Retirement Lockboxes

William F. Sharpe
Stanford University

CFA Society of San Francisco
January 31, 2008
Based on work with:

• Jason Scott and John Watson
  – Financial Engines’ Center for Retirement Research

• For more, see *Post-retirement Economics* at [www.wsharpe.com](http://www.wsharpe.com)
The Prototypical Problem

• An individual or family has $W$ dollars to finance retirement and
• Must choose a Retirement Financial Strategy, which includes decisions about:
  – Investment
  – Spending
  – Annuitization
Technologies Needed to find the Best Strategy

- Asset Pricing Theory
- Behavioral Economics
- Financial Engineering
- Operations Research
A Retirement Lockbox Strategy

• An analytical approach
  – Can provide greater clarity about the characteristics of traditional retirement financial strategies

• An actual approach
  – Can be tailored to provide better results for some retirees
  – Can provide better discipline to deal with problems associated with declining mental acuity
A Retirement Lockbox
Retirement Lockbox Characteristics

• Owner
  – Bill Sharpe

• Maturity Date
  – 2020

• Initial Investment
  – $20,000

• Investment Strategy
  – 60% Stocks, 40% Bonds, Rebalance annually

• Beneficiary
  – Monterey Institute of International Studies
Types of Retirement Lockboxes

• **Bequest**
  – Beneficiary gets the box if the owner is dead before the maturity date

• **Annuity**
  – An insurance company:
    • gets the box if the owner is dead before the maturity date
    • manages the investment strategy
    • matches the ending value in a pre-specified ratio if the owner is alive at the maturity date
A Retirement Lockbox Strategy
Individuals’ Performance When Making Financial Decisions

Home Equity Loan Interest Rates

"The Age of Reason: Financial Decisions Over the Lifecycle"
"The Age of Reason: Financial Decisions Over the Lifecycle"
The Simplest Possible Risky Capital Market

• Two periods
  – Now
  – Next year

• Two future *states of the world*
  – The market is up
  – The market is down

• Two securities
  – A riskless real bond
  – A portfolio of risky securities in market proportions
Capital Market Characteristics

**Bond**

- **0**: 1.00 with prob = 0.50
- **1.02** with prob = 0.50

**Market Portfolio**

- **0**: 1.00 with prob = 0.50
- **1.18** with prob = 0.50
- **0.94** with prob = 0.50
Desired Spending

\[ Spending \]

\[\begin{array}{c}
\text{0} \\
50.00 \\
55.80 \\
48.60 \\
\end{array}\]

\[\text{prob } = 0.50 \]

\[\text{prob } = 0.50 \]
Wealth, Financial Strategy and Desired Spending

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## Initial Wealth

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Market Portfolio Investment

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0 & 1.02 & 0.94 & 48.60 \\
\end{array}
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Wealth, Financial Strategy, Capital Markets and Spending

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Capital Market Characteristics

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Decisions → Spending

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Spending $\rightarrow$ Decisions

$x' = C^{-1}s$

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Lockbox, Period 1

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Desired Spending: Multiple Periods

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Dynamic Strategies

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| s | 50.00 | 55.60 | 49.20 | 63.84 | 53.76 | 52.80 | 47.04 |

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### Notes
- The table represents different dynamic strategies and their corresponding values for W, B0, M0, Bu, Mu, Bd, and Md.
- The values in the table are numerical and can represent costs or other metrics.
- The feedback values (s) are calculated based on the strategies and can be used to evaluate the effectiveness of each strategy.
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Lockbox Separation (1)

- A retirement financial strategy is **fully specified** if spending in each year can be determined for any scenario of market returns.
- A market is **complete** if any desired spending plan can be implemented with a retirement financial strategy.
- If the market is complete, any fully specified retirement financial strategy can be implemented with a lockbox strategy.
Lockbox Separation (2)

• If a market is not complete
  – it may or may not be possible to implement a given retirement financial plan with a lockbox strategy
  – or, if there is a comparable lockbox strategy it may incur added expense

• But many popular retirement financial plans have equal-cost lockbox counterparts

• Prime examples are the Fidelity Income Replacement Funds
The Fidelity Income Replacement Funds

• Horizon date
  – E.g. 2036

• Investment strategy
  – Time-dependent “glide path” asset allocation

• Spending Rule
  – Pre-specified time-dependent proportions of asset value
Fund Characteristics  
(from prospectus)

• “The Income Replacement Funds are designed for investors who seek to convert accumulated assets into regular payments over a defined period of time …

• The payment strategy for each Income Replacement Fund is designed to be implemented through a shareholder’s voluntary participation in the Smart Payment ProgramSM …

• Each Income Replacement Fund’s investment objective is intended to support the Smart Payment Program’s payment strategy …

• The income Replacement Funds are not designed for the accumulation of assets prior to retirement… [but they] do not provide a complete solution for a shareholder’s retirement income needs.”
Spending Rule

Annual Target Payment Rates

Years to Horizon

Percent of Fund Spent

0 10 20 30 40 50 60 70 80 90 100
Investment Strategy
Lockbox Equivalence

• Any strategy with a time-dependent proportional spending rule and a time-dependent investment strategy is equivalent to a lockbox strategy

