Pulse Lengthening Via Overcoupled Internal Second-Harmonic Generation*

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The paper discusses the use of internal second-harmonic generation as a means of lengthening the pulses of Q-switched lasers. The second-harmonic generator acts as both a nonlinear power-dependent loss and as an output coupler for the laser. The lengthening arises if the coupling to the second harmonic is greater than that necessary to maximize the peak second-harmonic power. It is shown that a pulse-lengthening factor of about 100 should be obtainable.

INTRODUCTION

In many nonlinear applications, pulses which are somewhat longer than may be obtained from normal Q-switched lasers are desirable. Such pulses would allow interactions to reach steady state and also reduce the possibility of various types of optical damage. Several authors have described techniques which make use of a nonlinearly increasing loss to achieve this pulse lengthening.[1-8] Such techniques have employed stimulated Rayleigh, Raman, and Brillouin scattering,[2,3] two-photon absorption,[4-7] and nonlinear free carrier generation[4,6] as the means of obtaining the nonlinear loss. In this paper we describe the use of second-harmonic generation as the internal loss mechanism.[4] In applications where the useful output is the second harmonic of the pumping laser, this approach is particularly advantageous because the pulse lengthening is achieved at no expense in output energy.

In the technique proposed here, the nonlinear doubling crystal is placed inside the laser cavity where it acts as both the output coupling for the laser and as the nonlinear loss. As the laser power builds up, the loss resulting from second-harmonic generation increases as the square of the fundamental power; peak power is reached when this loss has increased to where it is equal to the single-pass gain. Once this gain equals loss point is reached, the laser power may no longer increase, and the remaining population inversion is used in lengthening the pulse. If the coupling to the second harmonic is substantially greater than that necessary to optimumly couple, i.e., to produce the maximum peak power at the second-harmonic wavelength, then the majority of the inversion will be used to lengthen the pulse. For such output couplings there is a direct trade-off between pulse amplitude and length, and the energy of the second-harmonic pulse remains constant as a function of second-harmonic coupling.

THEORY

The rate equations[9] for a laser system with an additional term in the photon density equation representing the loss to second-harmonic generation are[10]

$$dN/dt=W-(\sigma c/LA)uN-N/\tau_{tot}, \quad (1)$$

$$du/dt=-u/\tau_c+(\sigma c/LA)N(u+1)-Ku^2, \quad (2)$$

where

- N Population inversion
- W Pumping rate

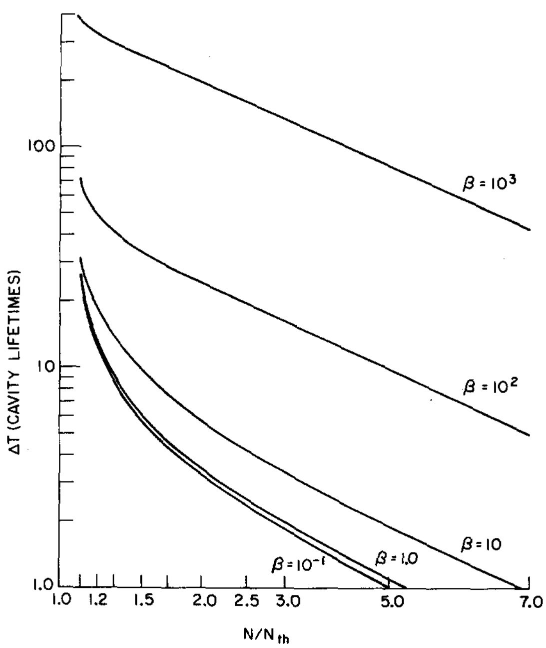

FIG. 1. Full width at half-amplitude of second-harmonic pulse versus initial inversion.

