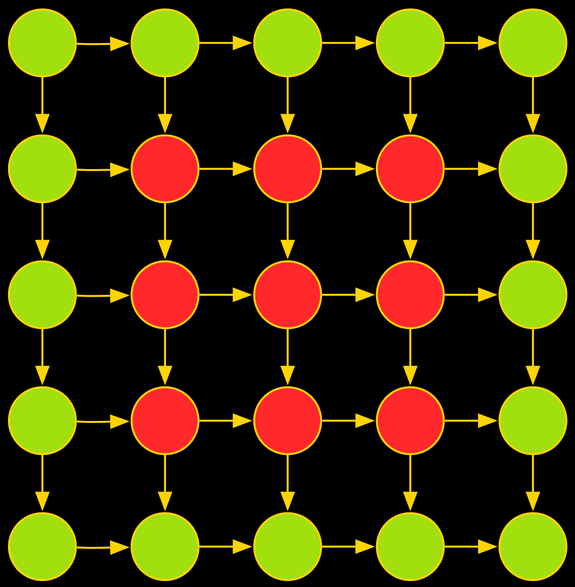

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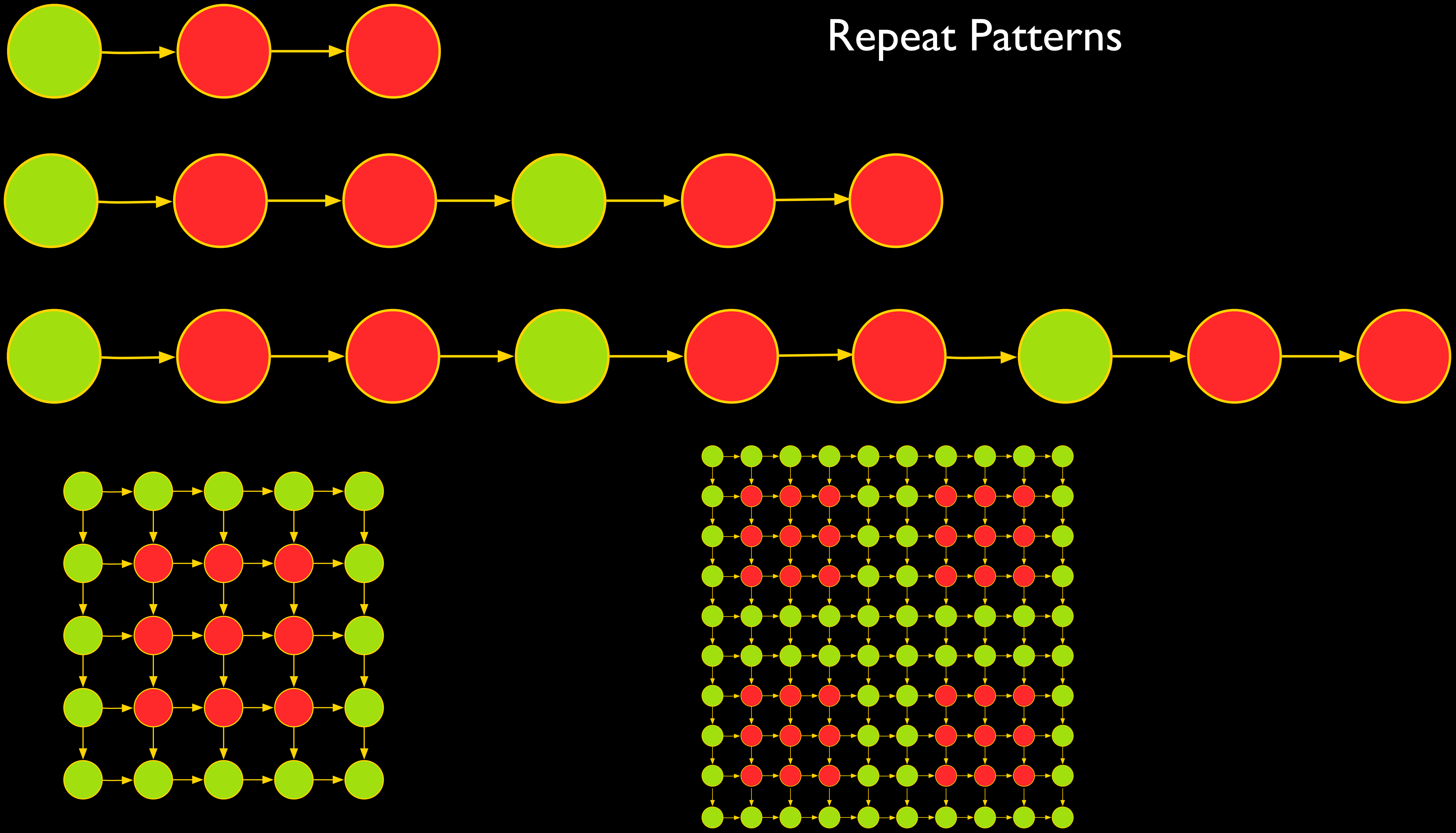

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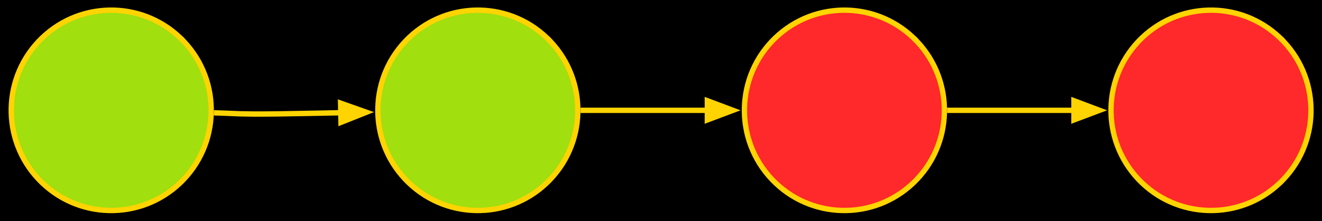
Proportionate Patterns



