A FIRST Encounter with Physics

Steven Bell

May 12, 2011

SVN Revision 6

Introduction

The purpose of this document is to give high school students a basic introduction to physics, using examples and applications from the FIRST Robotics Competition. It is intended to help students who have not yet taken physics to be able to solve some of the physics-related problems that come up in the FRC robot design process. A knowledge of algebra is required; understanding of geometry and basic trigonometry (specifically sine and cosine) will be helpful.

Perhaps this document will ultimately become a useful resource for learning physics. Who knows, perhaps when every high school in the country has a FRC team this will become the definitive physics text!

Contents

1 Units and calculations

We will use the metric system throughout this document, because it makes calculations much easier when we start using complex derived units. If you aren't familiar with the metric system (more correctly known as the SI system), look it up on Wikipedia.

1.1 Base units

There are four fundamental units we will deal with in this document:

- **length**, measured in meters (abbreviated m);
- **mass**, measured in grams (g);
- **time**, measured in seconds (s);
- and **electrical current**, measured in Amps (A).

1.2 Derived units

As you probably know, we often combine units to create new units for other things. For example, when we divide distance by time to get speed, we divide a distance unit (m) by a time unit (s) to get a unit for speed, $\frac{m}{s}$. If we were using miles to measure distance and hours to measure time, then we would measure speed in $\frac{miles}{hour}$, commonly abbreviated MPH. In the same way, when we multiply two distances (m) by each other, we get square meters (m^2). Or we might say that a 2 meter piece of aluminum stock has a mass of 6 kg, giving it a linear mass of $\frac{6\,\mathrm{kg}}{2\,\mathrm{m}} = 3\,\frac{\mathrm{kg}}{\mathrm{m}}$. Quite often, we give these units new names. For example, force is measured in $\frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2}$, which we call a Newton.

1.3 Significant figures

1.3.1 Counting significant figures

More often than not, physics deals with measurements. We measure a lot of different things, such as mass, force, speed, and current, and we use those measurements in our calculations. An important thing to keep in mind is that measurements are never perfectly precise. If I measure a piece of aluminum stock and tell you that it is 1 meter long, it might actually be 0.98 meters long, 1.0002 meters long, or even 1.3 meters long.

In physics and engineering, we have to take special care that we don't treat our numbers with more precision than they have. We define precision by the number of "significant figures" a number has. There are a few simple rules for counting significant figures:

- All leading zeros don't count. A measurement of 003 seconds is clearly not any more precise than a measurement of 3 seconds. More importantly, a

measurement of 3 milliseconds (0.003 seconds) does not have more significant figures than a measurement of 3 seconds.

- Trailing zeros before a decimal point don't count, unless they are followed by zeros after the decimal point. Because of the way we round numbers, "100 meters" could refer to 98 meters or 130 meters - just like 1 meter could refer to 0.98 and 1.3 meters. The two zeros at the end of 100 don't make it more precise.
- Trailing zeros after a decimal point do count. As you would assume, 2.000 meters is more precise than 2 meters.
- All non-zero digits count as significant figures.

The following table shows some examples of counting significant figures:

Measurement	Number of significant figures	
3.1415	5	All non-zero digits count
0.002	1	Leading zeros don't count
0.002010	4	Trailing zeros after a decimal point count
1000	1	Trailing zeros before a decimal point don't count, unless followed by figures after the decimal point
1000.0	5	

1.3.2 Math with significant figures

Math with significant figures boils down to three rules:

- When multiplying or dividing, the result must have the same number of significant figures as the least precise number in the equation. In the equation below, "0.3" only has one significant figure, so the answer must also have one significant figure.

$$0.54 \cdot 0.3 = 0.2$$

- When adding or subtracting, the result must have the *same precision* as the least precise value. In the equation $100 + 3$, the least precise value is 100, with the only significant digit in the 100s place. If we wrote the answer as 103, we would be implying that the 100 is precise to the ones place, which is not true. We have to round 103 to the nearest 100, so the correct answer is 100.
- Sometimes, numbers have perfect precision. For example, the radius of of a sprocket is exactly half of its diameter. In this case, "2" can be treated as "2.0000..." without being written that way.

These rules may be a little confusing at first, but they will make sense with practice.