• Each lockbox will have the same investment strategy and

• The initial amounts to be invested in the lockboxes can be computed from the pre-specified spending rates
Initial Lockbox Values (1)

• Let:
  \[ K_t = \text{the proportion spent in year } t \]
  \[ R_t = \text{the total return on investment in year } t \]
  \[(e.g. 1.02 \text{ for } 2\%)\]

• The amounts spent in the first three years will be:
  \[ Wk_0 \]
  \[(1-k_0)WR_1k_1 \]
  \[(1-k_0)WR_1(1-k_1) R_2k_2 \]
Initial Lockbox Values (2)

- Re-arranging:
  \[
  \{W_k_0\} \\
  \{W(1-k_0)k_1\} R_1 \\
  \{W(1-k_0)(1-k_1)k_2\} R_1R_2 
  \]

- But these are the ending values for lockboxes with the initial investments shown in the brackets \{ \}
  - investing these amounts in lockboxes will give the same spending plan as the original strategy
Percentages of Initial Wealth in Lockboxes
Rover: a Simple Income Replacement Fund

• Two assets
  – A riskless real bond
  – A market portfolio
    • (e.g. 60% Stocks, 40% Bonds)

• A glide path similar to that for equity funds in the Fidelity Income Replacement Funds

• A 30-year horizon

• Annual payment rates equal to those of the Fidelity Income Replacement Funds
Rover: Investment Strategy

![Graph showing investment strategy over time to horizon date. The graph illustrates a decreasing trend in the percent in market portfolio as time to horizon date increases.](image)
Rover: Percentages of Initial Wealth in Lockboxes
Capital Market Characteristics

- Riskless real return
  - 2 % per year

- Market portfolio real return
  - Lognormally distributed each year
  - Expected annual return
    - 6 % per year
  - Annual standard deviation of return
    - 12 % per year
  - No serial correlation from year to year
Monte Carlo Simulations

• 10,000 scenarios of 30 years each
• Returns for each lockbox are simulated
  – Results are the same as those for the original strategy
• The original set of scenarios is then used to evaluate alternative strategies
Rover: Spending in Year 30 per dollar invested in lockbox
Rover: Spending in Year 30: Strategy versus Market
Market Risk and Path Risk

- Market risk
  - Uncertainty about return due to uncertainty about cumulative market return

- Path risk
  - Uncertainty about return due to uncertainty about the path market returns will take

- In this setting, only market risk is rewarded with higher expected return
Minimizing Path Risk

- Sort all 10,000 amounts to be spent in the year from highest to lowest.
- Construct a strategy with the highest return in the scenario with the highest cumulative market return, the next highest return in the scenario with the next highest market return, and so on.
- This equal-distribution market strategy will have precisely the same distribution of spending with minimum path risk.
Rover: Spending in Year 30: Two Strategies versus Market
The Equal-distribution Market Strategy

• Provides returns almost the same as those from a constant-mix strategy rebalanced annually to give
  – 71% in the market portfolio
  – 29% in the riskless bond
• But it is cheaper to obtain these results since only market risk is taken
• Following such a constant-mix strategy with the funds in the lockbox will produce higher returns
  – In this case, over 11% better
Rover: Spending in Year 30: Three Strategies versus Market
Rover: Spending in Year 30: Glide Path versus Constant Mix

![Cumulative Returns, year 30: mean gain: 11.3%](chart)

- **Cumulative Returns, year 30: mean gain: 11.3%**
- **Axes:**
  - **Y-axis:** Probability of beating goal
  - **X-axis:** Goal
- **Graph:**
  - Blue line: Glide Path
  - Red line: Constant Mix
An Alternative Strategy

• Each lockbox follows a constant mix strategy
• The proportions invested in the market portfolio differ among boxes
• Boxes for later dates have more conservative asset allocations
• The distribution of outcomes for each year will be better than that for the original strategy
  – But the improvements will be greater for boxes with later dates
Is the Alternative Strategy Better?

• Probably for most retirees
• But it can provide more variation in spending from year to year
Percent Change in Spending for Two Strategies: Year 29 to Year 30
The Ultimate Goal

• To find the best retirement financial plan for a given retiree or retiree family
• This will depend on
  – Capital market characteristics
  – Personal preferences
Finding an Optimal Retirement Financial Strategy

Maximize:

\[ H(s) \]

Subject to:

\[ x(1) = W \]

\[ x = C^{-1}s \]

where \( H(s) \) is the investor’s happiness with spending plan \( s \)
Happiness and Future Spending

- If a strategy determined today is to be followed without change, the appropriate objective is to maximize the happiness a retiree gets today from contemplating future spending when he or she may be ill or have diminished mental capacity.

- Such a strategy allows a retiree to act in loco parentis for his or her future (possibly diminished) self.

- Key is representing a retiree’s personal preferences adequately.
Personal Preferences and Retirement Lockboxes

• Economists have an approach to formulating personal preferences in terms of utility functions.
• The goal is to maximize expected utility, taking probabilities of states of the world into account.
• Much more work needs to be done to adapt this framework to help solve the retirement financial problem.
• But it is likely that lockboxes can help as analytic constructs and, in some cases, in practice.
Are There Retirement Lockboxes in Your Future?