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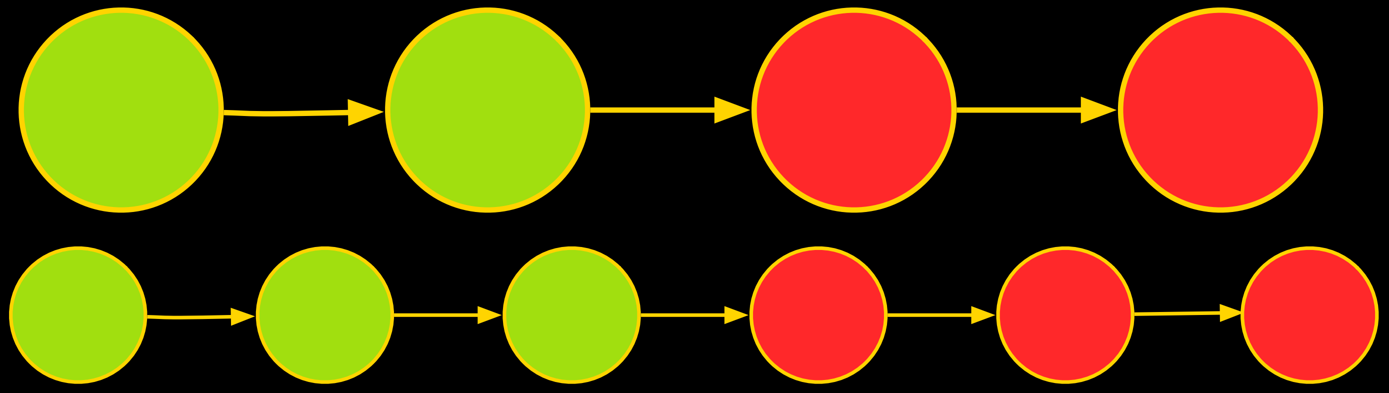

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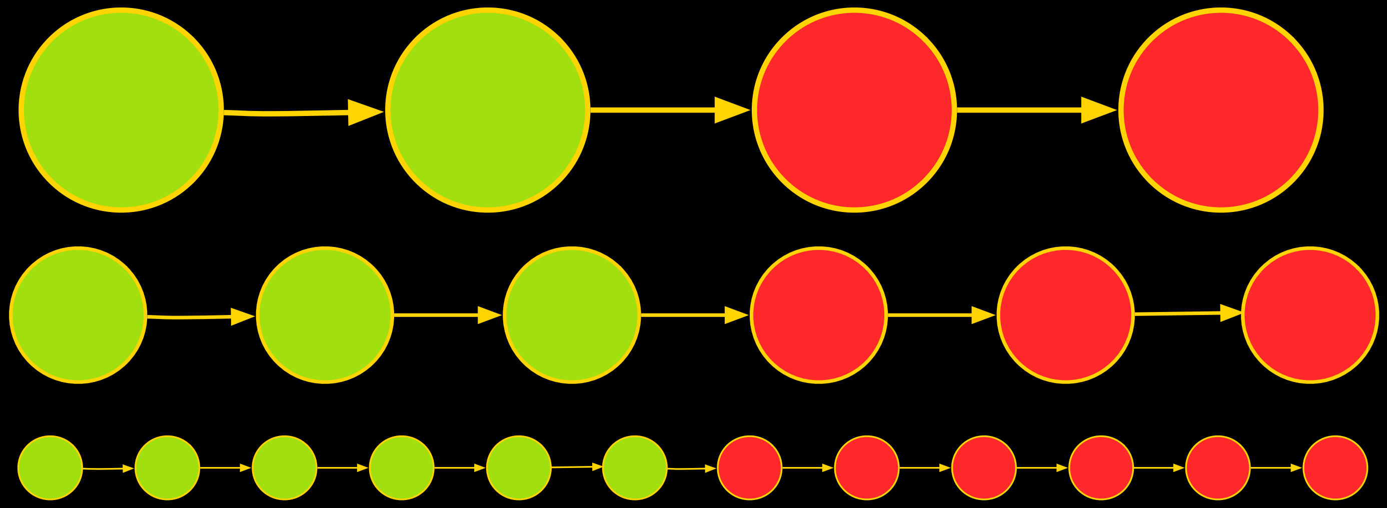

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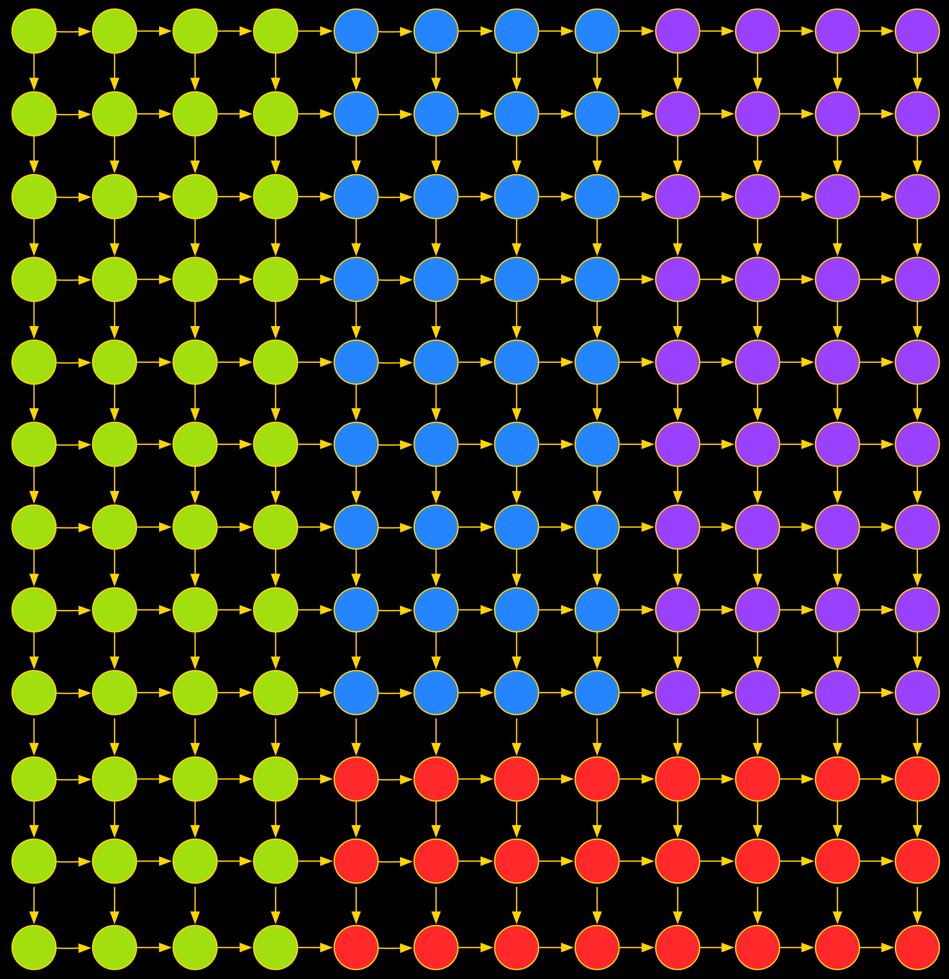