2 Gear ratios

2.1 Concepts: Force and Torque

Force is a specific term in physics that describes the amount of "push" on an object. When a force is exerted on an object - that is, when an object is pushed - it begins to accelerate. This is described by the famous equation

$$\mathbf{F} = m \cdot \mathbf{a} \tag{1}$$

where $\mathbf{F}$ is the force, m is the mass of the object, and $\mathbf{a}$ is the object's acceleration. It is important to note that in this equation, $\mathbf{F}$ is the sum of all forces acting on the object. When you stand on the floor, gravity exerts a downward force on your body, and the floor exerts an equal upward force on your body. As a result, the sum of the forces on your body equals 0, and your body doesn't move. In the SI system, force is measured in Newtons; a Newton is equivalent to a $\frac{kg \cdot m}{s^2}$, which is the units of acceleration $\left(\frac{m}{s^2}\right)$ multiplied by mass (kg).

When force is applied to the end of a lever, it produces torque. Mathematically, we say

$$\tau = \mathbf{r} \times \mathbf{F} \tag{2}$$

Here, the symbol $\times$ indicates the cross product of the two vectors, not multiplication. However, if we assume that the force is applied perpendicular to the lever arm, the cross product becomes

$$\tau = r \cdot F \tag{3}$$

which is a simple multiplication. Torque is measured in Newton-meters (abbreviated N-m).

2.2 Example: Arm Torque

Suppose we have a robot arm as shown in Figure 1. The arm is 1 meter long from the pivot to the center of the ball, and the ball has a mass of 5 kilograms. The acceleration due to gravity is 9.8 $\frac{m}{s^2}$. For the moment, we will make the (obviously false) assumption that the arm doesn't weigh anything, because it simplifies the problem. How much torque do we need to lift the arm?

The worst case will occur when the arm is horizontal and trying to lift the ball straight up, as shown in Figure 1. First, we find the downward force on the ball due to gravity, using Equation 1:

$$F = 5\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 49\,\frac{\mathrm{kg} \cdot \mathrm{m}}{\mathrm{s}^2} = 49\,\mathrm{N}$$

For the ball to not move, the forces on the ball must cancel out, which means that the arm must exert a force of 49 Newtons upward. We can use this information in Equation 3 to find the torque at the arm joint:

$$\tau = 1\,\mathrm{m} \cdot 49\,\mathrm{N} = 49\,\mathrm{N} \cdot \mathrm{m}$$

Of course, our goal is not to hold the ball still - we want to lift it! Thus, we actually need to provide more than 49 Newton-meters of torque.

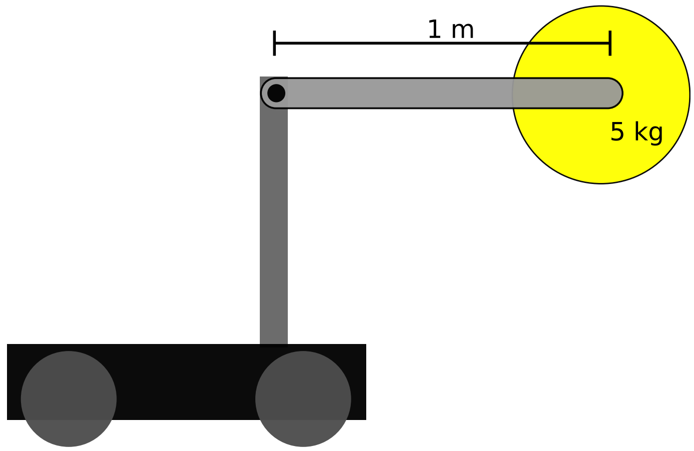

Figure 1: Example robot arm

2.3 Arm Torque, Part 2

In the previous section, we made the assumption that the arm was massless - it didn't weigh anything, and so it didn't require any torque to lift. Obviously, every arm weighs something, so it's important to include the arm in our torque calculations.

If we assume that the arm has a constant linear weight - that is, it doesn't get thinner or lighter towards one end - then we can take the total mass of the arm and act as if that mass is located at a point halfway along the length of the arm.[1] Suppose that in Figure 1 the arm weighs 8 kg. Thus, the downward force on the arm due to gravity is:

$$F = 8\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 78\,\mathrm{N}$$

The halfway point on the arm is $\frac{1\,\mathrm{m}}{2} = 0.5\,\mathrm{m}$, which means that the torque will be

$$\tau = 0.5\,\mathrm{m} \cdot 78\,\mathrm{N} = 39\,\mathrm{N} \cdot \mathrm{m}$$

We can calculate the total torque at the arm joint simply by adding the torque from the ball and the torque from the arm:

$$\tau_{total} = 49\,\mathrm{N} \cdot \mathrm{m} + 39\,\mathrm{N} \cdot \mathrm{m} = 88\,\mathrm{N} \cdot \mathrm{m}$$

If the arm has several sections of different size and weight, we can treat them each as separate pieces and calculate the torque for each one, just like we treated the ball and arm as separate pieces and calculated the total torque by adding them together.

