450D Political Methodology IV
TA Section 3

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October 13, 2017

Estimated time: 1hr, 15min

Goal Today

- Motivate and show the basics of MCMC.

Road map to those goals:

1. Motivation
2. Markov Chains
3. MCMC Algorithms
   - 3.1 Metropolis *
   - 3.2 Metropolis-Hastings *
   - 3.3 Gibbs sampler
   - 3.4 Hamiltonian
4. Application: Standard Normal
   - 4.1 Intuition
   - 4.2 Small step size
   - 4.3 Bigger step size
5. Briefly, next week's topics

Motivation

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- *Analytically*: we know what the mean and variance of the posterior distribution is and can state these exactly.
- *Simulation*: we have functions that allow us to draw random samples from these distributions and then compute the mean, variance, or any other quantity we are interested in.

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What is a Markov Chain?

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The conditional probability distribution is called the transition kernal, or the Markov kernel K.

$$\text{Transition distribution: } K(\theta_t|\theta_0, \cdots, \theta_{t-1}) = K(\theta_t|\theta_{t-1})$$

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I presume Doug will talk more about these conditions.

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- This transition distribution is Markovian, which means that it only depends on θ_{t-1} and not any previous values of θ beyond that.

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If these conditions hold, then the sampled parameters beyond a burn-in phase can be treated as random samples from the target density, which is the posterior density.

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3. Update θ^t to θ^* with probability α, otherwise keep θ^{t-1}.

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The reason why this algorithm is so powerful is that the proposal distribution can have any form and the invariant distribution of the resulting Markov Chain will still be the desired posterior distribution, $p(\theta|y)$.

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These are equivalent to the probability of moving forward, and moving backwards, respectively.

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5. If x^* is less likely than x_t, accept this update with probability equal to the ratio of their likelihoods
6. Update x_t based on the rules above and restart at 2

Drawing from a Standard Normal: Code

```
metro.sampler<-function (n, alpha, init){
  x <- init
  out <- c()  #a vector to hold each value of x
  out[1] <- x #store current value

  #now iterate
  for (i in 2:n) {
    can <- x + runif(1, -alpha, alpha) #step size
    aprob <- min(1, dnorm(can)/dnorm(x)) #prob of update
    x <- ifelse(runif(1) < aprob, can, x) #update x
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  }

  return(out)
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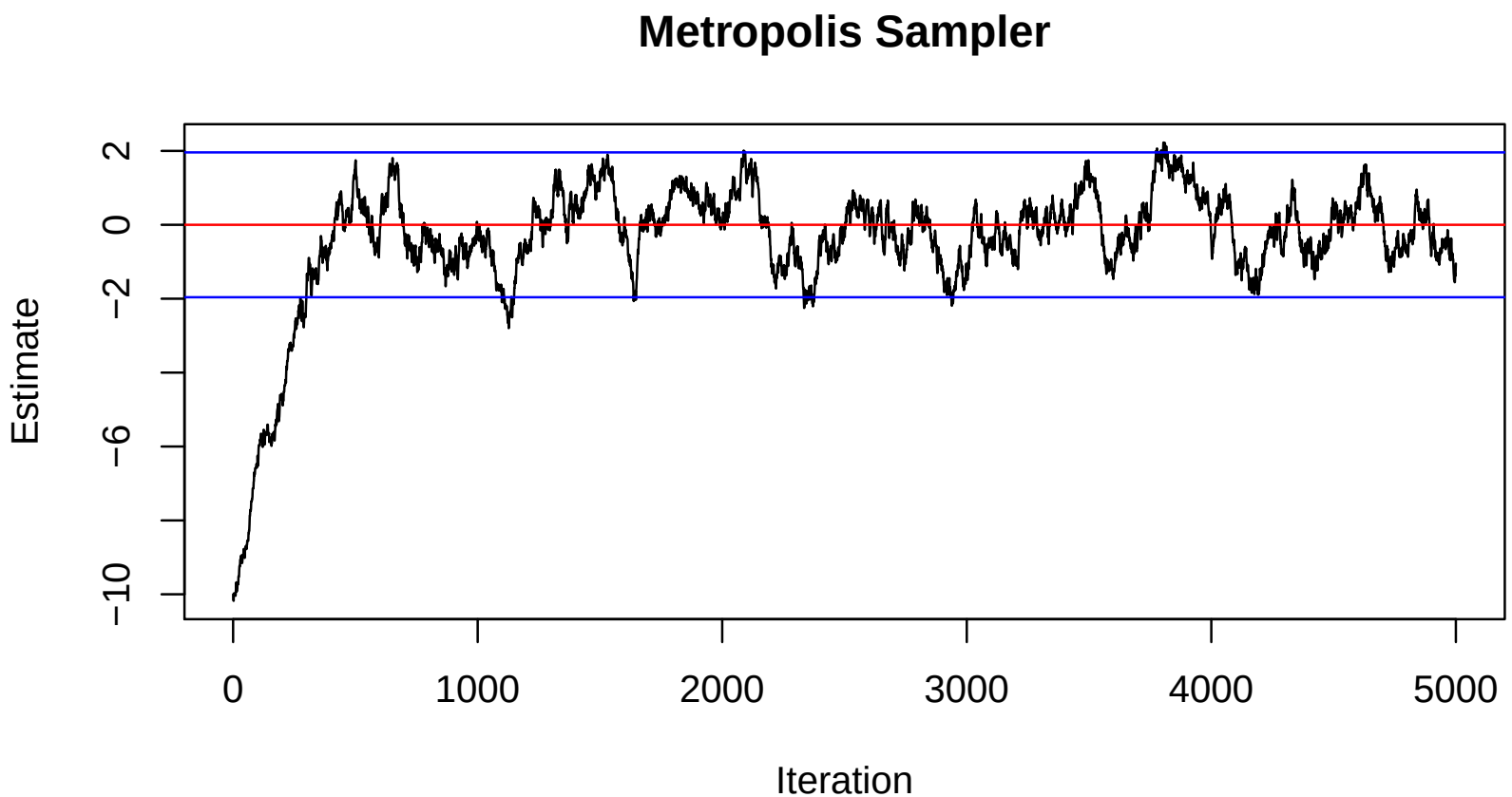
```

