Let $G(V, E)$ be a connected $d$-regular graph, $v_0 \in V(G)$, and assume that at each node, the ends of the edges incident with the node are labelled $1, 2, \cdots d$. A traverse sequence (for this graph, starting point, and labelling) is a sequence $(h_1, h_2, \cdots h_t) \subseteq \{1, \cdots d\}^t$ such that if we start a walk at $v_0$ and at the $i$'th step, we leave the current node through the edge labelled $h_i$, then we visit every node. A universal traverse sequence (for parameters $n$ and $d$) is a sequence which is a traverse sequence for every $d$-regular graph on $n$ nodes, every labelling of it, and every starting point. Prove the following:

For every $d \geq 2$ and $n \geq 3$, there exists a universal traverse sequence of length $O(d^2 n^3 \log n)$.

**Hint:** Use a probabilistic argument.