- u Total number of photons per pertinent cavity mode
- τ_c Photon lifetime
- τ_{tot} Total lifetime of upper state
- K Nonlinear coupling coefficient
- L Optical length of resonator
- A Beam area in laser material
- σ Cross section for the laser line of interest $=\lambda^2/4\pi^2\epsilon\tau_l\Delta\nu$

ϵ Relative dielectric constant
τ_l Radiative spontaneous lifetime of upper state
$\Delta\nu$ Full half-power linewidth.

In terms of the second-harmonic crystal parameters, K is given by

$$K = h\nu(c/L)^2(P_{SH}/P_F^2),$$
$$= (8\pi h/w^2)(\eta\nu)^3(dl)^2(c/L)^2,$$

where

P_F Internal fundamental power
P_{SH} Generated second-harmonic power
w Fundamental beam radius in second-harmonic crystal
η 377/refractive index of nonlinear crystal at second harmonic
ν Fundamental laser frequency in hertz
l Length of nonlinear crystal
d Nonlinear coefficient

and where near field focusing has been assumed.

For Q-switched operation,[11,12] the pumping term and the spontaneous emission terms can be neglected and Eqs. (1) and (2) may be normalized to yield

$$dn/dT = -\phi n, \tag{3}$$

$$d\phi/dT = \phi(n-1) - \beta\phi^2, \tag{4}$$

where[10]

$$n = N/N_{th} = (\sigma c/LA)\tau_c N,$$

$$\phi = (\sigma c/LA)\tau_c u,$$

$$T = t/\tau_c,$$

$$\beta = (LA/\sigma c)K.$$

We will define β as the normalized coupling parameter; it can also be written

$$\beta = \left(\frac{c}{L}\right) \times \left(\frac{\text{Second-harmonic conversion efficiency per fundamental power}}{\text{Single-pass gain per stored energy in inversion.}}\right),$$

where the second-harmonic conversion efficiency is defined as P_{SH}/P_F.

RESULTS

Figures 1–5 show normalized second-harmonic pulse length, peak second-harmonic power, total second-harmonic energy, fundamental pulse width, and peak fundamental power as a function of the normalized initial inversion n and the normalized coupling parameter β. The relations between the actual parameters and the normalized ones are[10]

$$P_{SH} = (h\nu LA/\sigma c\tau_c^2)\beta\phi^2,$$

$$P_{\text{Fund}} = (h\nu A/\sigma\tau_c)\phi,$$

$$E_{SH} = (h\nu LA/\sigma c\tau_c)\mathcal{E}_{SH}.$$

From Figs. 1 and 4 it is seen that until β is greater than 1, very little pulse lengthening is obtained. Note that Fig. 3 confirms that once the coupling exceeds approximately 10, the energy in the second-harmonic pulse remains constant, giving a direct trade-off between peak amplitude and pulse length. Figure 6 shows the computer-generated second-harmonic pulse shapes for $n_0 = 2.0$ and five different values of β. Figure 7 shows the value of β which maximizes the second-harmonic peak power for a given value of initial inversion. Unlike the cw case,[13,14] this optimum coupling for a Q-switched laser is a function of the initial inversion. Figure 8 shows the peak second-harmonic power that may be obtained from such an optimumly coupled laser.

The range of validity of these solutions is limited to the region where the generated second-harmonic power is nearly proportional to the square of the fundamental

