

MS&E 246: Lecture 11

Concluding remarks on subgame perfection

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Dynamic games

In our discussion of dynamic games of complete information, we studied two main types:

- Perfect information
- Imperfect information

In both cases, subgame perfect NE emerged as a natural way to capture “sequential rationality” (or credibility).

Possible problems

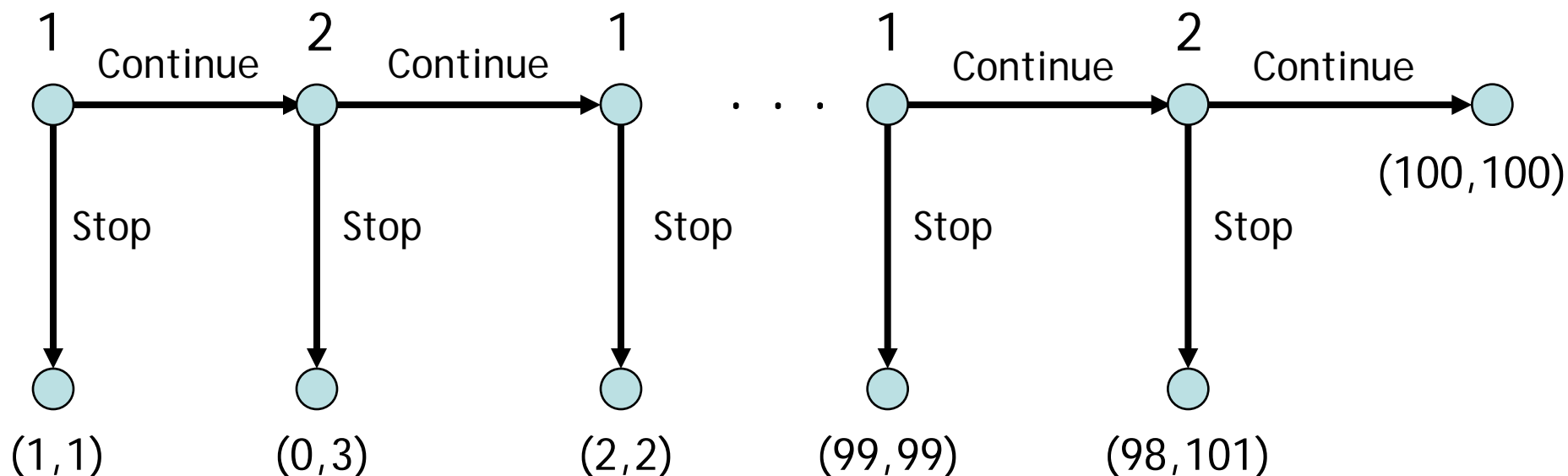
However, subgame perfection can give rise to two possible issues.

In some cases, it is overly restrictive as a predictive tool:
there are “not enough” SPNE.

In some cases, it is not useful as a predictive tool:
there are too many SPNE.

SPNE: overly restrictive?

Consider the following game:



(This is called the "centipede" game.)

SPNE: overly restrictive?

- In last information set, player 2 prefers to “stop” instead of “continue”
- Inductively, in each information set each player prefers to “stop” instead of “continue”
- Equilibrium payoffs: $(1, 1)$
- Is this a reasonable prediction of play?

SPNE: overly restrictive?

The centipede game reveals a key flaw in the definition of SPNE:

If play ever reaches a subgame *off* the equilibrium path of play, then rationality must have failed already.

But SPNE assumes rational behavior in *every subgame!*

SPNE: not restrictive enough?