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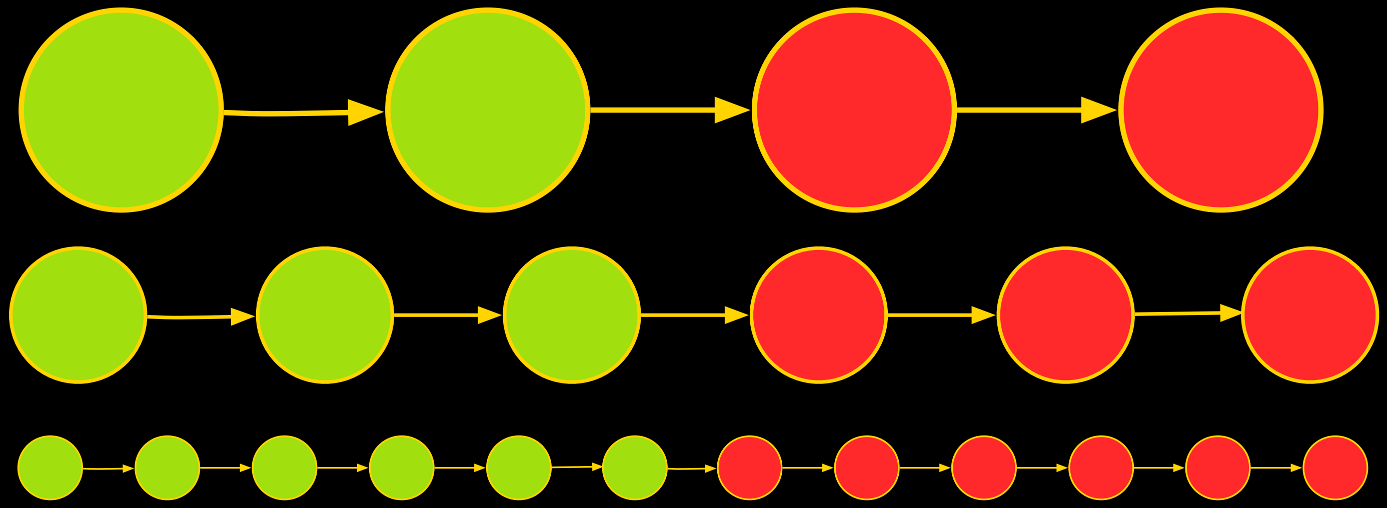

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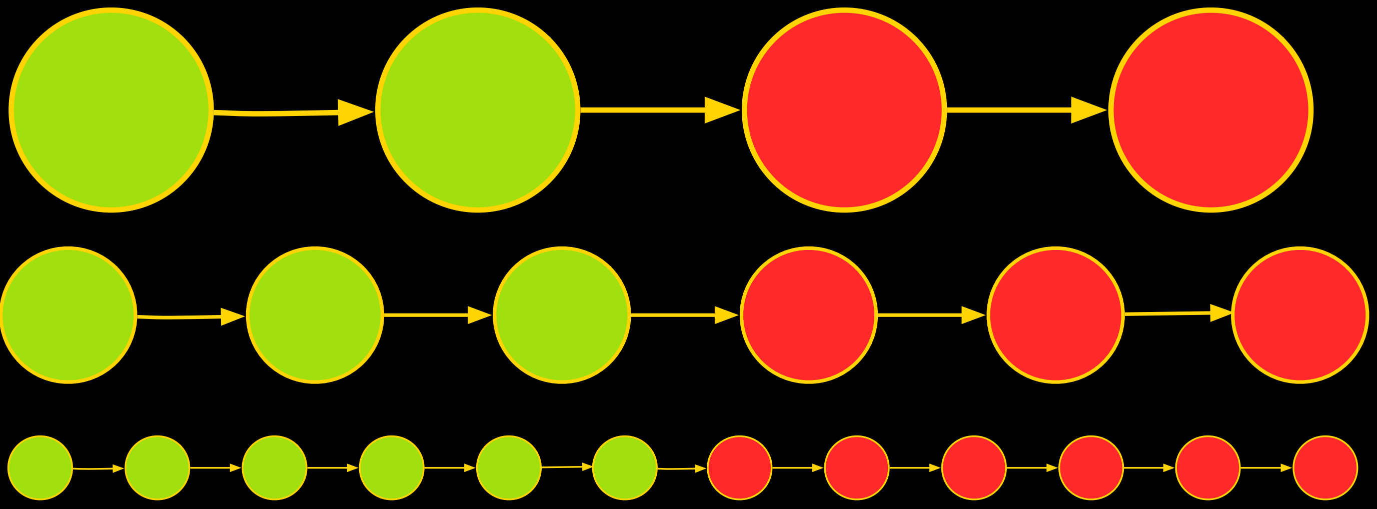