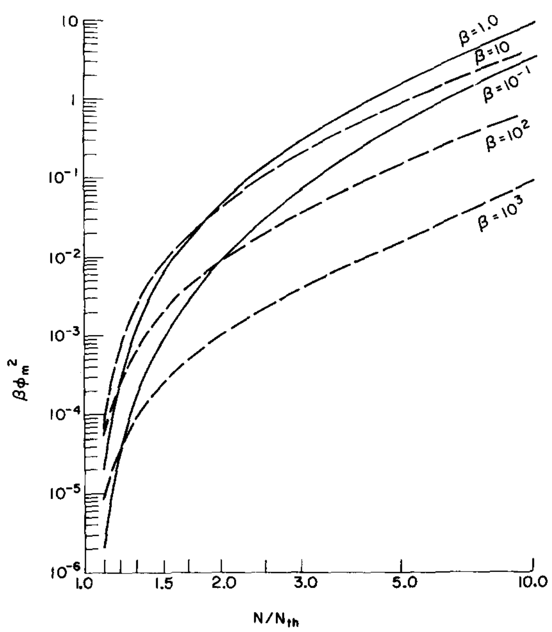

FIG. 2. Peak second-harmonic power versus initial inversion. This gives the total second-harmonic power generated and does not account for the fact that it is typically generated in both directions.

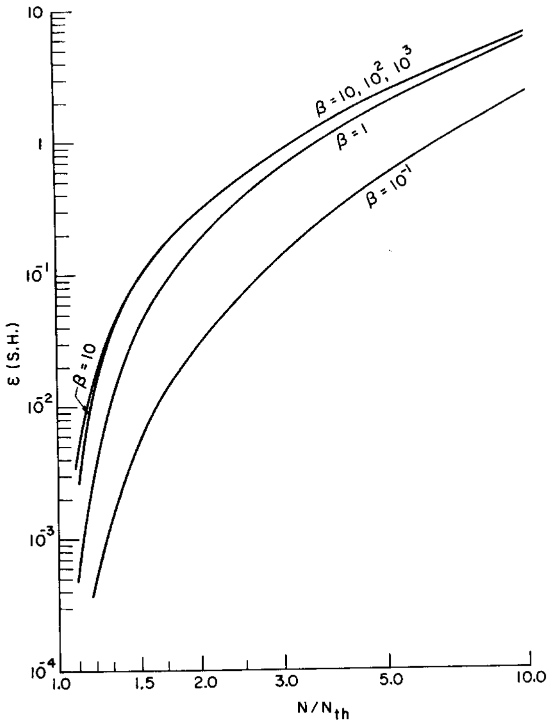

FIG. 3. Second-harmonic energy versus initial inversion.

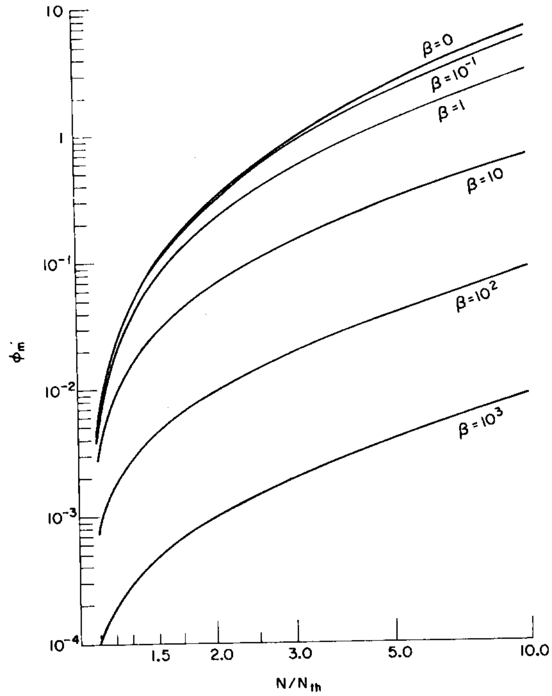

FIG. 5. Peak fundamental power internal to the laser cavity versus initial inversion.