We could replace a standard normal with any other distribution that we can write the PDF for.

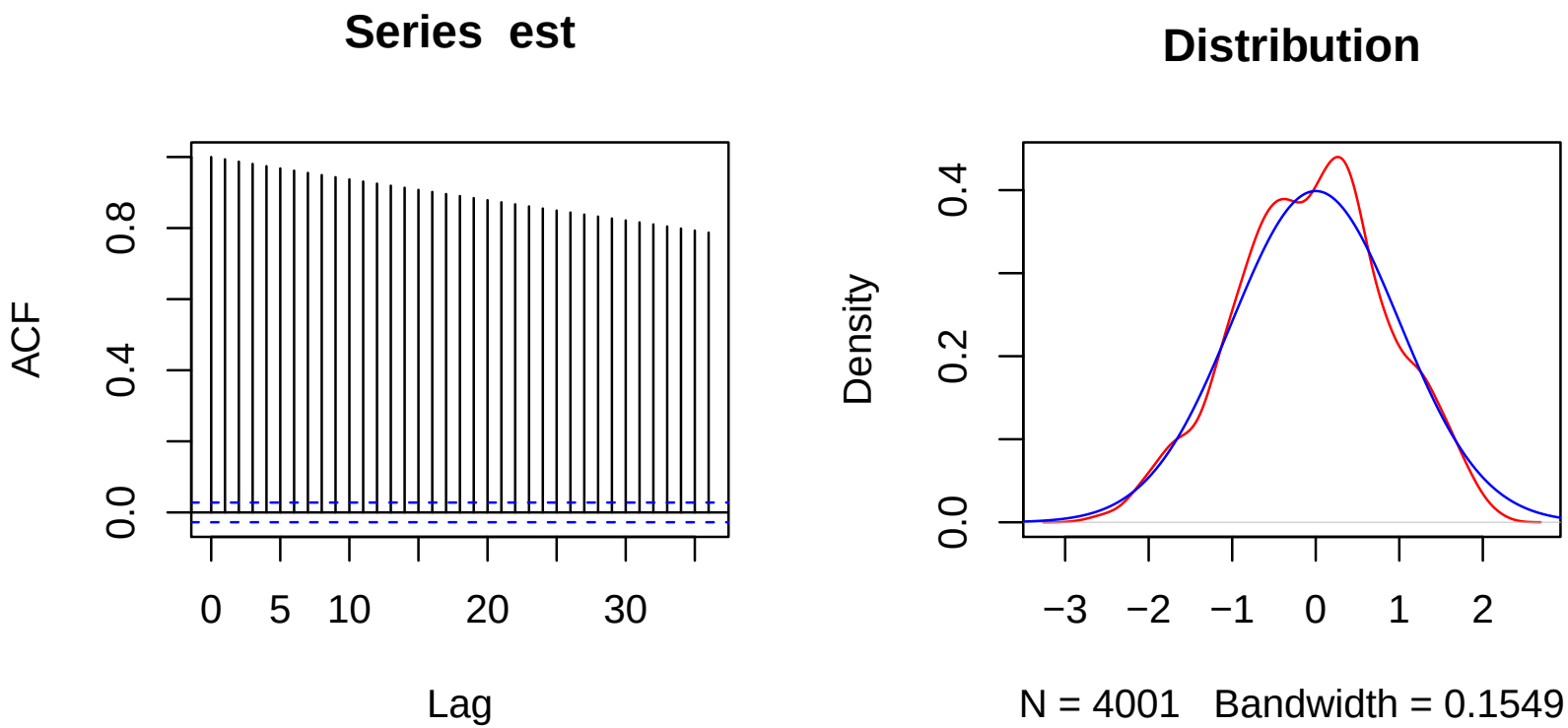
Drawing from a Standard Normal: Implement

```
m <- 5000
est <- metro.sampler(m, alpha=.25, init=-10)
plot(x=1:m, y=est, type="l", main="Metropolis Sampler", xlab="Iteration
abline(h=0, col="red")
abline(h=1.96, col="blue")
abline(h=-1.96, col="blue")
```