[1] This can derived using calculus. If r is the radius from the pivot point, g is the acceleration due to gravity, and μ is the mass of the arm per unit length, then the total torque is: $\int \mu g r \, dr$. This evaluates to $\frac{1}{2}\mu g r^2$, and since the the total mass of the arm (m) is μr, the torque is $\frac{1}{2} m g r$.

2.4 Gearing

Suppose now that we have to power the arm with a motor which can only provide 1 N-m of torque. Clearly, connecting the motor directly to the arm won't work, so what do we do? The answer is that we use a lever arm, in the form of a gear.

Imagine a gear that is 2 cm (0.02 m) in diameter, as in Figure 2.

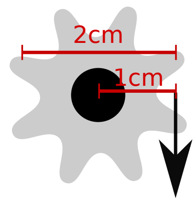

Figure 2: 2 cm gear

The length of the lever arm is the radius of the gear: $\frac{2\,\mathrm{cm}}{2} = 1\,\mathrm{cm}$. If we apply 1 N-m of torque to the center, the force shown by the arrow can be found by rearranging Equation 3:

$$F = \frac{\tau}{r}$$

$$F = \frac{1\,\mathrm{N} \cdot \mathrm{m}}{0.01\,\mathrm{m}} = 100\,\mathrm{N}$$

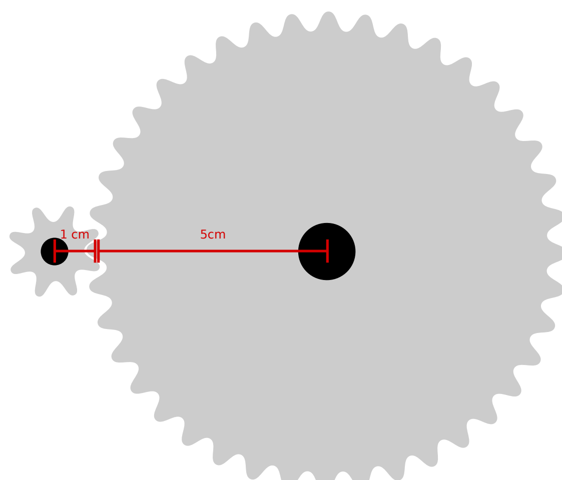

Figure 3: 2 cm gear driving 10 cm gear

In Figure 3, the gear teeth transmit the force to a second gear, 10 cm (0.10 m) in diameter. The torque on this gear will be

$$\tau = 0.05\,\mathrm{m} \cdot 100\,\mathrm{N} = 5\,\mathrm{N} \cdot \mathrm{m}$$

which is a big improvement! To get a torque greater than 88 N-m, we can use additional stages, as shown in Figure 4.

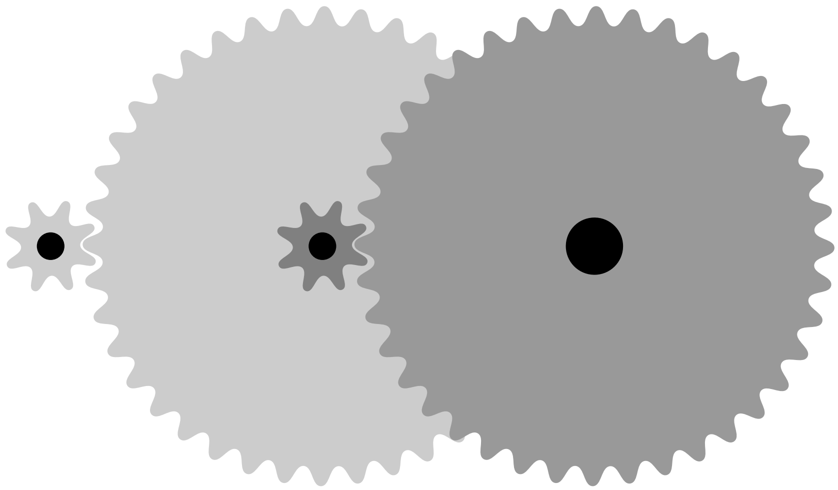

Figure 4: Two-stage gearing. The first small gear drives a large gear, which is fixed to the same shaft as a second small gear. The second gear drives another large gear, producing an overall gear ratio of 25:1.