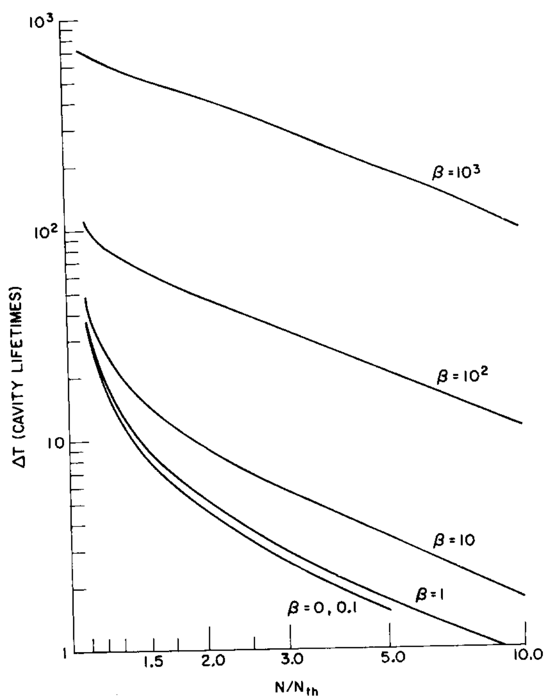

FIG. 4. Full width at half-amplitude of fundamental laser pulse versus initial inversion.

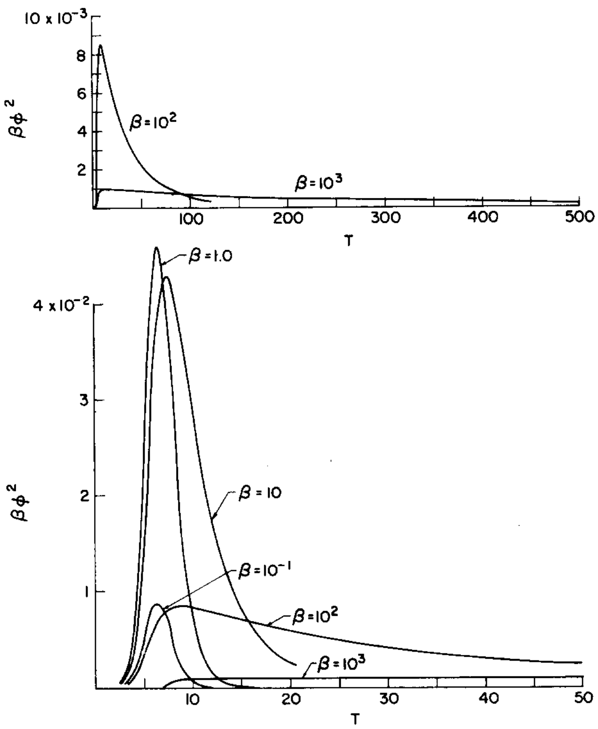

FIG. 6. Computer-generated waveforms for five values of the normalized coupling parameter β, and an initial inversion of 2.0. The upper figure shows the waveforms for $\beta = 10^2$ and 10^3 with an expanded time scale.

TABLE I. Transition cross sections and normalized coupling parameters for a few laser lines and second-harmonic crystals.

Laser system	σ (cm^2)	β				
		KDP[e,f]	α-HIO_3[e,g]	LIO_3[h,i,n]	$LiNbO_3$[j,k]	$Ba_2NaNb_5O_{15}$[l,m]
Nd^{3+}:YAG 0.946 μ	6.3×10^{-20} [a,b]	8.2	60	1.2×10^2	8.3×10^4	7.2×10^5
Nd^{3+}:YAG 1.064 μ	8.8×10^{-19} [b]	0.47	3.7	5.8	3.8×10^3	3.3×10^4
Nd^{3+}:YAG 1.318 μ	2.9×10^{-19} [c,b]	0.89	4.1	8.7	4.9×10^3	4.3×10^4
Ruby 0.694 μ	2.0×10^{-20} [d]	42	···	1.0×10^3	···	···

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power. For an exact large-signal solution, it would be necessary to replace the $\beta\phi^2$ term in Eq. (4) by

$$(c\tau_c/L)\phi \tanh^2[(\beta L/\tau_c c)\phi]^{1/2}.$$

However, if this is done, the equations do not normalize to a single parameter. If we require that the argument of the hyperbolic tangent function be less than 0.45, the error introduced to the differential equations by the small-amplitude approximation is less than 12%. This

