MS&E 246: Lecture 6
Dynamic games of perfect and complete information

Ramesh Johari

Outline

- Dynamic games
- Perfect information
- Game trees
- Strategies
- Backward induction

Dynamic games

Instead of playing simultaneously,

the rules dictate *when* players play,

and *what they know about the past* when they play.

Example

Consider the following game:

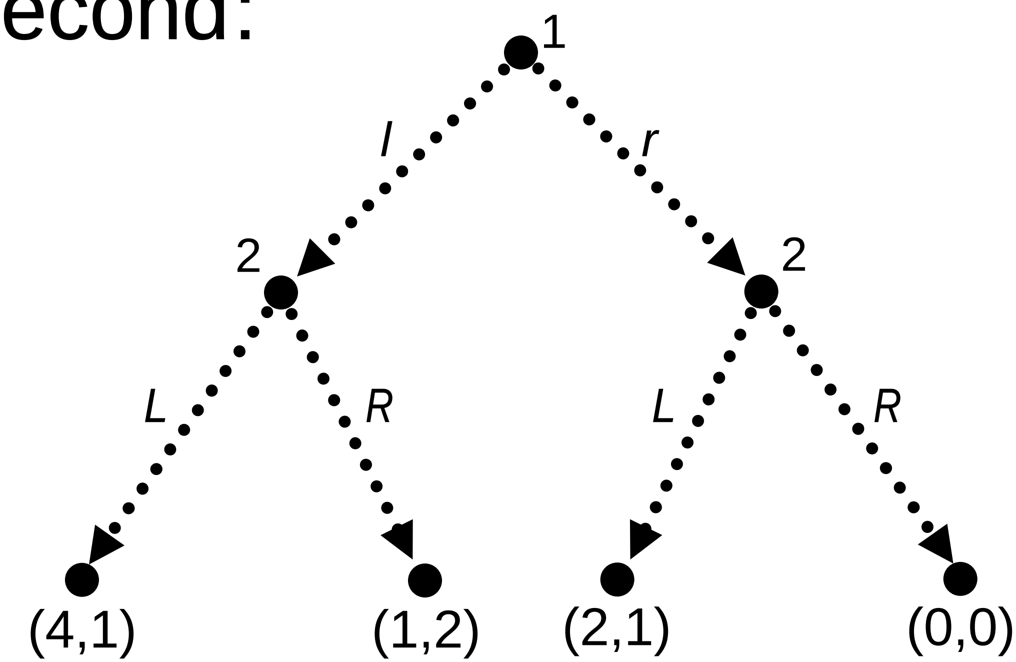
Player 2

		L	R
Player 1	l	(4,1)	(1,2)
	r	(2,1)	(0,0)

Only pure NE is (l, R).

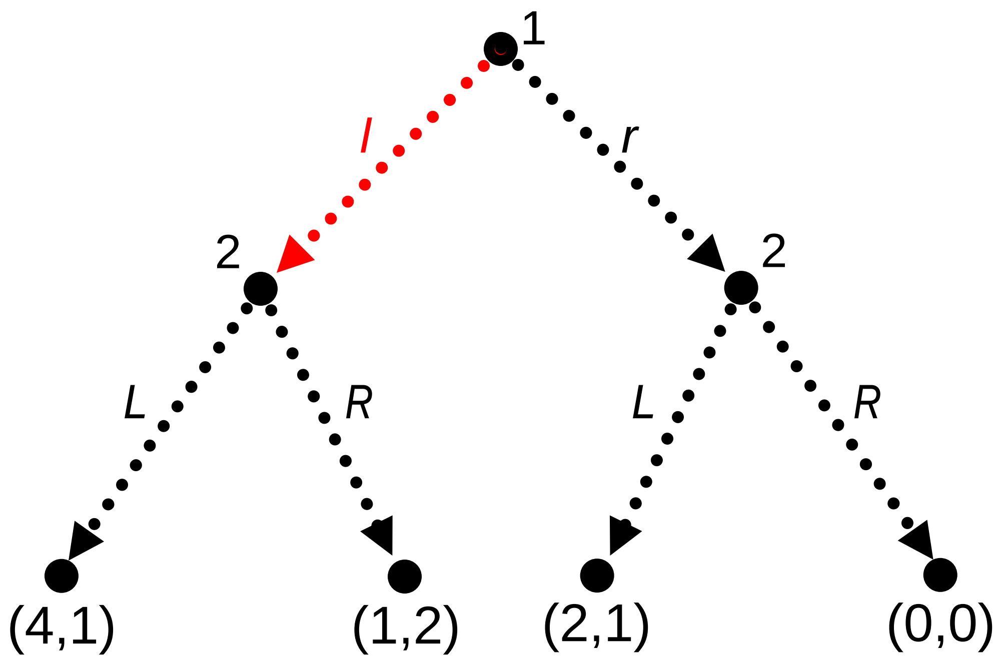
Example: dynamic game

Suppose player 1 moves first, and player 2 moves second:



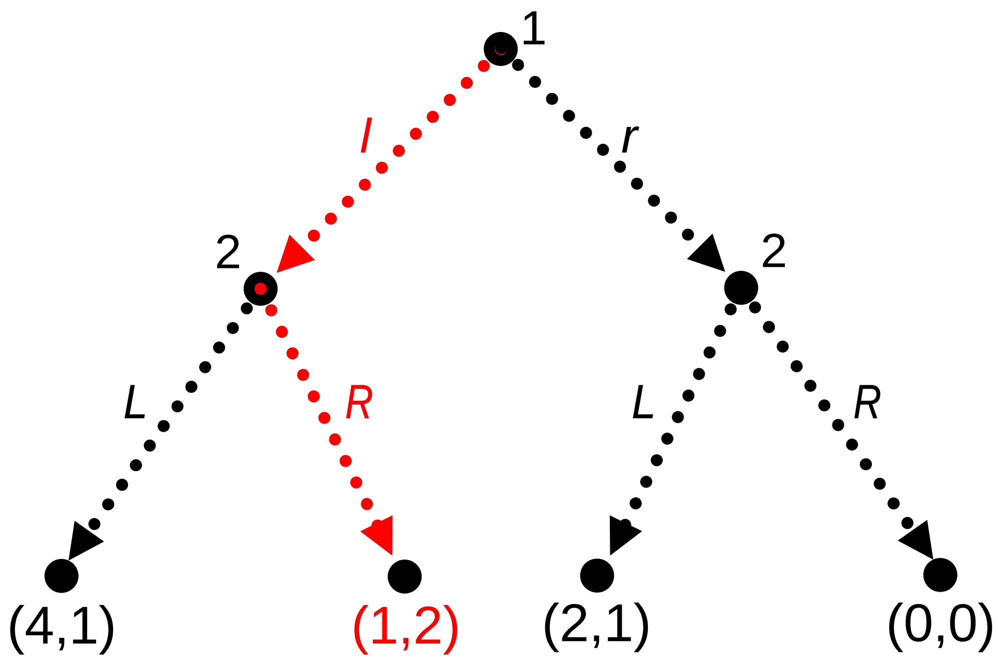
Example: dynamic game

If player 1 plays *l* ...



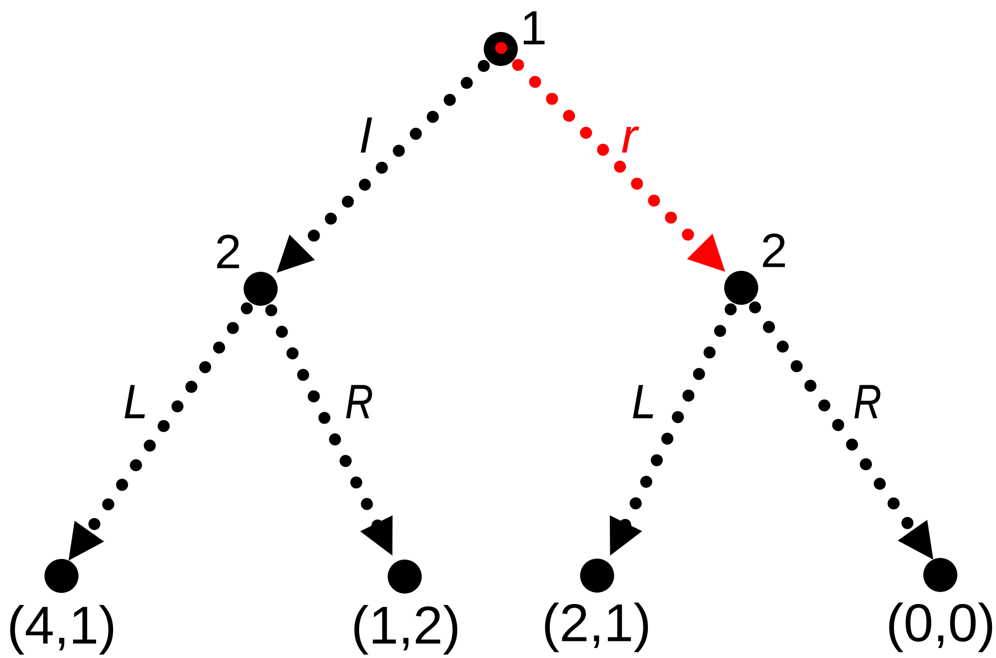
Example: dynamic game

...then player 2 plays $R \Rightarrow \Pi_1 = 1$.