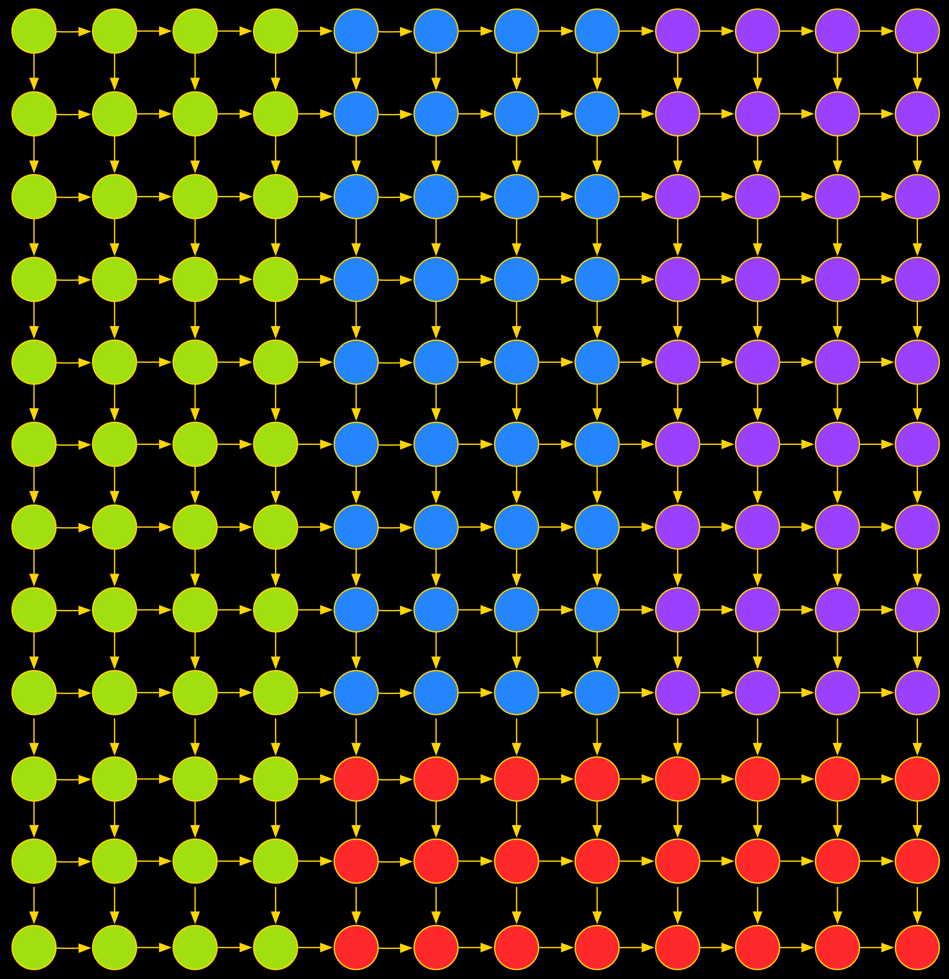

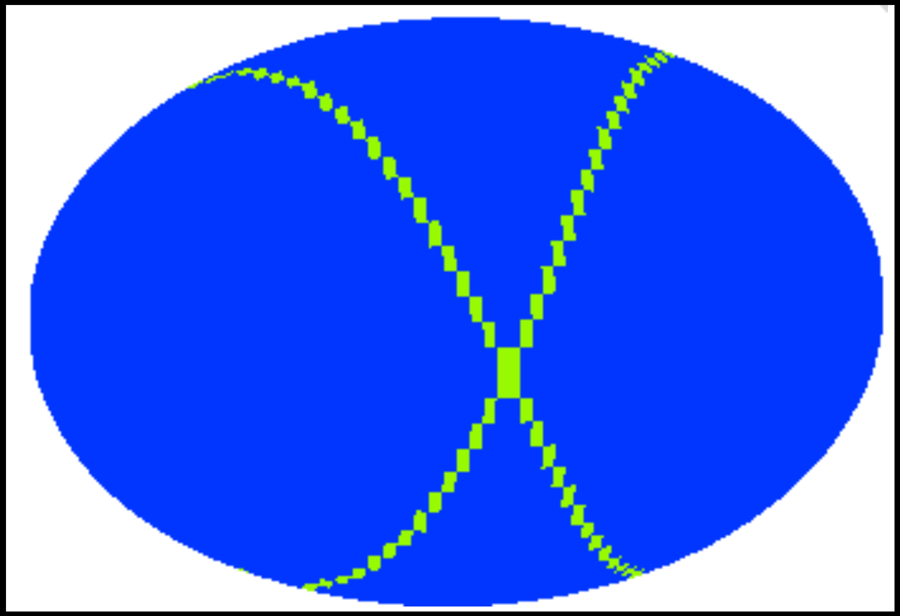

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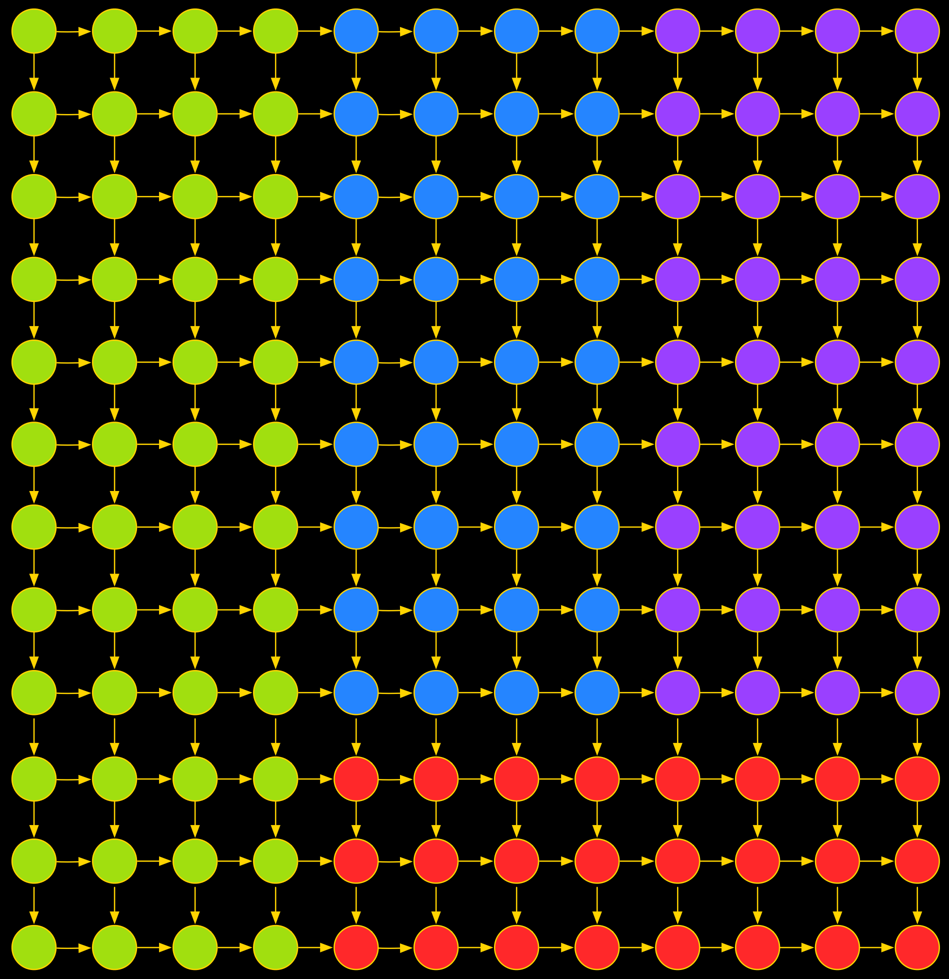