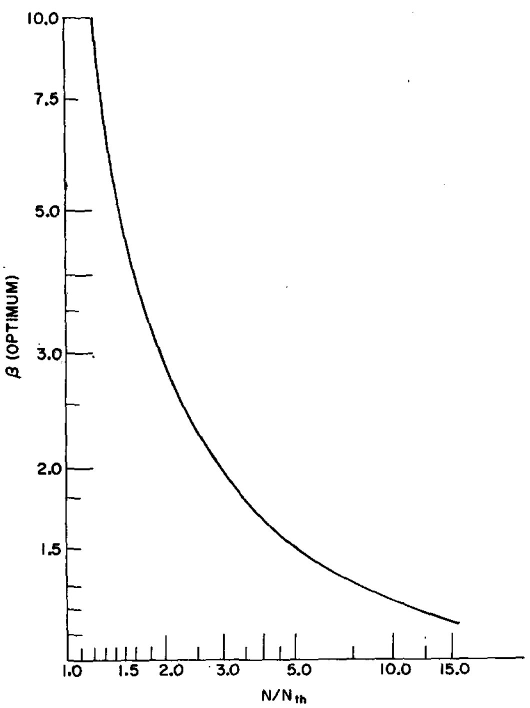

FIG. 7. Optimum coupling parameter for maximum second-harmonic peak power versus initial inversion.

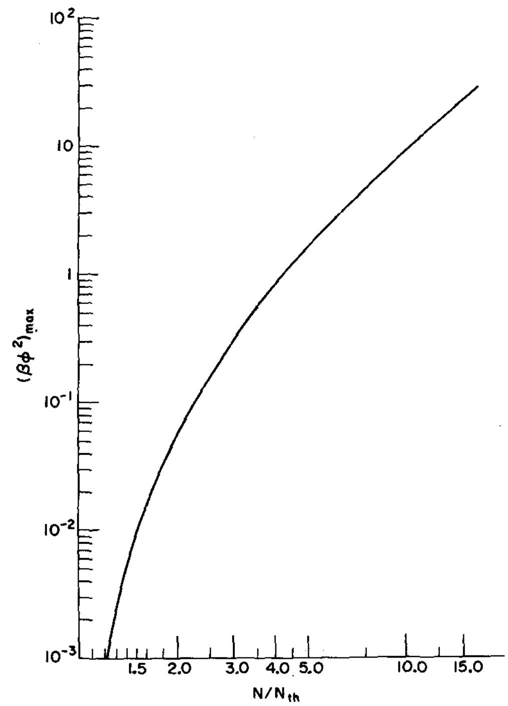

FIG. 8. Peak second-harmonic power versus initial inversion for optimum coupling.

condition can be written

$$(\beta L/\tau_c c)\alpha=\beta\phi\alpha<0.2,$$

where α is all single-pass cavity losses except that due to second-harmonic generation. Using this relation and Fig. 5, the regions of validity according to the above criteria can be obtained for any value of cavity loss, α.

PARAMETERS FOR SOME COMMON NONLINEAR CRYSTALS AND LASERS

Table I gives cross sections and normalized coupling parameters for a few of the common second-harmonic crystals and laser lines. For 90° phase matchable crystals such as $LiNbO_3$ and $Ba_2NaNb_5O_{15}$ the fundamental beam radius used in determining β was taken to be that which would confocally focus a $\frac{1}{2}$-cm crystal at the fundamental wavelength. For crystals such as KDP, LIO_3, and α-HIO_3 which exhibit walk-off, the fundamental beam size was chosen according to the criteria $w_0=\sqrt{2}\rho l$, where ρ is the walk-off angle in radians and l the crystal length. This criteria is somewhat conservative in terms of the total second-harmonic power which may be obtained from the laser, but it ensures a good Gaussian mode.[15] We have assumed a beam radius of 0.75 mm in the laser material and a cavity length of 60 cm; and all values of β are quoted without regard to damage thresholds.

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