Drawing from a Standard Normal: Diagnostics

```
par(mfrow=c(1,2))
acf(est)
plot(density(est[1000:m]), main="Distribution", col="red")
x <- seq(-5, 5, .01)
lines(x=x, y=dnorm(x), col="blue")
```



Notice that I'm dropping the first 1000 samples, *burn-in*.

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- As a result of this autocorrelation, the distribution of our estimates is not quite right.
- If we got the right distribution, we could say anything about it, not just the mean.
- Side note: some autocorrelation is actually helpful, we could have drawn similar values with a simple method called the "Accept-Reject" method which will not have autocorrelation but we will need more samples.

Drawing from a Standard Normal: Bigger steps

Exactly the same code but I changed the step size rom .25 to 1.5.

```
est <- metro.sampler(m, alpha=1.5, init=-10)
plot(x=1:m, y=est, type="l", main="Metropolis Sampler", xlab="Iteration
abline(h=0, col="red"); abline(h=1.96, col="blue"); abline(h=-1.96, col
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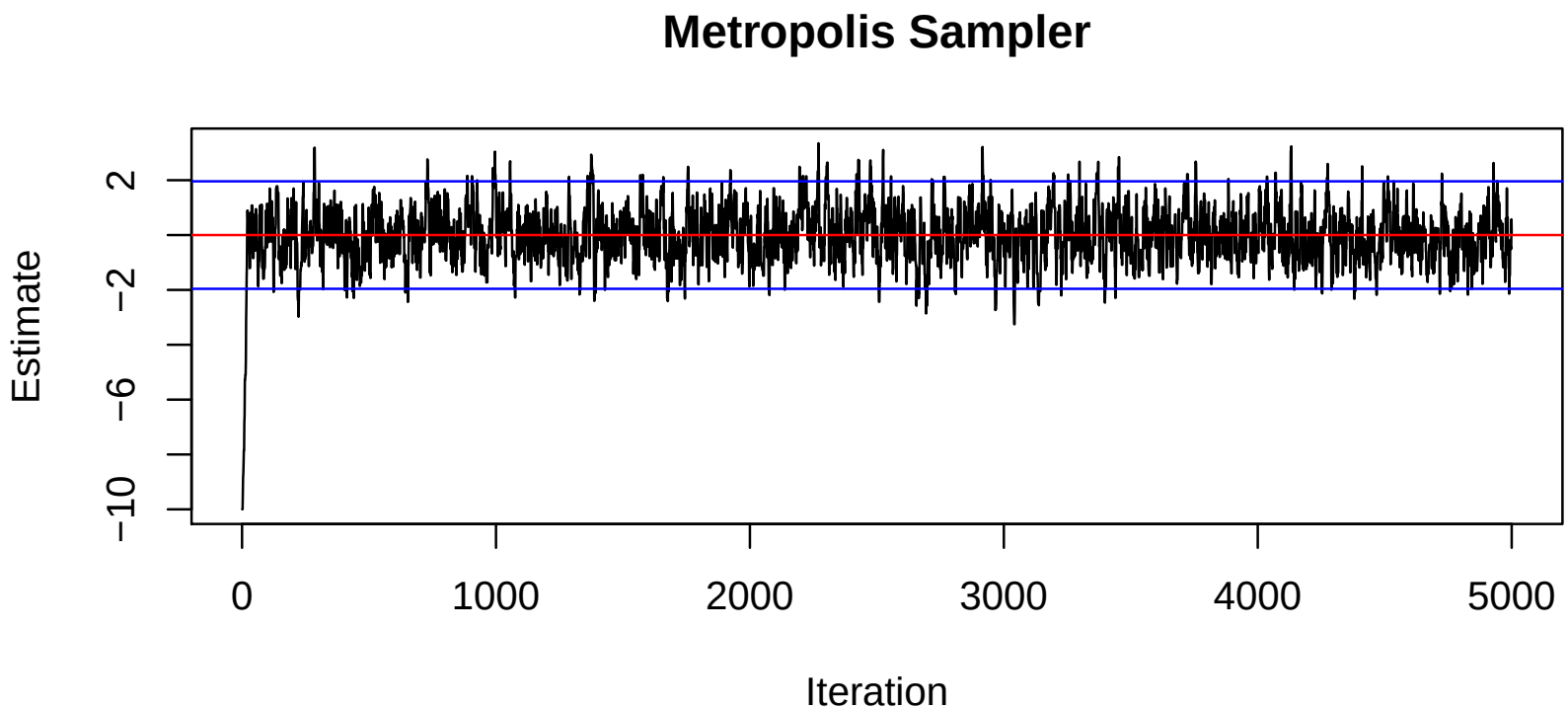

This plot looks much better, you want a "caterpillar" effect.

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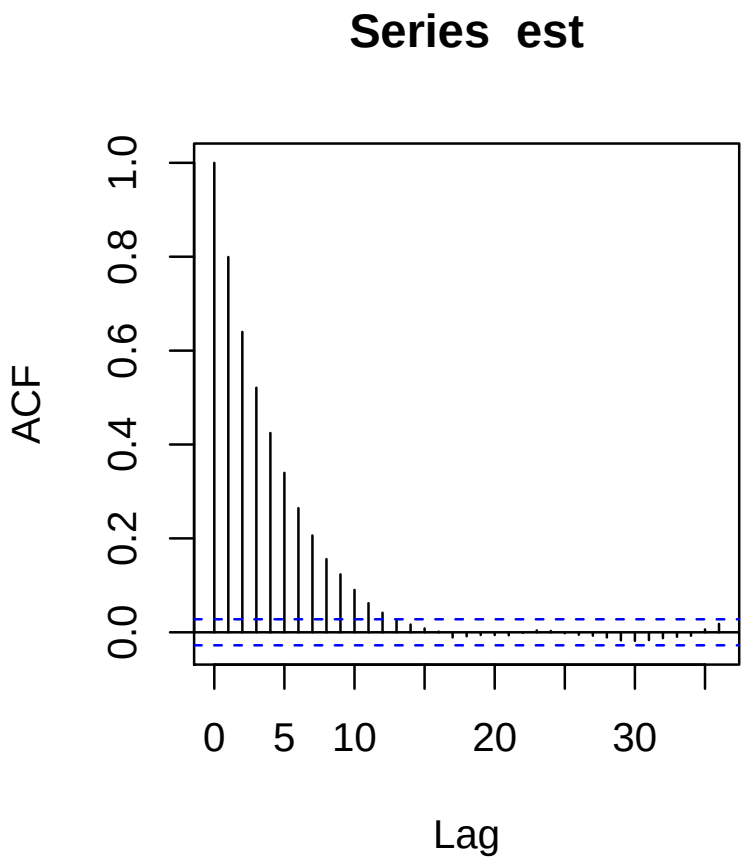


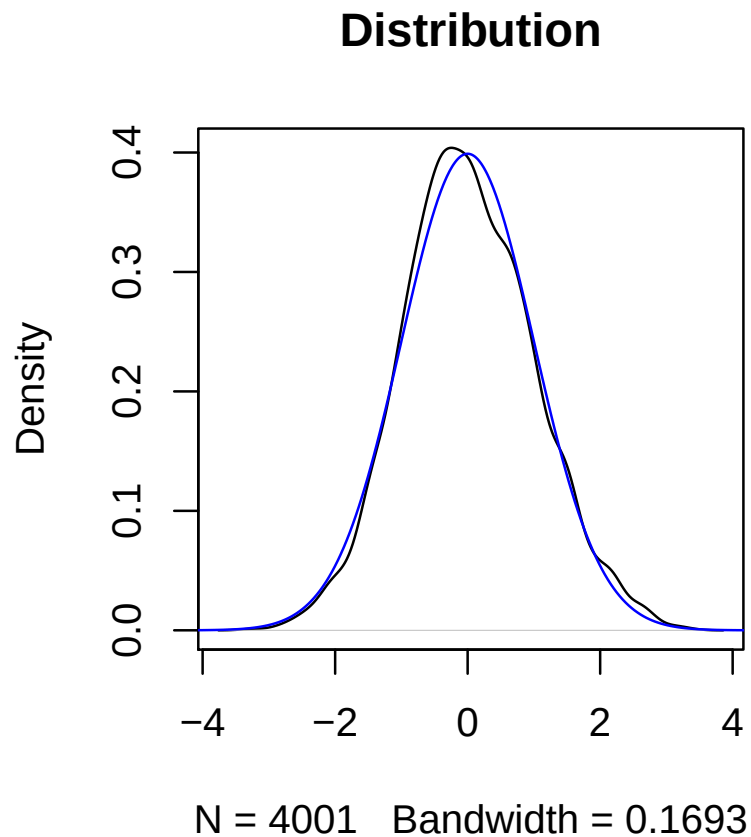
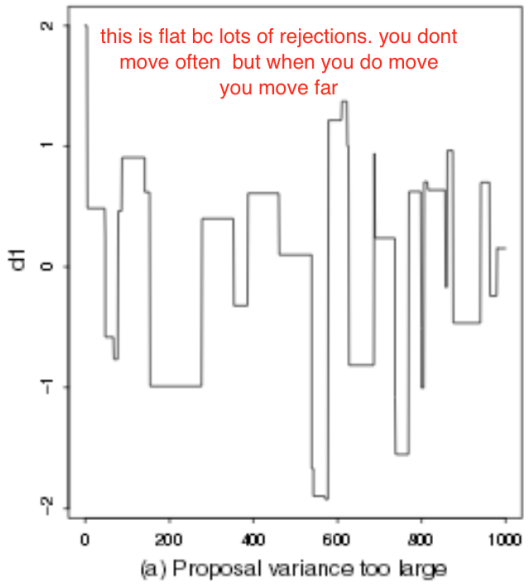


Figure: Step size and autocorrelation

too big!



