ICME QUALIFYING EXAM

DISCRETE MATHEMATICS AND ALGORITHMS

Let $G(V, E)$ be a connected d-regular graph, $v_0 \in V(G)$, and assume that at each node, the ends of the edges incident with the node are labelled $1, 2, \cdots d$. A traverse sequence (for this graph, starting point, and labelling) is a sequence $(h_1, h_2, \cdots h_t) \subseteq \{1, \cdots d\}^t$ such that if we start a walk at v_0 and at the i'th step, we leave the current node through the edge labelled h_i , then we visit every node. A universal traverse sequence (for parameters n and d) is a sequence which is a traverse sequence for every d-regular graph on n nodes, every labelling of it, and every starting point. Prove the following:

For every $d \geq 2$ and $n \geq 3$, there exists a universal traverse sequence of length $O(d^2 n^3 \log n)$.

Hint: Use a probabilistic argument.