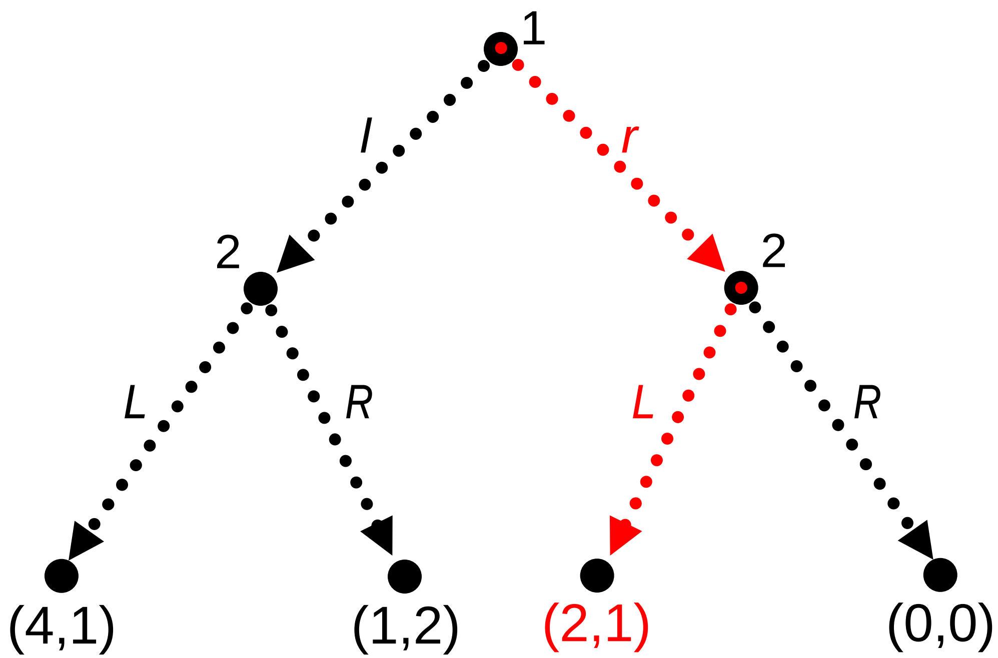
Example: dynamic game

If player 1 plays *r* ...



