

CME305 Sample Midterm I

1. Matchings and Independent Sets

Assume that you are given graph $G(V, E)$, a matching M in G and independent S in G . Show that

$$|M| + |S| \leq |V|.$$

2. Unique Minimum s-t Cut

Given a network $G(V, E, s, t)$, give a polynomial time algorithm to determine whether G has a unique minimum s-t cut.

3. Chinese Postman Problem

Imagine that you are a postman. You park your truck in your district, and you want to walk around delivering mail to every street in the district and then return to your truck. Also, you are efficient so you want to minimize the total number of streets that you have to visit.

This can be formulated as a graph problem: given a connected graph $G(V, E)$, find a closed walk of minimum length that traverses every edge at least once.

- (a) Give a polynomial time algorithm that gives a closed walk of length at most $2|E|$.
- (b) (Harder) Give a polynomial time algorithm that gives a closed walk of length at most $|E| + |V| - 1$.