MS&E 246: Lecture 16
Signaling games

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Signaling games

Signaling games are two-stage games where:

- Player 1 (with private information) moves first.
  His move is observed by Player 2.
- Player 2 (with no knowledge of Player 1's private information) moves second.
- Then payoffs are realized.

Dynamic games

Signaling games are a key example of *dynamic games of incomplete information.*

(i.e., a dynamic game where the entire structure is *not* common knowledge)

Signaling games

The formal description:

Stage 0:

Nature chooses a random variable t_1, observable only to Player 1, from a distribution P(t_1).

Signaling games

The formal description:

Stage 1:

Player 1 chooses an *action* a_1 from the set A_1.

Player 2 observes this choice of action.

(The action of Player 1 is also called a "message.")

Signaling games

The formal description:

Stage 2:

Player 2 chooses an action
a_2 from the set A_2.

Following Stage 2, payoffs are realized:

$\Pi_1(a_1, a_2 ; t_1)$; $\Pi_2(a_1, a_2 ; t_1)$.

Signaling games

Observations:

- The modeling approach follows Harsanyi’s method for static Bayesian games.
- Note that Player 2’s payoff depends on the type of player 1!
- When Player 2 moves first, and Player 1 moves second, it is called a *screening* game.

Application 1: Labor markets

A key application due to Spence (1973):

Player 1: worker

t_1 : intrinsic ability

a_1 : education decision

Player 2: firm(s)

a_2 : wage offered

Payoffs: Π_1 = net benefit

Π_2 = productivity

Application 2: Online auctions

Player 1: seller

t_1 : true quality of the good

a_1 : advertised quality

Player 2: buyer(s)

a_2 : bid offered

Payoffs: Π_1 = profit

Π_2 = net benefit

Application 3: Contracting

A model of Cachon and Lariviere (2001):

Player 1: manufacturer

t_1 : demand forecast

a_1 : declared demand forecast, contract offer

Player 2: supplier

a_2 : capacity built

Payoffs: Π_1 = profit of manufacturer

Π_2 = profit of supplier

A simple signaling game

Suppose there are two types for Player 1, and two actions for each player:

- $t_1 = H$ or $t_1 = L$

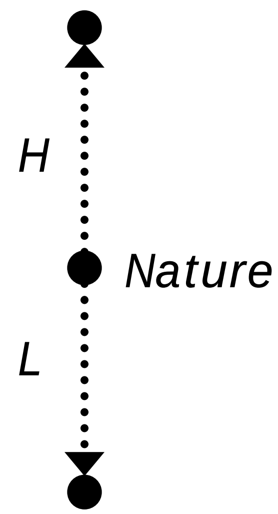
  Let p = P($t_1 = H$)
- $A_1 = \{ a, b \}$
- $A_2 = \{ A, B \}$

A simple signaling game

Nature moves first:

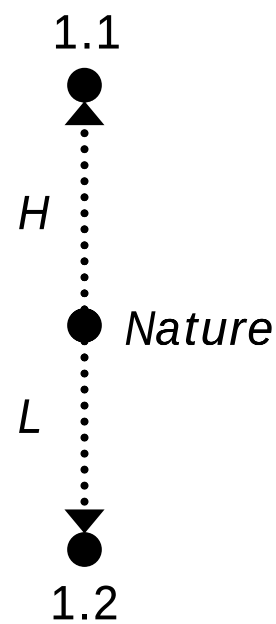
A simple signaling game

Nature moves first:



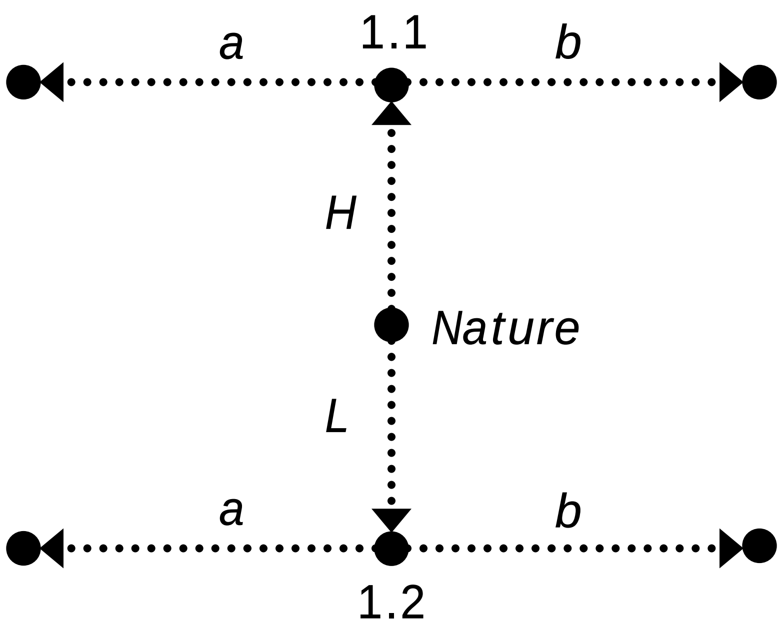
A simple signaling game

Player 1 moves second:



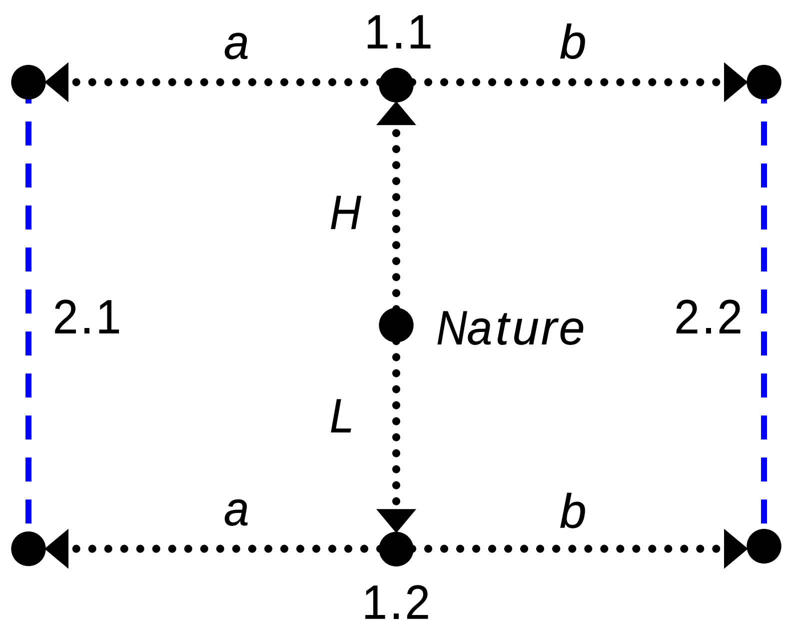
A simple signaling game

Player 1 moves second:



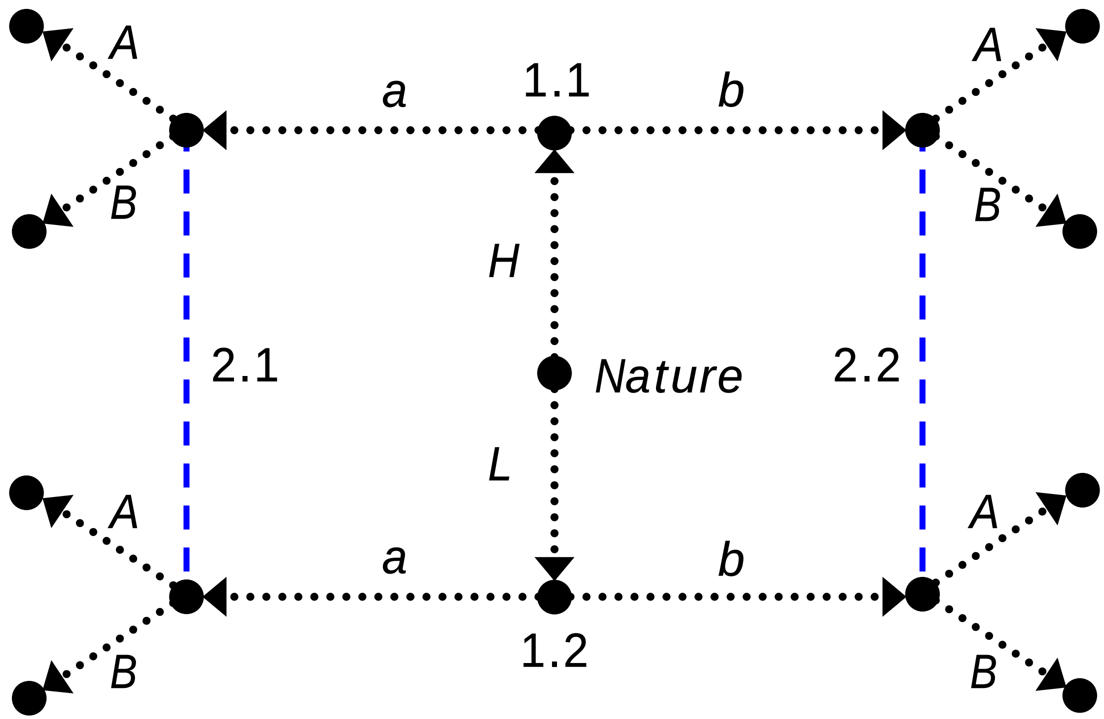
A simple signaling game

Player 2 observes Player 1's action:



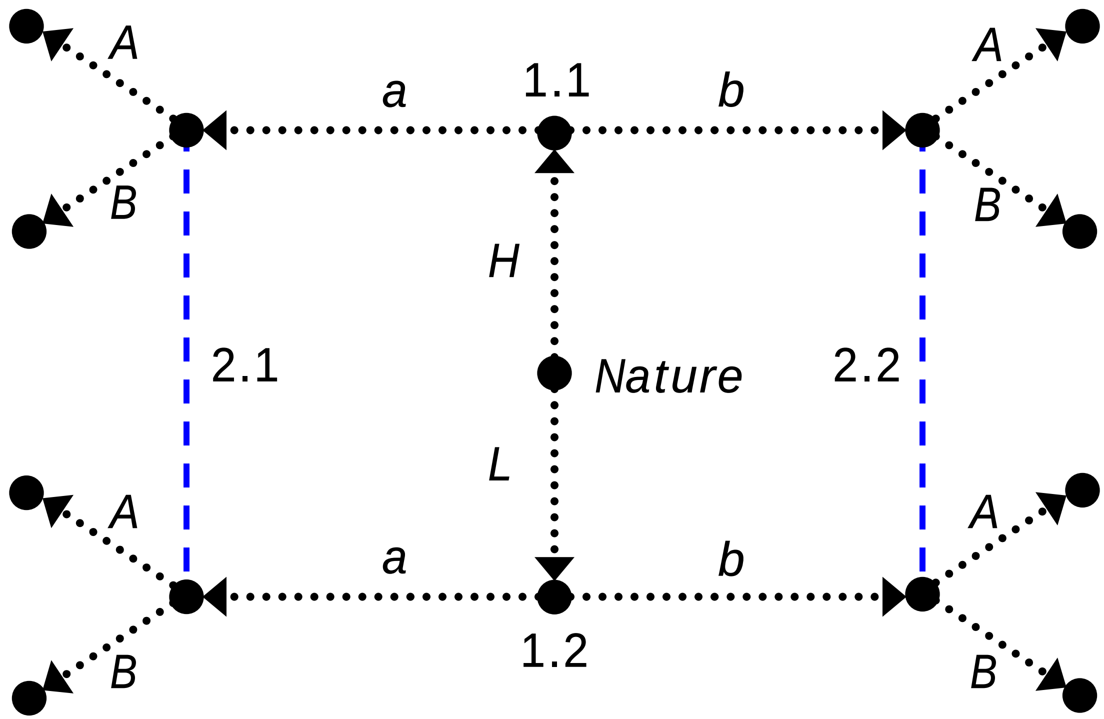
A simple signaling game

Player 2 moves:



A simple signaling game

Payoffs are realized: $\Pi_i(a_1, a_2; t_1)$



Perfect Bayesian equilibrium

Each player has 2 information sets, and 2 actions in each, so 4 strategies.

A PBE is a pair of strategies and beliefs such that:

-each players' beliefs are derived from strategies using Bayes' rule (if possible)

-each players' strategies maximize expected payoff given beliefs

Pooling vs. separating equilibria

When player 1 plays the *same* action, regardless of his type, it is called a *pooling strategy.*

When player 1 plays *different* actions, depending on his type, it is called a *separating strategy.*

Pooling vs. separating equilibria

In a *pooling equilibrium*,

Player 2 gains no information about t_1 from Player 1's message

$\Rightarrow P_2(t_1 = H \mid a_1) = P(t_1 = H) = p$

In a *separating equilibrium*,

Player 2 knows Player 1's type *exactly* from Player 1's message

$\Rightarrow P_2(t_1 = H \mid a_1) = 0 \text{ or } 1$

An eBay-like model

- Suppose seller has an item with quality either *H* (prob. p) or *L* (prob. 1 - p).
- Seller can advertise either *H* or *L*.
- Assume there are two bidders.
- Suppose that bidders always bid truthfully, given their beliefs.
  - (This would be the case if the seller used a second price auction.)

An eBay-like model

Suppose seller *always* advertises *high.*

Then: buyers will never "trust" the seller, and always bid expected valuation.

This is the pooling equilibrium:

$s_1(H) = s_1(L) = H$.

$s_B(H) = s_B(L) = p\,H + (1 - p)\,L$.

An eBay-like model

Is there any equilibrium where $s_1(t_1) = t_1$?

(In this case the seller is *truthful*.)

In this case the buyers bid:

$s_B(H) = H$, $s_B(L) = L$.

But if the buyers use this strategy,
the seller prefers to *always* advertise H!

An eBay-like model

Now suppose that if the seller *lies* when the true value is L, there is a cost c (in the form of lower reputation in future transactions).

If $H - c < L$, then the seller prefers to tell the truth $\Rightarrow$ separating equilibrium.

An eBay-like model

This example highlights the importance of *signaling costs*:

To achieve a separating equilibrium, there must be a difference in the costs of different messages.

(When there is no cost, the resulting message is called "cheap talk.")