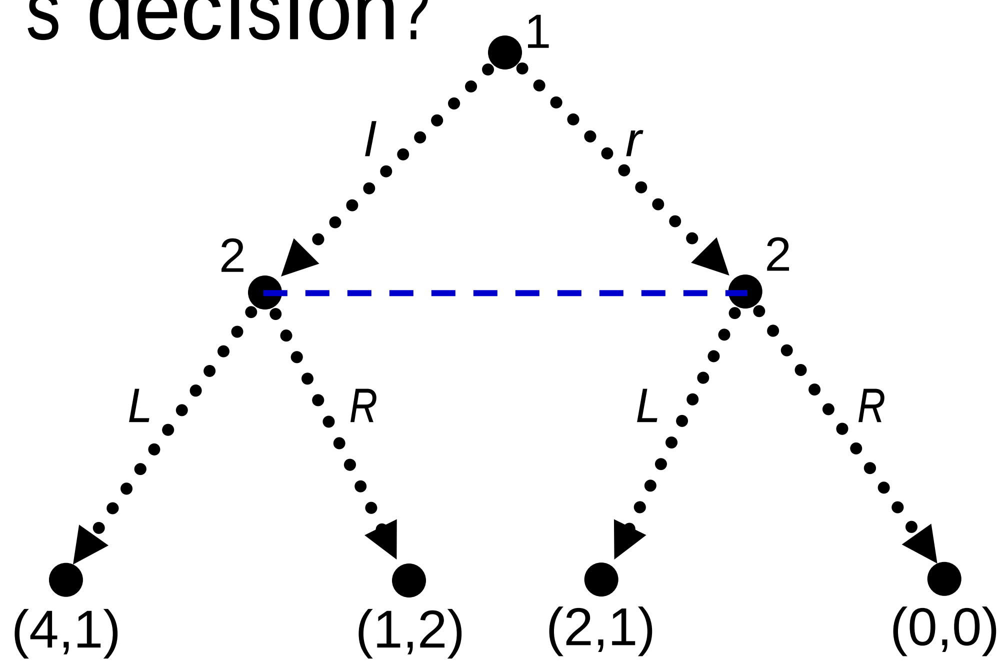
Example: dynamic game

...then player 2 plays *L* $\Rightarrow \Pi_1 = 2$!



Example: dynamic game

What if player 2 does not observe player 1's decision?



- - - -: *Player 2 cannot distinguish between these nodes*

Dynamics

What if player 2 does not observe player 1's decision?

Harder to predict what player 2 might do at stage 2.

Perfect information

In this lecture we will study (finite) dynamic games of *perfect information:*

These are games where all players observe the entire *history* of the game, and the game terminates in finitely many steps.

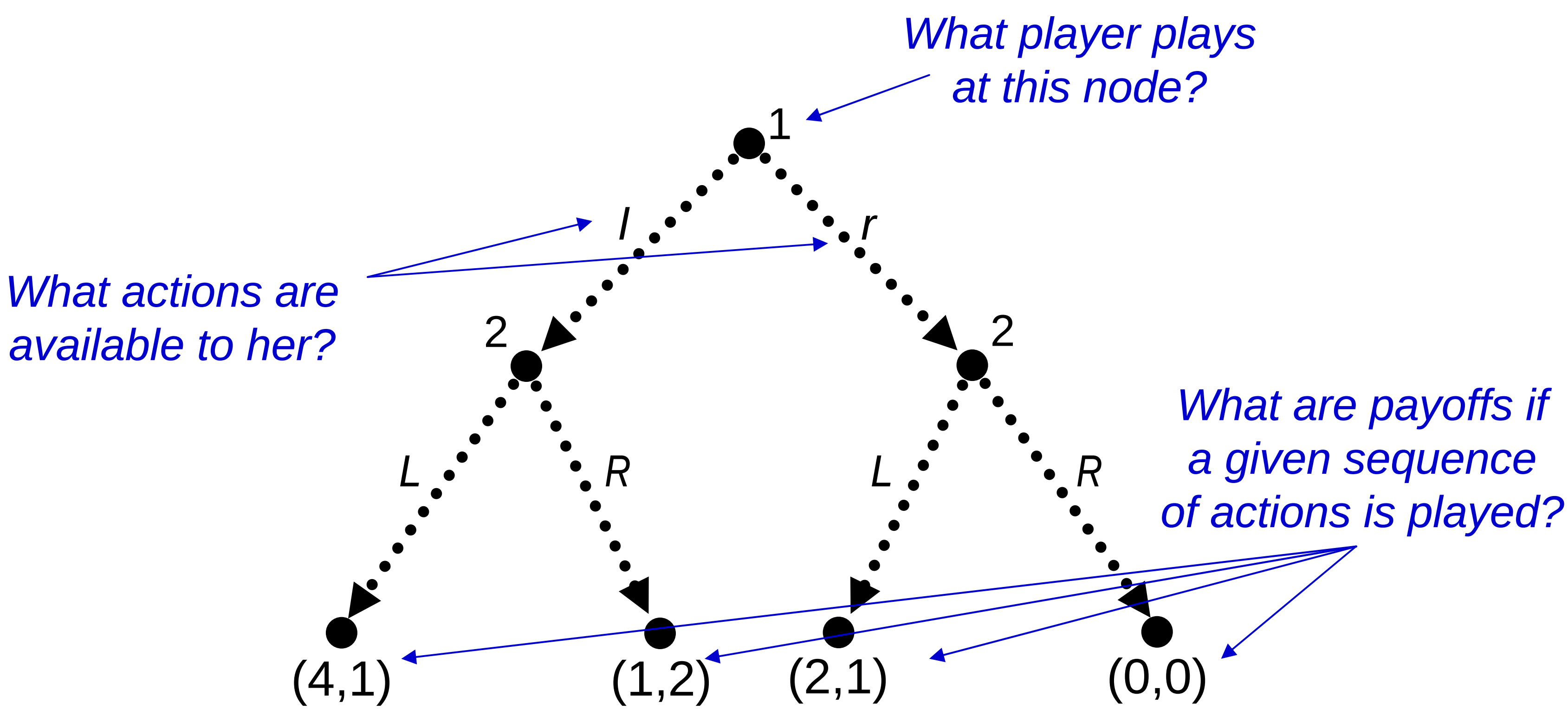
(NOTE: This is not complete information!)