As you've probably observed, the change in torque is

$$\tau_{out} = \tau_{in} \cdot \frac{\text{driven gear}}{\text{driving gear}} \tag{4}$$

For example, if an 8-tooth gear drives a 40-tooth gear, the output will have $\frac{40}{8} = 5$ times the torque. Notice, however, that we have to turn the 8-tooth gear around 5 times for the 40-tooth gear to turn around once. So if the motor spins at 1000 RPM, the output will spin at only $1000 \cdot \frac{8}{40} = 200$ RPM. This is referred to as a 5:1 gear ratio, since five turns of the input produces one turn of the output.

To summarize: when a small gear drives a larger gear, the result is slower but has more torque. This is referred to as "gearing down", because it results in a lower speed. When a large gear drives a smaller gear, the result is faster but has less torque. This is referred to as "gearing up". The same concept applies when using sprockets and chain or pulleys and belts. If a 12-tooth sprocket drives a 24 tooth sprocket, the output will have half the speed and twice torque.

3 Picking a Motor

Picking the right motors for each mechanism is an important step in building a high-performance robot. By carefully analyzing the system you're designing and the motors you have, you can squeeze the best performance out of each motor.

3.1 Concepts: Motor characteristics, Part 1

There are four important characteristics we need to know for every motor:

- Stall torque, the amount of torque the motor produces when stalled. This will always be the maximum torque the motor can produce.
- Stall current, the amount of electrical current the motor draws when stalled. This will always be the maximum current.
- Free speed, the speed at which the motor spins when nothing is connected to it. This will be the motor's maximum speed.
- Free current, the amount of current the motor draws when spinning freely. This will be the minimum current.

These values are typically given on the motor's datasheet, and are valid only for a particular voltage (most often 12 V). The Fisher-Price motor, for example, has the values shown in Table 1.

Stall Torque	Stall Current	Free Speed	Free Current
N-m	Amps	RPM	Amps
0.45	70	15,600	1.2

Table 1: Fisher-Price Motor characteristics (Taken from Appendix A)

The performance between these maximum and minimum values is linear, which makes them easy to work with. A plot of speed and current as a function of torque is shown in Figure 5.

3.2 Example: Kicker winch

Suppose we have a winch designed to pull back a kicking mechanism, which we need to drive with one of the kit motors. For now, let's select the Fisher-Price motor with the plastic gearbox. The cable wraps around a winch barrel which is 4 cm in diameter.

Looking at the table in Appendix A on page 17, the Fisher-Price motor by itself has a stall torque of 0.45 N-m. The gearbox has a 139:1 gear ratio, meaning that the motor turns around 139 times for each time the white output shaft turns around once. The output shaft torque is simply the input torque multiplied by the gear ratio:

$$\tau_{out} = 0.45\,\mathrm{N}\cdot\mathrm{m}\cdot 139 = 63\,\mathrm{N}\cdot\mathrm{m}$$

We can find the force on the cable by dividing by the length of the lever arm (the radius of the winch barrel):

$$F = \frac{63\,\mathrm{N}\cdot\mathrm{m}}{0.020\,\mathrm{m}} = 3150\,\mathrm{N}$$

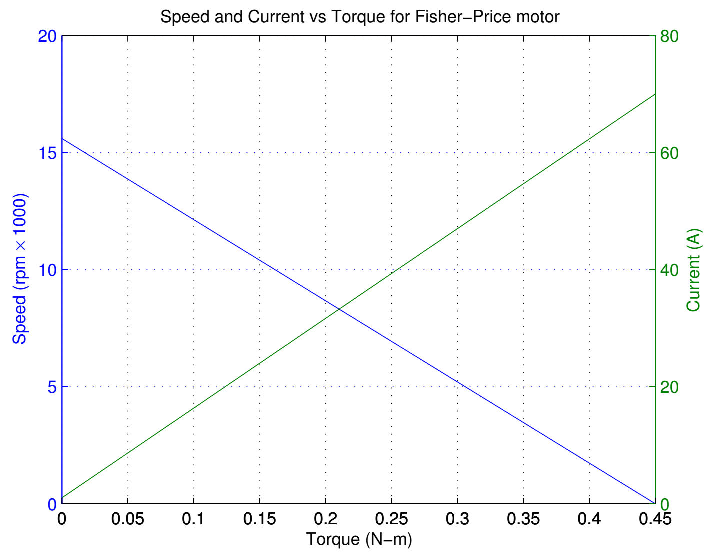

Figure 5: Speed and current as a function of torque for the Fisher-Price motor

That's equivalent to approximately 700 pounds, a LOT of force! Now, there's no way we'd actually get this much torque out of the motor, for several reasons:

