CME 305: Discrete Mathematics and Algorithms

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Midterm Review Session 02/10/15

1. Hamiltonian Paths and Cycles

   (a) Show that determining whether a graph contains a Hamiltonian Path is at least as hard as determining whether a graph contains a Hamiltonian Cycle

   (b) Show that determining whether a graph contains a Hamiltonian Cycle is at least as hard as determining whether a graph contains a Hamiltonian Path

   (c) Suppose you have a black box that takes input a graph and gives output YES if and only if the graph contains a Hamiltonian Cycle. Show how to use this black box to find a Hamiltonian Cycle.

2. Effective Resistances

   For each of the following graphs determine the effective resistance between the given pairs of nodes exactly, and determine bounds on the covering time of the graph.

   (a) Lollipop graph with node i in the clique and node j in the stick.

   (b) Barbel graph with nodes i and j in separate cliques.

   (c) Cycle graph with effective resistance as a function of the distance between i and j.

3. Integrality Problems as Max Flow

   (a) Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Assume that the dinner contingent has p families and that the ith family has q_i members. Assume also that there are t tables, and that the jth table has a seating capacity of b_j. Show how to find a satisfying assignment of people to tables in polynomial time.

   (b) You are given an $N \times N$ matrix A such that each entry of the matrix is a non-negative number. Further, the sum of the entries in any row or column is an integer. You are allowed to round each fractionally entry in the matrix i.e. to change each non-integer entry to either the next higher or next lower integer. Prove that there is a way of rounding each entry such that the row and column sums remain unchanged.

4. Greedy Max-Cut

   Consider the greedy algorithm that, given an ordering $v_1, \ldots, v_n$ of the vertices, assigns v_1 to set A, then greedily partitions the other vertices (by sequentially assigning each unassigned vertex v to either A or B according to whether v has more neighbors already assigned to B or more neighbors already assigned to A.) Assume that ties are broken by assigning the point to set A. Prove that the cut found by this greedy algorithm cuts at least $(|E| + |B|)/2$ edges, where $|B|$ is the size of set B at the end of the algorithm.

5. K-SAT

   Suppose you have a satisfiability problem where every clause contains at most K literals ($K > 3$). Reduce this problem to 3-SAT. Explain why in general this problem can not be reduced to 2-SAT.