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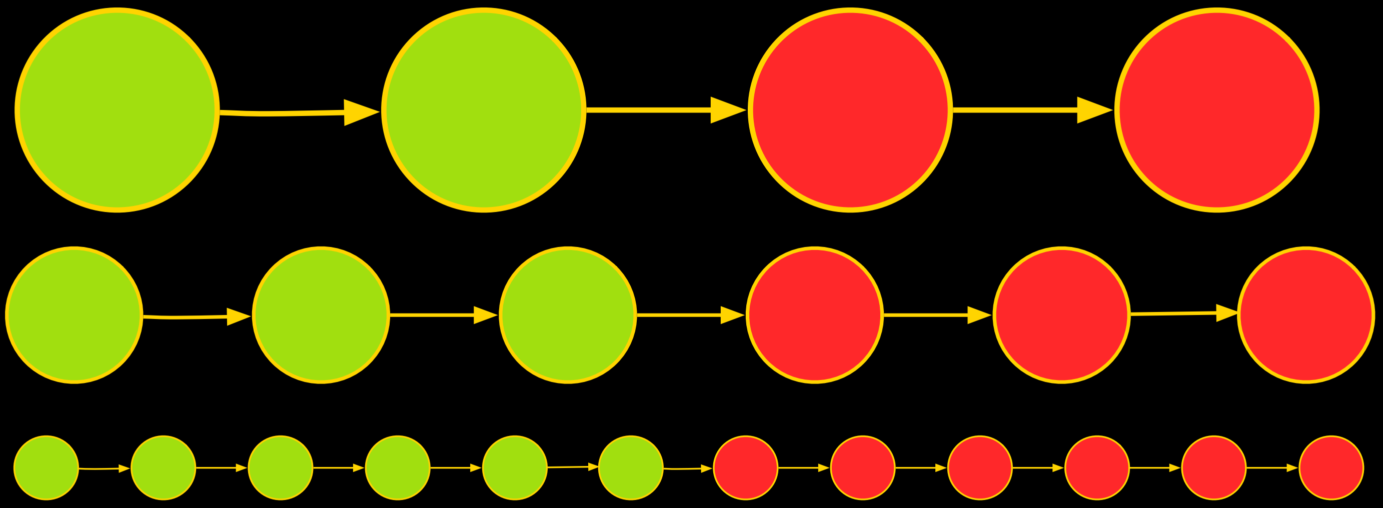

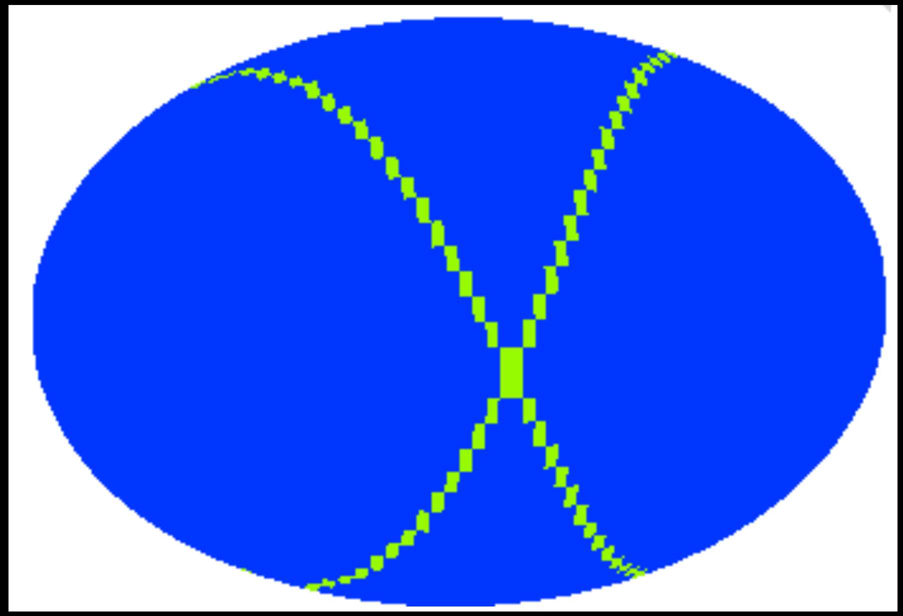

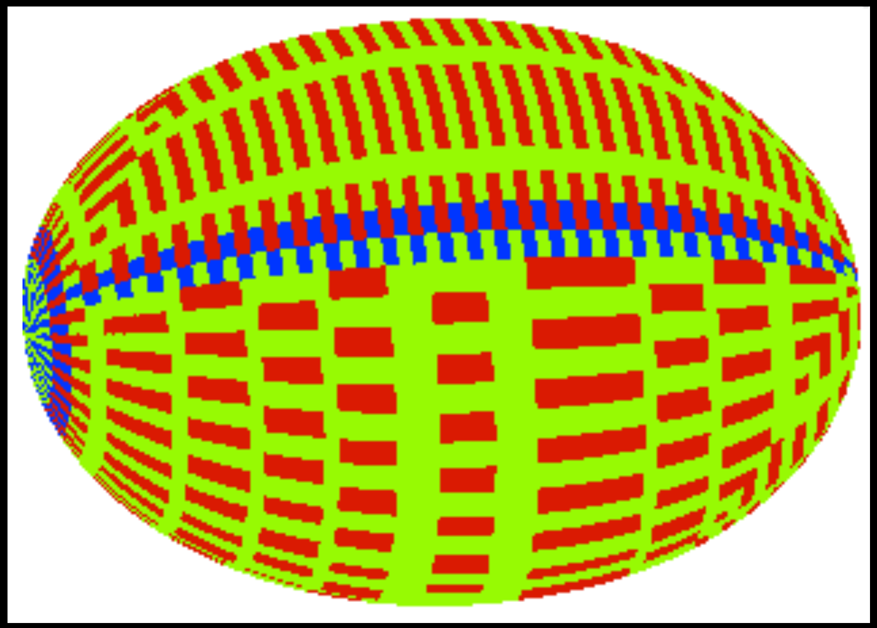

Other patterns

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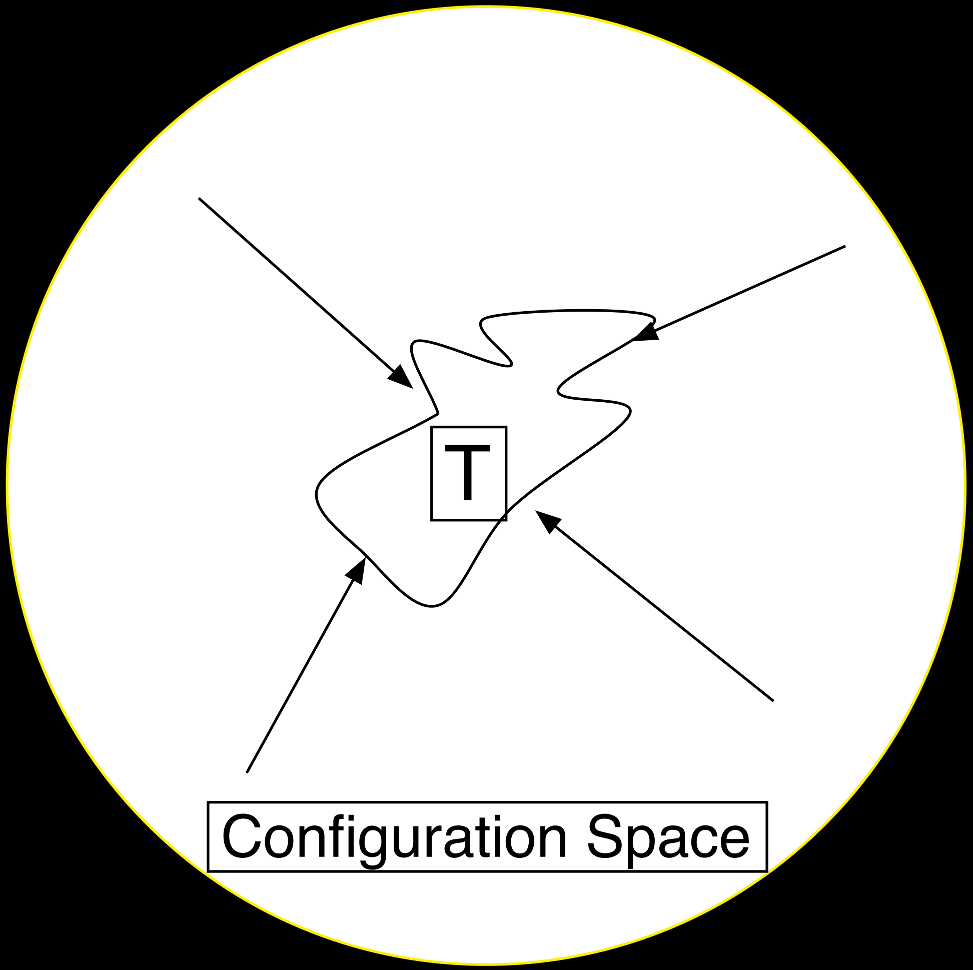