- We lose power to friction in the gearbox. For a single well-designed gearing stage, the loss is between 5% and 10%. The Fisher-Price gearbox uses cheap plastic gears and has 4 stages, so we'll estimate its overall efficiency at 60%.

- We don't want to run the motor anywhere near stall torque: when the motor stalls, all of the electrical energy going into it is converted into heat instead of motion. Within a few seconds, the motor will begin to smoke.

- At stall, the Fisher-Price motor will draw 70 amps, which will trip the breaker. We need to use less than 40 A, and would prefer to use even less than that. As mentioned earlier, torque and current are linearly related, so we can find the torque at a particular current using Equation 5.

$$\text{Torque} = \frac{\text{Desired current}}{\text{Stall current}} \cdot \text{Stall torque} \tag{5}$$

Let's redo the calculations, using a target current of 10 Amps and a gearbox efficiency of 60%:

$$\tau = \frac{10\,\mathrm{A}}{70\,\mathrm{A}} \cdot 0.45\,\mathrm{N}\cdot\mathrm{m} = 0.064\,\mathrm{N}\cdot\mathrm{m}$$

$$\tau_{out} = 0.064\,\mathrm{N}\cdot\mathrm{m} \cdot 139 \cdot \frac{60\%}{100\%} = 5.4\,\mathrm{N}\cdot\mathrm{m}$$

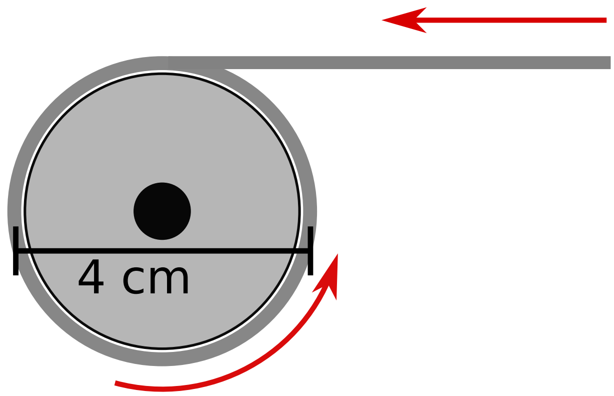

Figure 6: Diagram for kicker winch example

$$F = \frac{5.4\,\mathrm{N}\cdot\mathrm{m}}{0.020\,\mathrm{m}} = 270\,\mathrm{N}$$

This is equivalent to 61 pounds, which should be sufficient to drive our winch.

3.3 Concepts: Work and Power

In physics, energy is commonly defined as the ability to do work. You're probably aware that there are many forms of energy: chemical energy, heat, light, motion, and so forth. In the metric system, one unit for energy is the Joule, which is the energy required to exert 1 Newton of force for a distance of 1 meter.

Suppose we want to lift our robot 1.5 meters off the ground. How much energy does that require?

The downward force the robot exerts due to gravity is given by good old Equation 1:

$$\mathbf{F} = m \cdot \mathbf{a}$$

Because we're specifically dealing with weight, we often change two of the symbols in the equation:

$$w = m \cdot g \tag{6}$$

Here, w is the weight of the object (the downward force exerted on it) and g is the gravitational constant on Earth, roughly $9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$. As before, m is the mass of the robot. Assuming a 150 pound (68 kg) robot,

$$w = 68\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 670\,\mathrm{N}$$

Energy is given by force multiplied by the distance, which we will represent with the symbol x:

$$energy = F \cdot x \tag{7}$$

which gives us

$$energy = 670\,\mathrm{N} \cdot 1.5\,\mathrm{m} = 1.0\ \times 10^3\,\mathrm{J}$$

So it takes 1000 Joules to lift our robot. We often combine equation 6 and 7 to write

$$PE = m \cdot g \cdot h \tag{8}$$

where the term PE stands for "potential energy", and h stands for height.