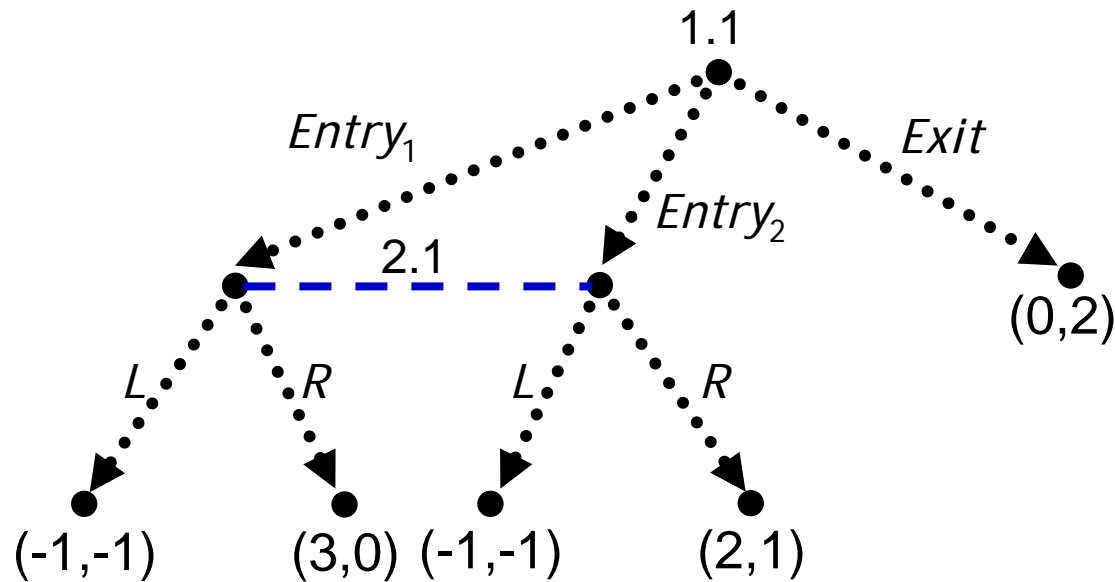
- In repeated games, we saw the folk theorem(s): with enough patience, any individually rational payoffs can be sustained by an SPNE.
- Too many equilibria for predictive use

SPNE: not restrictive enough?

Other problems can occur in situations where there are “not enough subgames” to rule out equilibria.

SPNE: not restrictive enough?

- Two firms
- First firm decides if/how to enter
- Second firm can choose to “fight”



Entry example

Note that this game only has *one* subgame.
Thus SPNE are *any* NE of strategic form.

| | | Firm 2 | |
|--------|---------------------------|----------|----------|
| | | <i>L</i> | <i>R</i> |
| Firm 1 | <i>Entry</i> ₁ | (-1, -1) | (3, 0) |
| | <i>Entry</i> ₂ | (-1, -1) | (2, 1) |
| | <i>Exit</i> | (0, 2) | (0, 2) |