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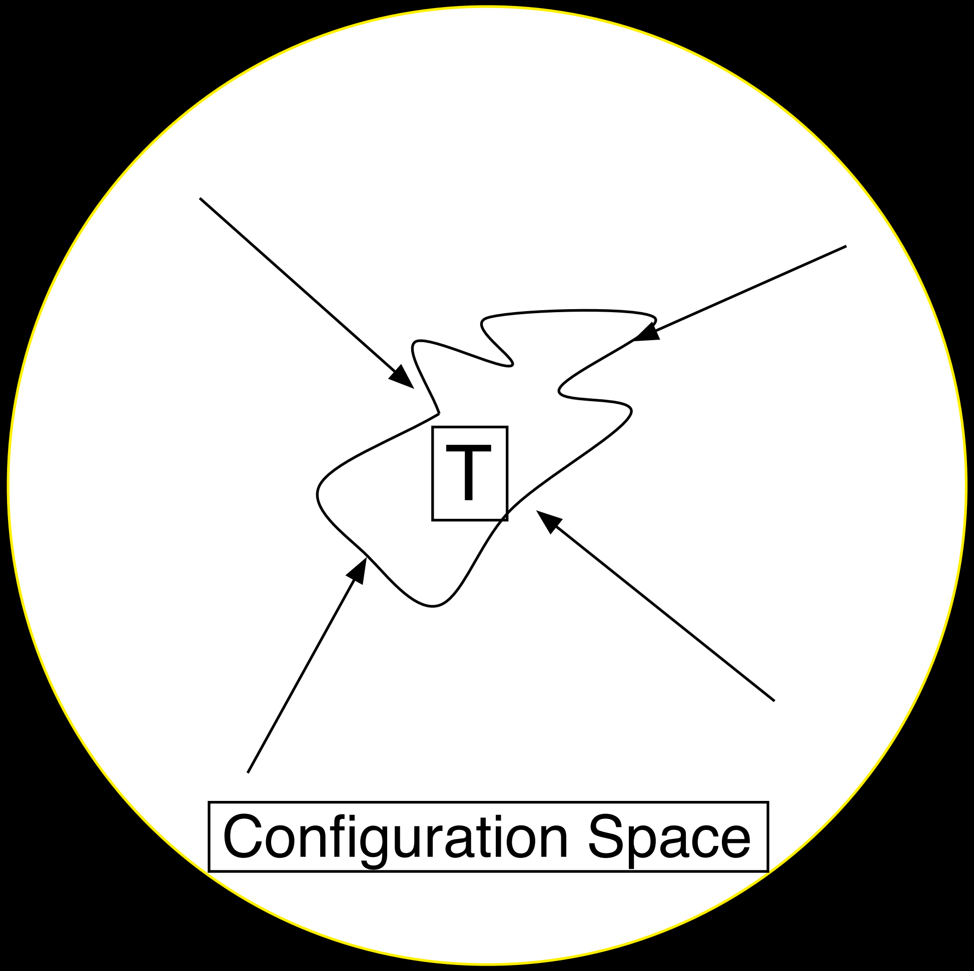

$\mathcal{G}$ Underlying Geometry

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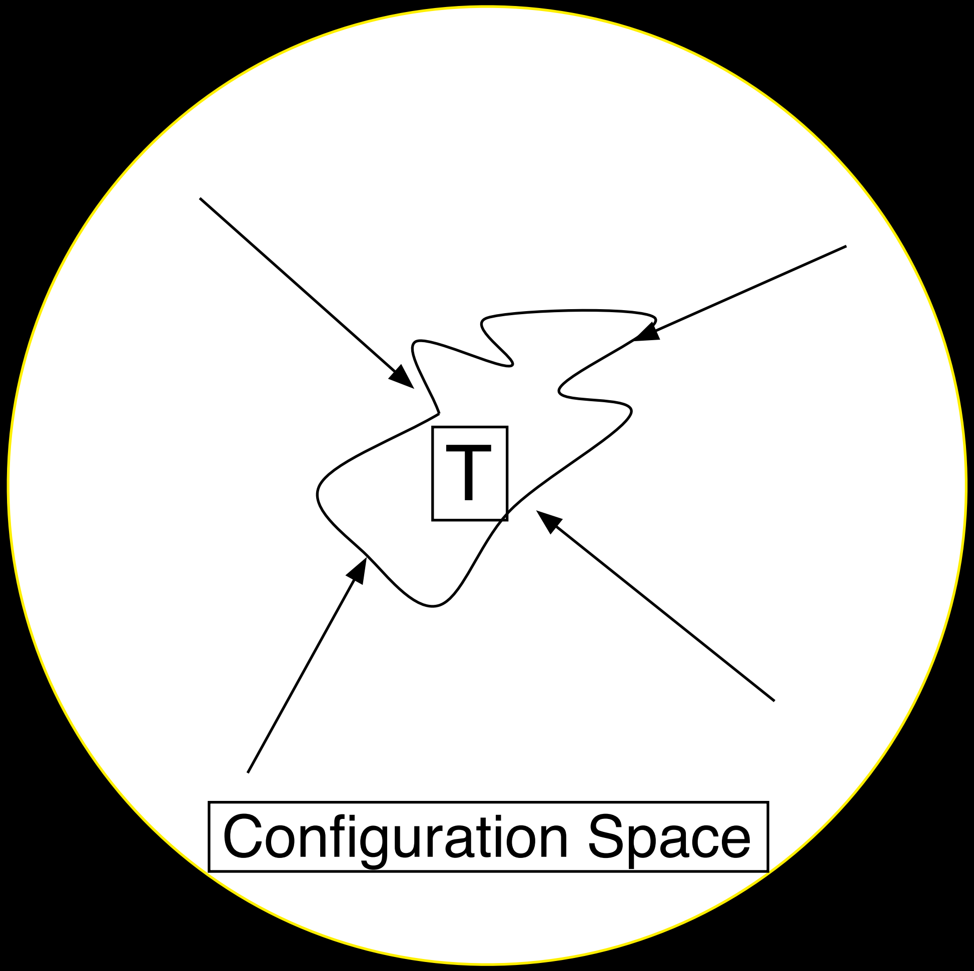