In everyday usage, we treat the terms "power" and "strength" as synonymous. In physics, however, power has a specific definition: the amount of work done in a specific amount of time. In other words, it's the number of Joules used in one second. This unit is named the watt (abbreviated W).

Suppose we want our lifting mechanism to do its job in 2.0 seconds. How much power is required?

We can use equation 9, where P is the power required, w is the work done, and Δt is the total time it takes to do the work.

$$P = \frac{w}{\Delta t} \tag{9}$$

Plugging in the values we have, we get:

$$P = \frac{1.0 \times 10^3\,\mathrm{J}}{2.0\,\mathrm{s}} = 5.0 \times 10^2\,\mathrm{W}$$

The concept of power is particularly important when choosing a motor: we can gear a motor down to get more torque, or gear it up to get more speed, but we can't do anything (that would be FIRST-legal) to increase the maximum power output of the motor.

3.4 Concepts: Motor characteristics, Part 2

3.4.1 Output power

How do we determine the motor's output power? It goes back the same physics of force and work.

As stated before, energy is given by force multiplied by distance, which we state formally with Equation 10. The variable W is work (the amount of energy used), $\mathbf{F}$ is the force applied, and $\mathbf{x}$ is the distance traveled.

$$W = \mathbf{F} \bullet \mathbf{x} \tag{10}$$

Imagine a lever arm attached to the motor. We prefer easy numbers, so we'll define it to be exactly 1 m long. The motor is spinning at some speed in RPM, which we'll call r. Every time the motor spins around, the tip of the lever arm travels in a circle with a circumference of

$$2 \cdot \pi \cdot r = 2 \cdot \pi\,\mathrm{m} \tag{11}$$

We can easily convert from RPM to revolutions per second:

$$RPS = \frac{RPM}{60} = \frac{r}{60}$$

which means that the tip of the lever arm travels

$$\frac{r}{60} \cdot 2 \cdot \pi \, \mathrm{m} \tag{12}$$

each second. Now we need to calculate the force at the end of the lever arm, which we can do by rearranging Equation 3:

$$F = \frac{\tau}{1\,\mathrm{m}} \tag{13}$$

Now we use Equation 10 to combine Equations 12 and 13:

$$W = \frac{\tau}{1\,\mathrm{m}} \cdot \frac{r}{60} \cdot 2 \cdot \pi \, \mathrm{m}$$

Because the distance we calculated in Equation 11 is the distance the arm moves in 1 second, we can divide by 1 second to get the output power in Watts.

$$P = \tau \cdot \frac{r}{60} \cdot 2 \cdot \pi \, \mathrm{W}$$

Notice that we have three variables in this equation: the torque, τ, the rotational speed, r, and the output power, P. However, we know that τ and r are related to each other. To find the power, we have to pick a value for one of them, determine the other using a motor curve chart, and then put these numbers into the equation.

3.4.2 Input power and efficiency

The input power is electrical, and thus it is given by Equation 14, where P is the power, I is the current, and V is the voltage.

$$P = I \cdot V \tag{14}$$

The efficiency of the motor is simply the percentage of the electrical input power that gets turned into mechanical output power. Mathematically,

$$\eta = \frac{P_{out}}{P_{in}}$$

Generally, we don't manually calculate input power, output power, and efficiency when we're working with a motor. Like good engineers, we make a computer do the hard work once, and then look it up in a table or graph. We'll do this in the following section.

3.4.3 Motor curves

The motor data discussed above is almost always plotted on a graph, known as the motor curve. Figure 5 is one example of a motor curve, which you may find on some datasheets. Figure 7 shows the same data, but also adds power and efficiency. Note that the maximum power occurs at exactly half of free speed,

but maximum efficiency occurs at a much higher speed. Most of the time we want to operate our motors between maximum efficiency and maximum power. Running motors close to stall is almost always a bad idea: the energy lost due to the motor's inefficiency gets converted into heat, causing the motor to heat up quickly. Because they are large and heavy, the CIM motors can handle a lot of heat. But the Fisher-Price and BaneBots motors have much less mass, and count on an internal fan to cool the motor. When one of these motors is stalled, the fan doesn't spin, and the motor heats up and burns out within a few seconds.