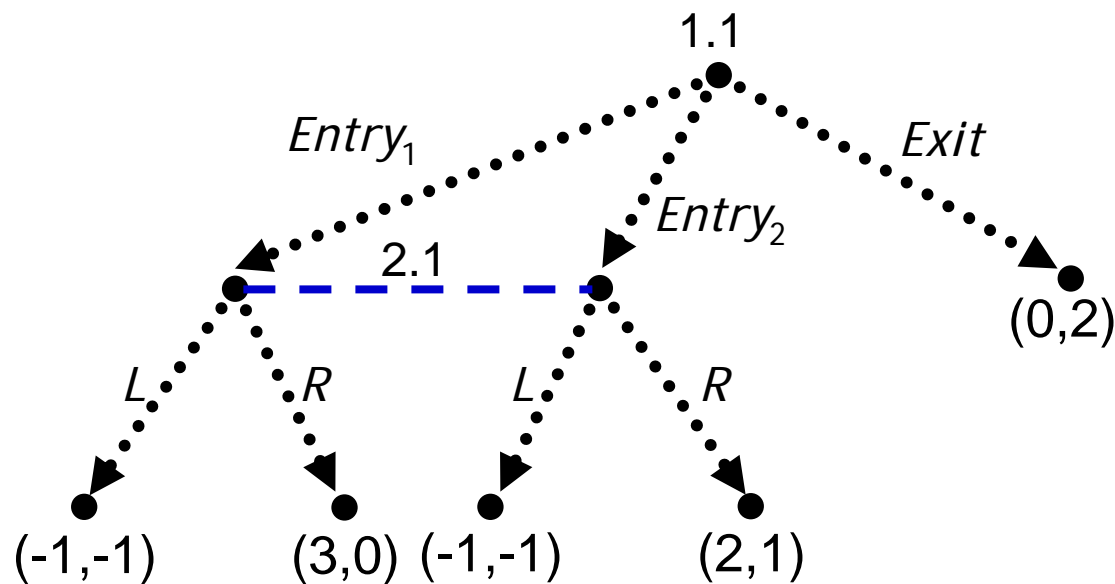
Entry example

Two pure NE of strategic form:
 $(Entry_1, R)$ and $(Exit, L)$

| | | Firm 2 | |
|--------|---------------------------|----------|----------|
| | | <i>L</i> | <i>R</i> |
| Firm 1 | <i>Entry</i> ₁ | (-1, -1) | (3, 0) |
| | <i>Entry</i> ₂ | (-1, -1) | (2, 1) |
| | <i>Exit</i> | (0, 2) | (0, 2) |

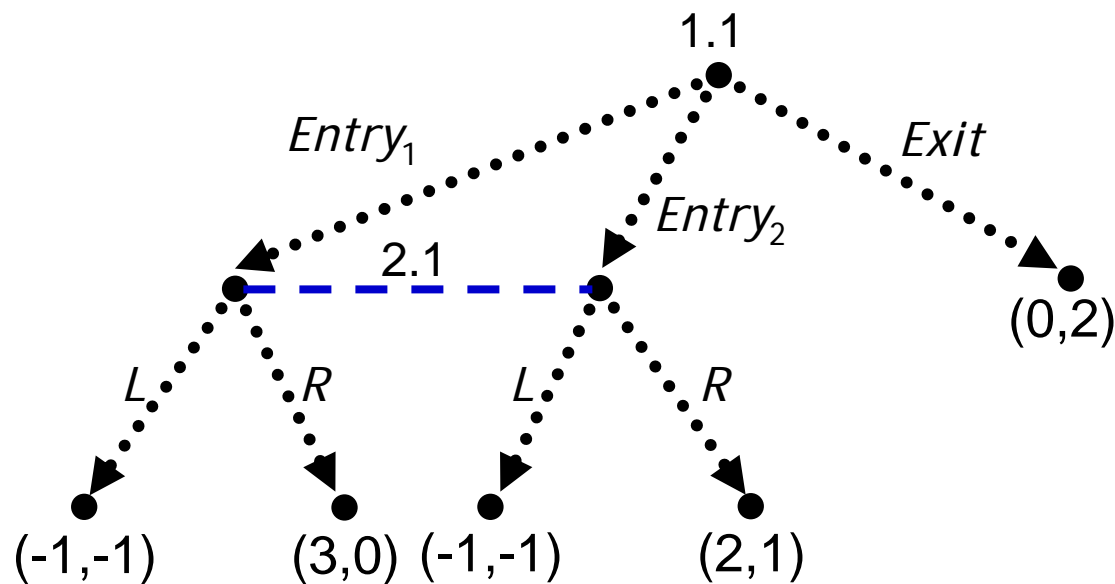
SPNE: not restrictive enough?

But firm 1 should “*know*” that if it chooses to enter, firm 2 will never “fight.”



SPNE: not restrictive enough?

So in this situation, there are again too many SPNE.



SPNE: not restrictive enough?

A solution to the problem of the entry game is to include *beliefs* as part of the solution concept:

Firm 2 should never fight, regardless of what it believes firm 1 played.

(We will study such an approach in the last part of the course.)