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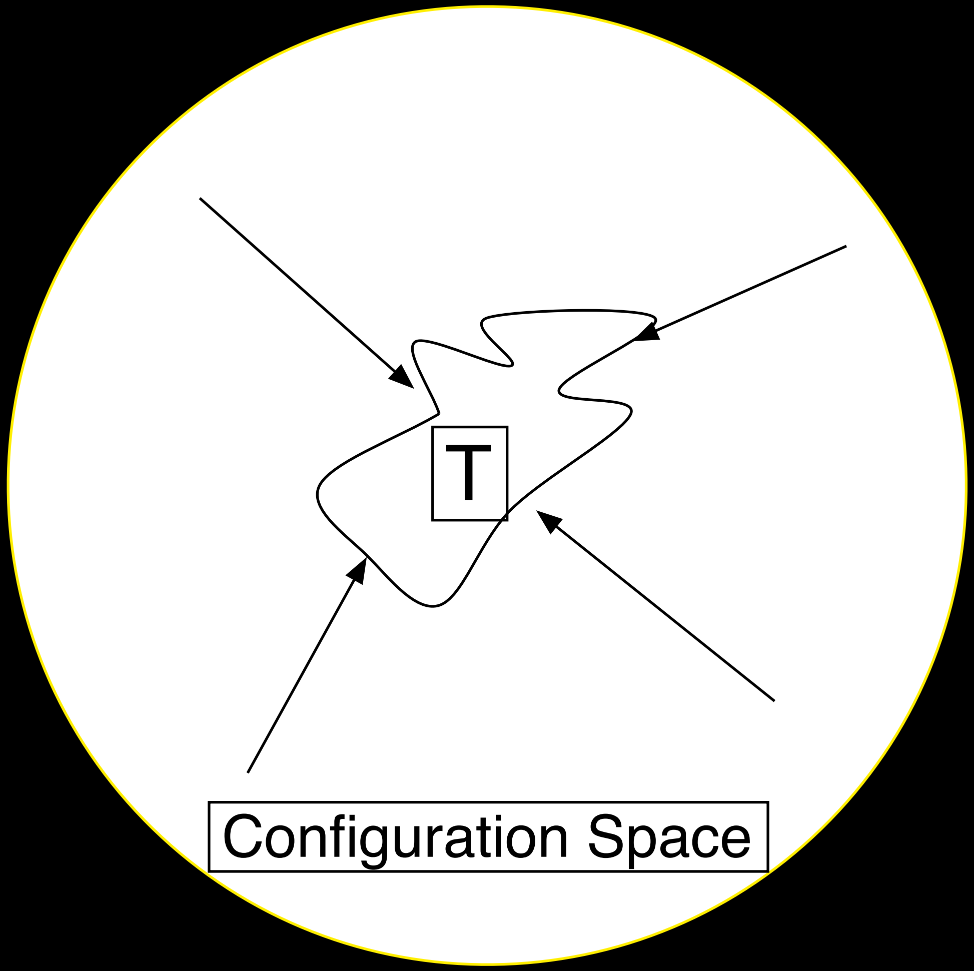

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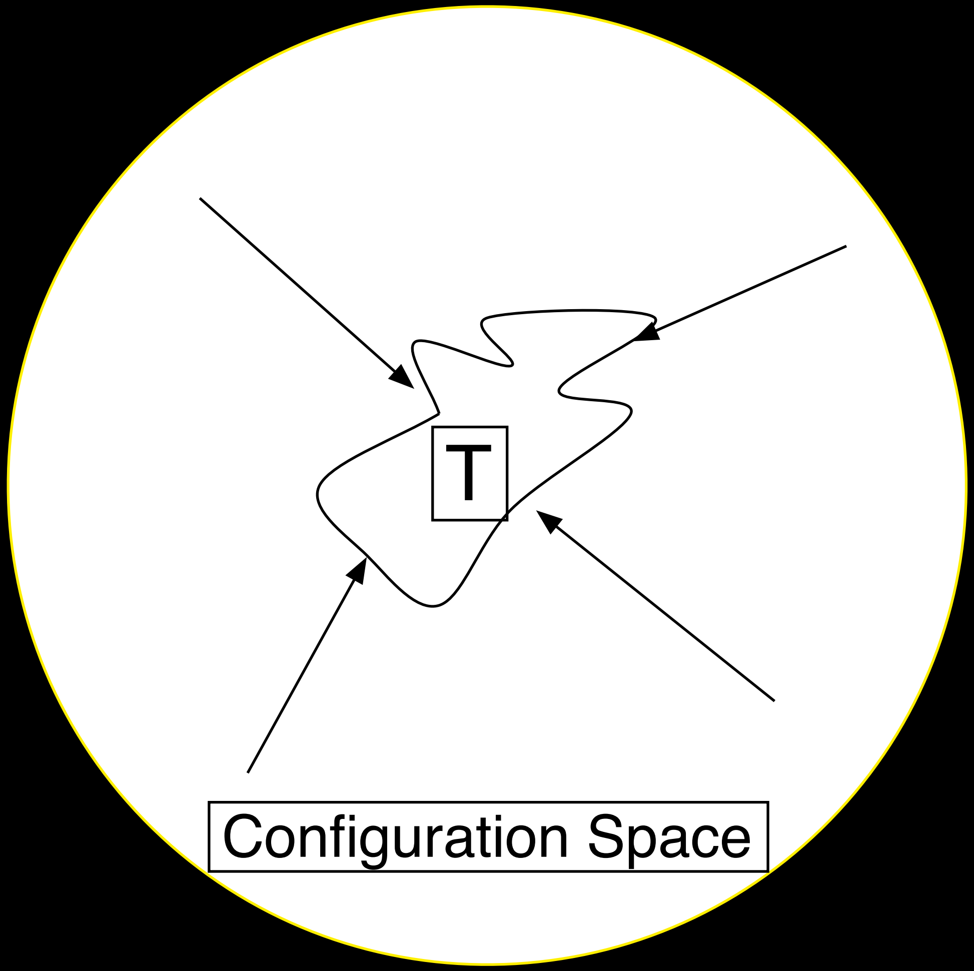

