Distributed Matrix Computations

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databricks
Spark
Outline

Data flow vs. traditional network programming
Spark computing engine
Optimization Example
Matrix Computations
Singular Value Decomposition
Current State of Spark Ecosystem
Data Flow Models

Restrict the programming interface so that the system can do more automatically

Express jobs as graphs of high-level operators
  » System picks how to split each operator into tasks and where to run each task
  » Run parts twice fault recovery

Biggest example: MapReduce
Spark Computing Engine

Extends a programming language with a distributed collection data-structure
  » “Resilient distributed datasets” (RDD)

Open source at Apache
  » Most active community in big data, with 50+ companies contributing

Clean APIs in Java, Scala, Python

Community: SparkR
Key Idea

Resilient Distributed Datasets (RDDs)
- Collections of objects across a cluster with user-controlled partitioning & storage (memory, disk, ...)
- Built via parallel transformations (map, filter, ...)
- The world only lets you make make RDDs such that they can be:

  Automatically rebuilt on failure
Optimization Example
Gradient Descent

data = spark.textFile(...).map(readPoint).cache()

w = numpy.random.rand(D)

for i in range(iterations):
    gradient = data.map(lambda p:
                        (1 / (1 + exp(-p.y * w.dot(p.x)))) * p.y * p.x
                      ).reduce(lambda a, b: a + b)
    w -= gradient

print "Final w: %s" % w
Optimization

At least two large classes of optimization problems humans can solve:

» Convex Programs

» Spectral Problems
Distributing Matrix Computations
Distributing Matrices

How to distribute a matrix across machines?

» By Entries (CoordinateMatrix)

» By Rows (RowMatrix)

» By Blocks (BlockMatrix)  As of version 1.3

All of Linear Algebra to be rebuilt using these partitioning schemes
Distributing Matrices

Even the simplest operations require thinking about communication e.g. multiplication

How many different matrix multiplies needed?

» At least one per pair of \{\text{Coordinate, Row, Block, LocalDense, LocalSparse}\} = 10

» More because multiplies not commutative
Singular Value Decomposition on Spark
Singular Value Decomposition
Singular Value Decomposition

Two cases

» Tall and Skinny

» Short and Fat (not really)

» Roughly Square

SVD method on RowMatrix takes care of which one to call.
Tall and Skinny SVD

- Given $m \times n$ matrix $A$, with $m \gg n$.
- We compute $A^T A$.
- $A^T A$ is $n \times n$, considerably smaller than $A$.
- $A^T A$ is dense.
- Holds dot products between all pairs of columns of $A$.

$$A = U\Sigma V^T \quad \quad A^T A = V\Sigma^2 V^T$$
Tall and Skinny SVD

\[ A^T A = V \Sigma^2 V^T \]

Gets us \( V \) and the singular values

\[ A = U \Sigma V^T \]

Gets us \( U \) by one matrix multiplication
Square SVD

ARPACK: Very mature Fortran77 package for computing eigenvalue decompositions

JNI interface available via netlib-java

Distributed using Spark – how?
Square SVD via ARPACK

Only interfaces with distributed matrix via matrix-vector multiplies

\[ K_n = \begin{bmatrix} b & Ab & A^2b & \cdots & A^{n-1}b \end{bmatrix} \]

The result of matrix-vector multiply is small.
The multiplication can be distributed.
Square SVD

<table>
<thead>
<tr>
<th>Matrix size</th>
<th>Number of nonzeros</th>
<th>Time per iteration (s)</th>
<th>Total time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>23,000,000 x 38,000</td>
<td>51,000,000</td>
<td>0.2</td>
<td>10</td>
</tr>
<tr>
<td>63,000,000 x 49,000</td>
<td>440,000,000</td>
<td>1</td>
<td>50</td>
</tr>
<tr>
<td>94,000,000 x 4,000</td>
<td>1,600,000,000</td>
<td>0.5</td>
<td>50</td>
</tr>
</tbody>
</table>

With 68 executors and 8GB memory in each, looking for the top 5 singular vectors
Communication-Efficient $A^T A$

All pairs similarity on Spark (DIMSUM)
All pairs Similarity

All pairs of cosine scores between n vectors
» Don’t want to brute force (n choose 2) m
» Essentially computes $A^TA$

Compute via DIMSUM
» Dimension Independent Similarity Computation using MapReduce
Intuition

Sample columns that have many non-zeros with lower probability.

On the flip side, columns that have fewer non-zeros are sampled with higher probability.

Results provably correct and independent of larger dimension, m.
Spark implementation

// Load and parse the data file.
val rows = sc.textFile(filename).map { line =>
    val values = line.split(' ').map(_.toDouble)
    Vectors.dense(values)
}
val mat = new RowMatrix(rows)

// Compute similar columns perfectly, with brute force.
val simsPerfect = mat.columnSimilarities()

// Compute similar columns with estimation using DIMSUM
val simsEstimate = mat.columnSimilarities(threshold)
Future of MLlib
Research Goal: General Distributed Optimization

Distribute CVX by backing CVXPY with PySpark

Easy-to-express distributable convex programs

Need to know less math to optimize complicated objectives

```python
from cvxpy import *

# Create two scalar optimization variables.
x = Variable()
y = Variable()

# Create two constraints.
constraints = [x + y == 1,
               x - y >= 1]

# Form objective.
obj = Minimize(square(x - y))

# Form and solve problem.
prob = Problem(obj, constraints)
prob.solve()  # Returns the optimal value.
print "status:" , prob.status
print "optimal value" , prob.value
print "optimal var" , x.value, y.value

status: optimal
optimal value 0.999999989323
optimal var 0.999999998248 1.75244914951e-09
```
A General Platform

Standard libraries included with Spark

Spark SQL structured
Spark Streaming real-time
GraphX graph
MLlib machine learning

Spark Core
Spark Community

Most active open source community in big data

200+ developers, 50+ companies contributing

Contributors in past year
Continuing Growth

Contributors per month to Spark

source: ohloh.net
Spark and ML

Spark has all its roots in research, so we hope to keep incorporating new ideas!