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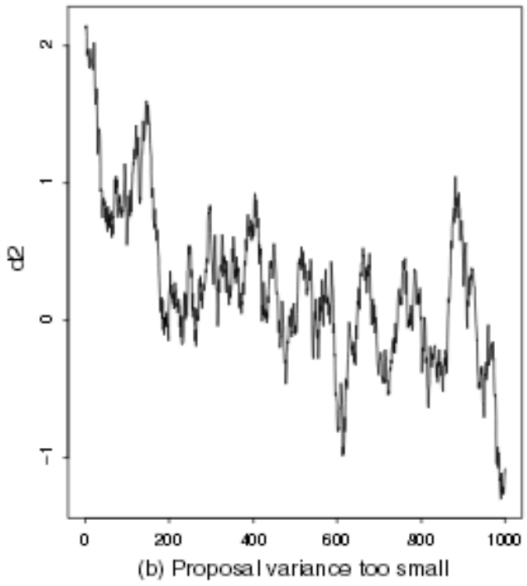
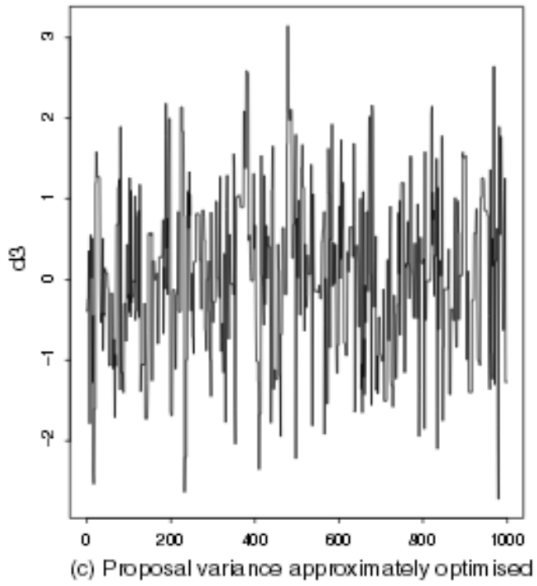
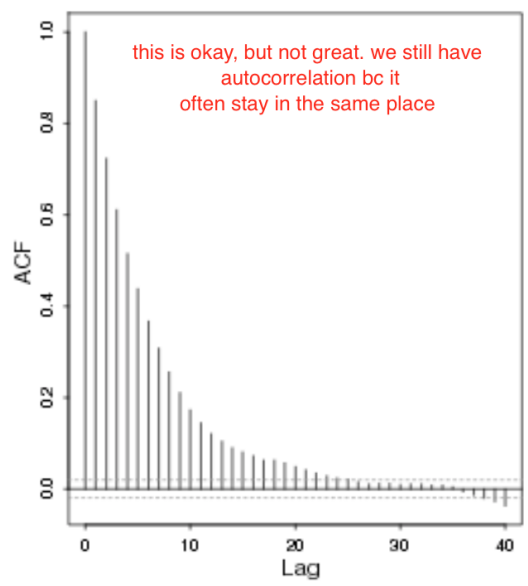
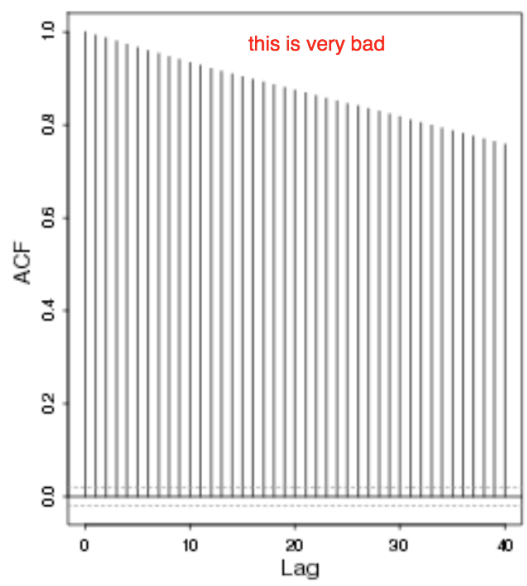


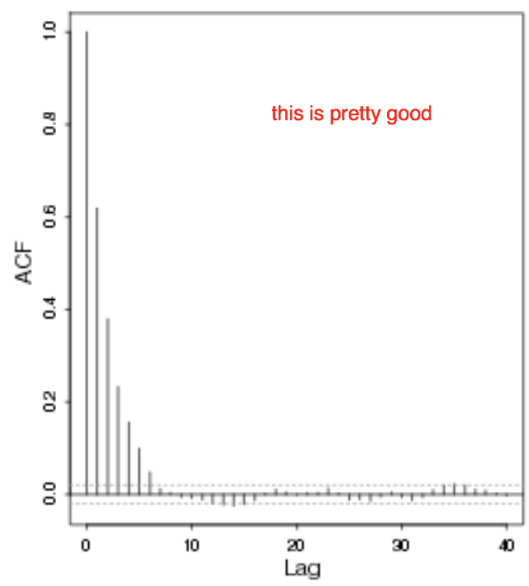
just right



these are autocorrelations between drags, want it to decrease quickly







Next week

Bayesian Poisson regression from scratch

Next week

Bayesian Poisson regression from scratch (and also not from scratch).