1. (30 points) Consider the following problem: Given \( n \) items with sizes \( a_1, a_2, \ldots, a_n \) all in \((0, 1]\), find a packing in unit size bins that minimizes the number of bins used.

(a) Prove that the following algorithm is a factor 2 approximation: Consider the items in an arbitrary order. In the \( i^{th} \) step, suppose you have a list of partially packed bins, say \( B_1, B_2, \ldots, B_k \). If possible, put \( a_i \) into any one of them. If \( a_i \) does not fit into any of these bins, open a new bin \( B_{k+1} \) and put \( a_i \) in it.

(b) Give an example on which the above algorithm does at least as bad as \( 5/3 \) of \( \text{OPT} \), where \( \text{OPT} \) is the number of bins in the optimal packing.

(c) Consider a modification of the algorithm in part (a). At the \( i^{th} \) step, suppose you have a list of partially packed bins, say \( B_1, B_2, \ldots, B_k \). You may only put \( a_i \) into bin \( B_k \). If \( a_i \) does not fit into bin \( B_k \), open a new bin \( B_{k+1} \) and put \( a_i \) in it. Prove that this modified algorithm also gives a factor 2 approximation.

2. (20 points) A simple graph \( G(V, E) \) is called Hamiltonian if it contains a cycle which visits all nodes exactly once. Prove that if every vertex has degree at least \( |V|/2 \), then \( G \) is Hamiltonian.