Game tree

Fundamental structure: the *game tree*.

1) Each non-leaf node v is identified with a unique player $I(v)$.
2) All edges out from a node v correspond to *actions* available to $I(v)$.
3) All leaves are labeled with the *payoffs* for all players.

Game tree



What player plays
at this node?
1
l
r
What actions are
available to her?
2
2
L
R
L
R
What are payoffs if
a given sequence
of actions is played?
(4,1)
(1,2)
(2,1)
(0,0)

Game trees and extensive form

Idea: At each node v, player $I(v)$ chooses an action; this leads to the next "stage."

The game tree is also called the *extensive form* of the game.

Strategies

For player 1 to "reason" about player 2, there must be a prediction of what player 2 would play *in any of his nodes.*

(See example at the beginning of lecture.)

Strategies

For player 1 to “reason” about player 2, there must be a prediction of what player 2 would play *in any of his nodes.*

Thus in a dynamic game, a strategy s_i is a *complete contingent plan*:

For each v such that $I(v) = i$, $s_i(v)$ specifies the *action* of player i at node v.

Backward induction

We solve finite games of perfect information using *backward induction.*

Idea: find "optimal" decisions for players from the bottom of the tree to the top.

Backward induction

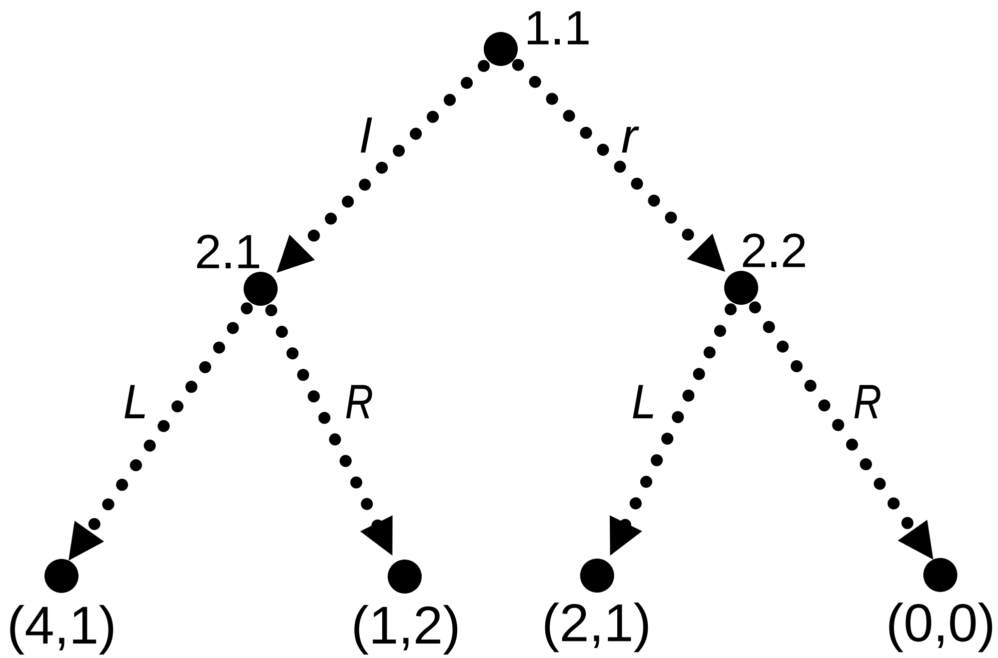
Formally: Suppose the game has L stages.

- Find the set of optimal actions for player $I(v)$ at each node v in stage L (possibly including mixed actions).
- Label each node v in stage L with payoffs from optimal actions, and remove any children.
- Return to (1) and repeat.