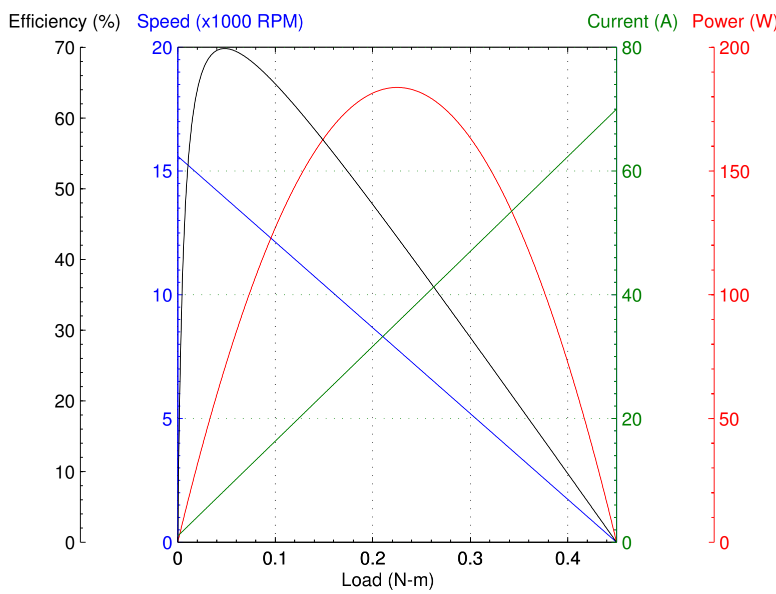

Figure 7: Motor curve for Fisher-Price 9015 series motor

4 Friction, Traction, and your Drivetrain

4.1 Concepts: Friction

As you probably know, friction is a force that resists motion whenever two surfaces are in contact. We can calculate the force due to friction using the equation

$$F_{friction} = F_N \cdot \mu \tag{15}$$

where F_N is the normal force (the force applied perpendicular to the surface) and μ is the friction coefficient. Typical robot wheels on carpet have friction coefficients between 0.6 and 1.0, but some wheels may have coefficients as high as 1.3.

In the case of a robot drivetrain, we often want to maximize $F_{friction}$ with the carpet, which we typically refer to as traction. The more frictional force we have against the carpet, the harder we can push. There are only two things we can do to increase traction: increase the normal force (F_N), or increase the coefficient of friction (μ). In the case of FRC, there is generally little we can do to increase F_N. Our robot's mass is limited by the rules, and in recent games, teams have not been allowed to lift field structures to increase the weight on their wheels.[2] The only way to increase μ is to use a "grippier" wheel material such as roughtop. Note that friction does not depend on the contact area between the surfaces, which means that theoretically your robot's traction does not depend on how many wheels you have.[3]

It is also important to note that every pair of surfaces has two values for μ: the static coefficient of friction, μ_s, and the kinetic coefficient of friction, μ_k. Static friction applies when the two surfaces are not moving relative to each other, kinetic friction applies as the surface slide. As you may have observed when sliding a heavy object, it takes more force to overcome the static friction than it does the kinetic friction. Mathematically, $\mu_s > \mu_k$, regardless of the surfaces.

4.2 Example: Traction-limited drivetrains

Because of the rough nature of FRC games, we generally want our drivetrains to be traction-limited. That is, we want our drivetrain to have enough torque so that when the robot gets into a shoving match, the wheels slip before the motors stall or a breaker trips. In this case, traction is the limit on our robot's pushing ability, and not our motors, hence the term "traction-limited".

Suppose we have a 150 lb (68 kg) robot using 8" (20.3 cm) wheels with $\mu_s = 1.0$. The normal force will be

$$F_N = w = 68\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} = 670\,\mathrm{N}$$

The total traction will be

$$F_{friction} = 670\,\mathrm{N} \cdot 1.0 = 670\,\mathrm{N}$$

Since the wheels have a diameter of 20.3 cm, they have a radius of 10.2 cm = 0.102 m. From this, we can find the total torque on the wheels:

$$\tau = 670\,\mathrm{N} \cdot 0.102\,\mathrm{m} = 68\,\mathrm{N} \cdot \mathrm{m}$$

[2] In Zone Zeal (2002), there were three mobile goals, each weighing around 82 kg (180 lbs). By lifting the goals, robots could increase the weight on their wheels and get better traction for the ensuing tug-of-war.

[3] In practice, you may get slightly more traction by having more wheels on the carpet, because the tread can dig into the carpet and grip somewhat like Velcro - but this is heavily debated. In situations where important parts of the field are covered with hard, slick surfaces such as Stack Attack (2003) and Lunacy (2009) the theoretical calculations are very close to reality.