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$\mathcal{S}$	Timing model

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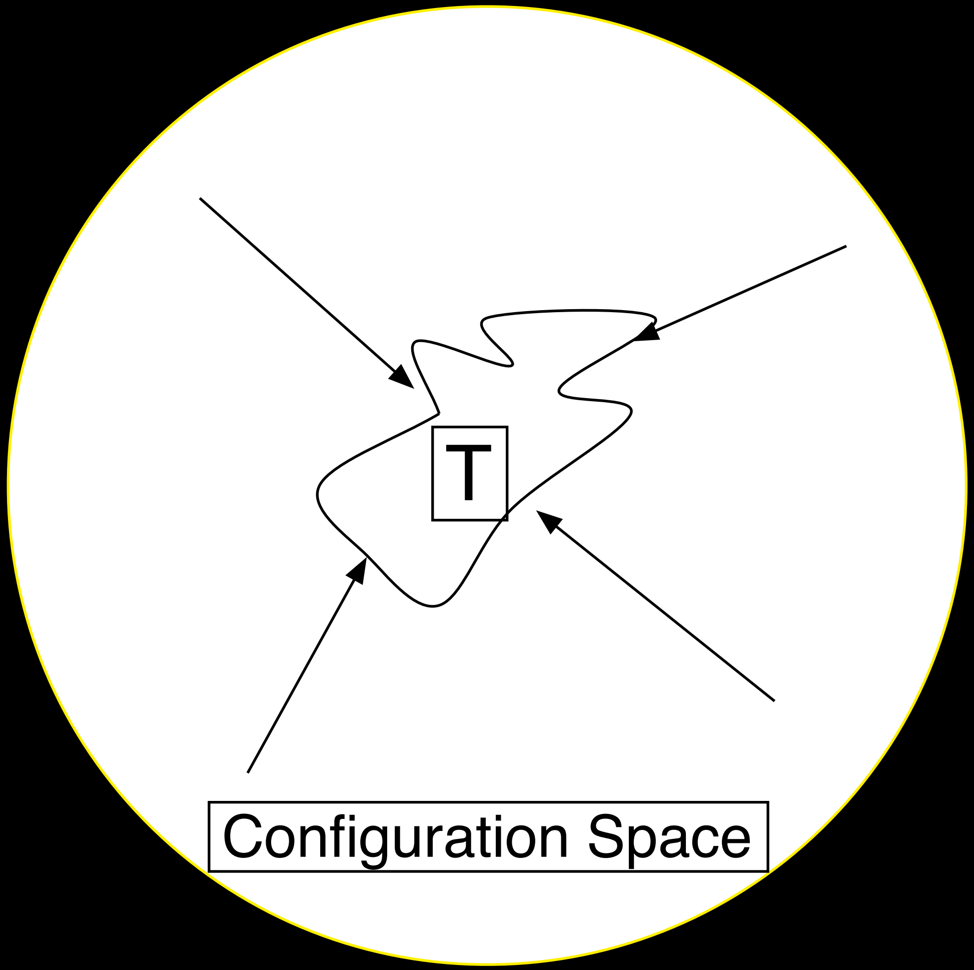

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T	Pattern

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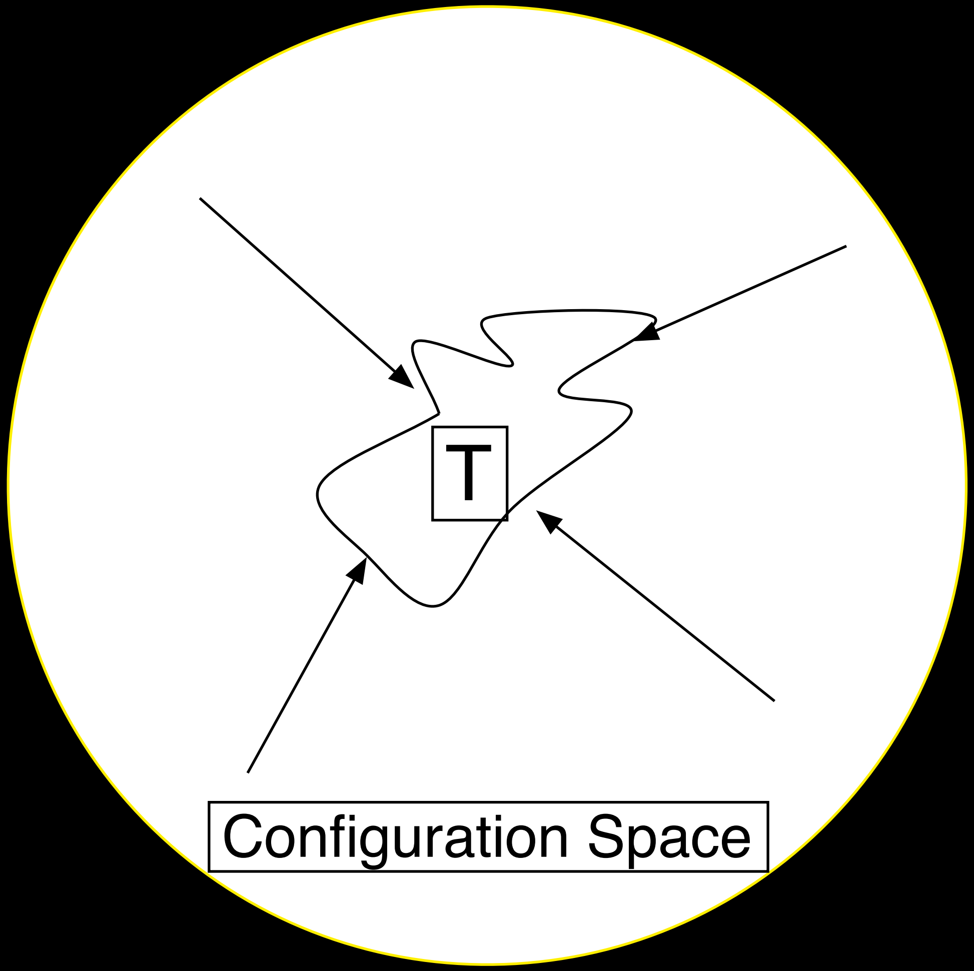

Disorder to Order

$\mathcal{G}$	Underlying Geometry
S	State set of size m
R	Communication radius
$\mathcal{S}$	Timing model
T	Pattern

$\Longrightarrow$ F Robust Solution

Other models: Amorphous Computing

Other models: Flocking & Sorting

Other models: Developmental Biology

Other models: Reconfigurable Robots

Other models: Pattern and Task Abstractions