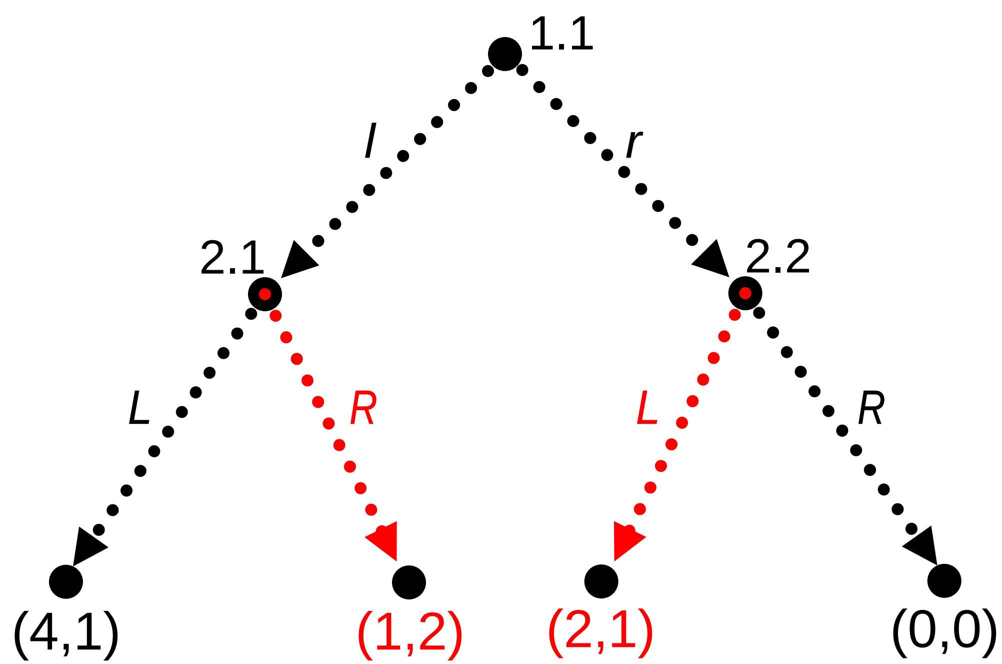
Backward induction

- Note this specifies *complete strategies* for all players,
  as well as the *path(s) of actual play.*
- Any finite dynamic game of perfect information has a backward induction solution.

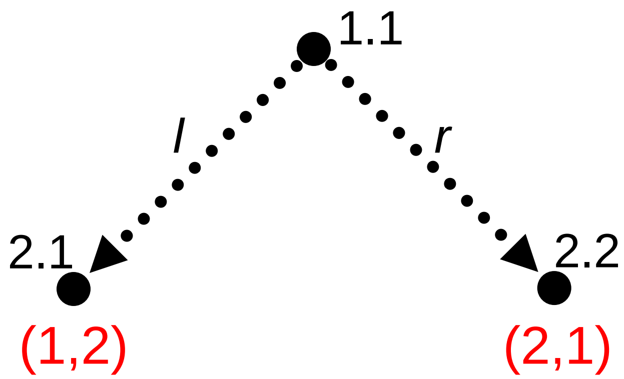
Example



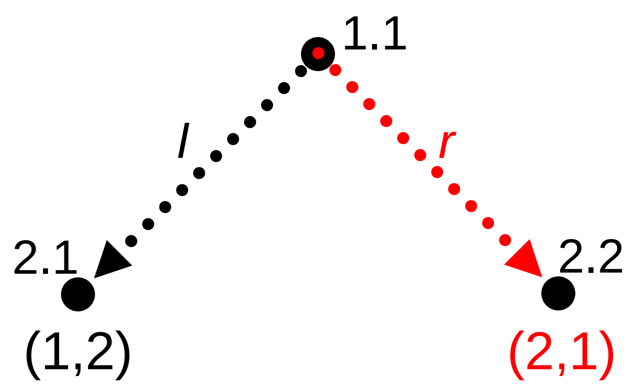
Example



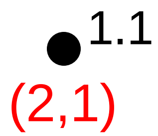
Example



Example



Example



Backward induction solution

Strategies:

Player 1 plays *r* at node 1.1.

Player 2 plays *R* at node 2.1, and plays *L* at node 2.2.

The *equilibrium path of play* is (*r*, *L*).

Strategic form

The strategic (or normal) form is:

Player 2

		LL	*LR*	*RL*	*RR*
Player 1	*l*	(4,1)	(4,1)	(1,2)	(1,2)
	r	(2,1)	(0,0)	(2,1)	(0,0)

Strategic form

What are all pure NE of the strategic form?

(*r*, *RL*) and (*l*, *RR*)

But (*l, RR*) is *not credible:*

Player 1 knows a rational player 2 would never play *R* in 2.2.

Strategic form

Any backward induction solution must be a NE of the strategic form,

but the converse does not necessarily hold.