Note that each wheel will only experience part of this torque; this is the total for all of the wheels touching the carpet.

If we use 4 CIM motors with two CIMple gearboxes having a 4.67:1 ratio, what sprocket ratio do we need to make the drivetrain traction-limited?

Each CIM will be on a 40 A breaker, so we can use equation 5 to find the maximum torque from each motor: 0.73 N-m. Multiplying by four motors gives us a total of 2.9 N-m. Coming out of the gearbox, we will have

$$4.67 \cdot 2.9\,\mathrm{N} \cdot \mathrm{m} = 14\,\mathrm{N} \cdot \mathrm{m}$$

of torque. We need a total of 68 N-m, so the ratio we need is

$$\frac{68\,\mathrm{N} \cdot \mathrm{m}}{14\,\mathrm{N} \cdot \mathrm{m}} = 4.9$$

How fast will this robot go? We can get an estimate for top speed by dividing the motor's free speed by the total gear/sprocket ratio:

$$\frac{5310\,\mathrm{RPM}}{4.67 \cdot 4.9} = 230\,\mathrm{RPM} = 3.8\,\frac{\mathrm{revolutions}}{\mathrm{second}}$$

$$3.8\,\frac{\mathrm{rev}}{\mathrm{s}} \cdot 0.203\,\mathrm{m} \cdot \pi = 2.4\,\frac{\mathrm{m}}{\mathrm{s}}$$

which is equivalent to 7.9 feet per second.

There are several things to notice about this calculation. First, we assumed that each motor had an absolute current limit of 40 A. This is a helpful estimate, but is not correct: the robot's main breaker trips at 120 A, so continuously running four motors at 40 Amps each is not possible. More importantly, breakers do not trip instantaneously: 40 A breakers can handle brief spikes of 60 to 80 A, and the main breaker takes several seconds to trip.

Second, note that the final calculated speed is somewhat slow. If you need higher speed, you have three options: You can add more motors on your drivetrain, increasing the torque available to slip your wheels. You can use a shifting transmission, so that you have a slow, traction-limited gear and a fast, torque-limited gear. Or you could ignore the problem and hope to avoid prolonged pushing matches. All three of these are valid options and are used by numerous teams.

Finally, we've conveniently ignored the gearbox and chain efficiency in these calculations, but they should be included in your analysis.

A Motor comparison table

Motor	Technical name	Max Power	Stall Torque	Stall Current	Free Speed	Free Current	Max Efficiency
		Watts	N-m	Amps	RPM	Amps	
CIM	FR-801-001	337	2.43	133	5,310	2.7	65%
Fisher-Price	9015	184	0.45	70	15,600	1.2	68%
Nippon-Denso	262100-3030	23	10.6	18.6	84	1.8	24%

Glossary

Amp The unit for electrical current. 1 Amp is equivalent to 1 Coloumb per second.

Efficiency The ratio of input power to output power. If a motor puts out 5 units of useful energy for every 10 units put into it, it has an efficiency of 50%.

Electrical current is the flow of electrons through a wire, measured in Amps.

Energy is TODO.

Force A description of the amount of push on an object, and is measured in Newtons. Force causes a change in motion, described by the equation $\mathbf{F} = m \cdot \mathbf{a}$.

Friction a force that opposes motion, caused by TODO surfaces.

Gearing down Using a small gear, sprocket, or pulley to drive a larger gear, sprocket, or pulley, resulting in lower speed but higher torque.

Gearing up Using a large gear, sprocket, or pulley to drive a smaller gear, sprocket, or pulley, resulting in higher speed but lower torque.

Joule One of the SI units for energy.

Loss is TODO.

Normal force is the force perpendicular (normal) to a surface. If you are standing on a level floor, the force of your body on the floor is perpendicular to the floor..

Power The rate at which energy is being used. Power is typically measured in Watts.

RPM Revolutions Per Minute, a measurement of rotational speed. 1 revolution is one 360-degree turn of a shaft.

Significant figure is TODO.

Stall When a motor can't produce enough torque to turn and stops moving. This is generally undesireable, and can cause some motors to burn out.

Watt The unit for energy, equivalent to 1 Joule per second.

Weight is the force due to an object's mass and gravity. Because weight is dependant on gravity, a 3kg rock would weigh more on Earth than on the moon.

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